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Analogue gravity in BEC and its gravity dual

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Outline

- **Introduction**
- **Holographic interpretation of acoustic black hole in fluid**
- **Analogue gravity in BEC vs. real gravity**
- **Summary**

Introduction

Analogue gravity is the phenomena that appeared in non-gravitational systems such as the condensed matter systems, which show many features resemble to real gravitational systems.

Typical features:

appearance of curved geometry, horizon and Hawking-Unruh effect, etc.

Many examples:

Acoustic black hole in fluid, Unruh, 1981; Visser, 1993

Classical refractive index , Reznik, 1997

Acoustic black hole in Bose–Einstein condensates, Barceló, Liberati and Visser, ’01, ’03

Slow light in fibre optics, Philbin, Kuklewicz, Robertson, Hill, König and Leonhardt, ’07

Graphene, Cortijo and Vozmediano, ’06

…

Acoustic black holes in fluid

Consider a d-dimensional convergent vorticity free (irrotational) neutral inviscid fluid, its dynamics are governed by

$$\rho\left(\partial_t \vec{v} + \vec{v}\cdot\nabla\vec{v}\right) = -\nabla P - \rho\nabla\Phi,$$

$$\partial_t \rho + \nabla\cdot(\rho\vec{v}) = 0.$$

The velocity can be written as $\vec{v} = \nabla\psi.$

Making the linearized perturbation (normal mode perturbation)

$$\xi = \bar{\xi} + \delta\xi \quad \text{and} \quad \psi = \bar{\psi} + \delta\psi,$$

where $\xi = \ln\rho.$ $\delta\psi$ is the phonon.

Then the eom of the phonon can be written as a KG eq.

$$\frac{1}{\sqrt{-\bar{g}}}\partial_\mu\left(\sqrt{-\bar{g}}\bar{g}^{\mu\nu}\partial_\nu\delta\psi\right) = 0$$

with the effective background metric (sound metric)

$$ds^2_{\rm ac} = \left(\frac{\bar{\rho}}{c_{\rm s}}\right)^{\frac{2}{d-2}} \left(-(c_{\rm s}^2 - \vec{\bar{v}}^2)d\tau^2 + \frac{\bar{v}_i \bar{v}_j}{c_{\rm s}^2 - \vec{\bar{v}}^2} dx^i dx^j + dx^i dx_i \right)$$

Without loss of generality, one can choose the fluid flowing in the $x^{d-1} \equiv z$ direction, i.e. $\bar{v}_i = \bar{v}_z$, then

$$ds^2_{\rm ac} = \left(\frac{\bar{\rho}}{c_{\rm s}}\right)^{\frac{2}{d-2}} \left(-(c_{\rm s}^2 - \bar{v}_z^2)d\tau^2 + \frac{c_{\rm s}^2}{c_{\rm s}^2 - \bar{v}_z^2} dz^2 + dx^a dx_a \right)$$

when $\bar{v}_z = c_s$, there is a coordinate singularity which is similar to that of the black hole horizon. Near $z_{\rm sh}$

$$\bar{v}_z = c_{\rm s} + \left.\frac{\partial \bar{v}_z}{\partial z}\right|_{z_{\rm sh}} (z - z_{\rm sh}) \equiv c_{\rm s} - \kappa(z - z_{\rm sh}),$$

when $z \leq z_{\rm sh}, \bar{v}_z \geq c_{\rm s}$,

the phonons cannot escape from the region $z \leq z_{\rm sh}$,

then $z_{\rm sh}$ is the location of the acoustic horizon.

The near horizon geometry of d-dim acoustic black hole is

$$ds_{\rm ac}^2 = \left(\frac{\bar\rho}{c_{\rm s}}\right)^{\frac{2}{d-2}} \left(-2\kappa c_{\rm s}(z - z_{\rm sh})d\tau^2 + \frac{c_{\rm s}}{2\kappa(z - z_{\rm sh})}dz^2 + dx^a dx_a\right),$$

where the temperature of the acoustic black hole (Hawking-like radiation) is

$$T_{\rm sh} = \left|\frac{\kappa}{2\pi}\right| = \frac{1}{2\pi}\left|\frac{\partial \bar v_z}{\partial z}\right|_{z_{\rm sh}}.$$

The above acoustic metric is just like the near horizon geometry of the static black brane, or the Rindler like metric.

Generalizations:

Acoustic black holes in viscous fluid (breaking the LT symm.)

Relativistic Acoustic black holes

Global acoustic black hole geometries (spherical, rotating…)

Acoustic black holes in viscous fluid (breaking the LT symm.)

$$\rho\left(\partial_t\vec{v}+\vec{v}\cdot\nabla\vec{v}\right) = -\nabla p+\eta\nabla^2\vec{v}+\left(\frac{d-3}{d-1}\eta+\zeta\right)\nabla(\nabla\cdot\vec{v})-\rho\nabla\Phi,$$
$$\partial_t\rho+\nabla\cdot(\rho\vec{v}) = 0.$$

The EoM of the phonon is (with the same acoustic metric as the inviscid case)

$$\Box\delta\psi = -\left(\frac{2d-4}{d-1}\eta+\zeta\right)\left(\frac{c_{\rm s}}{\bar{\rho}}\right)^{\frac{2}{d-2}}\frac{1}{c_{\rm s}^2}(\partial_t+\vec{\bar{v}}\cdot\nabla)\nabla^2\delta\psi,$$

In the Eikonal approximation $\delta\psi = a(x)\exp(-i\omega t+i\vec{k}\cdot\vec{x})$

$$\omega = \vec{k}\cdot\vec{\bar{v}}\pm\sqrt{c_{\rm s}^2\vec{k}^2-\left(\frac{(d-2)}{d-1}\nu+\frac{\mu}{2}\right)^2(\vec{k}^2)^2}-\frac{i}{2}\left(\frac{2(d-2)}{d-1}\nu+\mu\right)\vec{k}^2$$
$$= \vec{k}\cdot\vec{\bar{v}}\pm c_{\rm s}k-\frac{i}{2}\left(\frac{2(d-2)}{d-1}\nu+\mu\right)\vec{k}^2\mp\frac{1}{2c_{\rm s}}\left(\frac{(d-2)}{d-1}\nu+\frac{\mu}{2}\right)^2\vec{k}^2k+\mathcal{O}(k^4),$$

Relativistic Acoustic black holes

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + pP^{\mu\nu} - 2\eta\sigma^{\mu\nu} - \theta\zeta P^{\mu\nu},$$

$$\sigma^{\mu\nu} = P^{\mu\alpha}P^{\nu\beta}\left(\nabla_{(\alpha}u_{\beta)} - \frac{\theta}{d-1}P_{\alpha\beta}\right) \quad \text{and} \quad \theta = \nabla_\lambda u^\lambda$$

For the conformal fluids

$$\zeta = 0, \quad p = \frac{\epsilon}{d-1}, \quad \epsilon = \sigma T^d \quad \text{and} \quad c_s^2 = \frac{1}{d-1},$$

The acoustic black hole in the relativistic version is

$$ds_{\text{ac}}^2 = \left(\frac{\bar{T}^{d-2}}{c_\text{s}}\right)^{\frac{2}{d-2}} \left(-c_\text{s}^2 \bar{u}_\mu \bar{u}_\nu + P_{\mu\nu}\right) dx^\mu dx^\nu.$$

$$\Box\delta\psi = -\left(\frac{2d-4}{d-1}\right)\frac{\eta}{\bar{T}\bar{s}}\left(\frac{c_\text{s}}{\bar{T}^{d-2}}\right)^{\frac{2}{d-2}}\frac{1}{c_\text{s}^2}\bar{u}^\mu\partial_\mu(\partial_\lambda\partial^\lambda\delta\psi),$$

$$\omega = \pm c_\text{s} k - i\Gamma_\text{s}\vec{k}^2 + \mathcal{O}(k^3). \qquad \Gamma_\text{s} = \left(\frac{d-2}{d-1}\right)\frac{\eta}{\bar{T}\bar{s}} = \left(\frac{d-2}{d-1}\right)\frac{\eta}{\bar{\epsilon}+\bar{p}}$$

Although analogue gravity exhibits several features resembles to the real gravitational systems such as the emergent curved geometry and the Hawking-like radiation of the acoustic horizon, the underlying dynamics which govern the analogue gravity and the gravity are rather different.

Normal fluid:	**Navier-Stokes equation (conservation eq.)**
Dielectric material:	**Maxwell equation**
BEC:	**Gross-Pitaevskii equation**

Without further information such as connections between dynamics, it is just an analogy!

An interesting question is, can we say more beyond analogy?

Using the holographic approach, the analogous gravitational model can indeed be mapped to the real gravitational system.

Holographic interpretation of acoustic black hole in fluid

X.-H. Ge, **JRS,** Y. Tian, X. Wu and Y.-L. Zhang, 1508.01735, PRD 92, 084052 (2015);

A d-dim acoustic black hole on the cutoff surface was shown to be dual to a d+1-dim black hole in AAdS spacetime.

$$ds^2_{\rm ac} \sim \left(-2\alpha_{\rm c}\tilde{\hat{c}}_{\rm s}|\partial_z \bar{\tilde{u}}_z|\,(z - z_{\rm sh})\,d\tau^2 + \frac{\tilde{\hat{c}}_{\rm s}}{2\alpha_{\rm c}|\partial_z \bar{\tilde{u}}_z|\,(z - z_{\rm sh})}dz^2 + dx^a dx_a \right)$$

Connection between the dynamics

$$D_\nu \tilde{T}^\nu{}_\mu \propto D_\nu\left(\tilde{K}^\nu{}_\mu - \gamma^\nu{}_\mu \tilde{K}\right) = R_{BA}\gamma^B_\mu n^A \equiv R_{\mu A}n^A = G_{BA}\gamma^B_\mu n^A = 0,$$

$$D_\nu \delta\tilde{T}^\nu{}_\mu \propto \delta R_{\mu A}n^A = 0,$$

Connection between the temperatures;

$$\partial_z \ln T_{\rm H}|_{z_{\rm sh}} = \frac{4\pi\sqrt{f(r_{\rm c})}(1-\hat{c}^2_{\rm s})}{2(d-1)f(r_{\rm c})\alpha^2_{\rm c} - d(1-f(r_{\rm c}))(\hat{c}^2_{\rm s} + \frac{\alpha^2_{\rm c}}{\hat{c}^2_{\rm s}})}\frac{T_{\rm sh}}{\hat{c}_{\rm s}},$$

The phonon/scalar quasinormal mode duality;

$$\left(\partial_\mu \ln r_{\rm h} + \frac{\partial_\mu}{d-1}\right)\frac{P^{\nu\mu}\partial_\nu\delta\psi(t,z)}{\sqrt{-\partial_\alpha\psi\partial^\alpha\psi}} = -u^\mu\partial_\mu\left(\frac{Z_{\rm c}(r_{\rm c},t,z)}{\theta\mathfrak{s}'_2(r_{\rm c})r_{\rm h}}\right).$$

Analogue gravity in BEC vs. real gravity

A holographic model of BEC: consider the bulk

$$I=\frac{1}{16\pi G_{d+1}}\int d^{d+1}x\sqrt{-g}\left(-\frac{1}{2}D_\mu^*\phi D^\mu\phi-\frac{m^2}{2}\phi^*\phi-\lambda\left(\phi^*\phi\right)^2\right).$$

At finite temperature, the Hamiltonian density of the theory is

$$\mathcal{H}=\frac{1}{2}\left(\pi_1^2+\pi_2^2\right)+\frac{1}{2}\left(\mid\nabla\phi_1\mid^2+\mid\nabla\phi_2\mid^2\right)+\frac{m^2}{2}\left(\phi_1^2+\phi_2^2\right)+\lambda\left(\phi_1^2+\phi_2^2\right)^2,$$

where $\phi\equiv\phi_1+i\phi_2,\mid\nabla\phi_k\mid^2=g^{ij}\partial_i\phi_k\partial_j\phi_k$ (for k=1,2) and $\pi_k=\frac{\partial\phi_k}{\partial\tau}$ Is the conjugate momentum. The complex scalar field introduces an U(1) symmetry, with

$$j_\mu=i\sqrt{-g}\left(\phi^*\partial_\mu\phi-\phi\partial_\mu\phi^*\right)\quad\text{and}\quad Q=i\sqrt{-g}\left(\phi^*\partial_t\phi-\phi\partial_t\phi^*\right)$$

The effective action can be obtained from the path integral

$$Z=2\pi\mathcal{N}\int\mathcal{D}\phi_1\mathcal{D}\phi_2\exp\left(\int_0^\beta Nd\tau\int d^dx\sqrt{h}\mathcal{L}_{\rm eff}\right),$$

$$\mathcal{L}_{\rm eff}=-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi^*\partial_\nu\phi+i\frac{\mu}{2}g^{tt}\left(\phi^*\partial_t\phi-\phi\partial_t\phi^*\right)-\frac{1}{2}m^2\phi^*\phi-\frac{g^{tt}}{2}\mu^2\phi^*\phi-\lambda\left(\phi^*\phi\right)^2,$$

The eom of the scalar field is

$$\Box\phi - 2i\mu g^{tt}\partial_t\phi - \left(m^2 + g^{tt}\mu^2\right)\phi - 4\lambda\left(\phi^*\phi\right)\phi = 0.$$

Further making a global U(1) transformation $\phi = \psi e^{i\mu t}$

$$\Box\psi = m^2\psi + 4\lambda \mid \psi \mid^2 \psi.$$

Relativistic version of the Gross-Pitaevskii equation.

Consider the linearized perturbation $\psi = \psi_0(1 + \chi_1 + i\chi_2)$, around the condensate ψ_0

$$\frac{1}{\sqrt{-\bar{g}}}\partial_\mu\left(\sqrt{-\bar{g}}\bar{g}^{\mu\nu}\partial_\nu\chi_1\right) + \bar{g}^{\mu\nu}\left(a + 2 - \frac{a}{2}(d+1)\right)\partial_\nu \ln\psi_0\partial_\nu\chi_1 - 8\lambda\chi_1 \;=\; 0,$$

$$\frac{1}{\sqrt{-\bar{g}}}\partial_\mu\left(\sqrt{-\bar{g}}\bar{g}^{\mu\nu}\partial_\nu\chi_1\right) + \bar{g}^{\mu\nu}\left(a + 2 - \frac{a}{2}(d+1)\right)\partial_\nu \ln\psi_0\partial_\nu\chi_1 \;=\; 0,$$

in which $\bar{g}_{\mu\nu} = \psi_0^a g_{\mu\nu}$, and when $a = \frac{4}{d-1}$, the eoms become

$$\bar{\Box}\chi_1 \;=\; 8\lambda\psi_0^{\frac{2d-6}{d-1}}\chi_1, \qquad \text{massive mode}$$

$$\bar{\Box}\chi_2 \;=\; 0, \qquad \text{massless mode}$$

A simple case is d=3, then

$$\bar{R} = \psi_0^{-2} R - \frac{6}{\psi_0^3} \Box \psi_0.$$

Then the 0th order eom can be expressed as

$$\bar{R} + \left(6m^2 - R\right) \psi_0^{-2} + 24\lambda = 0.$$

If the original background spacetime is Minkowski

The acoustic geometry can be obtained by setting $\psi = \sqrt{\rho} e^{i\theta}$

$$g_{\mu\nu} = \rho \frac{c}{c_s} \left[\eta_{\mu\nu} + \left(1 - \frac{c_s^2}{c^2}\right) \frac{v_\mu v_\nu}{c^2} \right],$$

where $v^\mu = c u^\mu / ||u||$ is the 4-velocity and $u^\mu \equiv \frac{\hbar}{m} \eta^{\mu\nu} \partial_\nu \theta$, this model was viewed as the emergent Nordstrom gravity,

Belenchia, Liberati and Mohd, 2014

$$R + \Lambda = 24\pi \frac{G_N}{c^4} T,$$

An interesting case: the background spacetime is AdS, namely

$$R = -12/L^2.$$

4d conformal gravity

$$\begin{aligned} I &= \frac{\alpha}{32\pi G_4}\int\sqrt{-g}d^4x\left(C^{\alpha\beta\gamma\delta}C_{\alpha\beta\gamma\delta}\right) \\ &= \frac{\alpha}{32\pi G_4}\int\sqrt{-g}d^4x\left(R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta} - 2R^{\alpha\beta}R_{\alpha\beta} + \frac{R^2}{3}\right) \\ &= \frac{\alpha}{16\pi G_4}\int\sqrt{-g}d^4x\left(R^{\alpha\beta}R_{\alpha\beta} - \frac{1}{3}R^2\right) + \frac{\alpha\pi\chi}{4G_4}, \end{aligned}$$

It was shown that this action is equivalent to a 4d pure gravity with negative c.c. (+counterterms), Maldacena, 2011

4d critical gravity Lu and Pope, 2011

$$I = \frac{1}{16\pi G_4}\int\sqrt{-g}d^4x\left(R - 2\Lambda + \alpha R^{\mu\nu}R_{\mu\nu} + \beta R^2\right),$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} + E_{\mu\nu} = 0,$$

where

$$
\begin{aligned}
E_{\mu\nu} &= 2\alpha\left(R_{\mu\lambda}R^{\lambda}_{\nu} - \frac{1}{4}R^{\alpha\beta}R_{\alpha\beta}g_{\mu\nu}\right) + 2\beta R\left(R_{\mu\nu} - \frac{1}{4}Rg_{\mu\nu}\right) \\
&+\alpha\left(\Box R_{\mu\nu} + \frac{1}{2}\Box R g_{\mu\nu} - 2\nabla_{\lambda}\nabla_{(\mu}R^{\lambda}_{\nu)}\right) + 2\beta\left(g_{\mu\nu}\Box R - \nabla_{\mu}\nabla_{\nu}R\right),
\end{aligned}
$$

the linearized eom is

$$
\begin{aligned}
\mathcal{G}^{L}_{\mu\nu} &= R^{L}_{\mu\nu} - \tfrac{1}{2}R^{L}\, g_{\mu\nu} - \Lambda\, h_{\mu\nu}\,, \\
R^{L}_{\mu\nu} &= \nabla^{\lambda}\nabla_{(\mu}h_{\nu)\,\lambda} - \tfrac{1}{2}\Box h_{\mu\nu} - \tfrac{1}{2}\nabla_{\mu}\nabla_{\nu}h\,, \\
R^{L} &= \nabla^{\mu}\nabla^{\nu}h_{\mu\nu} - \Box h - \Lambda h\,.
\end{aligned}
$$

and

$$
\mathcal{G}^{L}_{\mu\nu} = -\tfrac{1}{2}\Box h_{\mu\nu} + \tfrac{1}{2}\nabla_{\mu}\nabla_{\nu}h + \frac{\Lambda}{3}\,h_{\mu\nu} + \frac{\Lambda}{6}\,h\,,
$$

the critical point is at $\alpha = -3\beta\,,$ then the curvature square term can be written as the square of the Weyl tensor $\frac{1}{2}\alpha C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma}$.

In the transverse traceless gauge,

$$\nabla^{\mu} h_{\mu\nu} = 0\,, \qquad g^{\mu\nu} h_{\mu\nu} = 0\,.$$

There are two graviton modes

$$\left(\Box - \frac{2\Lambda}{3}\right) h^{(m)}_{\mu\nu} = 0\,, \qquad \text{massless mode}$$

$$\left(\Box - \frac{4\Lambda}{3} - \frac{1}{3\beta}\right) h^{(M)}_{\mu\nu} = 0\,. \qquad \text{massive mode}$$

We suggest that they are related with

$$\bar{\Box}\chi_1 \;=\; 8\lambda\psi_0^{\frac{2d-6}{d-1}}\chi_1, \qquad \text{massive mode}$$

$$\bar{\Box}\chi_2 \;=\; 0, \qquad \text{massless mode}$$

Namely, χ_1 and χ_2 are related to the sound channel of $h^{(M)}_{\mu\nu}$ and $h^{(m)}_{\mu\nu}$ respectively.

Summary

An acoustic black hole formed in fluid can indeed be mapped to a bulk AdS black hole via the fluid/gravity correspondence.

The acoustic black hole formed in BEC might be connected with the critical gravity (requires further study).

This may shed light on testing the real gravitational phenomena using the experiments in the lab, e.g., Hawking radiation etc. And also the emergent gravity senario.

Many other interesting problems to be explore
Comparing the absorption cross sections;
Fluid with charge;
The deep connection between analogue gravity and the real gravitational systems
…

Thank You For Your Attention