Entropy in Spacetime and Topological Hair

Young-Hwan Hyun

In collaboration with

Yoonbai Kim

Sungkyunkwan University

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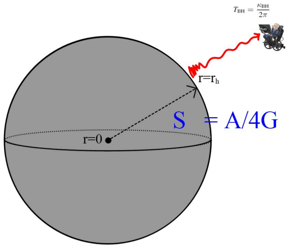
Toward the Quantum Gravity....

We have a clue,



$$S_{BH} = \frac{k_B c^3}{\hbar} \frac{A_{BH}}{4G}$$

[2] Motivation

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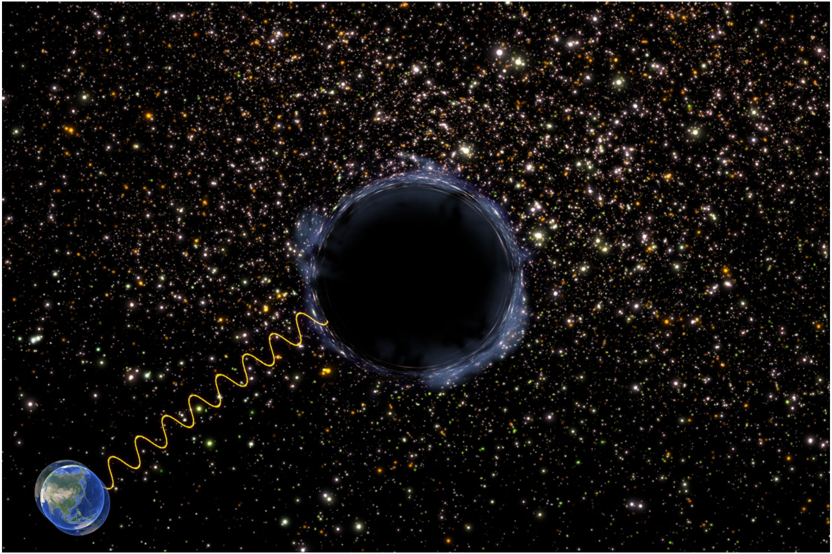

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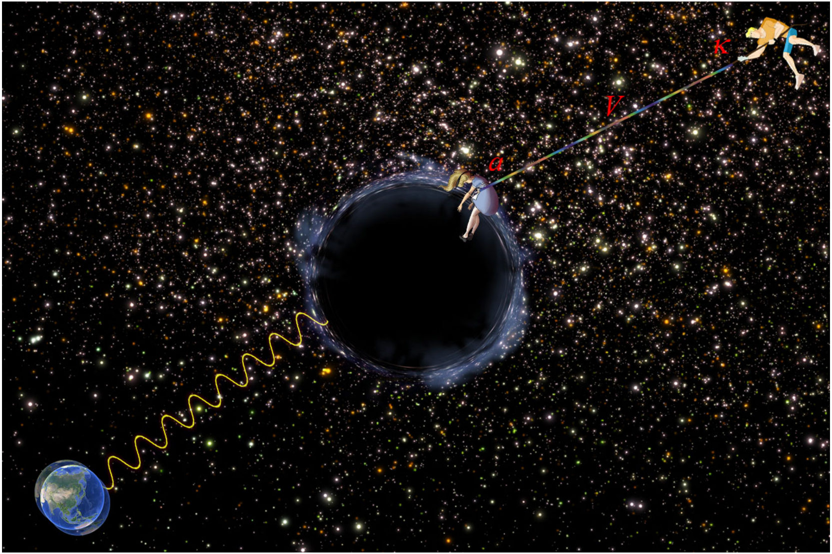

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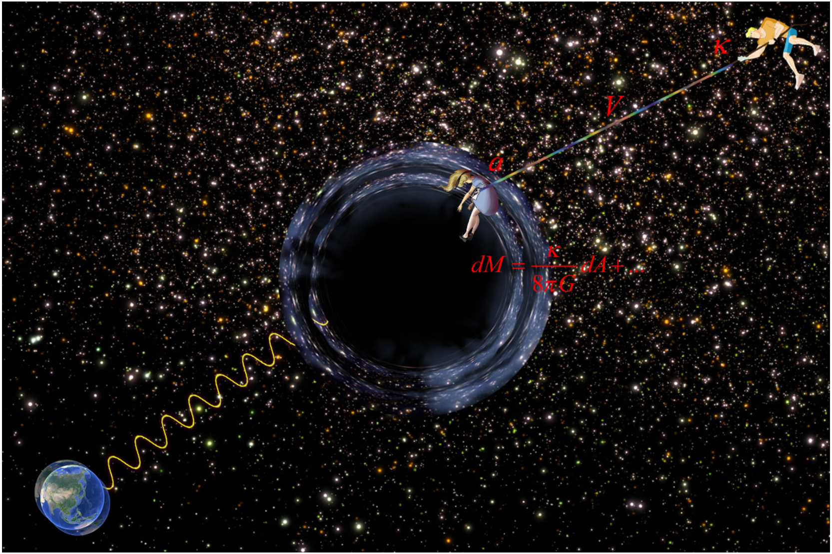

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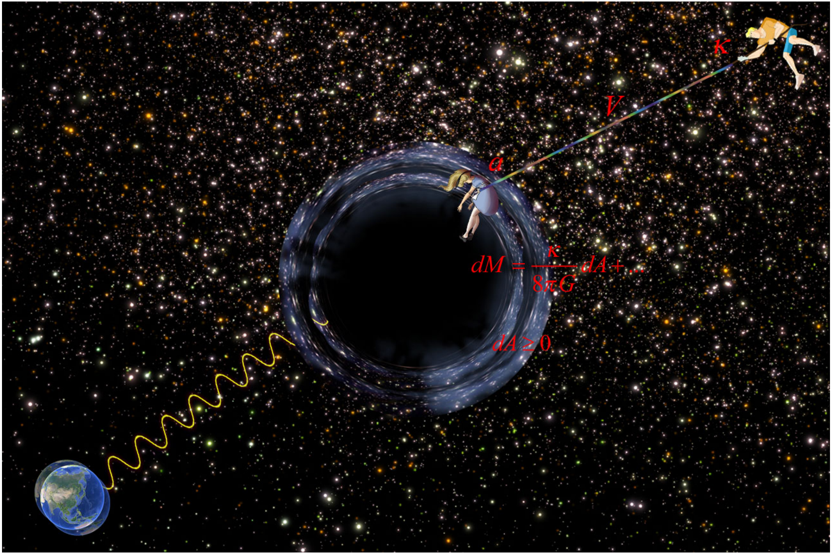

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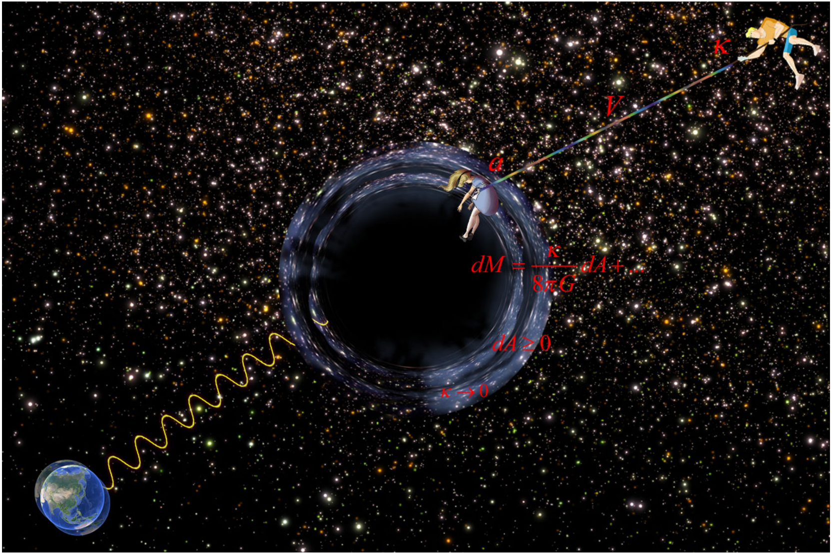

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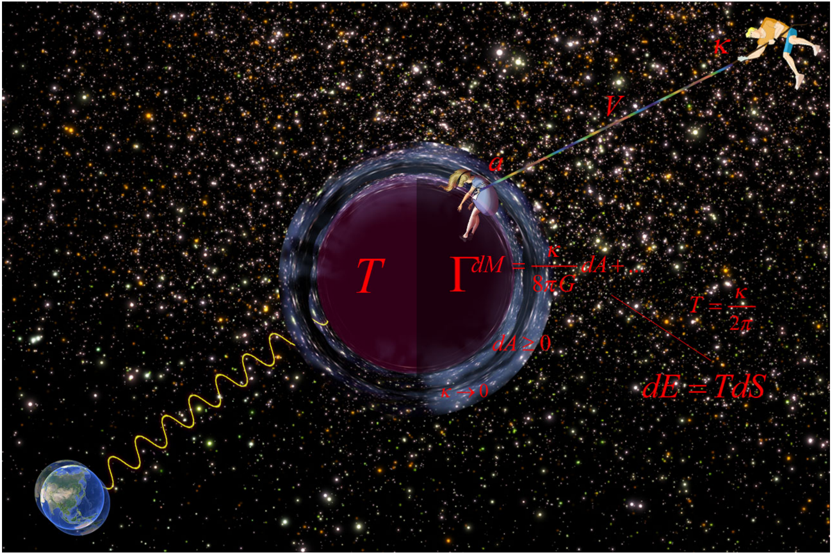

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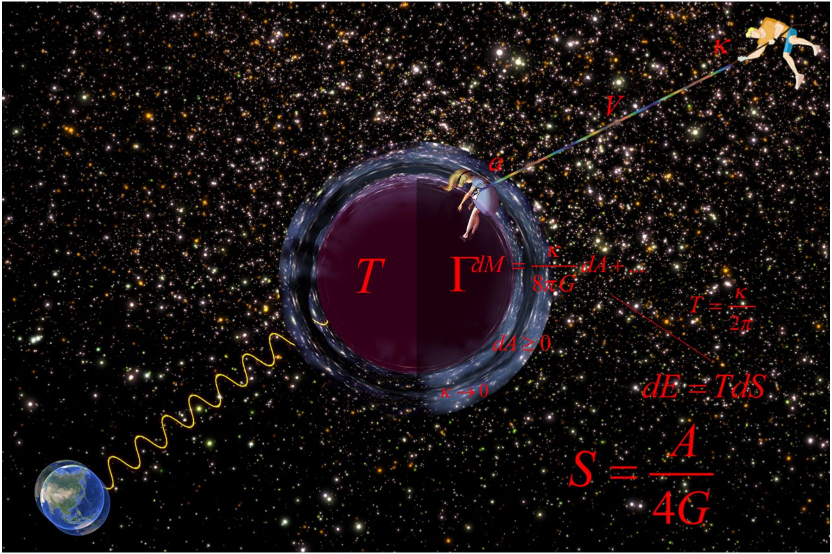

Problems in Black Hole Physics are,

- T? pure to mixed st. $\longrightarrow$ Unitarity problem
- Always $1/4G$? $\longrightarrow$ Universality problem

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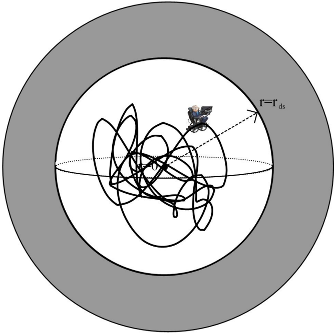

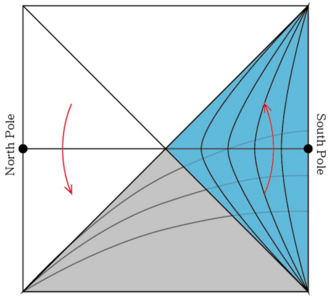

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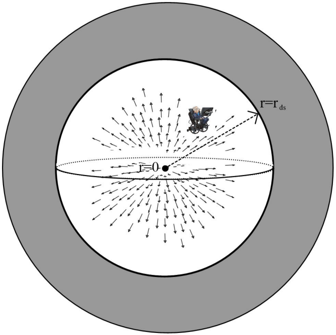
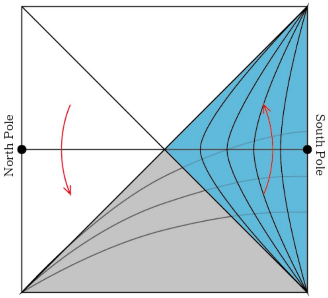




[5] Global Topological Soliton As a Non-trivial Matter Source

- $O(N-1)$ symmetry, a hedge hog shape.

$$\phi^i = \hat{r}^i \phi(r), \quad (i = 1, \cdots, d-2)$$

- Field potential : Higgs potential which is chosen in a minimal shape for supporting static global topological defect

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2$$

$\longrightarrow$ Energy behavior: $\rho \propto \frac{v^2}{r^2}$

$\longrightarrow$ Divergent energy when $r \to \infty$: Still Λ is dominant.

$\longrightarrow$ Not the finite energy case:
There exists a topological soliton solution
even in the higher dimension (Derrick-Hobart theorem)

$\longrightarrow$ It is the topologically stable non-trivial matter distribution.

[6] Model

- Action

$$S = \int d^d x \sqrt{-g} \left[\frac{1}{16\pi G}(R - 2\Lambda) - \left(\frac{g^{\mu\nu}}{2} \partial_\mu \phi^i \partial_\nu \phi^i + V(\phi) \right) \right]$$

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2$$

- Metric in the static coordinate

$$ds^2 = -e^{2\Omega(r)} A(r) dt^2 + \frac{dr^2}{A(r)} + r^2 d\Omega_{d-2}^2$$

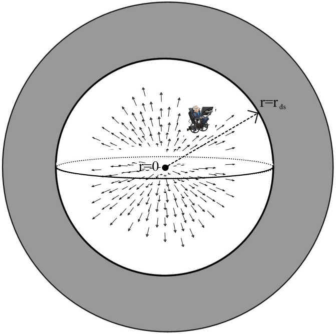
where

$$d\Omega_{d-2}^2 = d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \cdots + \sin^2\theta_1 \cdots \sin^2\theta_{d-3} d\theta_{d-2}^2$$

$$A(r) \equiv 1 - \Delta_{\text{dS}} - \left(\frac{r}{l}\right)^2 = 1 - \frac{2(\#)GM(r)}{r^{d-3}} - \left(\frac{r}{l}\right)^2$$

$$\Delta_{\text{dS}} = \frac{16\pi GM(r)}{(d-2)\Omega_{d-2} r^{d-3}}, \quad (\#) = \frac{8\pi}{(d-2)\Omega_{d-2}}$$

[7] Boundary Conditions



- Boundary condition:

$$\phi(0) = 0,\ \phi(r_{\rm h}) = v,\ M(0) = 0,\ \Omega(r_{\rm h}) = 0$$

[8] Solutions

- $r \longrightarrow 0$

$$\frac{\phi(r)}{v} \approx \phi_0 r + \cdots ,$$

$$A(r) \approx 1 - \left[\frac{1}{\lambda v^2 l^2} + \delta \frac{d-3}{d-2}\left(\frac{1}{\lambda v^2}\phi_0^2 + \frac{1}{2(d-1)}\right)\right](\sqrt{\lambda} v r)^2 + \cdots ,$$

$$\Omega(r) \approx \Omega_0 + \frac{\delta}{2}\frac{d-3}{d-2}\frac{\phi_0^2}{(\sqrt{\lambda} v)^2}(\sqrt{\lambda} v r)^2 + \cdots ,$$

$$\delta \equiv \frac{8\pi G v^2}{d-3}$$

- $r \longrightarrow r_{\rm h}$

$$\frac{\phi(r)}{v} \approx 1 - \frac{d-2}{2\left(1 + \frac{3-d}{\lambda v^2 l^2}\right)}\frac{1}{(\sqrt{\lambda} v r)^2} + \cdots ,$$

$$A(r) \approx -\left(\frac{r}{l}\right)^2 + 1 - \delta + \cdots ,$$

$$\Omega(r) \approx -\frac{(d-2)(d-3)}{4\left(1 + \frac{3-d}{\lambda v^2 l^2}\right)^2}\delta \frac{1}{(\sqrt{\lambda} v r)^4} + \cdots .$$

From the solutions, we find that

$$ds^2 = -\left[1 - \delta - \left(\frac{r}{l}\right)^2\right] dt^2 + \frac{dr^2}{1 - \delta - \left(\frac{r}{l}\right)^2} + r^2 d\Omega_{d-2}$$

$$\curvearrowleft \ t' = \sqrt{1-\delta}\, t, \ r' = r/\sqrt{1-\delta}$$

$$= -\left[1 - \left(\frac{r'}{l}\right)^2\right] dt'^2 + \frac{dr'^2}{1 - \left(\frac{r'}{l}\right)^2} + r'^2 (1-\delta) d\Omega^2_{d-2}.$$

[10] Thermodynamics Quantities

$$d(-E) + Pd(-V) = TdS$$

$$T_{\mathrm{dS}} = \frac{\kappa}{2\pi} = \frac{\sqrt{1-\delta}}{2\pi l} \ ,$$

$$E_{\delta\mathrm{dS}} \approx \Omega_{d-2} \frac{d-2}{16\pi G} l^{d-3} \delta (1-\delta)^{\frac{d-3}{2}} \ ,$$

$$P_{\delta\mathrm{dS}} = T^r{}_r \approx -\frac{d-2}{2} \frac{v^2}{r^2},$$

$$\Delta S_{\delta\mathrm{dS}} = \frac{A_{\mathrm{dS}}}{4G} \left(-\frac{d-2}{2} \right) (1-\delta)^{\frac{d-4}{2}} \mathrm{d}\delta$$

[11] Thermodynamics 1st Law

Then the entropy for the deformed system is given by

$$S_{\delta\mathrm{dS}} = S_{\mathrm{dS}} + \Delta S_{\delta\mathrm{dS}} = S_{\mathrm{dS}} + \int_{S(\delta=0)}^{S(\delta)} dS_{\delta\mathrm{dS}}$$

$$= \frac{A^{\mathrm{h}}_{\mathrm{dS}}}{4G}(1-\delta)^{\frac{d-2}{2}}$$

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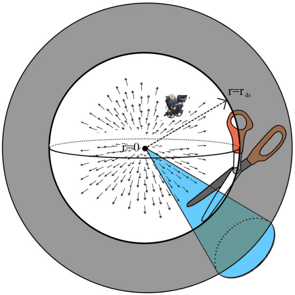
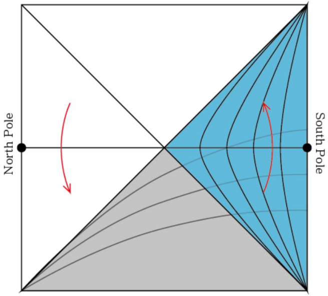
$$S_{\delta \mathrm{dS}} = \frac{A^{\mathrm{h}}_{\delta \mathrm{dS}}}{4G}$$

Exact!

[12] Result





(not exact⟶)

[13] Comparison with Previous Works (1)

- M. Aryal, L. H. Ford and A. Vilenkin, Cosmic strings and black holes, Phys. Rev. D 34, 2263 (1986).

The entropy is determined by the relation

$$dS = \frac{dE}{T} , \quad (26)$$

$$S = 4\pi b M^2 = \tfrac{1}{4} A . \quad (33)$$

- J. Jing, H. Yu and Y. Wang, Thermodynamics of a black hole with a global monopole, Phys. Lett. A 178, 59 (1993); H.-W. Yu, Black hole thermodynamics and global monopoles, Nucl. Phys. B 430, 427 (1994).

$$ds = dE/T , \quad (7)$$

$$S = \tfrac{1}{4} A . \quad (14)$$

[14] Comparison with Previous Works (2)

- Padmanabhan's thermodynamical identity:

  [T. Padmanabhan, CQG 19, 5387 (2002)]

$$ds^2 = -f(r)\mathrm{d}t^2 + \frac{1}{f(r)}\mathrm{d}r^2 + r^2 d\Omega_{d-2}$$

$$\Lsh \quad \pm T_{\mathrm{h}}\mathrm{d}S_{\mathrm{h}} - \mathrm{d}E_{\mathrm{h}} = T^t{}_t \mathrm{d}V_{\mathrm{h}}$$

- 1. Only varying matter content is introduced;

$$T_{\mathrm{h}}\mathrm{d}S_{\mathrm{h}} - \mathrm{d}\underbrace{E_{\mathrm{h}}}_{\int \rho dV} = \underbrace{T^t{}_t}_{-\rho}\mathrm{d}V_{\mathrm{h}}$$

$$\rightarrow \quad -\mathrm{d}\int \rho \mathrm{d}V \neq -\rho(r_{\mathrm{h}})A_{\mathrm{h}}\mathrm{d}r_{\mathrm{h}}$$

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$$\hookrightarrow \quad \pm T_{\mathrm{h}}\mathrm{d}S_{\mathrm{h}} - \mathrm{d}E_{\mathrm{h}} = T^t{}_t \mathrm{d}V_{\mathrm{h}}$$

- 2. equation of state

$$-\frac{d-2}{r}A\Omega' = 8\pi G(T^t{}_t - T^r{}_r)$$

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = [3A'\Omega' + A'' + 2A(\Omega'^2 + \Omega'')]^2 + \frac{2(A' + 2A\Omega')^2 + 2A'^2}{r^2} + \frac{4(1-A)^2}{r^4}$$

$$T^t{}_t = T^\theta{}_\theta = T^\phi{}_\phi = -\left[(d-2)\frac{1}{2r^2}\phi^2 + \frac{A}{2}\phi'^2 + V\right]$$

$$T^r{}_r = A\phi'^2 - \left[(d-2)\frac{1}{2r^2}\phi^2 + \frac{A}{2}\phi'^2 + V\right]$$

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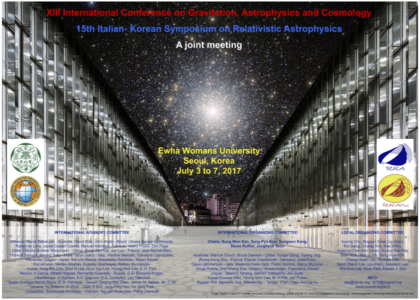
$$\hookrightarrow \quad \pm T_\mathrm{h}\mathrm{d}S_\mathrm{h} - \mathrm{d}E_\mathrm{h} = T^t{}_t \mathrm{d}V_\mathrm{h}$$

- 3. Horizon energy:

$$\begin{aligned}\pm T_\mathrm{h}\mathrm{d}S_\mathrm{h} - \mathrm{d}E_\mathrm{total} = {} & e^{\Omega(r_\mathrm{h})} P_\mathrm{h}\mathrm{d}V_\mathrm{h} \\ & - \frac{\Omega_{d-2}}{16\pi G}(d-2)e^{\Omega(r_\mathrm{h})}\left(\frac{\mathrm{d}\Omega(r_\mathrm{h})}{\mathrm{d}r_\mathrm{h}}\right) r_\mathrm{h}^{d-3}\mathrm{d}r_\mathrm{h} \\ & + \frac{\Omega_{d-2}}{16\pi G}(d-2)\mathrm{d}\left[\int_0^{r_\mathrm{h}} \mathrm{d}r e^{\Omega(r)}\Omega'(r,r_\mathrm{h}) r^{d-3}(1-A(r,r_\mathrm{h}))\right]\end{aligned}$$

[15] Conclusion

- We have a clue toward the Quantum Gravity. It's a Hawking radiation.
- In the black hole physics, there are Unitarity and Universality problems.
- We examined the Universality problem by looking at the change of the non-trivial matter configuration in the de Sitter spacetime.
- The area-entropy relation still holds in the limit of the small core radius of a global topological soliton. This also holds even in the other spacetime.
- The quantum gravitational degree of freedom related to the non-trivial matter source could be understood by investigating the nature of the deficit geometry.
- Global topological solitons of the hedgehog ansatz are non-BPS objects and interacting each other, the obtained exact area law after adding mutually interacting objects with topological stability is noteworthy and implies possible opening of a new arena to examine the nature of the quantum gravitational degree of freedom related to various nontrivial interacting matters.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Thank you!