Calibrated Entanglement Entropy

Eoin Ó Colgáin

International Conference on Gravitation

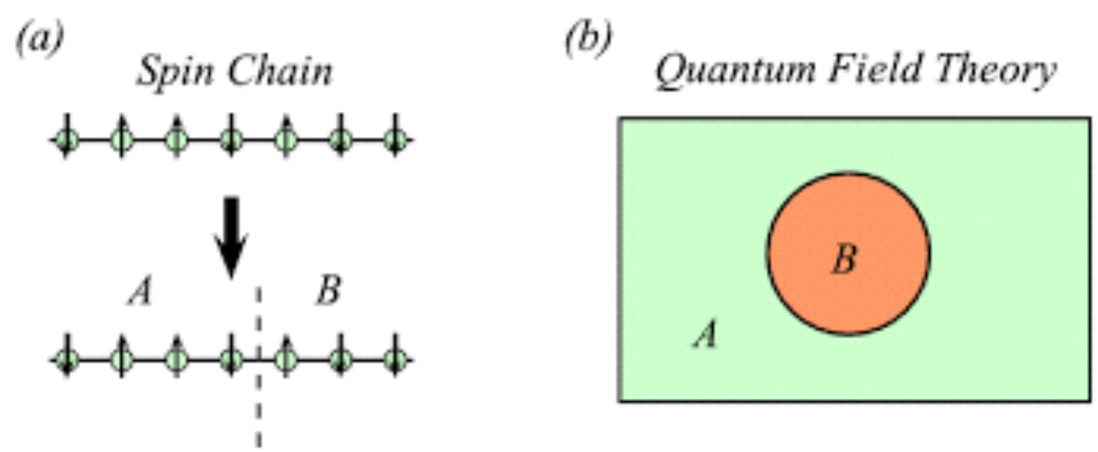
Ewha University, 3rd July 2017

with I. Bakhmatov, S. Deger, J. Gutowski, H. Yavartanoo

arXiv: 1610.05638, 1705.08319

APCTP
Asia Pacific Center for Theoretical Physics

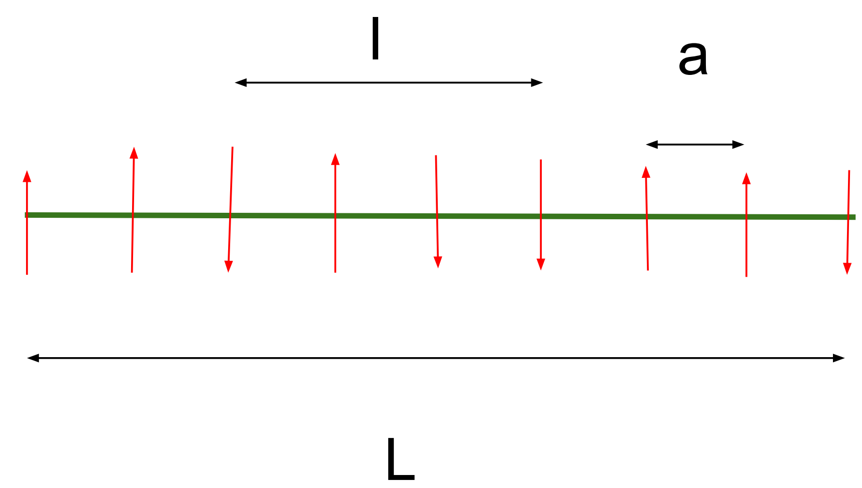
in spin chain, QFT, etc. can define von Neumann entropy



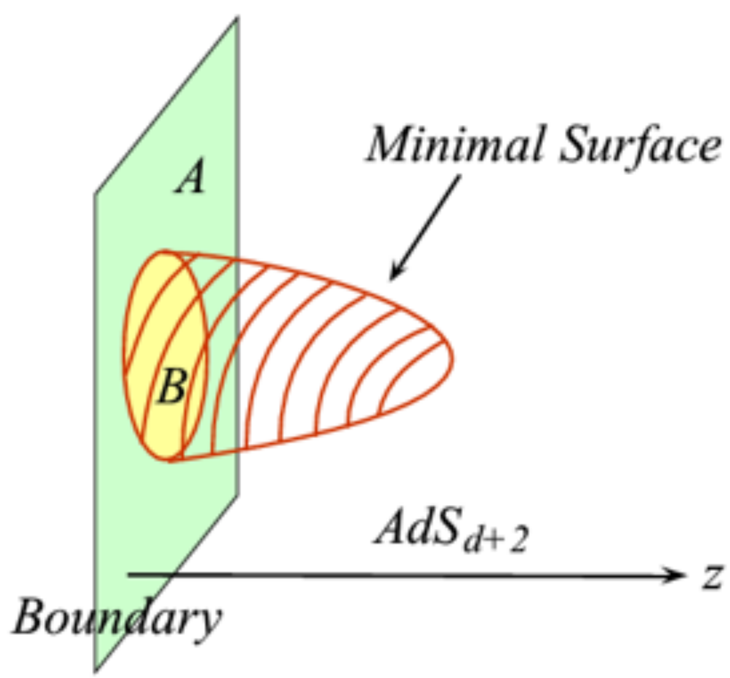
$$S_A = -\mathrm{tr}_A \rho_A \log \rho_A, \quad \rho_A = \mathrm{tr}_B |\Psi\rangle\langle\Psi|$$

1D system at criticality (2D CFT)

$$S_A = \frac{c}{3} \cdot \log\left(\frac{L}{\pi a} \sin\left(\frac{\pi l}{L}\right)\right)$$



Calabrese, Cardy; Holzhey, Larsen, Wilczek



$$S_A = \frac{\mathrm{Area}(\gamma_A)}{4G}$$

Ryu, Takayanagi

small catch - gets tricky quickly (in Einstein gravity)

$$\mathrm{d}s^2 = \frac{-\mathrm{d}x_0^2 + \mathrm{d}r^2 + r^2\mathrm{d}s^2(S^{d-1}) + \mathrm{d}z^2}{z^2}$$

$$rzz'' + (d-1)z(z')^3 + (d-1)zz' + dr(z')^2 + dr = 0$$

options: i) numerics or ii) start thinking put on high energy theory hat

for SUSY, there maybe life after Higgs

$$\nabla_\mu \epsilon = \frac{1}{2}\gamma_\mu \epsilon,$$

$$\gamma^\nu \nabla_{[\mu} \nabla_{\nu]} \epsilon = -\frac{1}{4} R_{\mu\nu} \gamma^\nu \epsilon, \quad R_{\mu[\nu_1 \nu_2 \nu_3]} = 0,$$

$$R_{\mu\nu} = -(d-1) g_{\mu\nu}$$

co-dimension 2 surface in AdS_3 = curve

curve: solve geodesic equation

trivial exercise due to $U(1)^2$ symmetry

BTZ BHs (locally AdS_3) all preserve SUSY locally

1. massless BTZ (Poincaré) - 2 SUSY

2. extremal BTZ - 1 SUSY

3. generic BTZ - 0 SUSY

Coussert, Henneaux; OC, Yavartanoo

$$\mathrm{d}s^2 = -(r^2 - m)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{(r^2 - m)} + r^2\mathrm{d}x^2,$$

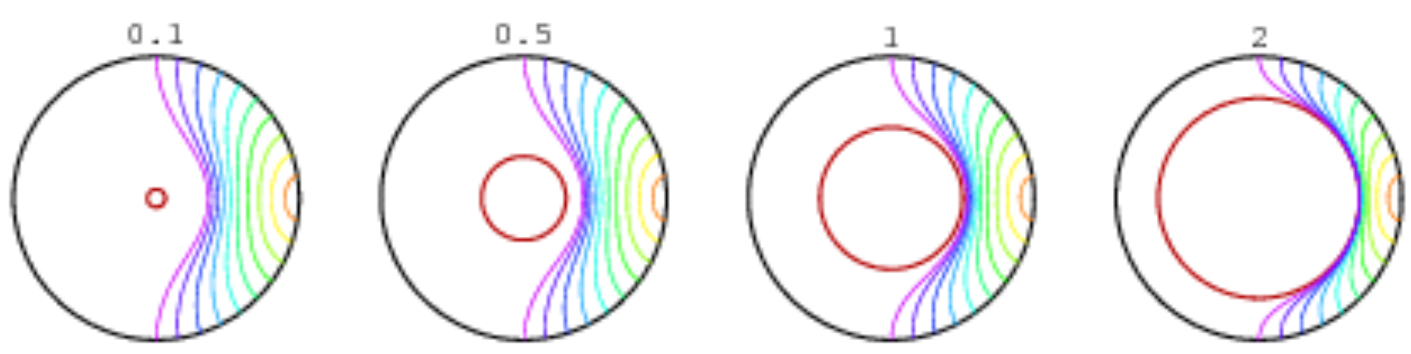
$$\epsilon_\pm = e^{\pm\frac{\rho}{2}\gamma_r} e^{\pm\frac{x\sqrt{m}}{2}\gamma_x} e^{\frac{t\sqrt{m}}{2}\gamma_{rt}} \eta_\pm, \quad r = \sqrt{m}\cosh\rho$$

projection condition

identify SUSY that "commute" with it

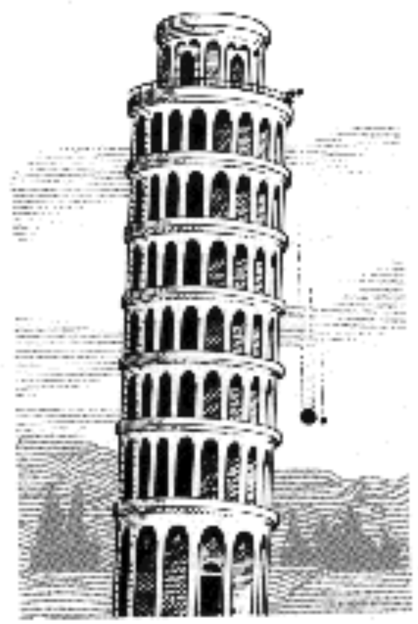
$$\Gamma\epsilon_\pm = \epsilon_\pm, \quad \Gamma \equiv \frac{\dot{x}^\mu E_\mu{}^\nu \gamma_\nu}{V}, \quad V = \sqrt{g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}$$

additional constant time condition



Hubeny, Rangamani, Takayanagi

all very nice, but doesn't work for general AdS_{d+2}



calibration (closed forms) - volume minimising cycles

$$D = 2n \text{ complex manifold } \mathcal{M}, \quad \varphi = \frac{1}{p!} J^p, \quad 1 \le p \le n$$

Kähler cycles - Calabi-Yau compactifications consider special Lagrangian (sLag) cycles

proposal: Bakhmatov, Deger, Gutowski, OC, Yavartanoo

$$S_A = \frac{1}{4G_3} \int_{\mathrm{sLag}} \mathrm{Re}(\varphi), \quad \varphi = e^{i\chi}\Omega$$

get 1st order equations - no SUSY

works for non-AdS holography

generalisations to higher dimensions

Example: massless BTZ

$$\mathrm{d}s_3^2 = \frac{1}{r^2}(-\mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}r^2) \quad \Rightarrow \quad \mathrm{d}s_2^2 = \frac{1}{r^2}(\mathrm{d}x^2 + \mathrm{d}r^2),$$

$$\varphi = e^{i\chi}\left(\frac{\mathrm{d}x}{r} + i\frac{\mathrm{d}r}{r}\right), \quad \mathrm{Im}(\varphi) = \mathrm{dRe}(\varphi) = 0$$

1st order PDE -> system of ODEs -> RT minimal surface

$$f \equiv \cos\chi, \quad c_1 = \frac{f}{r}, \quad c_2 = \sqrt{1-f^2} - f\frac{x}{r},$$

$$\left(x + \frac{c_2}{c_1}\right)^2 + r^2 = \frac{1}{c_1^2}$$

warped AdS$_3$ - consistent truncation of string theory

$$\mathcal{L} = R\,\mathrm{vol}_3 - 4\mathrm{d}U \wedge *_3\mathrm{d}U - 4e^{-4U} A \wedge *_3 A$$
$$+ 2e^{-4U}(2 - e^{-4U})\mathrm{vol}_3 - A \wedge F$$

Detournay, Guica; Karndumri, OC

$$\mathrm{d}s_3^2 = T_+^2 \mathrm{d}v^2 + 2\rho\,\mathrm{d}u\,\mathrm{d}v + \left[T_-^2 e^{4U} - \lambda^2\rho^2\right]\mathrm{d}u^2 + \frac{e^{4U}\mathrm{d}\rho^2}{4(\rho^2 - T_+^2 T_-^2)},$$
$$e^{4U} = 1 + \lambda^2 T_+^2, \quad A = \lambda\, e^{-2U}(T_+^2 \mathrm{d}v + \rho\,\mathrm{d}u).$$

$$\tilde{g}_{\mu\nu} = e^{-4U} g_{\mu\nu} + A_\mu A_\nu$$ auxiliary AdS$_3$

Compere, Guica, Rodriguez

HEE not geodesic, but particle trajectory Song, Wen, Xu

$$S = \frac{1}{4G_3} \int \mathrm{d}s \left[m\sqrt{g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu} + qA_\mu \dot{x}^\mu \right]$$

interesting - reduces to geodesic in auxiliary

interpretation as generalised calibration

Bakhmatov, Deger, Gutowski, OC, Yavartanoo

$$\mathrm{d}\ \mathrm{Re}(\Omega) = -\frac{q}{m} F$$

higher dimensional story

$$\varphi_{\text{disk}} = e^{i\chi}\left(\frac{\eta}{r}\right)^{p-1}\left(\frac{\mathrm{d}\eta}{r} + i\frac{\mathrm{d}r}{r}\right)\wedge \mathrm{vol}(S^{p-1})$$

$$0 = \sin\chi\mathrm{d}\eta + \cos\chi\mathrm{d}r = \partial_r\left(\cos\chi\frac{\eta^{p-1}}{r^p}\right) + \partial_\eta\left(\sin\chi\frac{\eta^{p-1}}{r^p}\right)$$

together imply RT equation

$$0 = \eta r r'' + (p-1)r(r')^3 + (p-1)rr' + p\eta(r')^2 + p\eta$$

calibrations show promise in application to holographic entanglement entropy

more to follow...

thank you