Black Holes in S3 and H3

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Outline

1. Motivation & Introduction

2. Black Hole Solutions in S3/H3
   with Static Perfect Fluid & Electric Field

3. Geodesics

4. Stability

5. Conclusions

Motivation

The spatial topology of the Universe is an unresolved problem.

Observational Data:

$$\Omega_k = 0.000 \pm 0.005 \; (95\%,\ \textit{Planck}\ \mathrm{TT{+}lowP{+}lensing{+}BAO})$$

It is never manifest if the Universe is flat, closed, or open.

There are trials for inflation models in closed/open universe

:- predict some peculiar feature distinguishable from FLAT models

:- but, still beyond the current observational resolution

We may consider an object such as a black hole in different spatial topologies.

Introduction (S3 and H3)

Metric

$$x_1^2 + x_2^2 + x_3^2 \pm x_4^2 = \pm R_0^2$$

Spatial Embedding, +: S3, -: H3

(t, r, θ, ϕ) coordinate system,

$$ds^2 = \mp dt^2 + \frac{dr^2}{1 - kr^2/R_0^2} + r^2 d\Omega_2^2$$

k=+1: S3, k=-1: H3

(t, χ, θ, ϕ) coordinate system

S3-I

$$r = R_0 \sin\chi, \qquad (r \le R_0),$$
$$ds^2 = -dt^2 + R_0^2 d\chi^2 + R_0^2 \sin^2\chi d\Omega_2^2.$$

S3-II

$$r = R_0 \cosh\chi, \qquad (R_0 \le r < \infty),$$
$$ds^2 = +dt^2 - R_0^2 d\chi^2 + R_0^2 \cosh^2\chi d\Omega_2^2.$$

H3

$$r = R_0 \sinh\chi, \qquad (0 \le r < \infty),$$
$$ds^2 = -dt^2 + R_0^2 d\chi^2 + R_0^2 \sinh^2\chi d\Omega_2^2.$$

Effective Energy-Momentum Tensor

$$\bar{G}^{\mu}_{\nu} = \mp\frac{1}{R_0^2}\mathrm{diag}(3,1,1,1) \quad \equiv 8\pi\bar{T}^{\mu}_{\nu}$$

: constant, -: S3 (ρ>0), +: H3 (ρ<0)

$$p = -\frac{1}{3}\rho = const.$$

: Eq. of State

R3: Schwarzschild BH

if M=0, Minkowski Space: spatially flat

$$ds^2 = -\left(1-\frac{2M}{r}\right)dt^2 + \left(1-\frac{2M}{r}\right)^{-1}dr^2 + r^2 d\Omega_2^2$$

Can we have such a black hole parameterized by MASS in S3/H3?

$$\bar{T}^{\mu}_{\nu} = \mp\frac{1}{8\pi R_0^2}\mathrm{diag}(3,1,1,1)$$

with effective EM tensor

$$ds^2 = -f(r)dt^2 + g(r)dr^2 + r^2 d\Omega_2^2.$$

search BH solution

NO such a solution exists

Black Holes in S3/H3 with Static Perfect Fluid & E-Field

Introduce "Static Perfect Fluid" & "Electric Field"

(t, r, θ, ϕ) coordinate system,

metric ansatz

$$ds^2 = -f(r)dt^2 + g(r)dr^2 + r^2 d\Omega_2^2.$$

Perfect Fluid

EM tensor

$$T^\mu_\nu = \mathrm{diag}[-\rho(r), p(r), p(r), p(r)],$$

Eq. of State

$$p(r) = -\frac{1}{3}\rho(r).$$

: require S3/H3 boundary condition

E-Field

EM tensor

$$\mathcal{T}^\mu_\nu = \frac{E^2(r)}{2f(r)g(r)}\mathrm{diag}(-1, -1, 1, 1).$$

Einstein Eqs.

$$G^0_0 = -\frac{1}{r^2} + \frac{1}{r^2 g} - \frac{g'}{rg^2} = 8\pi(T^0_0 + \mathcal{T}^0_0) = -8\pi\left[\rho(r) + \frac{E^2(r)}{2f(r)g(r)}\right],$$

$$G^1_1 = -\frac{1}{r^2} + \frac{1}{r^2 g} + \frac{f'}{rfg} = 8\pi(T^1_1 + \mathcal{T}^1_1) = 8\pi\left[p(r) - \frac{E^2(r)}{2f(r)g(r)}\right],$$

$$G^2_2 = G^3_3 = \frac{f'}{2rfg} - \frac{f'^2}{4f^2 g} - \frac{g'}{2rg^2} - \frac{f'g'}{4fg^2} + \frac{f''}{2fg} = 8\pi(T^2_2 + \mathcal{T}^2_2) = 8\pi\left[p(r) + \frac{E^2(r)}{2f(r)g(r)}\right]$$

Matter-Field Eq.

$$\nabla_\mu(T^{\mu\nu} + \mathcal{T}^{\mu\nu}) = 0,$$

$$\rho' + \frac{f'}{f}\rho + \frac{3E^2}{fg}\left(\frac{E'}{E} - \frac{f'}{2f} - \frac{g'}{2g} + \frac{2}{r}\right) = 0.$$

$$\nabla_\mu T^{\mu\nu} = 0, \qquad \nabla_\mu \mathcal{T}^{\mu\nu} = 0. \rightarrow \nabla_\mu \mathcal{F}^{\mu\nu} = 0$$

Solutions

$$\rho(r) = -\frac{3}{8\pi\alpha}\left\{1 \mp \frac{2\alpha|\beta|}{r}\left[\beta(r^2+\alpha)\right]^{1/2} + \frac{Q^2}{3}\left(\frac{1}{\alpha} + \frac{1}{2r^2}\right)\right\},$$

$$f(r) = \frac{\rho(r)}{\rho_c}, \qquad g^{-1}(r) = -\frac{8\pi}{3}(r^2+\alpha)\rho(r),$$

$$E(r) = \frac{Q}{3r^2\left[\beta(r^2+\alpha)\right]^{1/2}}, \qquad \text{where} \quad \rho_c = -\frac{9\beta}{8\pi}.$$

(i) Fluid BH: $Q = 0$.

(ii) RN BH: $\alpha \to \infty$, $\beta \to 0$, but $\alpha\beta =$ finite.

3 types of solutions

(t, χ, θ, ϕ) coordinate system

Define new parameters : $R_0^2 \equiv |\alpha|, \qquad K \equiv 2R_0^2 |\beta|^{3/2}.$

R_0^2 → **3-space curvature scale**

K → **Mass Parameter**

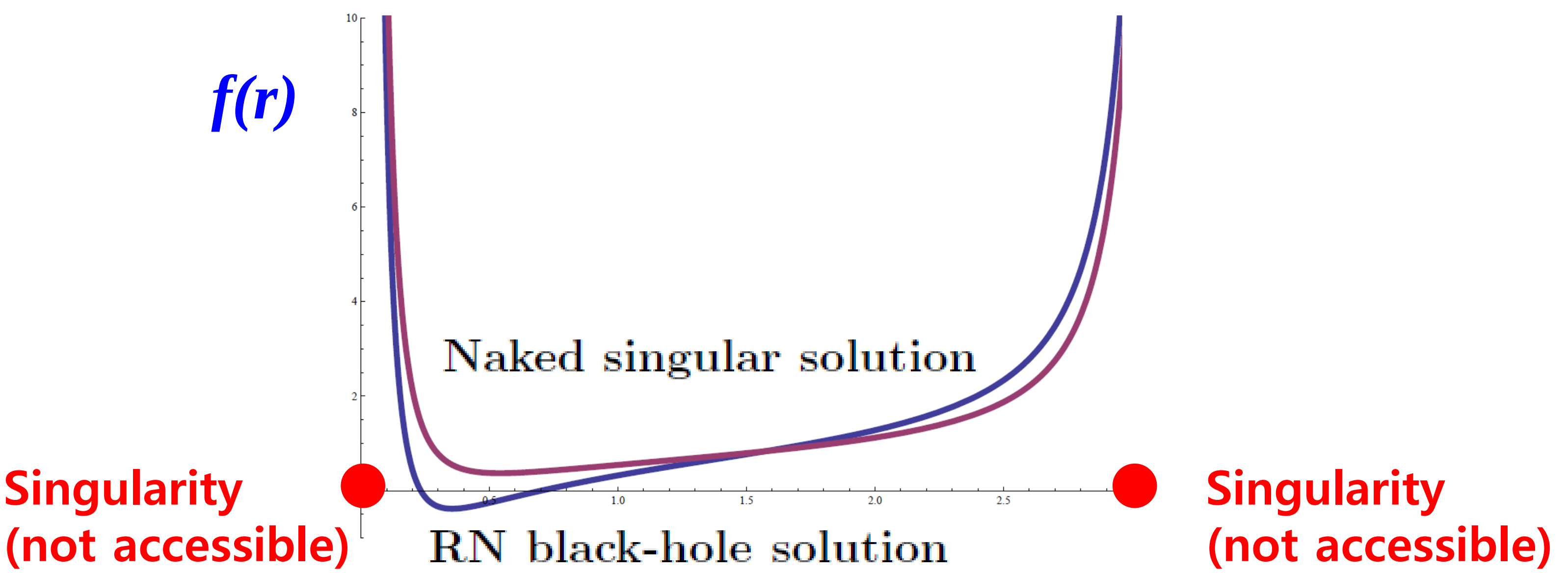
Class	$\rho(\chi)$	$f(\chi)$	$g(\chi)$
S_3-I	$\frac{3}{8\pi R_0^2}\left[1 - K\cot\chi - \frac{Q^2}{6R_0^2}(1-\cot^2\chi)\right]$	$\frac{\rho(\chi)}{\rho_c}, \quad (\rho_c > 0)$	$\frac{3}{8\pi\rho(\chi)}$
S_3-II	$\frac{3}{8\pi R_0^2}\left[1 \mp K\tanh\chi - \frac{Q^2}{6R_0^2}(1+\tanh^2\chi)\right]$	$\frac{\rho(\chi)}{\rho_c}, \quad (\rho_c < 0)$	$-\frac{3}{8\pi\rho(\chi)}$
H_3	$-\frac{3}{8\pi R_0^2}\left[1 \mp K\coth\chi + \frac{Q^2}{6R_0^2}(1+\coth^2\chi)\right]$	$\frac{\rho(\chi)}{\rho_c}, \quad (\rho_c < 0)$	$-\frac{3}{8\pi\rho(\chi)}$

Classification of Solutions

S3-I

$$ds^2 = -\frac{3}{8\pi R_0^2 \rho_c}\left[1 - K\cot\chi - \frac{Q^2}{6R_0^2}(1-\cot^2\chi)\right]dt^2$$
$$+\frac{R_0^2}{1 - K\cot\chi - (Q^2/6R_0^2)(1-\cot^2\chi)}d\chi^2 + R_0^2\sin^2\chi d\Omega_2^2.$$

$$\chi_h = \chi_\pm \equiv \cot^{-1}\left(\frac{3KR_0^2 \mp \sqrt{J_1}}{Q^2}\right), \quad \text{where} \quad J_1 = 9K^2R_0^4 - 6Q^2R_0^2 + Q^4.$$
: would-be horizon



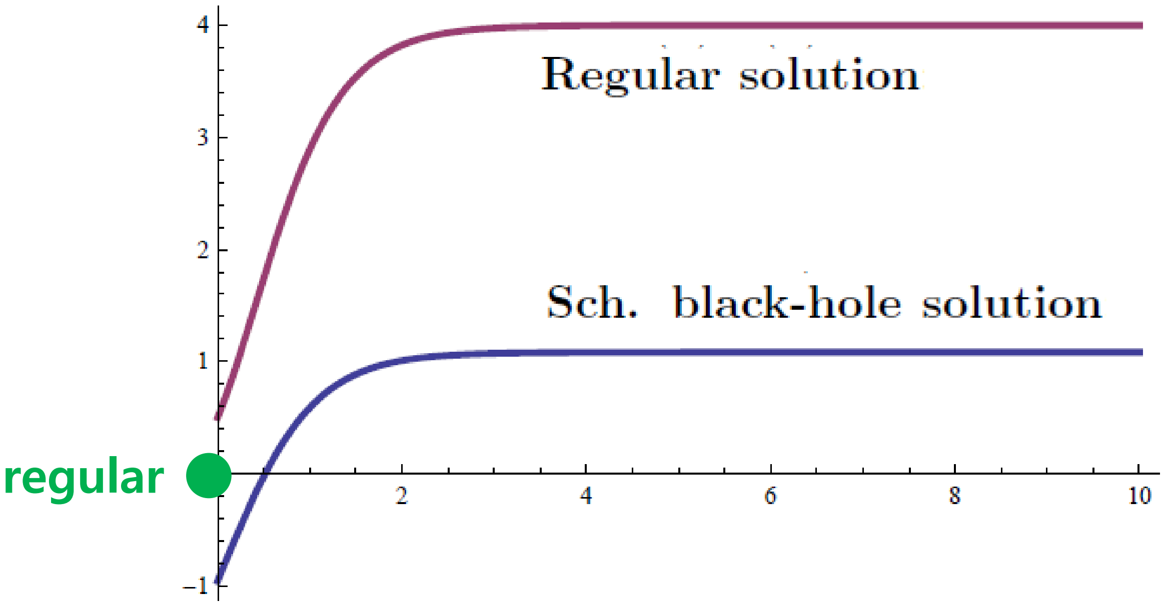
S3-II

$$ds^2 = -\frac{3}{8\pi R_0^2 \rho_c}\left[1 \ominus \oplus K \tanh\chi - \frac{Q^2}{6R_0^2}(1+\tanh^2\chi)\right] dt^2$$

$$+\frac{R_0^2}{-\left[1 \ominus \oplus K \tanh\chi - (Q^2/6R_0^2)(1+\tanh^2\chi)\right]} d\chi^2 + R_0^2 \cosh^2\chi d\Omega_2^2.$$

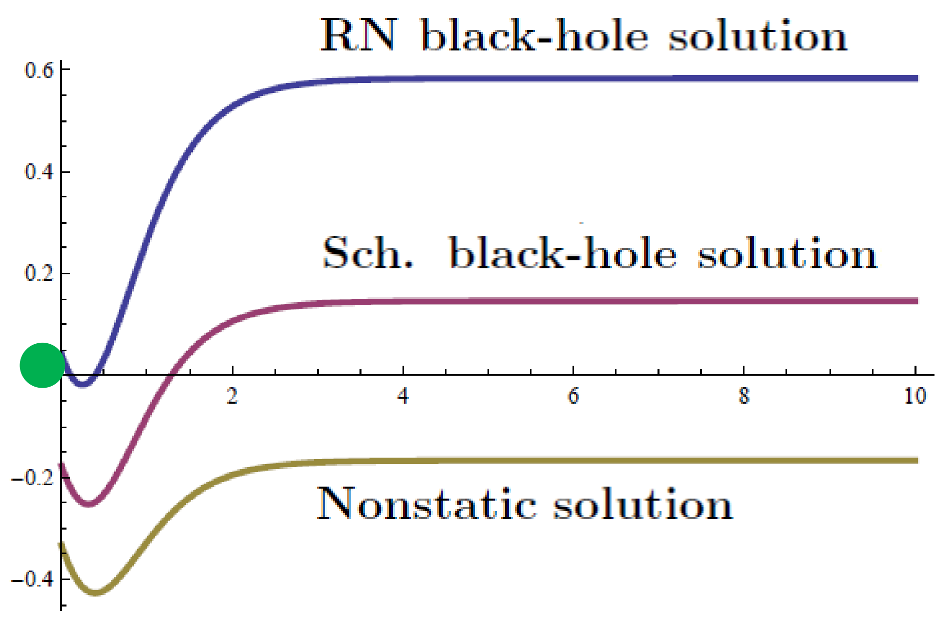
$$\chi_h = \chi_\pm \equiv \tanh^{-1}\left(\frac{\ominus \oplus 3KR_0^2 \pm \sqrt{J_2}}{Q^2}\right), \quad \text{where} \quad J_2 = 9K^2R_0^4 + 6Q^2R_0^2 - Q^4.$$

For the $\ominus$ solution

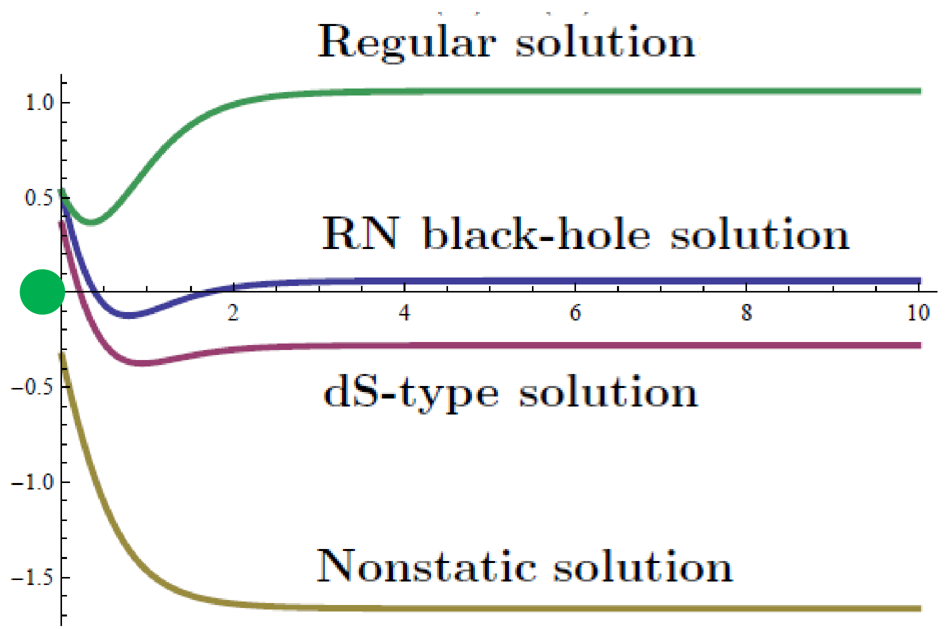


For the $\oplus$ solution

$J_2 > 0$ and $0 < K < 1$,



$J_2 > 0$ and $K > 1$,



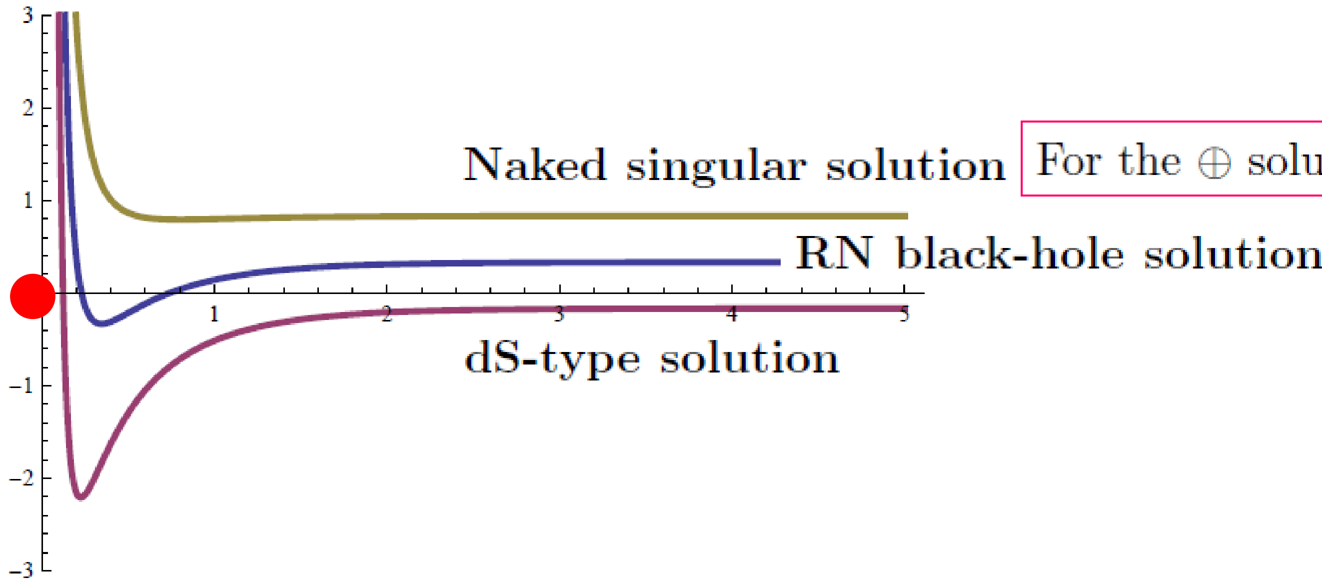
H3

$$ds^2 = -\frac{3}{8\pi R_0^2(-\rho_c)}\left[1 \ominus \oplus K \coth\chi + \frac{Q^2}{6R_0^2}(1+\coth^2\chi)\right] dt^2$$

$$+\frac{R_0^2}{1 \ominus \oplus K \coth\chi + (Q^2/6R_0^2)(1+\coth^2\chi)} d\chi^2 + R_0^2 \sinh^2\chi\, d\Omega_2^2.$$

$$\chi_h = \chi_\pm \equiv \coth^{-1}\left(\frac{\oplus\ominus 3KR_0^2 \mp \sqrt{J_3}}{Q^2}\right), \quad \text{where} \quad J_3 = 9K^2R_0^4 - 6Q^2R_0^2 - Q^4.$$

For the $\ominus$ solution



For the $\oplus$ solution

For simplicity, let us consider UNcharged case from now !

Q=0

S3-I

$r = R_0 \sin\chi \quad (0 \le \chi \le \pi).$

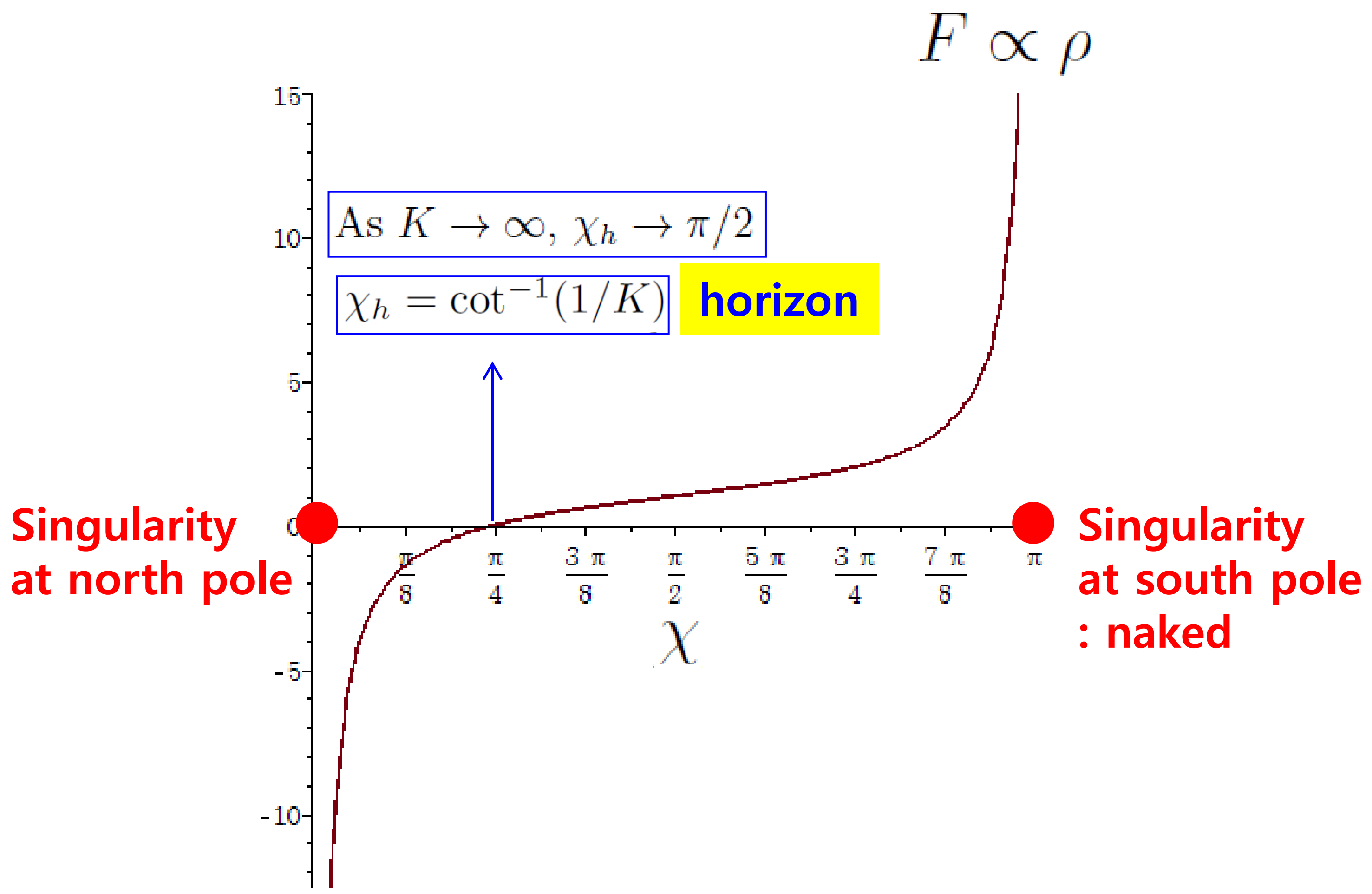
$\alpha < 0,\ \beta < 0,\ \text{and}\ 1 - 4\alpha^2\beta^3 > 0$

$$ds^2 = -\frac{3}{8\pi R_0^2 \rho_c}\underbrace{(1 - K\cot\chi)}_{\equiv F} dt^2 + \frac{R_0^2}{1 - K\cot\chi} d\chi^2 + R_0^2 \sin^2\chi d\Omega_2^2.$$

(Bronnikov & Zaslavskii 2008)

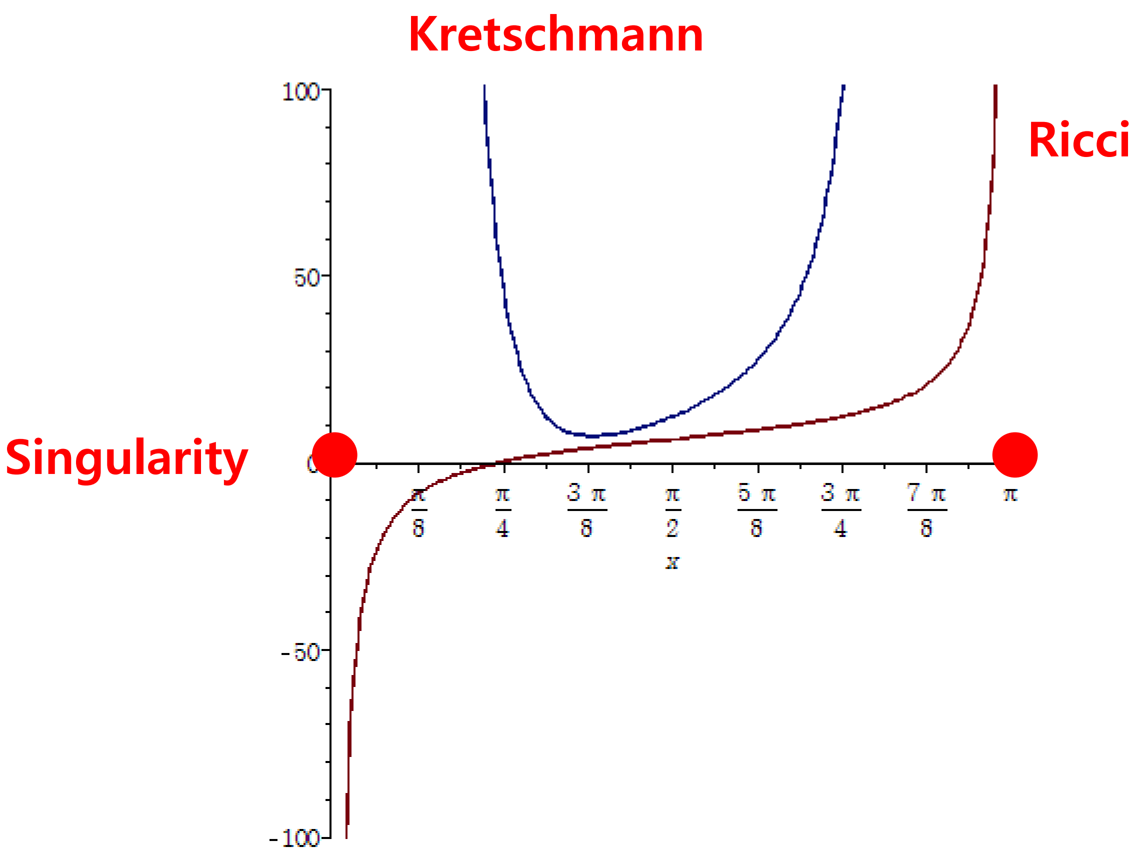
(1) Black Hole Solution

If K=0, it is S3

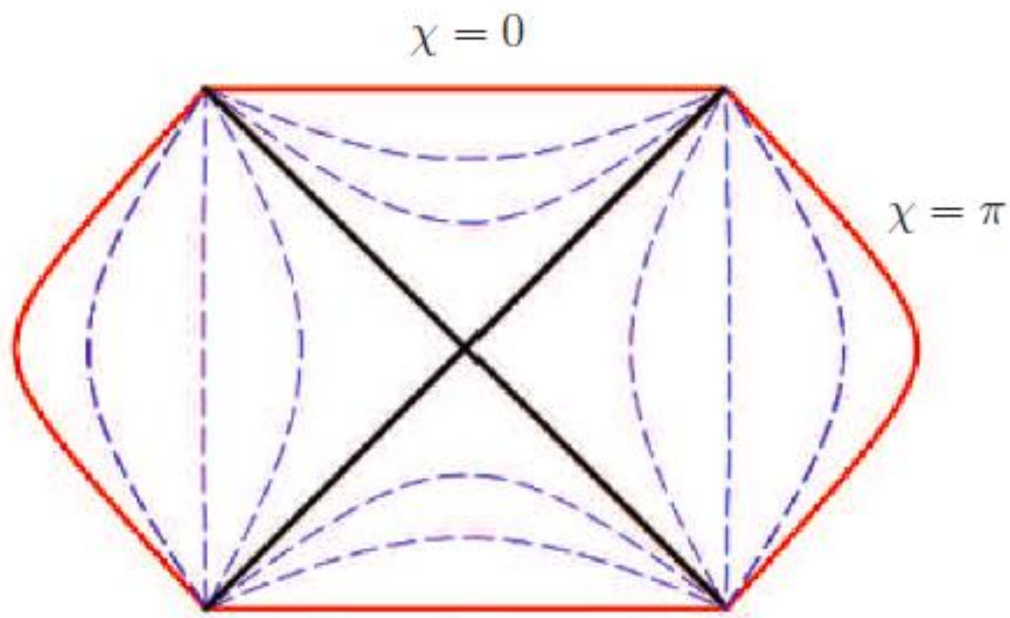


$$ds^2 = -\frac{3}{8\pi R_0^2 \rho_c}\left(1 - K\cot\chi\right)dt^2 + \frac{R_0^2}{1 - K\cot\chi}d\chi^2 + R_0^2 \sin^2\chi d\Omega_2^2.$$

Curvature



Kretschmann
Ricci
Singularity
100
50
0
-50
-100
π/8
π/4
3π/8
π/2
5π/8
3π/4
7π/8
π
x



Conformal diagram for S_3-I black hole. There are two singularities at $\chi = 0$ (north pole: center of the black hole) and at $\chi = \pi$ (south pole: naked). The solid diagonal lines represent the horizon at $\chi_h = \cot^{-1}(1/K)$. The dashed lines are the χ-constant lines. The t-constant lines are straight lines passing the center of the diagram (not shown). Outside the horizon, except the outgoing null rays, none of the geodesics can reach the naked singularity at the south pole.

S3-II

$r = R_0 \cosh\chi \quad (\chi \geq 0).$

$\alpha < 0,\ \beta > 0,\ \text{and}\ 1 - 4\alpha^2\beta^3 < 0.$

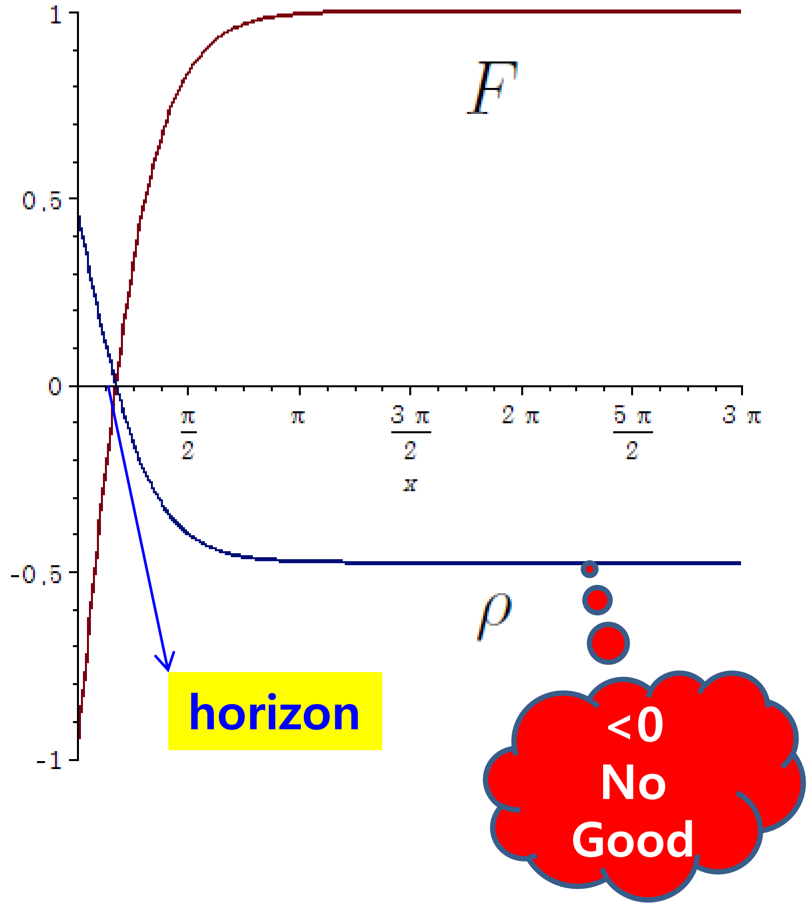
$$ds^2 = -\frac{3}{8\pi R_0^2 \rho_c}(1 \mp K\tanh\chi)\,dt^2 + \frac{R_0^2}{-(1 \mp K\tanh\chi)}d\chi^2 + R_0^2\cosh^2\chi\, d\Omega_2^2.$$

(1) Black Hole Solution: -, K>1

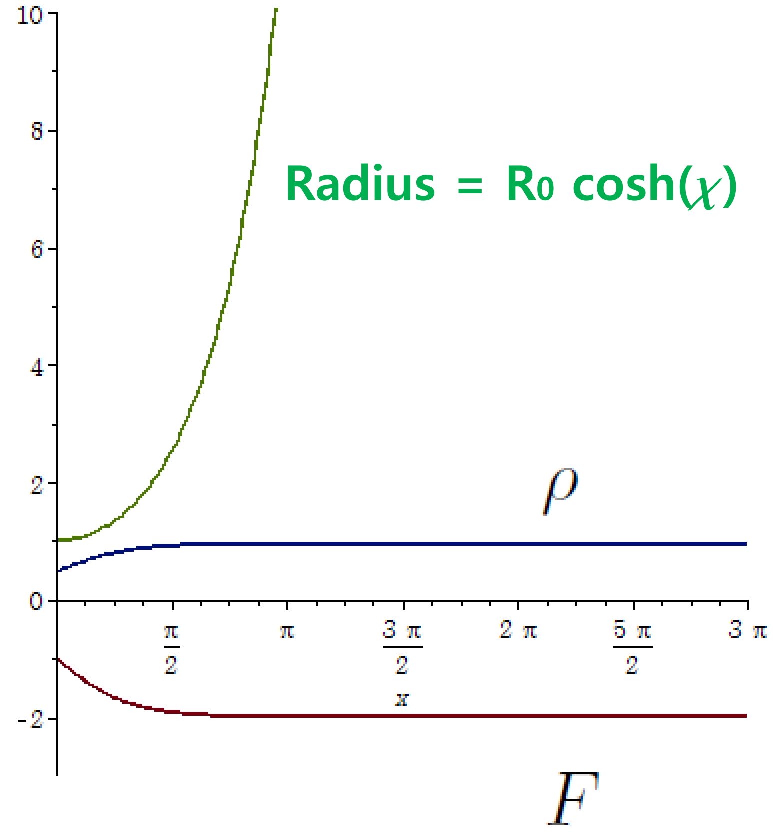
(2) Cosmological Solution: -, K<1, or +

χ: time coordinate
Role of t and χ is changed!!!
Finite size at χ=0
Expanding Universe from a finite size!

(1) Black Hole Solution: -, K>1

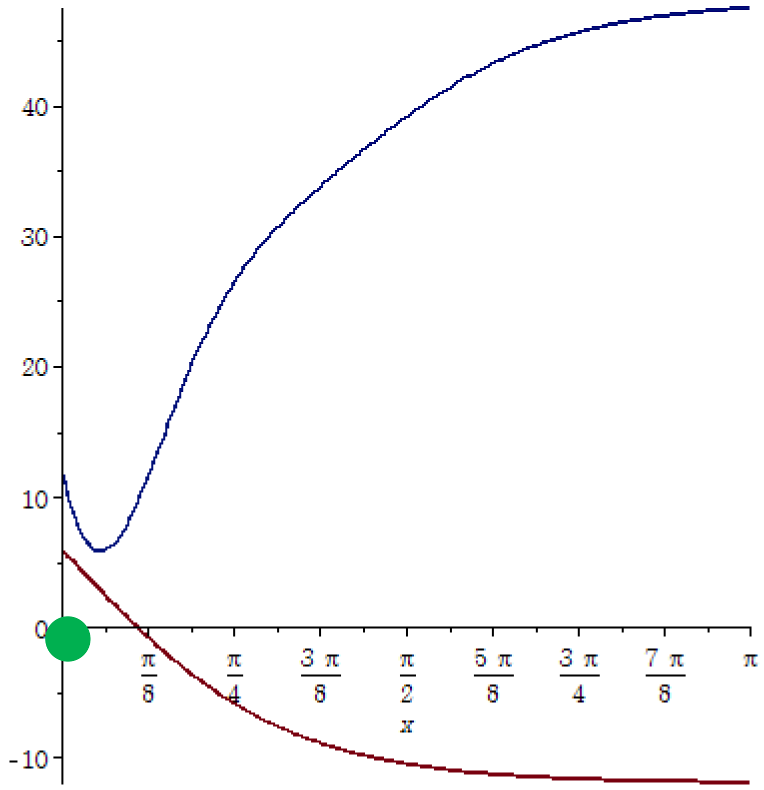


(2) Cosmological Solution: +

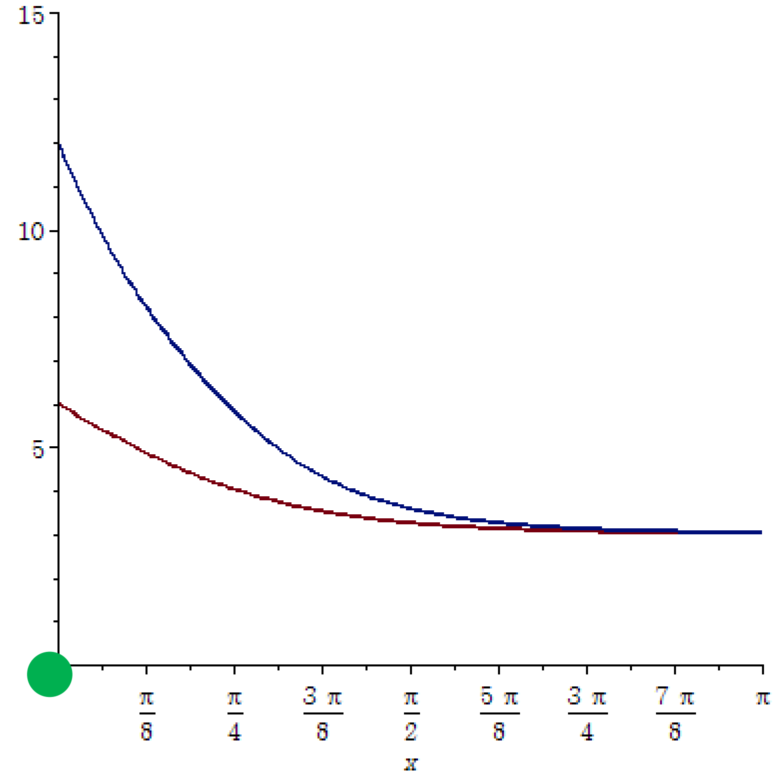


χ is time coordinate

Curvature



regular



H3

$r = R_0 \sinh\chi \quad (\chi \geq 0).$

$\alpha > 0,\ \beta > 0, \text{ and } 1 - 4\alpha^2\beta^3 > 0.$

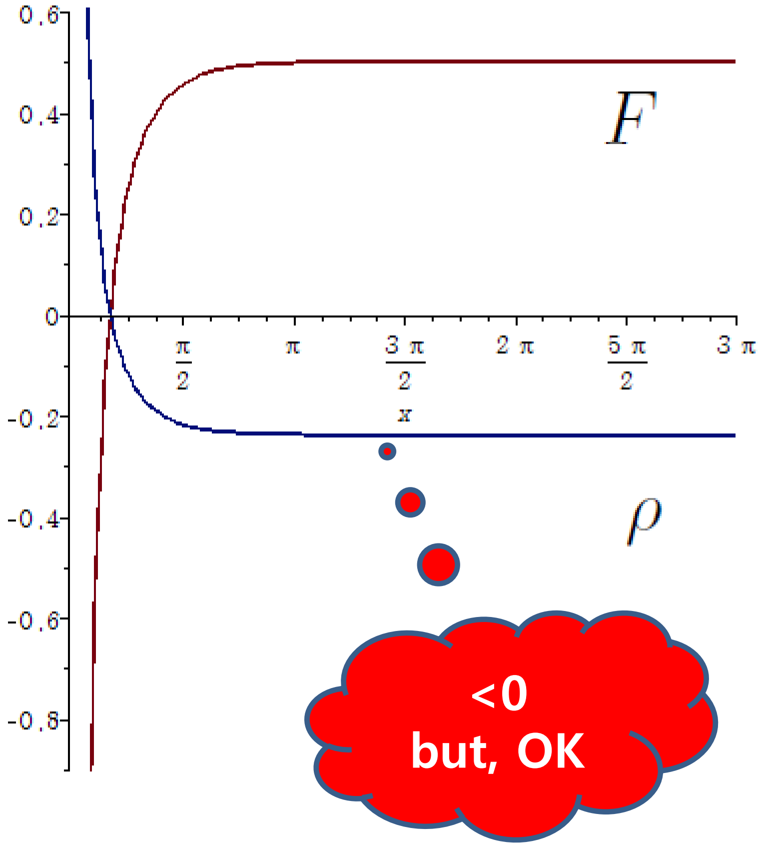
$$ds^2 = -\frac{3}{8\pi R_0^2(-\rho_c)}(1 \mp K\coth\chi)dt^2 + \frac{R_0^2}{1 \mp K\coth\chi}d\chi^2 + R_0^2\sinh^2\chi d\Omega_2^2.$$

(1) Black Hole Solution: -, K<1

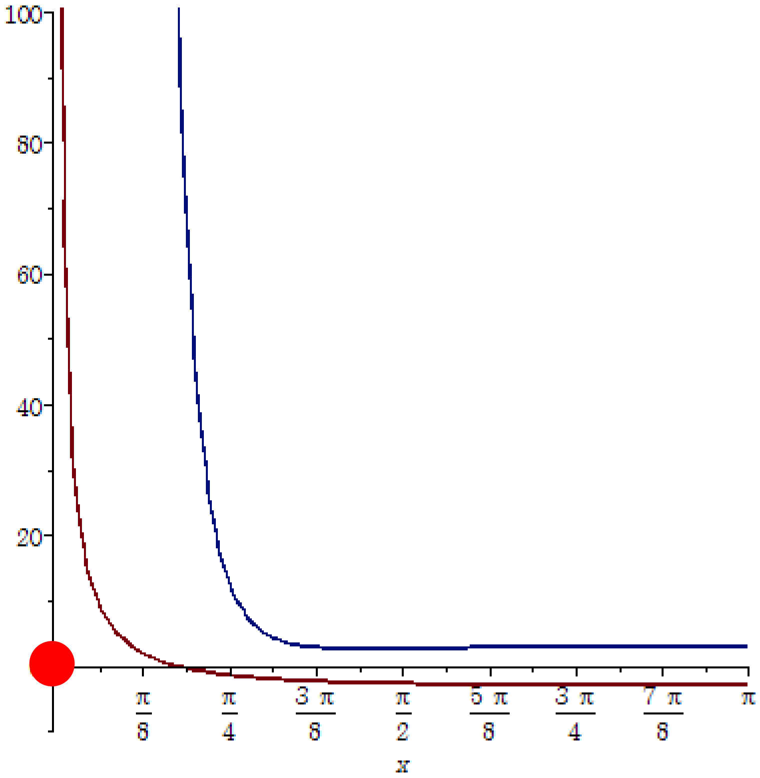
(2) Cosmological Solution: -, K>1 — **ρ>0 & ρ(0)=∞ : INITIAL SINGULARITY**

(3) Singular Static Solution: + — **ρ<0 & ρ(0)=-∞ : NOT Interesting**

(1) Black Hole Solution: -. K<1



Curvature



Singularity

Horizon Structure and Mass

Schwarzschild Black Hole

$$ds^2 = -\left(1-\frac{2M}{r}\right)dt^2 + \left(1-\frac{2M}{r}\right)^{-1}dr^2 + r^2 d\Omega_2^2$$

Near-horizon behavior

$$g^{-1}(r) = 1-\frac{2M}{r} = 1-\frac{r_{sh}}{r} = \frac{1}{r_{sh}}(r-r_{sh}) - \frac{1}{r_{sh}^2}(r-r_{sh})^2 + \cdots .$$

$$ds^2 = -f(r)dt^2 + g(r)dr^2 + r^2 d\Omega_2^2.$$

$$\rho(r) = -\frac{3}{8\pi\alpha}\left\{1 \mp \frac{2\alpha|\beta|}{r}\left[\beta(r^2+\alpha)\right]^{1/2}\right\},$$

$$f(r) = \frac{\rho(r)}{\rho_c}, \qquad g^{-1}(r) = -\frac{8\pi}{3}(r^2+\alpha)\rho(r).$$

$$g^{-1}(r) = \frac{1}{r_h}(r-r_h) + \cdots , \qquad r_h = \pm 2\alpha|\beta|\sqrt{\frac{\alpha\beta}{1-4\alpha^2\beta^3}},$$

: same structure with Sch. BH

Identifying r_h=r_sh, we can relate K with M:

$$K = \left(\frac{R_0^2}{4M^2}-1\right)^{-1/2}, \quad \left(-\frac{R_0^2}{4M^2}+1\right)^{-1/2}, \quad \left(\frac{R_0^2}{4M^2}+1\right)^{-1/2}.$$

S3-I   S3-II   H3

$$K = \left(\frac{R_0^2}{4M^2} - 1\right)^{-1/2}, \quad \left(-\frac{R_0^2}{4M^2} + 1\right)^{-1/2}, \quad \left(\frac{R_0^2}{4M^2} + 1\right)^{-1/2}.$$

S3-I　　S3-II　　H3

From this relation between K & M,

S3-I: **M_max = $R_0/2$**

→ K = ∞

→ horizon at equator: maximum size of S3-I BH

If M > M_max, S3-II black holes

Geodesics

$$ds^2 = -\frac{3}{8\pi R_0^2 \rho_c} F(\chi) dt^2 + g(\chi) d\chi^2 + R_0^2 b^2(\chi) d\Omega_2^2$$

$$t\text{-eq. :} \quad \frac{1}{F(\chi)} \frac{d}{d\lambda} \left[F(\chi) \frac{dt}{d\lambda} \right] = 0,$$
$$\phi\text{-eq. :} \quad \frac{1}{b^2(\chi)} \frac{d}{d\lambda} \left[b^2(\chi) \frac{d\phi}{d\lambda} \right] = 0.$$

$$F(\chi) \frac{dt}{d\lambda} = \text{const.} \equiv E, \qquad b^2(\chi) \frac{d\phi}{d\lambda} = \text{const.} \equiv L.$$

On the $\theta = \pi/2$ plane, the χ-equation becomes

$$g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} = -\varepsilon,$$

$\varepsilon = 1, 0$ for timelike and null geodesics,

$$\frac{1}{2} \left(\frac{d\chi}{d\lambda} \right)^2 + V(\chi) = \frac{3E^2}{16\pi R_0^4 |\rho_c|} \equiv \tilde{E}^2,$$

$$V(\phi) = \frac{1}{2} F(\chi) \left[\frac{L^2}{b^2(\chi)} + \frac{\varepsilon}{R_0^2} \right].$$

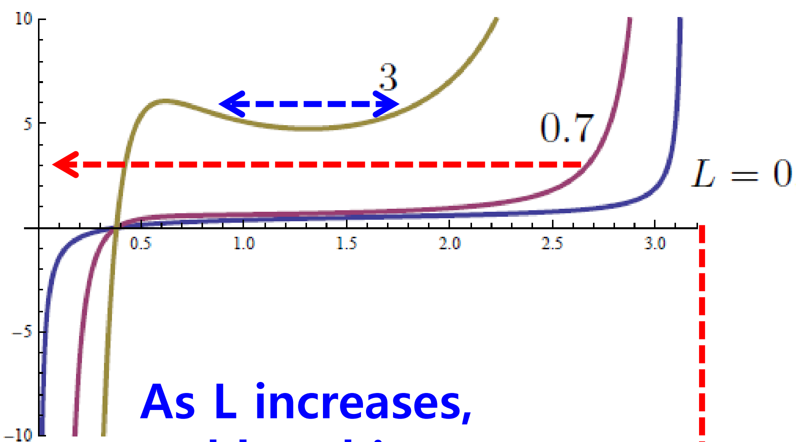
Effective Potential

S3-I

$$F(\chi) = 1 - K\cot\chi, \qquad b(\chi) = \sin\chi,$$

$$V(\phi) = \frac{1}{2}(1 - K\cot\chi)\left(\frac{L^2}{\sin^2\chi} + \frac{\varepsilon}{R_0^2}\right).$$

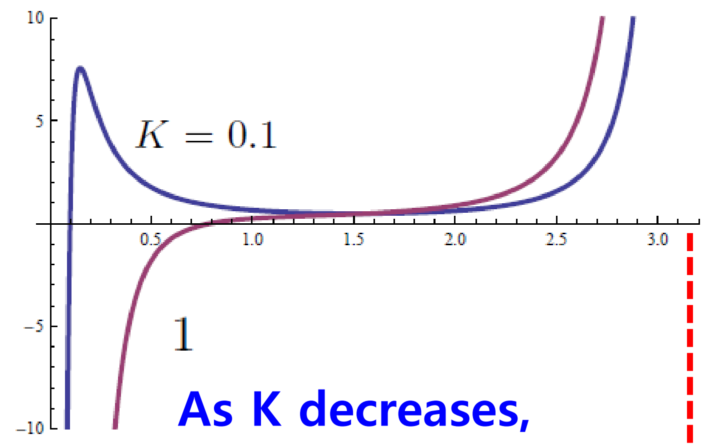
Timelike



As L increases, stable orbit appears.

(a) S_3 timelike geodesics for $R_0 = 1$ and $K = 0.4$:

Null



As K decreases, stable orbit appears.

(b) S_3 null geodesics for $R_0 = 1$ and $L = 1$:

NO geodesic can reach the singularity at south pole!!!

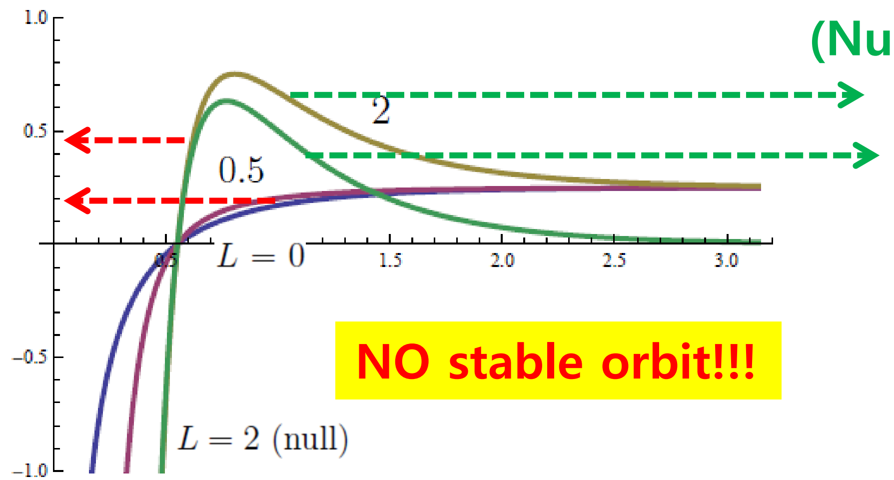
H3

$$F(\chi) = 1 - K\coth\chi, \qquad b(\chi) = \sinh\chi,$$

$$V(\phi) = \frac{1}{2}(1 - K\coth\chi)\left(\frac{L^2}{\sinh^2\chi} + \frac{\varepsilon}{R_0^2}\right).$$

Similar with Sch. BH

As L increases,
bouncing orbit appears.
(Null: always)



can escape

NO stable orbit!!!

(c) H_3 timelike and null geodesics for $R_0 = 1$ and $K = 0.5$:

S3-II : similar to H3

Stability

Spherical Scalar Perturbations

$$ds^2 = -f(t,\chi)dt^2 + g(t,\chi)d\chi^2 + R_0^2 b^2(\chi)d\Omega_2^2,$$

$$f(t,\chi) = f_0(\chi) + \epsilon f_1(t,\chi),$$
$$g(t,\chi) = R_0^2\left[g_0(\chi) + \epsilon g_1(t,\chi)\right],$$

$$f_0(\chi) = \frac{\rho_0(\chi)}{\rho_c} = \frac{3}{8\pi R_0^2\rho_c}F(\chi),$$
$$g_0(\chi) = \frac{s}{F(\chi)},$$

background

s=+1: S3-I
s=-1: S3-II, H3

$$T^{\mu\nu} = (\rho+p)u^\mu u^\nu + pg^{\mu\nu},$$

$$\rho(t,\chi) = \rho_0(\chi) + \epsilon\rho_1(t,\chi),$$

source of perturbation

$$u^\mu = [u^0(t,\chi), u^1(t,\chi), 0, 0].$$

normalization $u^\mu u_\mu = -1$

$1/\sqrt{f_0(\chi)}$ $\quad -f_1/(2f_0^{3/2})$ from normalization

$$u^0(t,\chi) = u_0^0(\chi) + \epsilon u_1^0(t,\chi),$$
$$u^1(t,\chi) = u_0^1(\chi) + \epsilon u_1^1(t,\chi).$$

$$-\sqrt{\frac{2\pi R_0^2\rho_c}{3}}\frac{\dot{g}_1 b' F}{s^2 b\sqrt{F}}.$$

from Einstein's eq.: (0,1)

comoving fluid

After manipulating the Einstein equations, one finally gets the single master perturbation equation.
If we introduce the perturbation in the form,

$$g_1(t,\chi) = e^{i\omega t}\varphi(\chi),$$

the perturbation equation becomes

$$-F^2\varphi'' - \left(3FF' + 2F^2\frac{b''}{b'}\right)\varphi' + \left[\frac{\omega^2}{S} - 2FF'' - FF'\left(5\frac{b''}{b'} - \frac{b'}{b}\right) - 2F^2\left(\frac{b'''}{b'} - \frac{b''^2}{b'^2} + \frac{b''}{b} - \frac{b'^2}{b^2}\right)\right]\varphi = 0,$$

For all cases $S = 1/(8\pi R_0^4 \rho_c s) > 0$

We rescale the radial coordinate and the amplitude function by

$$z = \int_0^\chi \frac{d\chi}{2F(\chi)},$$
$$\Phi(z) = N\frac{F(\chi)b'(\chi)}{z}\varphi(\chi),$$

The perturbation equation is cast into the Schrodinger-type equation,

$$\left[-\frac{1}{2}\frac{d^2}{dz^2} - \frac{1}{z}\frac{d}{dz} + V(z)\right]\Phi(z) = -\frac{\omega^2}{S}\Phi(z) = -8\pi R_0^4|\rho_c|\omega^2\Phi(z),$$

eigenvalue

$$V[z(\chi)] = F^2\left[-\frac{F''}{F} + \left(\frac{F'}{F}\right)^2 - \frac{F'}{F}\left(2\frac{b''}{b'} - \frac{b'}{b}\right) - \frac{b'''}{b'} + 2\left(\frac{b''}{b'}\right)^2 - 2\frac{b''}{b} + 2\left(\frac{b'}{b}\right)^2\right]$$

: effective potential

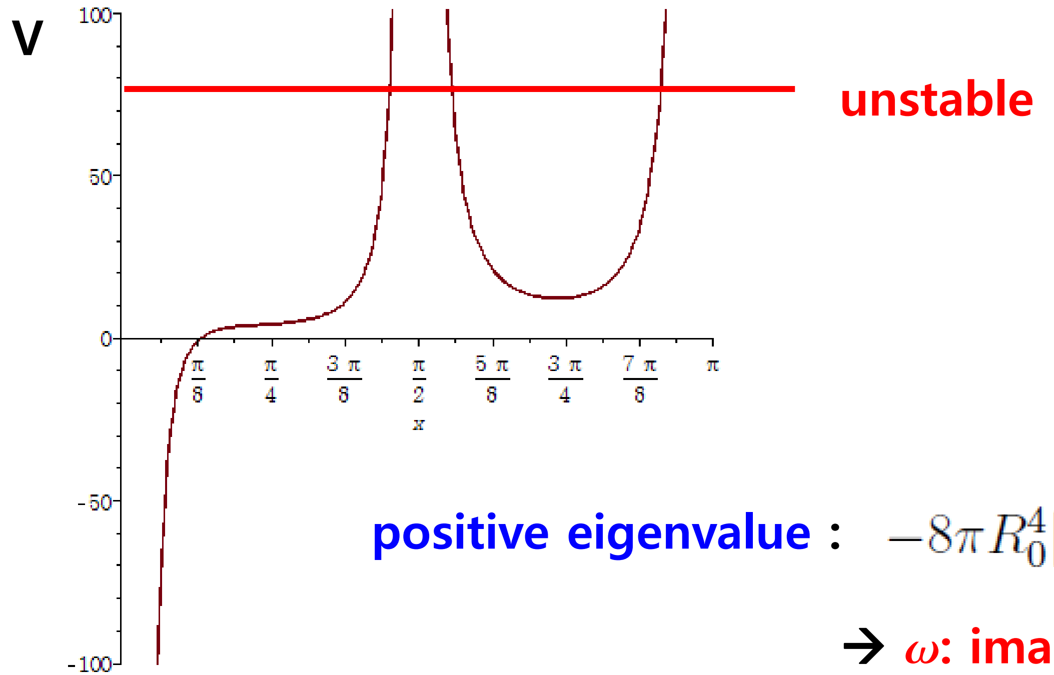
Unconditionally UNSTABLE !!!

$$\left[-\frac{1}{2}\frac{d^2}{dz^2} - \frac{1}{z}\frac{d}{dz} + V(z)\right]\Phi(z) = -\frac{\omega^2}{S}\Phi(z) = -8\pi R_0^4|\rho_c|\omega^2\Phi(z),$$

eigenvalue

since this is negative, there exists always unstable modes for any type of V

S3-I

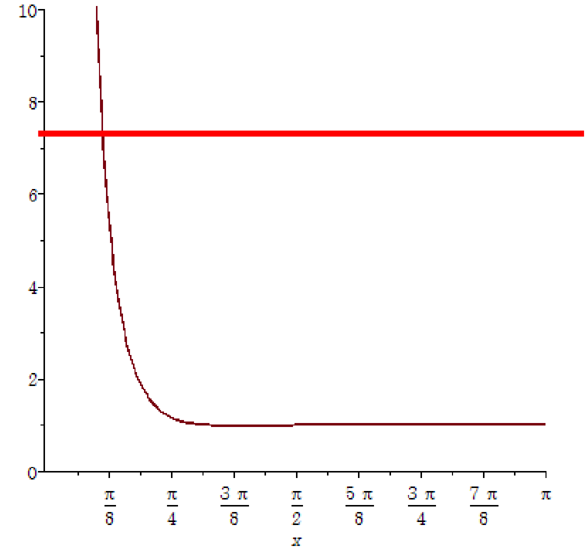


positive eigenvalue : $-8\pi R_0^4|\rho_c|\omega^2 > 0$

→ ω: imaginary

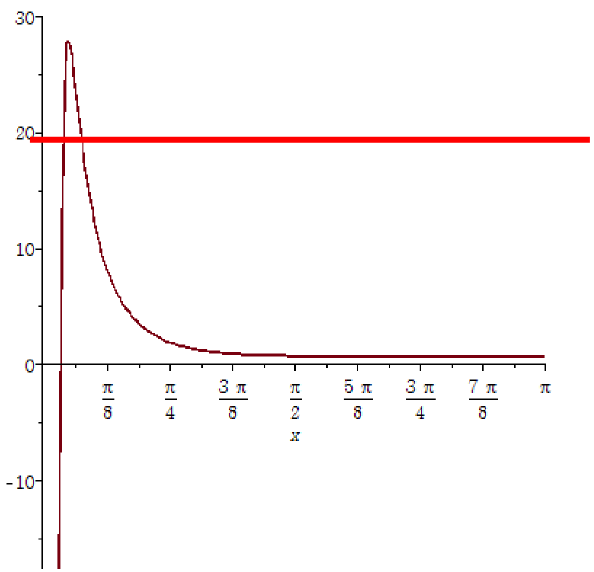
→ UNSTABLE!!

S3-II



unstable

H3



unstable

Fate of Black Holes

1) BH may collapse: may leave some cosmological remnants

2) BH may remain while BACKGROUND expands
   :- since the b.g. matter is perfect fluid,
      the instability may imply the Friedmann expansion
   :- BH may sustain its nature in expanding b.g. Universe

Other type of metric perturbations

$$S^2(t) \equiv 1 + \epsilon a(t)$$: scale factor

$$ds^2 = -f_0[\chi, \kappa(t)]dt^2 + S^2(t)\{g_0[\chi, \kappa(t)]d\chi^2 + R_0^2 b^2(\chi)d\Omega_2^2\}\,.$$

$$K \to K + \epsilon\kappa(t)$$: horizon location

Solutions:

$$a(t) = a_1 t + a_0, \qquad \kappa(t) = \kappa_1 t + \kappa_0.$$

:- Both are linear in "t"

:- Their evolutions are independent

:- Depending on I.C., the Direction of Evolution is determined

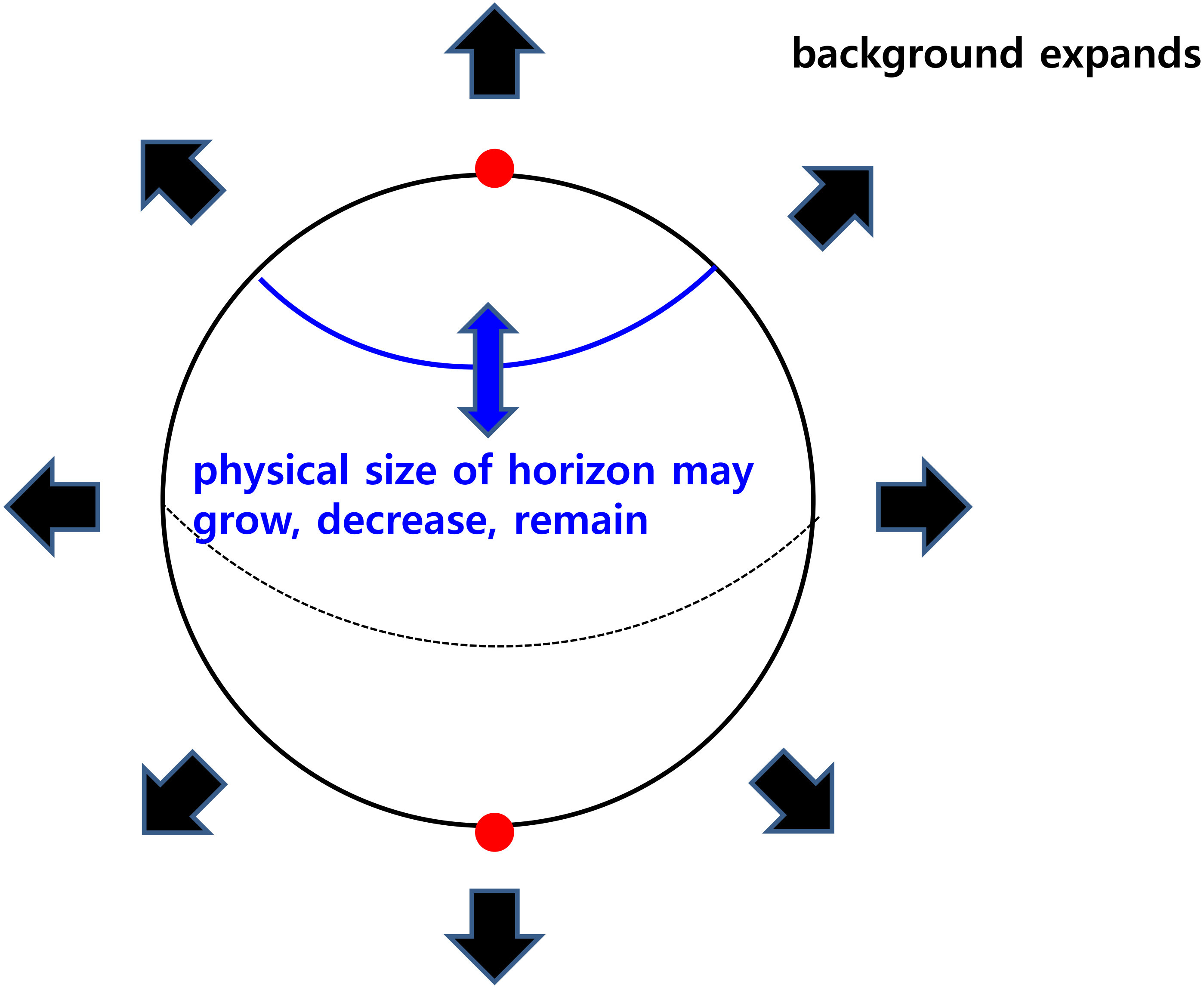
e.g.) impose $\rho_1=0$ at the horizon (S3-I black hole)

→ means NO energy-flow through the horizon: v=0

→ preserves "positivity of energy" density in both regions

→ Coordinate of horizon decreases

→ BUT, "physical size of horizon" is UNCHANGED !!!


background expands
physical size of horizon may
grow, decrease, remain

Conclusions

1) We found black-hole solution with static perfect fluid & electric field in spatial topology S3 & H3

For Charged Case,

i) S3-I
   (a) RN black-hole solution
   (b) naked singular solution

ii) S3-II
   (a) RN black-hole solution
   (b) Schwartzchilld black-hole solution
   (c) dS-type solution
   (d) regular solution
   (e) nonstatic cosmological solution

iii) H3
   (a) RN black-hole solution
   (b) dS-type solution
   (c) naked singular solution

For Pure Fluid Case,

i) S3-I

(a) black-hole solution

:- singularities at both poles

:- the one is hidden by horizon, the other is naked

:- the naked singularity is visible to the observer

:- however, observer never falls into the naked singularity

:- observer may fall into BH, or can have a stable orbit

ii) S3-II

(a) black-hole solution

:- not very interesting b/c $\rho<0$ in the regular region

(b) cosmological solution

:- expanding from a finite size

iii) H3

(a) black-hole solution : single singularity behind horizon, no stable orbit

(b) cosmological solution: initial singularity

(c) regular solution: central singularity

2) Stability :- Unconditionally unstable

:- However, black hole can remain its feature in an expanding background