Extended vector-tensor theories

Atsushi NARUKO (Tohoku U. ← TiTech)

in collaboration with

Rampei KIMURA (TiTech) & Daisuke YOSHIDA (McGill)

based on : 1608.07066 (published in JCAP)

vector field theory
on **curved** spacetime
with **degenerate** kinetic matrix

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goal

✓ Recently, in the field of cosmology or gravitation,
degenerate theories have been intensively studied.

- **what** is **degenerate** theory/kinetic matrix ?
- **how** can **degenerate** theories be constructed ?

✓ Without introducing field theory on curved spacetime,
we can (mostly) understand the essential part of
degenerate theory in terms of analytical mechanics.

motivation ①

✓ to understand/explain primordial&current accelerated expansion of the universe

- Λ ?? (why so small ? why that value ?)
- $\Lambda[\phi]$?? ⇒ **scalar**-tensor theory

  e.g. canonical, k-essence, Horndeski, Beyond Horn, …

- $g^{\mu\nu}$?? (change of gravity law) = **tensor** theory

  e.g. (dRGT) massive gravity, bi-gravity…

⇒ decoupling limit of massive gravity (or bi-gravity)

= described by **scalar** & **vector** fields

motivation ②

✓ unique prediction from vector-field during inflation

$$\mathcal{L} = (GR) + (scalar) - \frac{1}{4} f^2(\phi) F_{\mu\nu} F^{\mu\nu}$$

Watanabe, Kanno Soda. (2009)

★ first (**healthy**) counter example for cosmic no-hair conjecture

★ predict **statistically anisotropic** power spectrum :

$$P(\vec{k}) = P(k)\left[1 + g_*(\hat{k} \cdot \hat{v})^2\right]$$

v : privileged direction

$$g_* = 0.002^{+0.016}_{-0.016}$$

Kim & Komatsu (2013)

vector field theory

on **curved** spacetime

with **degenerate** kinetic matrix

degenerate theory

or

degenerate kinetic matrix

$\Leftrightarrow$ **magic** to introduce the kinetic term

for ***non-dynamical*** d.o.f.(s)

non-degenerate system

✓ two fields ***without degeneracy***

$$\mathcal{L} = \frac{1}{2}\,\dot{\phi}_1^2 + \frac{1}{2}\,\dot{\phi}_2^2$$

✓ conjugate mom. (π_1, π_2) ♻ $(\dot{\phi}_1\,,\ \dot{\phi}_2)$ ***invertible !***

$$\pi_1 \equiv \frac{\delta \mathcal{L}}{\delta \dot{\phi}_1} = \dot{\phi}_1\,, \qquad \pi_2 \equiv \frac{\delta \mathcal{L}}{\delta \dot{\phi}_2} = \dot{\phi}_2\,,$$

$$\begin{pmatrix} \pi_1 \\ \pi_2 \end{pmatrix} = \mathcal{K} \begin{pmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \end{pmatrix}$$ with $$\det|\mathcal{K}| \neq 0$$

⇔ ***no primary constraint*** in the language of Hamiltonian analysis

degenerate system

✓ two fields ***with degeneracy***

$$\mathcal{L} = \frac{1}{2}\dot{\phi}_1^2 + \frac{1}{2}\dot{\phi}_2^2 + \dot{\phi}_1\,\dot{\phi}_2 \quad \rightarrow \quad \frac{1}{2}\dot{\Phi}^2 \qquad \Phi \equiv \phi_1 + \phi_2$$

✓ conjugate mom. (π_1, π_2) $(\dot{\phi}_1\,,\ \dot{\phi}_2)$ ***non-invertible !!***

$$\pi_1 \equiv \frac{\delta\mathcal{L}}{\delta\dot{\phi}_1} = \dot{\phi}_1 + \dot{\phi}_2\,, \qquad \pi_2 \equiv \frac{\delta\mathcal{L}}{\delta\dot{\phi}_2} = \dot{\phi}_1 + \dot{\phi}_2 = \pi_1$$

$$\begin{pmatrix}\pi_1\\ \pi_2\end{pmatrix} = \mathcal{K}\begin{pmatrix}\dot{\phi}_1\\ \dot{\phi}_2\end{pmatrix} \quad \text{with} \quad \det|\mathcal{K}| \;\sout{\neq 0}\; = 0$$

⇔ ***a primary constraint*** in the language of Hamiltonian analysis

vector field theory

on **curved** spacetime

with **degenerate** kinetic matrix

Maxwell & Proca

- ✓ A_μ have 4 components in 4D = (in maximum) **4 d.o.f.s**
- ✓ In Maxwell theory, **A0 is non-dynamical** (no kinetic term)

$$-F_{\mu\nu}F^{\mu\nu} \sim \vec{E}^2 - \vec{B}^2 \sim \dot{A}_i^2 - A_{[i\,,j]}^2$$

(gauge sym. kills longitudinal mode → 2 d.o.f.s)

- ✓ **Proca theory** (+m^2A^2, no gauge sym.) ⇔ 3 d.o.f.s

In Maxwell & Proca, **A0 is non-dynamical = no kinetic term**

→ **with magic (degeneracy)**, kinetic term for **A0 ??**

Kimura, **AN**, Yoshida [1608.07066 (2016)]

Extended vector-tensor

✓ action with **two first derivative of Aμ** & 4D general covariance

$$\mathcal{L} = f(Y)\,R + C^{\mu\nu\rho\sigma}\,(\nabla_\mu A_\nu)\,(\nabla_\rho A_\sigma)$$

— sym.

— asym.

$$C^{\mu\nu\rho\sigma} = \alpha_1 g^{\mu(\rho}g^{\sigma)\nu} + \alpha_2 g^{\mu\nu}g^{\rho\sigma} + \frac{1}{2}\alpha_3(A^\mu A^\nu g^{\rho\sigma} + A^\rho A^\sigma g^{\mu\nu})$$
$$+\frac{1}{2}\alpha_4(A^\mu A^{(\rho}g^{\sigma)\nu} + A^\nu A^{(\rho}g^{\sigma)\mu}) + \alpha_5 A^\mu A^\nu A^\rho A^\sigma + \alpha_6 g^{\mu[\rho}g^{\sigma]\nu}$$
$$+\frac{1}{2}\alpha_7(A^\mu A^{[\rho}g^{\sigma]\nu} - A^\nu A^{[\rho}g^{\sigma]\mu}) + \frac{1}{4}\alpha_8(A^\mu A^\rho g^{\nu\sigma} - A^\nu A^\sigma g^{\mu\rho}) + \frac{1}{2}\alpha_9\varepsilon^{\mu\nu\rho\sigma}.$$

R : Ricci scalar (4D)   f & αi : functions of $Y = A_\mu A^\mu$

☝ C is ***not symmetric*** under μ ↔ ν & ρ ↔ σ

(cf. $\nabla_\mu\nabla_\nu\phi$: $A_\mu \to \nabla_\mu\phi$)

degeneracy condition

✓ Action after ADM decomposition (separate time & space) :

$$\mathcal{L}_{\rm kin} = \mathcal{A}\dot{A}_*^2 + 2\mathcal{B}^i \dot{A}_* \dot{\hat{A}}_\mu + 2\mathcal{C}^{\mu\nu} \dot{A}_* K_{\mu\nu} + \mathcal{D}^{\mu\nu} \dot{\hat{A}}_\mu \dot{\hat{A}}_\nu + 2\mathcal{E}^{\mu\nu\rho} \dot{\hat{A}}_\mu K_{\nu\rho} + \mathcal{F}^{\mu\nu\rho\sigma} K_{\mu\nu} K_{\rho\sigma} ,$$

kinetic term for A0

$$A_* \, (= n^\mu A_\mu) \sim A_0$$

☝ generic f & α_i ⇒ **6 d.o.f.s** = 4 (A_μ) + 2 (GW)

✓ **degeneracy** cond. ⇔ making **A0 non-dynamical** :

$$0 = |\mathcal{M}_{\rm kin}| = \mathcal{D}_0 + \mathcal{D}_2 \, A_*^2 + \mathcal{D}_4 \, A_*^4$$

$$0 = \mathcal{D}_0 \propto (\alpha_1 + \alpha_2) \, F(\alpha_i \, , f)$$

→ **case A** : $\alpha_1 + \alpha_2 = 0$

→ **case B** : F = 0 (f ≠ 0)

→ **case C** : F = 0 (f = 0)

$$\mathcal{L}_{\rm kin} \to (\dot{A}_* + \dot{\hat{A}}_\mu)^2 + (\dot{A}_* + K_{\mu\nu})^2$$

example of degenerate theory

$$\mathcal{L} = f(Y)\,R + C^{\mu\nu\rho\sigma}\,(\nabla_\mu A_\nu)\,(\nabla_\rho A_\sigma)$$

$$\begin{aligned}C^{\mu\nu\rho\sigma} &= \alpha_1 g^{\mu(\rho}g^{\sigma)\nu} + \alpha_2 g^{\mu\nu}g^{\rho\sigma} + \frac{1}{2}\alpha_3(A^\mu A^\nu g^{\rho\sigma} + A^\rho A^\sigma g^{\mu\nu}) \\ &+\frac{1}{2}\alpha_4(A^\mu A^{(\rho}g^{\sigma)\nu} + A^\nu A^{(\rho}g^{\sigma)\mu}) + \alpha_5 A^\mu A^\nu A^\rho A^\sigma + \alpha_6 g^{\mu[\rho}g^{\sigma]\nu} \\ &+\frac{1}{2}\alpha_7(A^\mu A^{[\rho}g^{\sigma]\nu} - A^\nu A^{[\rho}g^{\sigma]\mu}) + \frac{1}{4}\alpha_8(A^\mu A^\rho g^{\nu\sigma} - A^\nu A^\sigma g^{\mu\rho}) + \frac{1}{2}\alpha_9\varepsilon^{\mu\nu\rho\sigma}.\end{aligned}$$

✓ an example :

$$f = 1 \quad 2\alpha_6 + Y\alpha_7 = 0 \quad \alpha_1 = \frac{-8(2\alpha_2 + Y\alpha_3) - Y(4 + 4Y\alpha_2 - Y^2\alpha_3)\alpha_8}{2Y^2\alpha_8}$$

$$\alpha_4 = \frac{4(1 + Y\alpha_2)}{Y^2} - \alpha_3 + \frac{8(2\alpha_2 + Y\alpha_3)}{Y^3\alpha_8} + \alpha_8 - \frac{Y^2\alpha_8^2}{8}$$

$$\alpha_5 = \frac{-2 + Y^2\alpha_3}{Y^3} - \frac{4(2\alpha_2 + Y\alpha_3)}{Y^4\alpha_8} - \frac{\alpha_8}{Y} + \frac{Y\alpha_8^2}{8} + \frac{12(2\alpha_2 + Y\alpha_3)}{Y^2(Y^2\alpha_8 - 8)}$$

$g_{\mu\nu}$ & A_μ transformations

- ✓ metric & vector field transformations :

$$g_{\mu\nu} \to \Omega(Y) g_{\mu\nu} + \Gamma(Y) A_\mu A_\nu \quad \& \quad A_\mu \to \Upsilon(Y) A_\mu$$

☝ classification is stable : case A → case A

(as far as the tr. is regular) $\alpha_1+\alpha_2=0$

- ✓ Rewriting Maxwell theory [$g_{\mu\nu} \to g_{\mu\nu} - 2A_\mu A_\nu$] :

$$R - F_{\mu\nu}^2 \;\to\; \sqrt{1-2Y}\,R - \frac{1}{\sqrt{1-2Y}}\left[(\nabla_\mu A^\mu)^2 - (\nabla_\mu A_\nu)^2\right]$$

  - ✓ no U(1) gauge symmetry : $A_\mu \to A_\mu + \nabla_\mu \psi$
  - ✓ same # of d.o.f.s → new gauge symmetry ??

summary

- ✓ We have constructed degenerate vector-tensor theory that includes two first derivative of vector field.
- ✓ New theory for massive vector field includes 5 d.o.f.s = 3 massive vector in A_μ & 2 GW in $g_{\mu\nu}$
- ✓ Applying transformations of metric & vector field, we have investigated the stability of classification
- ✓ no instabilities in vector theory (?) $\Leftrightarrow$ scalar theory

Thank you very much
for your attention