Gravitational Radiation: From test particle to comparable masses

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[1] R. Ruffini, J. Rueda, J. Rodriguez

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- Numerical Relativity: Strong field, comparable masses nd high velocity [Pretorius, F. 2005].

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- Solve the equations of motion of test particle with gravitational backreaction and contrast our result with Ori and Thorne 2000[1].
- Formulate Newtonian two-body-like problem and compute gravitational waveforms and constrast with Numerical Relativity.

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Teukolsky's Formalism

- Hereafter we use the Boyer-Linquist coordinates (t, r, θ, φ).
- Study gauge invariant curvature perturbations $\longrightarrow$ Newman-Penrose scalars. Master equation for perturbations [Teukolsky, S., 1973]. For spin $s = -2$:

$$\psi_4 = C_{\alpha\beta\mu\nu} n^\alpha \bar{m}^\beta n^\nu \bar{m}^\nu \to \frac{1}{2}(\ddot{h}_+ - i\ddot{h}_\times), r \to \infty. \tag{1}$$

- Separation of variable:

$$\psi_4 = (r - a\cos\theta)^{-4} \int d\omega\, e^{-i\omega t} \sum_{lm} {}_sS_{lm\omega}(\theta) e^{im\phi} R_{lm\omega}(r), \tag{2}$$

- The spheroidal harmonics satisfy the polar equation:

$$({}_s\hat{L}^2 + a^2\omega^2\cos^2\theta - 2a\omega\cos\theta + \mathcal{E}_{lm})\,{}_sS_{lm} = 0 \tag{3}$$

Radial Equation

- The radial function satisfies:

$$\Delta^2 \frac{d}{dr}\left(\frac{1}{\Delta}\frac{R_{lm\omega}}{dr}\right) - V(r)R_{lm\omega} = \mathcal{T}_{lm\omega}, \tag{4}$$

where

$$V = \frac{K^2 + 4i(r-M)K}{\Delta} + 8i\omega r + \mathcal{E}_{lm} - 2am\omega + (a\omega)^2. \tag{5}$$

- The source T is obtained from the projections of the energy-momentum tensor into the Newman-Penrose null-tetrad [2] and from this it is obtained $T_{lm\omega}$:

$$4\pi\Sigma T = \int d\omega e^{-i\omega t} \sum_{lm} {}_sS_{lm\omega}(\theta) e^{im\varphi} T_{lm\omega}(r) \tag{6}$$

[2] Breuer, R. A., Gravitational Perturbation Theory and Synchrotron Radiation, 1975

Sasaki-Nakamura Equation

- The potential in the original Teukolsky radial equation is long-ranged.
- Sasaki-Nakamura found a change of variables that makes the potential and the source short-ranged[3]:

$$\frac{d^2}{dr^*}X_{lm\omega} - \mathscr{F}(r)\frac{d}{dr^*}X_{lm\omega} - \mathscr{U}(r)X_{lm\omega} = \mathscr{S}_{lm\omega} \qquad (7)$$

where r^* is the Kerr "tortoise" coordinate.

- Solve the equations and compute the gravitational wave radiation using the Green's function technique as described in Rodriguez, J. F., Rueda, J & Ruffini, R, 2017.

[3] Sasaki, M. & Nakamura, T., Prog. Theor. Phys, 67, 1788 (1982)

Gravitational Radiation

- The solution of the radial equation with source is:

$$R_{lm} = Z^H_{lm\omega} R^\infty_{lm\omega} + Z^\infty_{lm\omega} R^H_{lm\omega} \tag{8}$$

- The complex numbers $Z^H_{lm\omega}$ is given by:

$$Z^H_{lm\omega} = \frac{1}{2i\omega A^{\text{in}}_{lm\omega}} \int_{r_+}^{r} dr' \frac{R^H_{lm\omega} \mathcal{T}_{lm\omega}}{\Delta}, \tag{9}$$

where $\Delta = r^2 - 2rM + a^2$, and the complex number A^{in}_{lm} is obtained from the wronskian of the two independent solutions.

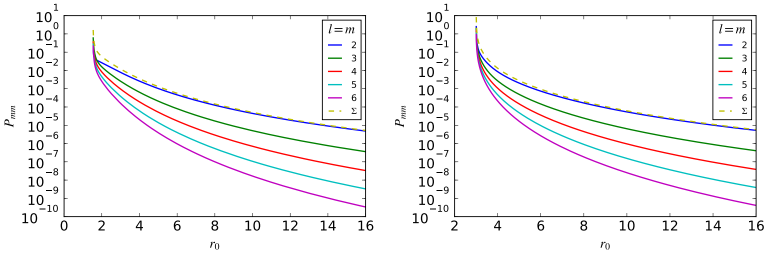
- The fluxes of energy and angular momentum to ∞ due to GW are given in terms of $Z^{H}_{lm\omega}$:

$$\frac{dE}{dt} = \sum_{l,m}^{\infty} \frac{|\tilde{Z}^{H}_{lm\omega}|^2}{4\pi\omega_m^2}, \tag{10}$$

$$\frac{dJ}{dt} = \sum_{l,m}^{\infty} \frac{m|\tilde{Z}^{H}_{lm\omega}|^2}{4\pi\omega_m^3}, \tag{11}$$

- In the case of circular orbits $\omega_m = m\Omega$, where Ω is the orbital angular velocity.

Energy Flux



(a) Kerr, $a = 0.9075$

(b) Schwarzschild

Figure 1: Energy flux to infinity due Gravitational Wave radiation.

Non-conservative dynamics

- The Hamiltonian of the test particle of mass m in the field of the Kerr black hole of mass M is given by

$$H = -P_t = -N^i P_i + N \sqrt{m^2 + \gamma^{ij} P_i P_j}, \tag{12}$$

where $N = 1/\sqrt{-g^{00}}$, $N^i = -g^{ti}/g^{tt}$, $\gamma^{ij} = g^{ij} - g^{ti} g^{tj}/g^{tt}$.

- Dynamical equations are:

$$\dot{r} = \frac{\partial H}{\partial p_r} \qquad \Omega \equiv \dot{\phi} = \frac{\partial H}{\partial L}, \tag{13}$$

$$\dot{P}_r = -\frac{\partial H}{\partial r} + \mathcal{F}_r^{\rm nc} \qquad \dot{P}_\phi = -\mathcal{F}_\phi^{\rm nc} \tag{14}$$

where $\mathcal{F}_r^{\rm nc}$ and $\mathcal{F}_\phi^{\rm nc}$ are the radial and azimuthal non-conservative radiation-reaction forces.

Non-conservative force

- For quasi-circular, adiabatic motion we have:

$$\mathcal{F}_r^{\text{nc}} = 0, \tag{15}$$

$$\mathcal{F}_\phi^{\text{nc}} = -\frac{1}{\Omega}\frac{dE}{dt}. \tag{16}$$

- The energy flux works only for quasi-circular orbits. We used it up to LSO.
- The dynamics includes the contribution of the radial momentum which is not negligible near the LSO.

Plunge

- Study the effective potential.
- For $L > L_{\text{LSO}}$, a small decrease in L due t radiation makes the particle to change from one minimum to the next.
- At $L = L_{\text{LSO}}$ the particle reaches LSO, for $L < L_{\text{LSO}}$, the effective potential has no minima, particle falls.
- The effective potential of Schwarzschild and Kerr is different:

$$V(r = r_+, L = L_{\text{LSO}}) = \begin{cases} 0 & \text{Schwarzschild} \\ \alpha \neq 0 & \text{Kerr} \end{cases}$$

- The particle's effective potential with LSO in Kerr spacetime is flat $\Longrightarrow \Delta E \approx 0$ during the plunge.
- The plunge is geodesic i.e. with constant E and L.

Effective potential

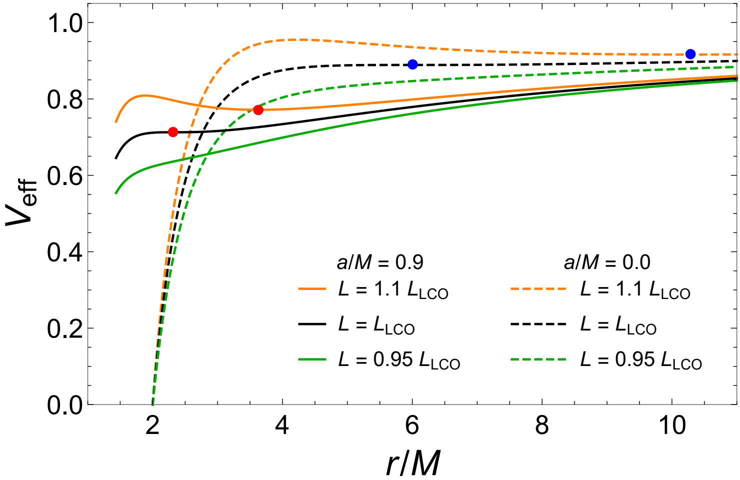


Figure 2: Effective potential for selected values of L and E for the case of Schwarzschild and Kerr. [Rodriguez, Rueda & Ruffini arXiv:1706.06440]

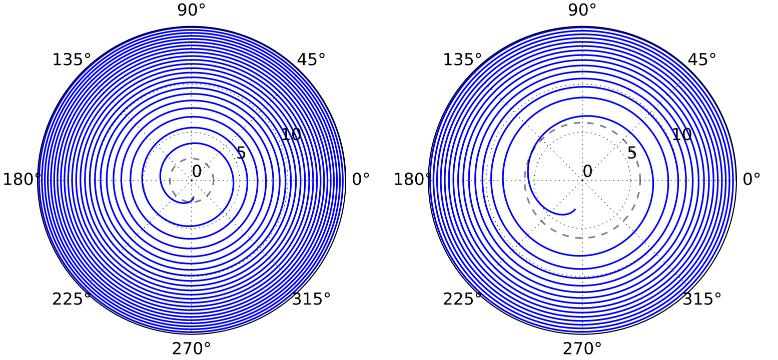
Test particle extrapolation

- We make the extrapolation based on the Newtonian two-body problem.

$$m \mapsto \mu, M_{\text{BH}} \mapsto M = m_1 + m_2 \tag{17}$$

- Angular momentum conservation $\rightsquigarrow$ the “background” space has angular momentum. The normalized angular momentum is the one of the final black hole.

Trajectories



(a) Kerr, $a = 0.9075$ (b) Schwarzschild

Figure 3: Trajectories for $\nu = 0.25$ i.e. $m_1 = m_2$. See [Rodriguez, Rueda & Ruffini `arXiv:1706:06440`].

Dynamics

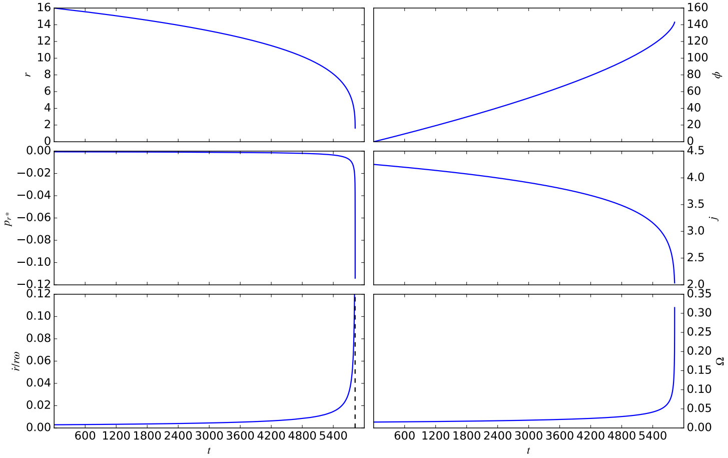


Figure 4: Dynamics of test particle formalism for $\nu = 0.25, a/M_f = 0.9075$

“Test particle” waveforms

- The gravitational waveform can be constructed from circularized waves [Rodriguez, Rueda & Ruffini `arXiv:1706.07704`]:

$$\frac{1}{2}(h_+ - i h_\times) = -\frac{1}{R}\sum_{l,m}\frac{Z^H_{lm}}{\omega_m^2}\,{}_{-2}S_{lm}(\Theta)e^{im\Phi}e^{-i\omega_m(t-R^*)}, \qquad (18)$$

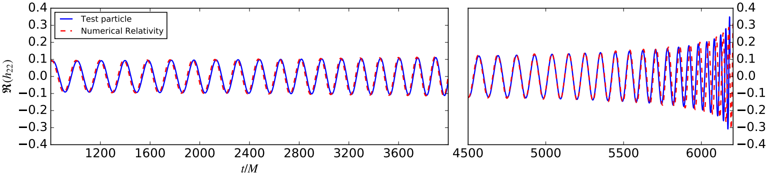
  where R is the distance to the observer, Θ is the angle between the axis of rotation and the observer, Φ is the azimuthal coordinate of the orbiting body at $t = 0$; R^* is the Kerr “tortoise”.

- Along the dynamics the wave frequency changes with time.

- The complex number $Z_{lm\omega}$ evolves with time, inducing a variable wave amplitude and phase shift.

$$\omega_m(t - r^*) \mapsto m\phi(t - r^*).$$

“Test particle” vs Numerical Relativity



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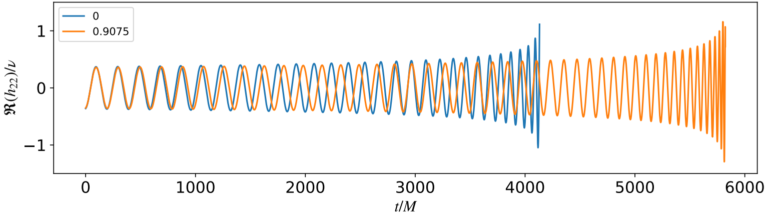

(b) Schwarzschild

Figure 5: Gravitational waveforms. For test particle waveforms $\nu = 0.25$ $a/M_f = 0.9075$. The NR simulation is BBH:0230 with the same parameters.

Conclusion

- We have studied two different cases of emission of gravitational waves in the strong-field limit of falling body into an already formed black.
- We have obtained the transition from the adiabatic regime to the plunge using the Hamiltonian with additive forces. The plunge can be treated as geodesic.
- We have used the Newtonian two-body like problem to compute the waveform of the in-spiral of two black holes.
- We have compared NR relativity waveforms and test particles waveforms and found general agreement between NR nd the current approach.