Effect of a Magnetic Field on Schwinger Mechanism in de Sitter Spacetime

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Outline

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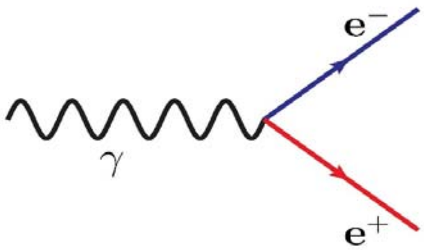
Introduction: Pair Creation

Schwinger effect: the creation of pairs out of the vacuum under the presence of a strong electric field background in the Minkowski spacetime.

Was initially discovered in the pioneers' works: [F. Sauter, Z. Phys. 69, 742 (1931); J. S. Schwinger, Phys. Rev. 82, 664 (1951)].

The pair creation rate is proportional to:

$$\Gamma \propto \exp\left(-\frac{\pi E_c}{E}\right), \qquad E_c = \frac{m^2}{e} \simeq 10^{18} V/m$$



Hence, despite tremendous efforts on the experimental point of view, it has never been detected so far.

What else can create pairs? A strong gravity field [1].

The tool to study is quantum field theory in curved spacetime [2].

To detect this effect a new idea is developing past years: considering Schwinger effect in astrophysical and cosmological contexts where huge background fields could naturally be present.

1. L. PARKER, PHYS. REV. LETT. 21, 562 (1986).
2. L. PARKER AND D. TOMS, QUANTUM FIELD THEORY IN CURVED SPACETIME, (CAMBRIDGE UNIVERSITY PRESS, 2009).

Introduction: de sitter

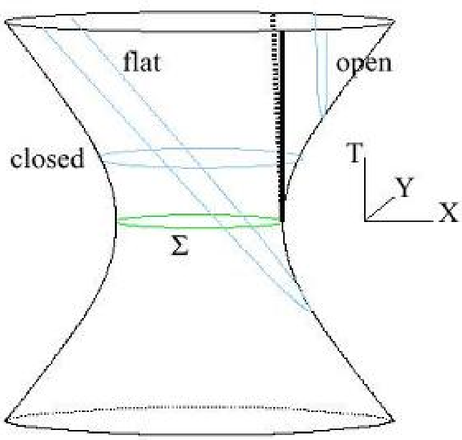
In the early universe to model the gravitational field we consider the de Sitter (dS) spacetime.

dS might describe both the early stage of inflation and the late stage of acceleration of the expansion of the Universe.

In the Poincare path, the dS metric reads:

$$ds^2 = \Omega^2(\tau)\left(d\tau^2 - dx^2 - dy^2 - dz^2\right), \qquad \Omega(\tau) = \frac{-1}{\tau H},$$

$$\tau \in (-\infty, 0), \qquad (x, y, z) \in \mathbb{R}^3$$



This portion of dS is conformal to a portion of Minkowski spacetime.

Introduction: Existing Literature

Considering **a constant electric field** background in dS, the **semi-classical pair creation rate** of Schwinger **scalars** has been investigated:

- In 1+1D dS: Fröb et al, JCAP. 04, 009 (2014).

- In 1+3D dS: Kobayashi and Afshordi, JHEP, 10, 166 (2014).

- In 1+dD dS: Bavarsad, Stahl and Xue, PRD 94, 104011 (2016).

The right quantity to describe Schwinger effect in curved spacetime is the induced current.

Indeed, it is not plagued by the need of the notion of particle in the adiabatic future which allows one to explore a broader parameter space.

The renormalized in-vacuum expectation value of the **induced current** operator of the **scalar field** coupled to a **constant electric** background:

In 1+1D dS: Fröb et al, JCAP. 04, 009 (2014).

In 1+3D dS: Kobayashi and Afshordi, JHEP, 10, 166 (2014).

In 1+2D dS: Bavarsad, Stahl and Xue, PRD 94, 104011 (2016).

In 1+3D dS with another renormalization scheme: Hayashinaka and Yokoyama, JCAP 07, 012 (2016).

Aiming at checking if there is a difference between boson and fermion **the equivalent problem for Dirac particles** has been investigated.

- Semi-classical pair creation rate and the renormalized induced current in 1+1D dS:

Stahl, Strobel, and Xue, PRD 93, 025004 (2016).

- Semi-classical pair creation rate in 1+3D dS: Stahl, Strobel [arXiv:1507.01401].

- Renormalized Induced current in 1+3D dS: Hayashinaka, Fujita, and Yokoyama, JCAP 07, 010 (2016).

In this work, we propose to take one step back and **to add the presence of a constant magnetic field background** to the already present dS and electric backgrounds.

Preliminaries

To study the Schwinger effect we start from the action of a complex scalar field coupled to an electromagnetic field

$$S = \int d^4x \sqrt{|g|} \left\{ (\partial_\mu + ieA_\mu)\varphi(\partial_\nu - ieA_\nu)\varphi^* - (m^2 + \xi R)\varphi\varphi^* - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right\}$$

We assume: the gravitational and electromagnetic field are background fields whereas the scalar field is dynamical (and quantized).

The dS has a constant scalar curvature, hence for simplicity, **we set:**

$$\xi = 0$$

The dS metric in conformal coordinates has been written as

$$ds^2 = \Omega^2(\tau)\left(d\tau^2 - dx^2 - dy^2 - dz^2\right), \qquad \Omega(\tau) = \frac{-1}{\tau H}, \qquad (x, y, z) \in \mathbb{R}^3$$

The vector potential describing a **constant electric and magnetic fields parallel** to each other in the conformal metric is given by

$$A_\mu = -\frac{E}{H^2 \tau}\delta_\mu^3 + eBy\delta_\mu^1$$

The Klein-Gordon (KG) equation then reads form the action

$$\left[\partial_0^2 - \frac{2}{\tau}\partial_0 - \left(\partial_1 + ieBy\right)^2 - \partial_2^2 - \left(\partial_3 - \frac{ieE}{H^2\tau}\right)^2 + \frac{m^2}{H^2\tau^2}\right]\varphi(x) = 0$$

Using the ansatz

$$\varphi(x) = \Omega^{-1}(\tau)e^{\pm i\mathbf{k}_{\not y}.\mathbf{x}}h^{\pm}(y)f^{\pm}(\tau), \qquad \mathbf{k}_{\not y} = (k_x, 0, k_z)$$

Where + and - denote the positive and negative frequency solutions of KG equation respectively,

we decouple the spatial and time dependent parts of KG equation as

$$\frac{d^2h^{\pm}(y)}{dy^2} - \left(eBy \pm k_x\right)^2 h^{\pm}(y) = -sh^{\pm}(y),$$

$$\frac{d^2f^{\pm}(\tau)}{d\tau^2} + \left[\left(\frac{eE}{H^2\tau} \mp k_z\right)^2 + \frac{m^2}{H^2\tau^2} - \frac{2}{\tau^2}\right]f^{\pm}(\tau) = -sf^{\pm}(\tau)$$

The harmonic wave function is a Landau state

$$h_n(y_\pm) = \sqrt{\frac{\sqrt{eB}}{\sqrt{\pi} n! 2^n}} e^{\frac{-y_\pm^2}{2}} H_n(y_\pm), \qquad y_\pm = \sqrt{eB}\, y \pm \frac{k_x}{\sqrt{eB}}, \qquad s = (2n+1)eB, \qquad n \in \mathbb{W}$$

Mode functions:

The positive and negative frequency solutions with desired asymptotic forms (mode functions in Minkowski spacetime) at early times ($\tau \to -\infty$)

$$U_{\text{in}}(x) = (2k)^{\frac{-1}{2}} e^{\frac{i\kappa\pi}{2}} \Omega^{-1} e^{+i\mathbf{k.x}} h(y_+) W_{\kappa,\gamma}(2ip),$$

$$V_{\text{in}}(x) = (2k)^{\frac{-1}{2}} e^{\frac{-i\kappa\pi}{2}} \Omega^{-1} e^{-i\mathbf{k.x}} h(y_-) W_{\kappa,\gamma}(-2ip),$$

The positive and negative frequency solutions with desired asymptotic forms (mode functions in Minkowski spacetime) at late times $(\tau \to 0)$

$$U_{\text{out}}(x) = (-4i\gamma k)^{\frac{-1}{2}} e^{\frac{i\gamma\pi}{2}} \Omega^{-1} e^{+i\mathbf{k.x}} h(y_+) M_{\kappa,\gamma}(2ip),$$

$$V_{\text{out}}(x) = (-4i\lambda k)^{\frac{-1}{2}} e^{\frac{i\gamma\pi}{2}} \Omega^{-1} e^{-i\mathbf{k.x}} h(y_-) M_{\kappa,\gamma}(-2ip),$$

where W, M are the Whittaker functions.

The parameters have been defined as:

$$k=\sqrt{k_z^2+(2n+1)eB}, \qquad r=\frac{k_z}{k}, \qquad p=-k\tau, \qquad \mathbf{p}_y=-\mathbf{k}_y\tau,$$

$$\lambda=\frac{eE}{H^2}, \qquad \mu=\frac{m}{H}, \qquad \rho=\sqrt{\lambda^2+\mu^2}, \qquad \gamma=\sqrt{\frac{9}{4}-\rho^2}, \qquad \kappa=i\lambda r$$

The Bogoliubov coefficients are defined

$$\mathcal{A}=\left(U_{\text{out}},U_{\text{in}}\right), \qquad \mathcal{B}=\left(U_{\text{out}},V_{\text{in}}\right),$$

$$\mathcal{A}=(2\pi)^2\delta^2(\mathbf{k}_y-\mathbf{k}_y^{'})\delta_{n,n'}\alpha, \qquad \mathcal{B}=(2\pi)^2\delta^2(\mathbf{k}_y-\mathbf{k}_y^{'})\delta_{n,n'}\beta,$$

$$\alpha=\frac{\left(2|\gamma|\right)^{\frac{1}{2}}\Gamma\left(2\gamma\right)}{\Gamma\left(\frac{1}{2}+\gamma+\kappa\right)}e^{\frac{i\pi(\kappa-\gamma)}{2}}, \qquad \beta=-i\frac{\left(2|\gamma|\right)^{\frac{1}{2}}\Gamma\left(-2\gamma\right)}{\Gamma\left(\frac{1}{2}-\gamma+\kappa\right)}e^{\frac{i\pi(\kappa+\gamma)}{2}}, \qquad |\alpha|^2-|\beta|^2=1$$

Schwinger Effect

The usual quantity describing the Schwinger effect is the pair creation or decay rate which is derived from the Bogoliubov coefficients

$$\frac{N}{\sqrt{|g|}TV} = \frac{1}{\Omega^4 T L_y} \sum_{n=0} \int \frac{dk_z}{(2\pi)} \int \frac{dk_x}{(2\pi)} |\beta|^2$$

The Bogoliubov coefficient β is independent of the momentum k_x which determines the position of the center of the Gaussian wave pocket on y axis by

$$y = \frac{k_x}{eB} \quad \Rightarrow \quad \int \frac{dk_x}{(2\pi)} = \frac{eBL_y}{(2\pi)}$$

In order to transform the k_z integral into a τ integral, we can use the semi-classical relation [1,2]

$$k_z \sim -\frac{|\gamma|}{\tau}, \qquad \frac{(eE)^2}{H^4} + \frac{m^2}{H^2} \gg 1$$

which estimates the time when most of the particles are created. Then

$$\int \frac{dk_z}{(2\pi)} \to \frac{H^2 |\gamma|}{(2\pi)} \int \Omega^2(\tau) d\tau$$

1. FRÖB ET AL, JCAP. 04, 009 (2014).
2. KOBAYASHI AND AFSHORDI, JHEP, 10, 166 (2014).

Using the following parameterization of the decay rate

$$\Gamma = \Omega^2 \frac{N}{\sqrt{|g|}TV}$$

$$\Gamma = \left(\frac{H^2 |\gamma|}{2\pi}\right)\left(\frac{eB}{2\pi}\right)\sum_{n=0}\left[\frac{e^{-2\pi|\kappa|}-1}{e^{2\pi|\gamma|}-e^{-2\pi|\gamma|}}+\frac{1}{e^{2\pi|\gamma|}-1}\right]$$

with

$$|\kappa| = \frac{\lambda |\gamma|}{\sqrt{|\gamma|^2 + (2n+1)eB\tau^2}}$$

$$\Gamma = \left(\frac{H^2|\gamma|}{2\pi}\right)\left(\frac{eB}{2\pi}\right)\sum_{n=0}\left[\frac{e^{-2\pi\kappa}-1}{e^{2\pi|\gamma|}-e^{-2\pi|\gamma|}}+\frac{1}{e^{2\pi|\gamma|}-1}\right]$$

- There is a term independent of the Landau levels whose sum apparently gives a diverging factor. However, using the Riemann zeta function $\zeta(0)=\frac{-1}{2}$ gives

$$\Gamma = \left(\frac{H^2|\gamma|}{2\pi}\right)\left(\frac{eB}{2\pi}\right)\frac{1}{e^{4\pi|\gamma|}-1}\left[\frac{1}{2}+\sum_{n=0}e^{2\pi(|\gamma|-|\kappa|)}\right]$$

- In the limit $E=0$ the first term in the square bracket vanishes and the second term is the dS radiation with a Gibbons-Hawking temperature

$$\Gamma = \frac{1}{2}\left(\frac{H^2|\gamma|}{2\pi}\right)\left(\frac{eB}{2\pi}\right)\frac{1}{e^{2\pi|\gamma|}-1}$$

$$\Gamma = \left(\frac{H^2|\gamma|}{2\pi}\right)\left(\frac{eB}{2\pi}\right)\sum_{n=0}\left[\frac{e^{-2\pi\kappa}-1}{e^{2\pi|\gamma|}-e^{-2\pi|\gamma|}}+\frac{1}{e^{2\pi|\gamma|}-1}\right]$$

- In the Minkowski spacetime limit $(H = 0)$ gives the Schwinger formula in scalar QED

$$\Gamma = \frac{1}{2}\left(\frac{eE}{2\pi}\right)\left(\frac{eB}{2\pi}\right)\frac{e^{\frac{-\pi m^2}{|eE|}}}{\sinh\left(\frac{\pi B}{E}\right)}$$

Induced current

Now I wish to investigate the **in vacuum expectation value of the current operator** which is referred to as the **induced current**

$$j^{\mu} = \frac{ie}{2} g^{\mu\nu} \left(\{(\partial_{\nu}\varphi + ieA_{\nu}\varphi), \varphi^{*}\} - \{(\partial_{\nu}\varphi^{*} - ieA_{\nu}\varphi^{*}), \varphi\} \right)$$

The scalar field operator and in vacuum state

$$\varphi = \sum_{n=0} \int \frac{d^2 k_y}{(2\pi)^2} \left[U_{\mathrm{in}} a_{\mathrm{in}} + V_{\mathrm{in}} b_{\mathrm{in}}^{\dagger} \right], \qquad a_{\mathrm{in}} \left| 0 \right\rangle = 0$$

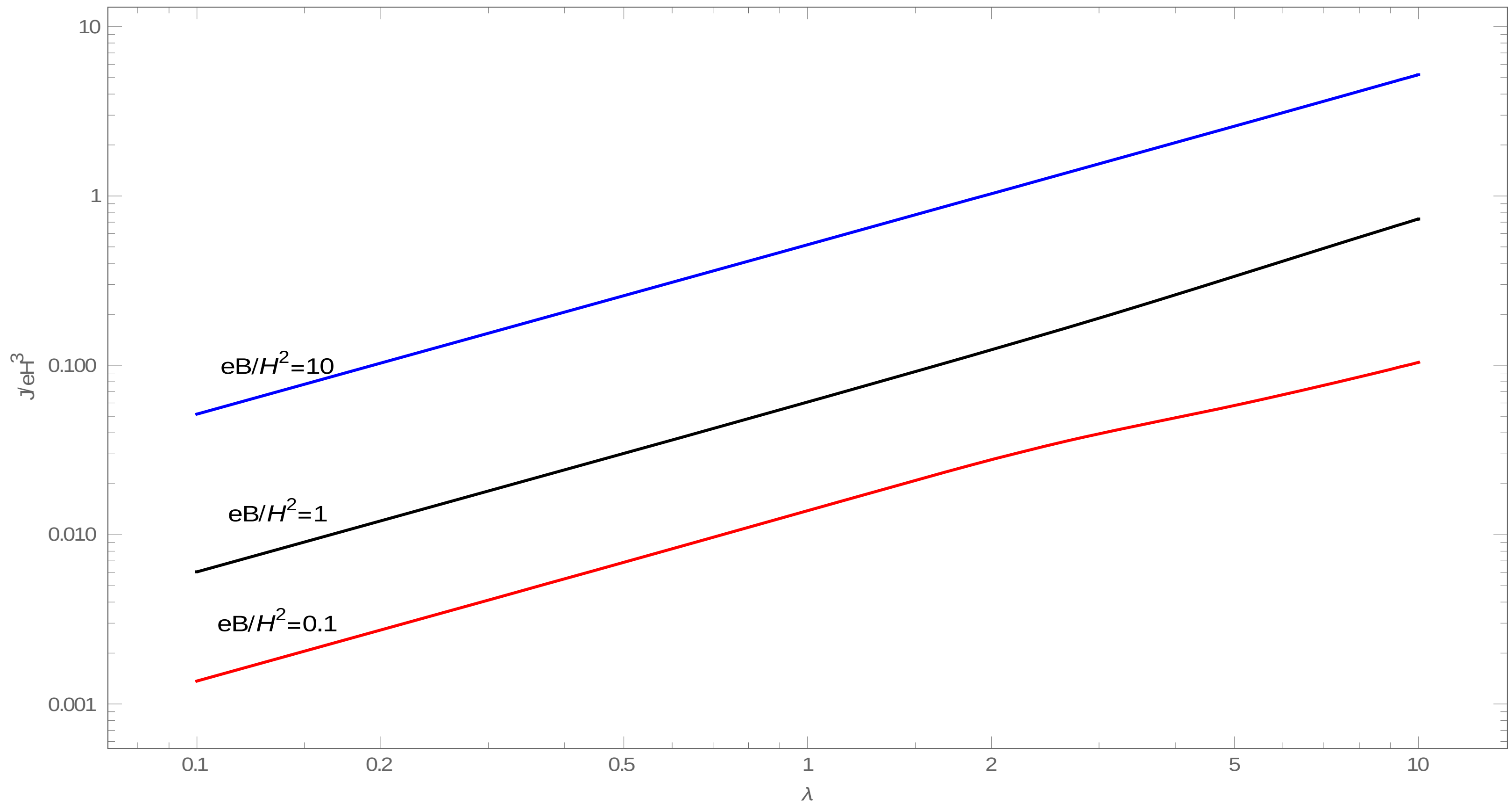
By the symmetry the time-like and the perpendicular components of the current vanish

$$\langle j^0 \rangle = \langle j^1 \rangle = \langle j^2 \rangle = 0$$

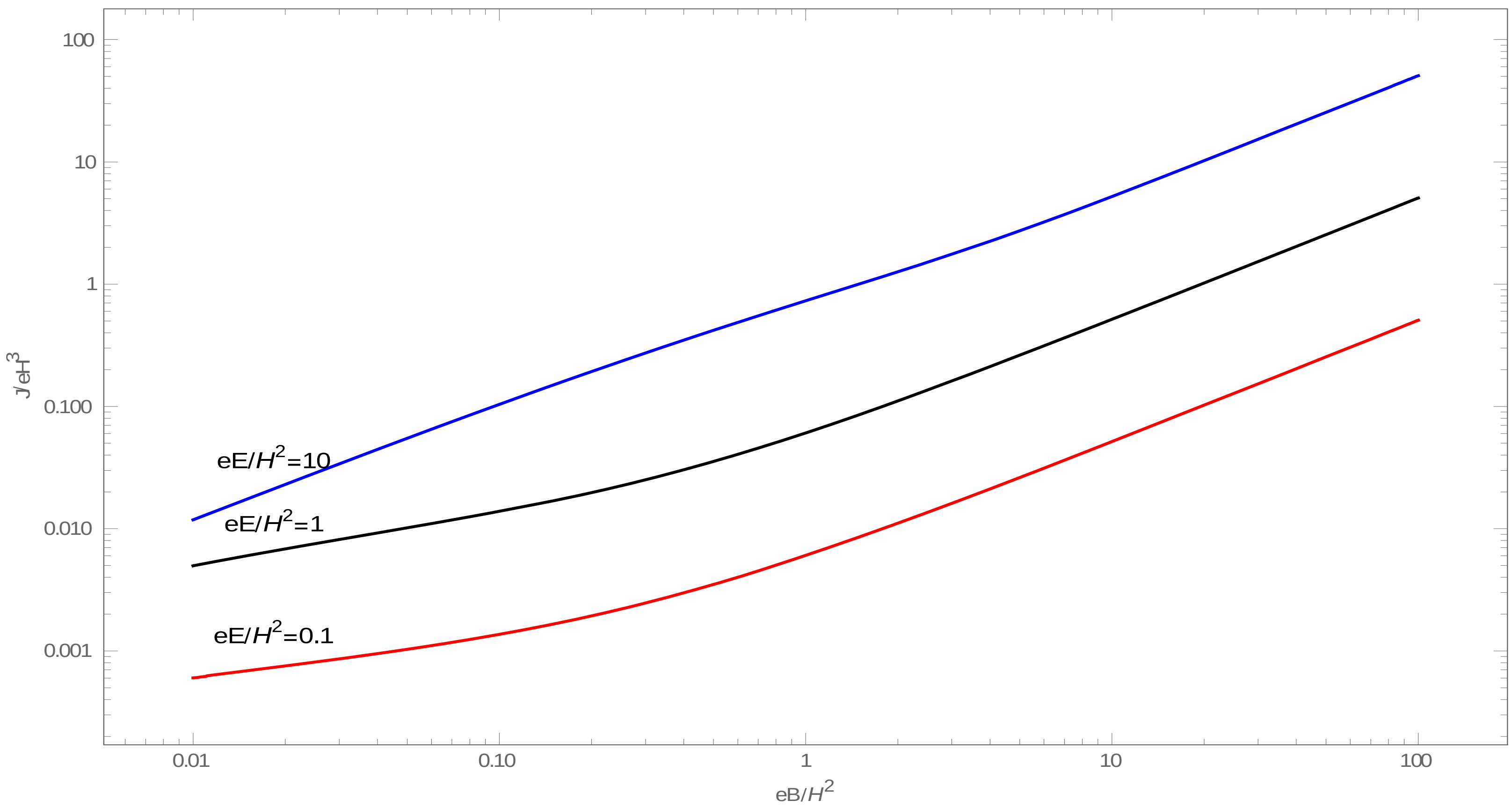
It is easily seen that the **only non-vanishing component of the current** is the component **parallel to the electric field background**

$$\langle j^3 \rangle = \frac{eH}{2\pi}\left(\frac{eB}{2\pi}\right)\sum_{n=0}\int_{-\infty}^{+\infty}\frac{dp_z}{p}(rp+\lambda)e^{-\pi\lambda r}\left|W_{i\lambda r,\gamma}(-2ip)\right|^2$$

$$p = \sqrt{p_z^2 + (2n+1)eB\tau^2}, \qquad \lambda = \frac{eE}{H^2}, \qquad \mu = \frac{m}{H}, \qquad \gamma = \sqrt{\frac{9}{4} - \lambda^2 - \mu^2}$$



For different values of eB/H^2, the induced current J/eH^3 is plotted as a function of eE/H^2 in the lowest Landau state $n = 0$ with $\frac{m}{H} = 1$



For different values of eE/H^2, the induced current J/eH^3 is plotted as a function of eB/H^2 in the lowest Landau state $n = 0$ with $\frac{m}{H} = 1$

We will now analytically investigate the limiting behavior of the induced current

Limiting Behavior in Weak Magnetic Field Regime

In the weak magnetic field regime the relation $\frac{eB}{H^2} \ll \min\left\{1, \frac{eE}{H^2}, \frac{m}{H}\right\}$ is satisfied.

Using the similar integration procedure introduced in [Fröb et al, JCAP. 04, 009 (2014)] and [Kobayashi and Afshordi, JHEP, 10, 166 (2014)],

and **the zeta function regularization**

$$J = \frac{eH}{2\pi}\left(\frac{eB}{2\pi}\right)\left(\frac{\gamma \sinh(2\pi\lambda)}{\sin(2\pi\gamma)} + \lambda\right)$$

$$J = \frac{eH}{2\pi}\left(\frac{eB}{2\pi}\right)\left(\frac{\gamma \sinh(2\pi\lambda)}{\sin(2\pi\gamma)} + \lambda\right)$$

In the strong electric field regime: $\frac{eE}{H^2} \gg \max\left\{1, \frac{m}{H}\right\}$ the leading order term obtained

$$J \simeq \frac{e^3 EB}{4\pi^2 H} e^{\frac{-\pi m^2}{|eE|}}$$

In the IR regime $\frac{eE}{H^2} \ll 1$ and $\frac{m}{H} \ll 1$ the leading order term obtained

$$J \simeq \left(\frac{9e^3 H^3}{8\pi^2}\right)\left(\frac{EB}{(eE)^2 + (mH)^2}\right)$$

In this regime the current and conductivity are increasing as the electric field is decreasing (IRHC) [Fröb et al, JCAP. 04, 009 (2014)].

Strong Magnetic Field Regime

In the strong magnetic field regime the relation $\frac{eB}{H^2} \gg \max\{1, \frac{eE}{H^2}, \frac{m}{H}\}$ is satisfied.

The leading order term of the current obtained

$$J \sim \frac{e^3 EB}{8\pi^2 H}$$

Conclusions

- We obtained the pair production rate, which agrees with the known Schwinger result in Minkowski spacetime and with the Hawking radiation in the limit of zero electric field in dS.
- Using the zeta function regularization scheme we calculate the induced current and examine the effect of magnetic field on the vacuum expectation value of the current operator.
- We find that in the strong electric E or magnetic field B the current responds as $E.B$, instead in the infrared regime, it responds as B/E which leads to a phenomenon of infrared hyperconductivity.
- Those results of the induced current would be important for discussing the cosmic magnetic field evolution.

Thank you for your attention.