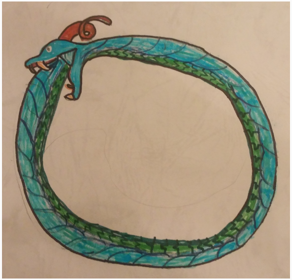
Stringy Gravity & Solution to Dark Problems



Uroboros : An ancient Egyptian symbol for a serpent which eats its own tail

Jeong-Hyuck Park **박 정 혁 (朴 廷 爀)** **Sogang University**

13th International Conference on Gravitation, Astrophysics, and Cosmology
15th Italian-Korean Symposium on Relativistic Astrophysics
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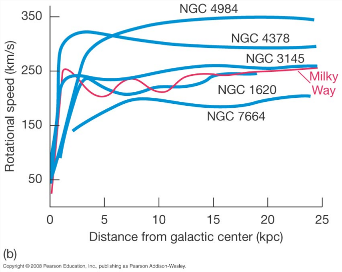
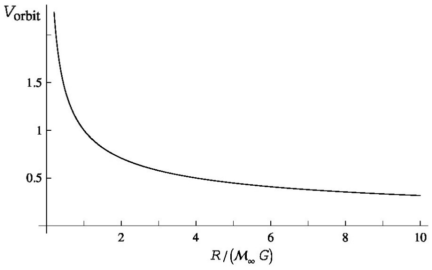
Prologue

Dark Matter Problem





Galaxy rotation curves : observation

Keplerian $\sqrt{MG/R}$ fall-off : GR or Newton

- The galaxy rotation curve is a plot of the orbital velocities of visible stars versus their radial distance from the galactic center.
- While Einstein gravity (GR), with Schwarzschild solution, predicts the Keplerian (inverse square root) monotonic fall-off of the velocities, $V_{\mathrm{orbit}} = \sqrt{MG/R}$, observations however show rather 'flat' (~ 200 km/s) curves after a fairly rapid rise.
- The resolution of the discrepancy may call for 'dark matter', or modifications of the law of gravity (MOND), or perhaps both, as we shall see shortly in the string theory extension of General Relativity, *i.e.* **Stringy Gravity**.

– While mathematics is about dimensionless numbers, *e.g.* $\mathbb{N}$, $\mathbb{R}$, $\mathbb{C}$, physical quantities are generically dimensionful,

$$[R] = \text{Length}\,, \qquad [M] = \text{Mass}\,, \quad \textit{etc}.$$

– Physical laws given by mathematical formulas must be consistent with the dimensions of the physical quantities.

- In natural unit, $c \equiv 1$, the orbital velocity is dimensionless. Further, $R/(MG) \equiv \mathbf{x}$ is a dimensionless radial variable normalized by mass.

- The Keplerian orbital velocity reads then in terms of the two dimensionless variables,

$$V_{\text{orbit}} = \sqrt{1/\mathbf{x}}\,.$$

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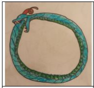
Main Message & Reference of This Talk

⋆ Stringy Gravity modifies the orbital velocity formula at 'short' distance scale, while it becomes asymptotically Keplerian at infinity in terms of $\mathbf{x}$,

$$V_{\mathrm{orbit}} = f(\mathbf{x}) \neq \sqrt{1/\mathbf{x}} \quad : \quad f(\mathbf{x}) \to \sqrt{1/\mathbf{x}} \quad \text{as} \quad \mathbf{x} \to \infty \,.$$

The resulting rotation curves generically feature a maximum at short $\mathbf{x}$, and this may solve the dark matter/energy problems.

⋆ The observations of stars and galaxies far away, or the dark matter/energy problems, are actually revealing the mathematically short-distance nature of Gravity:
Long distance divided by far heavier mass gives small values of $\mathbf{x} = R/(MG)$.

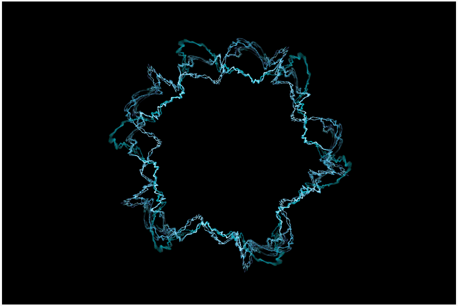
	Electron ($R \simeq 0$)	Proton	Hydrogen Atom	Billiard Ball	Earth	Solar System ($1\mathrm{AU}/M_{\odot}G$)	Milky Way (visible)	Galaxy Cluster	Universe ($M \propto R^3$)
$R/(MG)$	0^+	7.1×10^{38}	2.0×10^{43}	2.4×10^{26}	1.4×10^{9}	1.0×10^{8}	1.5×10^{6}	$\sim 10^{5}$	0^+

'Uroboros' spectrum of the dimensionless Radial variable normalized by Mass in natural units.
The orbital speed of rotation curves is also dimensionless, and depends on the single variable, $R/(MG)$.

Reference: *"The rotation curve of a point particle in stringy gravity"*

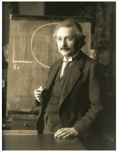
Sung Moon Ko, JHP and Minwoo Suh 1606.09307 [JCAP]

Stringy Gravity



Stringy Gravity is the ‘unambiguous’ extension of General Relativity, dictated entirely by the Symmetry Principle from string theory.

Ever since Einstein formulated General Relativity (GR), by employing the differential geometry *a la* Riemann, the Riemannian metric, $g_{\mu\nu}$, has been privileged to be the only geometric and hence gravitational field.



- Diffeomorphisms : $\quad \partial_\mu \quad \longrightarrow \quad \nabla_\mu = \partial_\mu + \Gamma_\mu$
- $\nabla_\lambda g_{\mu\nu} = 0,\ \Gamma^\lambda_{[\mu\nu]} = 0 \quad \longrightarrow \quad \Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\rho}(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\mu\rho} - \partial_\rho g_{\mu\nu})$
- Curvature : $\quad [\nabla_\mu, \nabla_\nu] \quad \longrightarrow \quad R^\kappa{}_{\lambda\mu\nu} \quad \longrightarrow \quad R$
- Vielbein ("square-root" of metric) : $\quad g_{\mu\nu} = e_\mu{}^p e_\nu{}^q \eta_{pq}$
- Local Lorentz symmetry of locally inertial frames :

$$\mathcal{D}_\mu = \nabla_\mu + \omega_\mu\,, \qquad \mathcal{D}_\lambda e_\mu{}^p = \nabla_\lambda e_\mu{}^p + \omega_\lambda{}^p{}_q e_\mu{}^q = 0 \quad \longrightarrow \quad \omega_{\lambda pq} = e_p{}^\mu \nabla_\lambda e_{\mu q}$$

- All other fields are meant to be 'extra' matters.
- The coupling of GR to matters, *e.g.* to the Standard Model, are then 'minimally' determined through the explicitly appearing metric and covariant derivatives in Lagrangians, which ensure both diffeomorphisms and local Lorentz symmetry.

⋆ *Symmetry dictates interaction.* C. N. Yang

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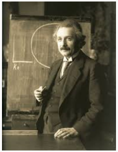

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- On the other hand, string theory suggests us to put a skew-symmetric two-form gauge potential, $B_{\mu\nu}$, and a scalar dilaton, ϕ, on an equal footing along with the metric,

$$\int \mathrm{d}^D x \, \sqrt{-g} e^{-2\phi} \left(R_g + 4\partial_\mu \phi \partial^\mu \phi - \tfrac{1}{12} H_{\lambda\mu\nu} H^{\lambda\mu\nu} \right) \qquad \text{where} \qquad H = \mathrm{d}B \, ,$$

  - Forming the massless sector of closed strings, they are ubiquitous in all string theories.
  - Further, a nontrivial symmetry of string theory, called T-duality which forms the group, $\mathbf{O}(D,D)$, transforms them to one another: Namely, $\{g_{\mu\nu}, B_{\mu\nu}, \phi\}$ forms a multiplet of T-duality. Buscher

- **String theory suggests us to view the whole massless sector of closed strings as the gravitational unity, or $\{g_{\mu\nu}, B_{\mu\nu}, \phi\}$ as the gravitational trinity.**
  - Riemannian geometry is for 'particle' theory.
  - 'String' theory requires a novel differential geometry, beyond Riemann.

- The conventional treatment of the closed string massless sector is 'organized' in terms of Riemannian geometry:

$$\int \mathrm{d}^D x \, \sqrt{-g} e^{-2\phi} \left(R_g + 4\partial_\mu \phi \partial^\mu \phi - \tfrac{1}{12} H_{\lambda\mu\nu} H^{\lambda\mu\nu} \right) .$$

  - In this conventional description, the Riemannian metric provides the background geometry, while the dilaton and the B-field are viewed as *'matter'* living on it.
  - The diffeomorphisms and the B-field gauge symmetry are manifest, *e.g.*

$$\delta B_{\mu\nu} = \mathcal{L}_\xi B_{\mu\nu} + \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu \,, \qquad \delta H_{\lambda\mu\nu} = \mathcal{L}_\xi H_{\lambda\mu\nu} \,.$$

    But, $\mathbf{O}(D, D)$ T-duality symmetry mixing the massless sector is secretly hidden.

  - There is also much ambiguity to occur, when we try to couple the closed string massless sector, especially ϕ and $B_{\mu\nu}$, to other matters, or the Standard Model, *e.g.* the choice of the frame, string (Jordan) or Einstein.

- Thus, Riemannian geometry fails to provide the unifying geometric description of the closed string massless sector.

- This talk is all about the action above, or completely new formulation of it into **Stringy Gravity** (the string theory extension of GR), and its physical implications especially to Dark Matter/Dark Energy problems.

- In Stringy Gravity, the above entire action is going to be identified as an integral of a single scalar curvature of stringy differential geometry beyond Riemann.

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- In Stringy Gravity, the above entire action is going to be identified as an integral of a single scalar curvature of stringy differential geometry beyond Riemann.

Due to limited time, I am not going to explain the details of Stringy Gravity.

Rather I will try to sketch the essence of it in a compact manner.

Stringy Gravity has emerged through the development of so called Double Field Theory (DFT) *a la* Siegel 1993 as well as Hull-Zwiebach 2009.

And my group including, Imtak Jeon and Kanghoon Lee, has contributed to the identification of the underlying **Stringy Differential Geometry**:

- **Stringy differential geometry, beyond Riemann** — **Jeon, Lee, JHP** — **1105.6294** — **PRD**
- **Stringy Unification of IIA and IIB Supergravities under $\mathcal{N}=2$ $D=10$ Supersymmetric Double Field Theory** — **Jeon, Lee, JHP, Yoonji Suh** — **1210.5078** — **PLB**
- **Comments on double field theory and diffeomorphisms** — **JHP** — **1304.5946** — **JHEP**
- **Covariant action for a string in doubled yet gauged spacetime** — **Lee, JHP** — **1307.8377** — **NPB**
- **Green-Schwarz superstring on doubled-yet-gauged spacetime** — **JHP** — **1609.04265** — **JHEP**
- **Standard Model as a Double Field Theory** — **Kangsin Choi, JHP** — **1506.05277** — **PRL**

- *Notation*

  - The word *double* in Double Field Theory (DFT) refers to the usage of doubled coordinates,
  $$x^{A}\quad:\qquad A=1,2,\cdots,D+D\,,$$
  for the description of D-dimensional physical spacetime. In this talk, $D\equiv 4$.

  - Throughout the talk, the captial letters, $A,B,C,\cdots$,, denote the $\mathbf{O}(D,D)$ vector indices, which can be freely raised or lowered by the constant $\mathbf{O}(D,D)$ invariant metric (with its inverse),
  $$\mathcal{J}_{AB}=\left(\begin{array}{cc}\mathbf{0}&\mathbf{1}\\ \mathbf{1}&\mathbf{0}\end{array}\right).$$

  - The constant $\mathbf{O}(D,D)$ metric, $\mathcal{J}_{AB}$, naturally decomposes the doubled coordinates into two parts,
  $$x^{A}=(\tilde{x}_{\mu},x^{\nu})\,,\qquad\qquad\partial_{A}=(\tilde{\partial}^{\mu},\partial_{\nu})\,,$$
  where μ, ν are D-dimensional curved indices.

- *Doubled-yet-gauged spacetime*

Stringy Gravity adopts a doubled-yet-gauged coordinate system: the doubled coordinates are 'gauged' by an equivalence relation,

$$x^A \sim x^A + \Delta^A(x)\,,$$

such that each equivalence class, or gauge orbit in $\mathbb{R}^{D+D}$, represents a single physical point in $\mathbb{R}^D$.

In the above, Δ^A is an arbitrary derivative-index-valued $\mathbf{O}(D,D)$ vector. This means that its superscript index must be identifiable as that of derivative, $\partial^A = \mathcal{J}^{AB}\partial_B$.

For example, with arbitrary functions, Φ_1, Φ_2 belonging to the theory, $\Delta^A = \Phi_1\partial^A\Phi_2$.

The equivalence relation can be realized by requiring that *all* the fields/functions in Stringy Gravity should be invariant under the coordinate gauge symmetry shift,

$$\Phi(x+\Delta) = \Phi(x) \quad \Longleftrightarrow \quad \Delta^A\partial_A = 0\,.$$

This invariance is equivalent to the 'section condition' in DFT, $\partial_A\partial^A = 2\,\partial_\mu\tilde{\partial}^\mu = 0$, which can be generically solved, up to $\mathbf{O}(D,D)$ rotations, by letting $\tilde{\partial}^\mu \equiv 0$, and hence

$(\tilde{x}_\mu\,, x^\nu) \sim (\tilde{x}_\mu + \Phi_1\partial_\mu\Phi_2\,, x^\nu)$: $\tilde{x}_\mu$ coordinates are gauged and x^ν's form a section.

$\mathbf{O}(D,D)$ transformations then rotate the gauged directions (and the section).

- *Doubled-yet-gauged spacetime* JHP 2013

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- ***Diffeomorphisms***

Diffeomorphism symmetry in doubled-yet-gauged spacetime is generated by so called the 'generalized Lie derivative, Siegel 1993

$$\hat{\mathcal{L}}_\xi T_{A_1\cdots A_n} := \xi^B\partial_B T_{A_1\cdots A_n} + \omega\partial_B\xi^B T_{A_1\cdots A_n} + \sum_{i=1}^{n}(\partial_{A_i}\xi_B - \partial_B\xi_{A_i})T_{A_1\cdots A_{i-1}}{}^B{}_{A_{i+1}\cdots A_n}\,,$$

where ω is the weight of the tensor density.

– It unifies the Riemannian diffeomorphisms and the B-field gauge symmetry as $\xi^A = (\lambda_\mu, \xi^\nu)$, and form a closed algebra thanks to the section condition (Coordinate Gauge Symmetry).

\# Unlike $\mathbf{O}(D,D)$ rotations, diffeomorphisms leave the gauged directions of the doubled coordinates, *e.g.* $\{\tilde{x}_\mu\}$, invariant, and preserve the section, $\{x^\nu\}$. Thus, diffemorphisms and $\mathbf{O}(D,D)$ rotations differ intrinsically.

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- *Fundamental fields : building blocks of Stringy Gravity*

The geometric and hence gravitational fields in Stringy Gravity consist of $\{d, P_{AB}\}$, *i.e.* a dilaton, d, and a symmetric projection matrix,

$$P_{AB} = P_{BA} \qquad P_A{}^B P_B{}^C = P_A{}^C \,.$$

They represent the massless sector of closed strings.

– The complementary, orthogonal projector, $\bar{P}_{AB}$, follows

$$\bar{P}_{AB} = \bar{P}_{BA} := \mathcal{J}_{AB} - P_{AB} \,, \qquad \bar{P}_A{}^B \bar{P}_B{}^C = \bar{P}_A{}^C \,, \qquad P_A{}^B \bar{P}_B{}^C = 0 \,.$$

– The difference of the two projectors sets the DFT-metric,

$$\mathcal{H}_{AB} = P_{AB} - \bar{P}_{AB} \,,$$

and hence

$$P_{AB} = \tfrac{1}{2}(\mathcal{J}_{AB} + \mathcal{H}_{AB}) \,, \qquad \bar{P}_{AB} = \tfrac{1}{2}(\mathcal{J}_{AB} - \mathcal{H}_{AB}) \,.$$

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$$e^{-2d} \,.$$

Naturally the cosmological constant term in Stringy Gravity should be $e^{-2d}\Lambda_{\rm SG}$.

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- <u>*Parametrizations: Riemannian vs. non-Riemannian*</u>

With the choice of the section, $\tilde{\partial}^{\mu} \equiv 0$, which solves the Coordinate Gauge Symmetry constraint, the $\mathbf{O}(D,D)$ covariant fundamental fields can be generically parametrized by the conventional (Riemannian) variables:

$$e^{-2d} \equiv \sqrt{|g|}e^{-2\phi}\,, \qquad\qquad \mathcal{H}_{AB} \equiv \begin{pmatrix} g^{-1} & -g^{-1}B \\ Bg^{-1} & g - Bg^{-1}B \end{pmatrix}.$$

⋆ However, the above is not the most general parametrization.

  – Stringy Gravity encompasses novel geometries which do not allow any Riemannian interpretation, *i.e.* non-Riemannian spacetime, *e.g.*

$$\mathcal{H}_{AB} = \mathcal{J}_{AB} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Lee-JHP 2013, JHP 2016

⋆ **Stringy Gravity stands on its own feet, to be formulated through** $\{P_{AB}\,, \bar{P}_{AB}\,, d\}$.

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- *Covariant derivative & Curvatures* Jeon-Lee-JHP 2010, 2011

  - Covariant derivatives and curvatures have been constructed.
  - In particular, the string theory extension of the Christoffel connection has been uniquely identified:

    $\Gamma_{CAB}=2\big(P\partial_C P\bar{P}\big)_{[AB]}+2\Big(\bar{P}_{[A}{}^{D}\bar{P}_{B]}{}^{E}-P_{[A}{}^{D}P_{B]}{}^{E}\Big)\partial_D P_{EC}-\frac{4}{D-1}\Big(\bar{P}_{C[A}\bar{P}_{B]}{}^{D}+P_{C[A}P_{B]}{}^{D}\Big)\Big(\partial_D d+(P\partial^E P\bar{P})_{[ED]}\Big)$

    which ensures the compatibility with the massless sector of closed strings,

$$\nabla_A P_{BC}=\nabla_A\bar{P}_{BC}=\nabla_A d=0\,.$$

  - It turns out that there are scalar curvature and two-indexed "Ricci" curvature,

$$S_0\,,\qquad\qquad P_A{}^C\bar{P}_B{}^D S_{CD}\,,$$

    from which conserved "Einstein" curvature can be also constructed,

$$G_{AB}:=4P_{[A}{}^C\bar{P}_{B]}{}^D S_{CD}-\tfrac{1}{2}\mathcal{J}_{AB}S_0\,,\qquad\qquad \nabla_A G^{AB}=0\,.$$

    Yet, there seems no four-indexed "Riemann-like" curvature in Stringy Gravity.

  - Extension to vielbein formalism has been also completed,

$$P_{AB}=V_A{}^p V_B{}^q\eta_{pq}\,,\qquad\qquad \bar{P}_{AB}=\bar{V}_A{}^{\bar{p}}\bar{V}_B{}^{\bar{q}}\bar{\eta}_{\bar{p}\bar{q}}\,,$$

    which manifests the *twofold* local Lorentz symmetries of Stringy Gravity:

$$\mathbf{Spin}(1,D{-}1)_L\times\mathbf{Spin}(D{-}1,1)_R\,.$$

- *Covariant derivative & Curvatures*

  - Covariant derivatives and curvatures have been constructed.
  - In particular, the string theory extension of the Christoffel connection has been uniquely identified:

    $$\Gamma_{CAB}=2\big(P\partial_C P\bar{P}\big)_{[AB]}+2\Big(\bar{P}_{[A}{}^{D}\bar{P}_{B]}{}^{E}-P_{[A}{}^{D}P_{B]}{}^{E}\Big)\partial_D P_{EC}-\tfrac{4}{D-1}\Big(\bar{P}_{C[A}\bar{P}_{B]}{}^{D}+P_{C[A}P_{B]}{}^{D}\Big)\Big(\partial_D d+(P\partial^E P\bar{P})_{[ED]}\Big)$$

    which ensures the compatibility with the massless sector of closed strings,

    $$\nabla_A P_{BC}=\nabla_A\bar{P}_{BC}=\nabla_A d=0\,.$$

  - It turns out that there are scalar curvature and two-indexed "Ricci" curvature,

    $$S_0\,,\qquad\qquad P_A{}^C\bar{P}_B{}^D S_{CD}\,,$$

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    which manifests the *twofold* local Lorentz symmetries of Stringy Gravity:

    $$\mathbf{Spin}(1,D{-}1)_L\times\mathbf{Spin}(D{-}1,1)_R\,.$$

- *Coupling to the Standard Model* Choi-JHP 2015 [PRL]

$D=4$ **Stringy Gravity naturally, or minimally, couples to the Standard Model in particle physics, dictated by Symmetry Principle:**

- $\mathbf{O}(4,4)$ T-duality
- Twofold local Lorentz symmetry, $\mathbf{Spin}(1,3)_L \times \mathbf{Spin}(3,1)_R$
- Doubled-yet-gauged diffeomorphisms
- $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ gauge symmetry

$$\mathcal{L}_{\mathrm{SM}} = e^{-2d}\left[\begin{array}{l} \frac{1}{16\pi G_N}S_0 \\ +\sum_{\mathcal{V}} P^{AB}\bar{P}^{CD}\mathrm{Tr}(\mathcal{F}_{AC}\mathcal{F}_{BD}) + \sum_{\psi}\bar{\psi}\gamma^a\mathcal{D}_a\psi + \sum_{\psi'}\bar{\psi}'\bar{\gamma}^{\bar{a}}\mathcal{D}_{\bar{a}}\psi' \\ -\mathcal{H}^{AB}(\mathcal{D}_A\phi)^\dagger\mathcal{D}_B\phi - V(\phi) + y_d\,\bar{q}\cdot\phi\, d + y_u\,\bar{q}\cdot\tilde{\phi}\, u + y_e\,\bar{l}'\cdot\phi\, e' \end{array}\right]$$

While coupling to SM, one has to decide the spin group for each fermion, as it a prediction of Stringy Gravity that **the spin group is intrinsically twofold:**

$$\mathbf{Spin}(1,3)_L \quad \textbf{vs.} \quad \mathbf{Spin}(3,1)_R\,.$$

Conjecture: **the quarks and the leptons may belong to the distinct spin groups.**

- *Proper length in doubled-yet-gauged spacetime*

In doubled-yet-gauged spacetime, the *proper length* is defined by a path integral:

$$\text{Length} := -\ln\left[\int \mathcal{D}\mathcal{A}\ \exp\left(-\int\sqrt{Dx^M Dx^N \mathcal{H}_{MN}}\right)\right],$$

where $Dx^M = \mathrm{d}x^M - \mathcal{A}^M$ is a gauged infinitesimal one-form with the auxiliary gauge potential, $\mathcal{A}^M$, satisfying the same properties as the Coordinate Gauge Symmetry generator, *i.e.* derivative-index-valued vector, such as $\mathcal{A}^M\partial_M = 0$, $\mathcal{A}_M\mathcal{A}^M = 0$. Essentially, half of the components are trivial, *e.g.* with $\tilde{\partial}_\mu \equiv 0$,

$$\mathcal{A}^M = A_\lambda \partial^M x^\lambda = (A_\mu\,,\,0)\,, \qquad Dx^M = (\mathrm{d}\tilde{x}_\mu - A_\mu\,,\,\mathrm{d}x^\nu)\,.$$

And for Riemannian DFT-metric, we get a useful relation,

$$Dx^M Dx^N \mathcal{H}_{MN} \equiv \mathrm{d}x^\mu \mathrm{d}x^\nu g_{\mu\nu} + (\mathrm{d}\tilde{x}_\mu - A_\mu + \mathrm{d}x^\rho B_{\rho\mu})(\mathrm{d}\tilde{x}_\nu - A_\nu + \mathrm{d}x^\sigma B_{\sigma\nu})\,g^{\mu\nu}\,.$$

Hence, after integrating out the gauge potential, A_μ, the above $\mathbf{O}(D,D)$ covariant path integral definition of the proper length reduces to the conventional one,

$$\text{Length} \implies \int\sqrt{\mathrm{d}x^\mu \mathrm{d}x^\nu g_{\mu\nu}}\,.$$

Apparently, being $\tilde{x}_\mu$-independent and depending on x^μ only,

$$\left|x_1^A - x_2^A\right| = \left|x_1^\mu - x_2^\mu\right|\,,$$

it measures the distance between two gauge orbits rather than two points in $\mathbb{R}^{D+D}$, which is of course a desired feature.

- *Doubled-yet-gauged point particle action* Ko-JHP-Suh 2016

The definition of the proper length readily gives

$$\mathcal{S}_{\mathrm{particle}} = \int \mathrm{d}\tau \, \left[e^{-1} D_\tau x^A D_\tau x^B \mathcal{H}_{AB}(x) - \tfrac{1}{4} m^2 e \right] ,$$

where e is an einbein and m is the mass of the particle.

With the Riemannian DFT-metric substituted, after integrating out e and $\mathcal{A}^A$, the above action reduces to the conventional one in string frame:

$$S_{\mathrm{particle}} \equiv \int \mathrm{d}\tau \ \ -m\sqrt{-\dot{x}^\mu \dot{x}^\nu g_{\mu\nu}} \, .$$

This implies that the particle follows the geodesic path defined in the string frame.

This preferred choice of the frame, *i.e.* **String frame (Jordan) over Einstein frame,** is due to the Fundamental Symmetry Principle: $\mathbf{O}(D, D)$ & Coordinate Gauge Symmetry.

\# Newton mechanics can be also formulated in the doubled-yet-gauged Euclidean space,

$$\mathcal{L}_{\mathrm{Newton}} = \tfrac{1}{2} m \, D_t x^M D_t x^N \, \delta_{MN} \, - \, V(x) \, ,$$

where $M, N = 1, 2, \cdots, 6$, and the potential, $V(x)$, satisfies the section condition.

* **The formalism has been successfully applied to Superstring Theory to construct**

  - *$D = 10$ Maximally Supersymmetric Stringy Gravity* — Jeon-Lee-JHP-Suh 2012

$$\mathcal{L} = e^{-2d}\Big[\tfrac{1}{8}S_0 + \tfrac{1}{2}\mathrm{Tr}(\mathcal{F}\bar{\mathcal{F}}) + i\bar{\rho}\mathcal{F}\rho' + i\bar{\psi}_{\bar{p}}\gamma_q\mathcal{F}\bar{\gamma}^{\bar{p}}\psi'^{q} + i\tfrac{1}{2}\bar{\rho}\gamma^p\mathcal{D}_p\rho - i\tfrac{1}{2}\bar{\rho}'\bar{\gamma}^{\bar{p}}\mathcal{D}_{\bar{p}}\rho'$$
$$-i\bar{\psi}^{\bar{p}}\mathcal{D}_{\bar{p}}\rho - i\tfrac{1}{2}\bar{\psi}^{\bar{p}}\gamma^q\mathcal{D}_q\psi_{\bar{p}} + i\bar{\psi}'^{p}\mathcal{D}_p\rho' + i\tfrac{1}{2}\bar{\psi}'^{p}\bar{\gamma}^{\bar{q}}\mathcal{D}_{\bar{q}}\psi'_{p}\Big]$$

  - *Doubled-yet-gauged Green-Schwarz superstring* — JHP 2016

$$\mathcal{S} = \frac{1}{4\pi\alpha'}\int \mathrm{d}^2\sigma\ -\tfrac{1}{2}\sqrt{-h}h^{ij}\Pi_i^M\Pi_j^N\mathcal{H}_{MN} - \epsilon^{ij}D_iX^M\left(\mathcal{A}_{jM} - i\Sigma_{jM}\right)\,,$$

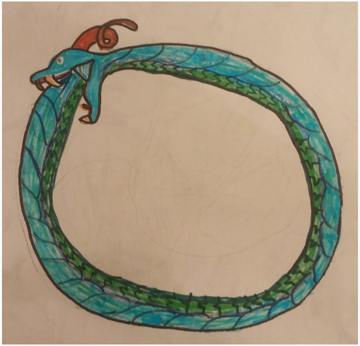
where with $i, j = 0, 1$, we set $\Pi_i^M := D_iX^M - i\Sigma_i^M$ and $\Sigma_i^M := \bar{\theta}\gamma^M\partial_i\theta + \bar{\theta}'\bar{\gamma}^M\partial_i\theta'$.

They enjoy all the desired symmetries :

- **$\mathbf{O}(D,D)$ T-duality**
- **Doubled-yet-gauged diffeomorphisms**
- **twofold Lorentz symmetry, Spin$(1,9)_L \times$ Spin$(9,1)_R$** $\Rightarrow$ **Unification of IIA & IIB**
- **Maximal 16+16 SUSY & kappa symmetries (full order)**
- **Worldsheet diffeomorphisms plus Weyl symmetry**
- **Coordinate gauge symmetry :** $X^M \ \sim\ X^M + \Delta^A,\ \ \Delta^A\partial_A = 0$

* **String theory is better formulated on doubled-yet-gauged spacetime.**

Gravitational Implication



Stringy Gravity modifies GR at 'short' distance in terms of x $= R/(MG)$, and may solve the DM/DE problems in Uroboros manner.

Ko-JHP-Suh 1606.09307 [JCAP]

- *Darkness of Stringy Gravity*

i) Point-like particles couple to the string metric only,

$$\int \mathrm{d}\tau \left[e^{-1} D_\tau x^A D_\tau x^B \mathcal{H}_{AB}(x) - \tfrac{1}{4} m^2 e \right] \quad \Longrightarrow \quad \int \mathrm{d}\tau \; - m\sqrt{-\dot{x}^\mu \dot{x}^\nu g_{\mu\nu}}\,.$$

Hence, the string dilaton, ϕ, and B-field are ***dark*** to point particles.

ii) Each SM fermion couples to Stringy Gravity as

$$e^{-2d}\,\bar{\psi}\gamma^A \mathcal{D}_A \psi \;\equiv\; \frac{1}{\sqrt{2}}\sqrt{-g}\,\bar{\chi}\gamma^\mu \left(\partial_\mu \chi + \frac{1}{4}\omega_{\mu pq}\gamma^{pq}\chi + \frac{1}{24} H_{\mu pq}\gamma^{pq}\chi \right)$$

c.f. Coimbra-Strickland-Constable-Waldram

where $\chi \equiv e^{-\phi}\psi$. This field redefinition removes the string dilaton, ϕ, completely.

- The string dilaton, ϕ, is ***dark*** to the SM fermions, χ ;
- Like F1, χ can source the H-flux, and seems to remember it stringy origin!

iii) Each SM gauge boson couples to Stringy Gravity as

$$e^{-2d}\,\mathrm{Tr}\left(P^{AB}\bar{P}^{CD}\mathcal{F}_{AC}\mathcal{F}_{BD} \right) \equiv -\tfrac{1}{4}\sqrt{-g}e^{-2\phi}\,\mathrm{Tr}\left(g^{\kappa\lambda}g^{\mu\nu}F_{\kappa\mu}F_{\lambda\nu} \right)$$

- B-field, or 'axion' (dual scalar), is ***dark*** to the gauge bosons;
- Standard Model gauge bosons can source the string dilaton, ϕ.

- *Spherical symmetry in doubled-yet-gauged spacetime*

While ϕ and B-field are 'dark' to point particles, the self-interaction of the massless closed string sector, together with its coupling to the Standard Model, should let Stringy Gravity modify General Relativity.

This motivates us to look for spherically symmetric vacua of Stringy Gravity, especially $D=4$.

– Spherical solutions should admit three Killing vectors, ξ_a^A, $a=1,2,3$,

$$\hat{\mathcal{L}}_{\xi_a}\mathcal{H}_{MN}=0 \qquad \Longleftrightarrow \qquad (P\nabla)_M(\bar{P}\xi_a)_N-(\bar{P}\nabla)_N(P\xi_a)_M=0$$
$$\hat{\mathcal{L}}_{\xi_a}(e^{-2d})=0 \qquad \Longleftrightarrow \qquad \nabla_M\xi_a^M=0$$

which form an $\mathbf{so}(3)$ algebra in terms of the $\mathbf{C}$-bracket,

$$[\xi_a,\xi_b]_{\mathbf{C}}=\sum_c \epsilon_{abc}\xi_c\,.$$

JHP-Rey-Rim-Sakatani 2015

- *Asymptotically flat spherical vacuum:*

$$e^{2\phi}=\gamma_+\left(\frac{r-\alpha}{r+\beta}\right)^{\frac{b}{\sqrt{a^2+b^2}}}+\gamma_-\left(\frac{r-\alpha}{r+\beta}\right)^{\frac{-b}{\sqrt{a^2+b^2}}}\,,\qquad B_{(2)}=h\cos\vartheta\,\mathrm{d}t\wedge\mathrm{d}\varphi\,,$$

$$\mathrm{d}s^2=e^{2\phi}\left[-\left(\frac{r-\alpha}{r+\beta}\right)^{\frac{a}{\sqrt{a^2+b^2}}}\mathrm{d}t^2+\left(\frac{r-\alpha}{r+\beta}\right)^{\frac{-a}{\sqrt{a^2+b^2}}}\left(\mathrm{d}r^2+(r-\alpha)(r+\beta)\mathrm{d}\Omega^2\right)\right]\,,$$

where a,b,h ($h^2\leq b^2$) are three free parameters and

$$\alpha=\frac{a}{a+b}\sqrt{a^2+b^2}\,,\qquad \beta=\frac{b}{a+b}\sqrt{a^2+b^2}\,,\qquad \gamma_\pm=\frac{1}{2}\left(1\pm\sqrt{1-h^2/b^2}\right)\,.$$

In particular, the special case of $b=h=0$ corresponds to the Schwarzschild geometry.

– This is a rederivation of the solution by Burgess-Myers-Quevedo (1994) who generated the above solution by applying S-duality to the scalar-gravity solution of Fischer (1948), Janis-Newman-Winicour (1968). It solves the familiar action,

$$\int \mathrm{d}^4x\,\sqrt{-|g|}\,e^{-2\phi}\left(R+4\,|\mathrm{d}\phi|^2-\tfrac{1}{12}\,|\mathrm{d}B|^2\right)\,.$$

– Equivalently, it solves the EOMs of $D=4$ pure Stringy Gravity: the “Einstein” curvatuture vanishes,

$$G_{AB}=0\,.$$

Thus, within the framework of Stringy Gravity, it should be identified as **the vacuum solution**, in analogy with the Schwarzschild solution in GR.

Further, although it would be naked-singular from GR point of view, within Stringy Gravity it can be regular: no $\mathbf{O}(4,4)$ covariant curvature diverges.

- *Asymptotically flat spherical vacuum:*

$$e^{2\phi}=\gamma_{+}\left(\frac{r-\alpha}{r+\beta}\right)^{\frac{b}{\sqrt{a^2+b^2}}}+\gamma_{-}\left(\frac{r-\alpha}{r+\beta}\right)^{\frac{-b}{\sqrt{a^2+b^2}}}\,,\qquad B_{(2)}=h\cos\vartheta\,\mathrm{d}t\wedge\mathrm{d}\varphi\,,$$

$$\mathrm{d}s^2=e^{2\phi}\left[-\left(\frac{r-\alpha}{r+\beta}\right)^{\frac{a}{\sqrt{a^2+b^2}}}\mathrm{d}t^2+\left(\frac{r-\alpha}{r+\beta}\right)^{\frac{-a}{\sqrt{a^2+b^2}}}\left(\mathrm{d}r^2+(r-\alpha)(r+\beta)\mathrm{d}\Omega^2\right)\right]\,,$$

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  Further, although it would be naked-singular from GR point of view, within Stringy Gravity it can be regular: no $\mathbf{O}(4,4)$ covariant curvature diverges.

- *Orbital velocity*

Given the exact spherical solution, we define 'proper' radius, $R := \sqrt{g_{\vartheta\vartheta}(r)}$, which converts the string metric into a canonical form,

$$\mathrm{d}s^2 = g_{tt}\mathrm{d}t^2 + g_{RR}\mathrm{d}R^2 + R^2\left(\mathrm{d}\vartheta^2 + \sin^2\vartheta\,\mathrm{d}\varphi^2\right) .$$

We then compute the 'orbital velocity' of circular geodesics,

$$V_{\mathrm{orbit}} = \left|R\frac{\mathrm{d}\varphi}{\mathrm{d}t}\right| = \left[-\tfrac{1}{2}R\frac{\mathrm{d}g_{tt}}{\mathrm{d}R}\right]^{\frac{1}{2}} ,$$

as a function of $R/(MG)$ which is a dimensionless radial variable normalized by 'asymptotic' mass (Komar mass[1]),

$$MG := \lim_{R\to\infty}(RV^2_{\mathrm{orbit}}) = \tfrac{1}{2}\left(a + b\sqrt{1 - h^2/b^2}\right) .$$

⋆ **Stringy Gravity reduces to Newton Gravity at spatial infinity,**

$$g_{tt} \;\to\; -1 + \frac{2MG}{R} , \qquad V_{\mathrm{orbit}} \;\to\; \sqrt{\frac{MG}{R}} \qquad \textbf{as} \quad R \to \infty .$$

⋆ Yet, **Stringy Gravity modifies GR at 'short' distance, in terms of $R/(MG)$.** Generically $(b \neq 0)$, the orbital velocity is not monotonic: it features a maximum.

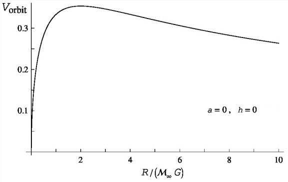
[1] *c.f.* ADM mass *a la* Wald, $\mathcal{Q}[\partial_t] = \frac{1}{4}\left[a+\left(\frac{a-b}{a+b}\right)\sqrt{a^2+b^2}\right]$ JHP-Rey-Rim-Sakatani, Blair 2015

- *Rotation curves*

– Specifically, if $b=0$ (and hence $h=0$), the solution reduces to the Schwarzschild metric, resulting in the Keplerian orbital velocity, $V_{\rm orbit}=\sqrt{\frac{MG}{R}}$.

– As long as $b\neq 0$, **rotation curves feature a maximum** and thus non-Keplerian over a finite range, while becoming asymptotically Keplerian at infinity.

For example, if $a=h=0$ and $b=2MG$, we reproduce the renowned orbital velocity formula, $V_{\rm orbit}=\sqrt{\frac{MR}{(R+2MG)^2}}$, by Hernquist :

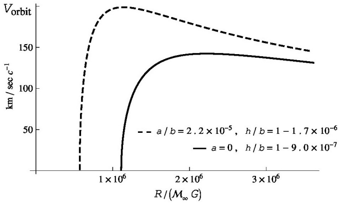


The orbital velocity in Hernquist model assumes its maximum, $\frac{1}{2\sqrt{2}}$, about 35% of the speed of light, at $R=2MG$.

However, this value seems too high compared to observations of galaxies.

– **More interesting cases turn out to include nontrivial H-flux ($h\neq 0$ and hence $b\neq 0$).**

– By tuning the variable, it is possible to make the maximal velocity arbitrarily small, such as about $150\,\mathrm{km/s}\,c^{-1}$, comparable to observations:



Rotation curves in Stringy Gravity
(dimensionless, nonexhaustive).

For sufficiently small $R/(MG)$, the gravitational force can be repulsive.

Rescaling the horizontal axis, $R/(MG) \to R$, rotation curves oscillate.

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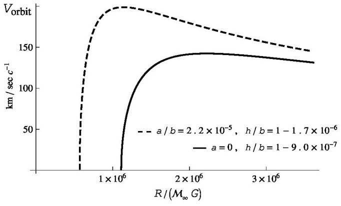

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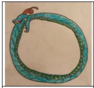
- *Uroboros spectrum of $R/(MG)$*

	Electron ($R \simeq 0$)	Proton	Hydrogen Atom	Billiard Ball	Earth	Solar System ($1\mathrm{AU}/M_\odot G$)	Milky Way (visible)	Galaxy Cluster	Universe ($M \propto R^3$)
$R/(MG)$	0^+	7.1×10^{38}	2.0×10^{43}	2.4×10^{26}	1.4×10^{9}	1.0×10^{8}	1.5×10^{6}	$\sim 10^{5}$	0^+

'Uroboros' spectrum of the dimensionless Radial variable normalized by Mass in natural units.
The orbital speed of rotation curves is also dimensionless, and depends on the single variable, $R/(MG)$.

– The observations of stars and galaxies far away, or the dark matter and the dark energy problems, are revealing the short-distance nature of gravity!

– The repulsive gravitational force at very short-distance, $R/(MG) \to 0^+$, may be responsible for the acceleration of the Universe.

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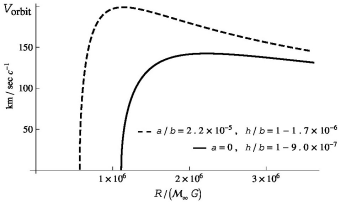

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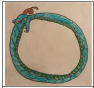	Electron $(R \simeq 0)$	Proton	Hydrogen Atom	Billiard Ball	Earth	Solar System $(1\mathrm{AU}/M_\odot G)$	Milky Way (visible)	Galaxy Cluster	Universe $(M \propto R^3)$
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Thank you.

This talk is based on works in collaborations with Imtak Jeon, Kanghoon Lee, Soo-Jong Rey, Yuho Sakatani, Yoonji Suh, Wonyoung Cho, Jose Fernández-Melgarejo, Woohyun Rim, Sung Moon Ko, Charles Melby-Thompson, Rene Meyér, Minwoo Suh, Kang-Sin Choi, Chris Blair, Emanuel Malek and Xavier Bekaert.

- *Differential geometry with a projection: Application to double field theory* — 1011.1324 JHEP
- **Stringy differential geometry, beyond Riemann** — **1105.6294 PRD**
- *Incorporation of fermions into double field theory* — 1109.2035 JHEP
- *Ramond-Ramond Cohomology and O(D,D) T-duality* — 1206.3478 JHEP
- *Supersymmetric Double Field Theory: Stringy Reformulation of Supergravity* — 1112.0069 PRD
- **Stringy Unification of IIA and IIB Supergravities under** $\mathcal{N}=2$ $D=10$ **Supersymmetric Double Field Theory** — **1210.5078 PLB**
- *Supersymmetric gauged Double Field Theory: Systematic derivation by virtue of 'Twist'* — 1505.01301 JHEP
- **Comments on double field theory and diffeomorphisms** — **1304.5946 JHEP**
- *Covariant action for a string in doubled yet gauged spacetime* — 1307.8377 NPB
- **Green-Schwarz superstring on doubled-yet-gauged spacetime** — **1609.04265 JHEP**
- *Double field formulation of Yang-Mills theory* — 1102.0419 PLB
- **Standard Model as a Double Field Theory** — **1506.05277 PRL**
- **The rotation curve of a point particle in stringy gravity** — **1606.09307 JCAP**
- **O**(D,D) *Covariant Noether Currents and Global Charges in Double Field Theory* — 1507.07545 JHEP
- *Dynamics of Perturbations in Double Field Theory & Non-Relativistic String Theory* — 1508.01121 JHEP
- *Higher Spin Double Field Theory: A Proposal* — 1605.00403 JHEP
- *U-geometry: SL(5)* $\Rightarrow$ **U-gravity: SL(N)** — 1302.1652 JHEP/**1402.5027 JHEP**
- *M-theory and Type IIB from a Duality Manifest Action* — 1311.5109 JHEP