Quantum Gravity Effects in Cosmology

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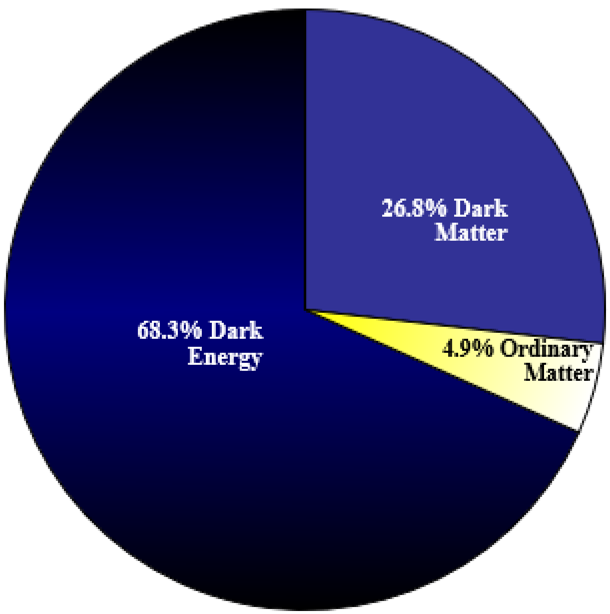
Outline

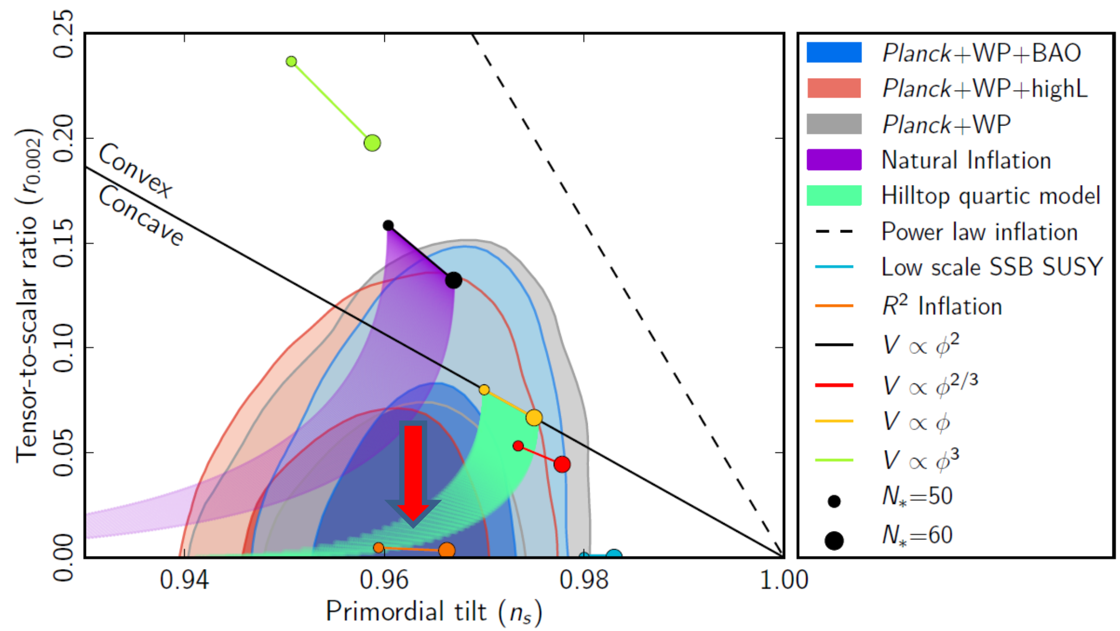
- Motivation
- Semiclassical Quantum Cosmology (QC)
- Gibbons-Hawking Temperature corrected by QC
- Conclusion

Motivation

Dark Energy Dominated Universe

Planck 2013 Results: [arXiv:1303.5082v2]





Starobinsky inflation model ($R + \alpha R^2$) [PLB 91 ('80)]: a de Sitter-type acceleration

Effect of Quantum Gravity in Cosmological Observations?

- Wheeler-DeWitt equation for FRW universe with scale factor $a = e^{\alpha}$, inflaton ϕ and Fourier-modes f_k of ϕ-fluctuations has the wavefunction $\Psi(\alpha, \phi, f_k)$; Assume the ϕ-derivatives to be much smaller than the α-derivatives (a slow-roll approximation); Born-Oppenheimer interpretation
- Quantum cosmology corrected power spectrum [Kiefer, Kramer, PRL 108 ('12); 1st prize in 2012 essay competition of Gravity Research Foundation]

$$\Delta^2_{(1)}(k) = \Delta^2_{(0)}(k)\left(1 - \frac{43.56}{k^3}\frac{H^2}{m_P^2}\right)^{-3/2}\left(1 - \frac{189.18}{k^3}\frac{H^2}{m_P^2}\right)$$

- Suppression of power spectrum at large scales and weaker upper bound on H than tensor-to-scalar ratio.
- Any violations of Einstein Equivalence Principle at cosmic scale hint quantum gravity effects [Wei-Tou Ni's talk].

de Sitter (dS) Space

- The Friedmann equation with a cosmological constant (dS)

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = H_\Lambda^2$$

- The periodic solution in a Euclidean time ($k = \pm 1$ for closed /open geometry)

$$a_{k=1}(\tau) = \frac{1}{H_\Lambda}\cos(H_\Lambda \tau), \quad a_{k=1}(t) = \frac{1}{H_\Lambda}\cosh(H_\Lambda t)$$

$$a_{k=-1}(\tau) = \frac{1}{iH_\Lambda}\sin(H_\Lambda \tau), \quad a_{k=-1}(t) = \frac{1}{H_\Lambda}\sinh(H_\Lambda t)$$

- The period is the inverse Gibbons-Hawking (GH) temperature (as measured by a test probe)

$$\beta = \frac{1}{T_{GH}} = \frac{2\pi}{H_\Lambda}$$

The 1st Law of BH Thermodynamics for FRW Universe [Cai, SPK, JHEP 02 ('05)]

- The 1st law gives one Einstein equation for FRW universe

$$\left.\begin{array}{l} -dE = TdS \quad , T = \dfrac{1}{2\pi \tilde{r}_A} \quad , S = \dfrac{S}{4G} \\ -dE = A(\rho + p) H \tilde{r}_A dt \end{array}\right\} \Rightarrow \quad \dot{H} - \frac{k}{a^2} = -\frac{8\pi G}{3}(\rho + p)$$

- The continuity equation

$$\dot{\rho} + 3H(\rho + p) = 0$$

- BH thermodynamics ≡ Einstein equation in nonstationary spacetimes

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho$$

Semiclassical Quantum Cosmology

Emergence of CG from QG & Quantum Effects

Quantum Gravity/Cosmology	
$\hat{G}_{\mu\nu} = 8\pi G \hat{T}_{\mu\nu}$	WDW, HH wave function, tunneling wave function

$$G = 1/m_{\mathrm{P}}^2 << 1$$

Semiclassical Quantum Gravity/Cosmology*	
$G^{\mathrm{C}}_{\mu\nu} + G^{\mathrm{Q}}_{\mu\nu}[G] = 8\pi G \left\langle \hat{T}_{\mu\nu} \right\rangle$	QFT in curved spacetime, Hawking radiation, pair production

$$\hbar << 1$$

Classical Gravity/Cosmology	
$G^{\mathrm{C}}_{\mu\nu} + G^{\mathrm{Q}}_{\mu\nu}[G] = 8\pi G \left(T^{\mathrm{C}}_{\mu\nu} + T^{\mathrm{Q}}_{\mu\nu}[\hbar] \right)$	inflationary models, dark energy

Quantum Cosmology

- Supermetric for FRW geometry and a minimal scalar

$$ds^2 = -da^2 + a^2 d\phi^2$$

- Hamiltonian constraint and Wheeler-DeWitt equation

$$H(a,\phi) = -\underbrace{\left(\pi_a^2 + V_G(a)\right)}_{\text{gravity part}} + \underbrace{\frac{1}{a^2}\left(\pi_\phi^2 + 2a^6 V(\phi)\right)}_{\text{matter part}} = 0$$

$$\left[-\nabla^2 - V_G(a) + 2a^4 V(\phi)\right]\Psi(a,\phi) = 0$$

$$\nabla^2 = -a^{-1}\frac{\partial}{\partial a}\left(a\frac{\partial}{\partial a}\right) + \frac{1}{a^2}\frac{\partial^2}{\partial \phi^2}, \quad V_G(a) = ka^2 - 2\Lambda a^4$$

- The universe scatters from an initial surface to a final one in superspace of 3-geometries. A prescription of the boundary condition?

Quantum Universes in the Superspace

- Single-field inflation model

$$V(\phi) = \lambda_{2p}\phi^{2p}/(2p)$$

- Symanzik scaling-law for eigenstates [SPK, PRD 46 ('92)]

$$H_M(\phi,a)\Phi_n(\phi,a) = E_n(a)\Phi_n(\phi,a)$$

$$E_n(a) = \left(\lambda_{2p}a^6/p\right)^{1/(p+1)}\varepsilon_n$$

$$\Phi_n(\phi,a) = \left(\lambda_{2p}a^6/p\right)^{1/4(p+1)}F_n\left(\left(\lambda_{2p}a^6/p\right)^{1/(p+1)}\phi\right)$$

- Coupling matrix among energy eigenfunctions

$$\frac{\partial}{\partial a}\vec{\Phi}(\phi,a) = \Omega(a)\vec{\Phi}(\phi,a)$$

$$\Omega_{mn}(a) = \left(3/4(p+1)a\right)\left(\varepsilon_m - \varepsilon_n\right)\int d\varsigma F_m(\varsigma)F_n(\varsigma)\varsigma^2$$

Quantum Universes in the Superspace

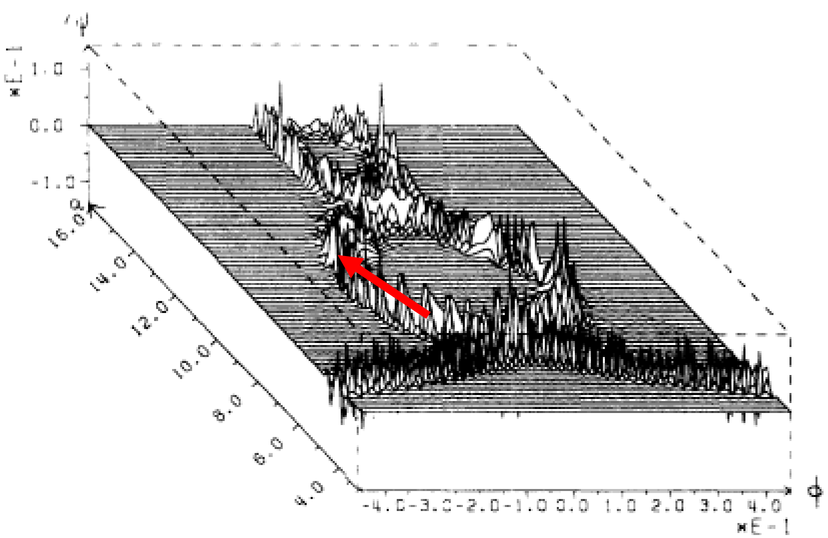
- The two-component wave function [SPK, PRD 46 ('92)]

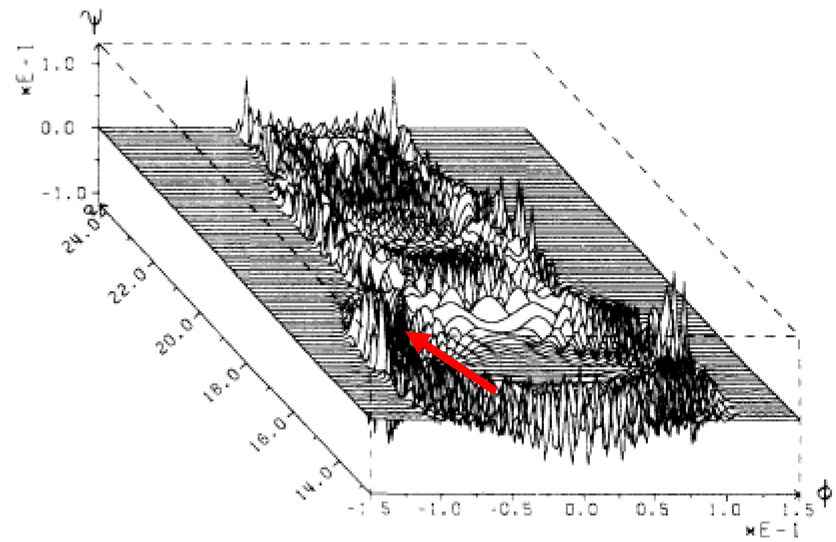
$$\begin{pmatrix} \Psi(a,\phi) \\ \partial\Psi(a,\phi)/\partial a \end{pmatrix} = \begin{pmatrix} \vec{\Phi}^T(\phi,a) & 0 \\ 0 & \vec{\Phi}^T(\phi,a) \end{pmatrix}$$

$$\times T\exp\left[\int \begin{pmatrix} \Omega(a') & I \\ V_G(a') - E/a'^2 & \Omega(a') \end{pmatrix} da'\right] \begin{pmatrix} \vec{\psi}(a_0) \\ d\vec{\psi}(a_0)/da_0 \end{pmatrix}$$

- Off-diagonal components are gravitational part equation only with $V_G(a) - E/a^2$.
- Continuous transitions among energy eigenfunctions.

Wave Packet for FRW with a Minimal Scalar





A closed universe (k=1), m = 6, and n =120 (harmonic quantum number) [Kiefer, PRD 38 ('88)]; red arrow denotes the tangential direction for a directional derivative in semiclassical gravity.

de Broglie-Bohm Pilot-Wave Theory & Born-Oppenheimer Idea

- Wave functions are peaked around wave packets and allow de Broglie-Bohm pilot-wave theory

$$\left[-\frac{\hbar^2}{2M}\nabla^2 - MV_{\mathrm{G}}(h_a) + \hat{H}(\phi, -i\frac{\delta}{\delta\phi}, h_a)\right]\Psi(h_a, \phi) = 0 , \quad (h_a = h_{ij})$$

- Apply Born-Oppenheimer idea that separates a slow moving massive particle (M: Planck mass squared) from a fast moving light particle (matter field, perturbations) and then expand quantum state for fast moving variable by a certain basis to be determined

$$|\Psi(h_a, \phi)\rangle = \psi(h_a)|\Phi(\phi, h_a)\rangle$$

$$|\Phi(\phi, h_a)\rangle = \sum_k c_k(h_a)|\Phi_k(\phi, h_a)\rangle$$

Semiclassical Quantum Gravity

[SPK, PRD 52 ('95); CQG 13 ('96); PRD 55 ('97)]

- Apply de Broglie-Bohm pilot-wave theory to gravity part only

$$\psi(h_a) = F(h_a) e^{iS(h_a)/\hbar}$$

- Then, in a semiclassical regime, WDW equation is equivalent to

$$\frac{1}{2M}(\nabla S)^2 - MV_G(h_a) + H_{nn} - \frac{\hbar^2}{2M}\frac{\nabla^2 F}{F} - \frac{\hbar^2}{M}\mathrm{Re}(Q_{nn}) = 0$$

$$\frac{1}{2}\nabla^2 S + \frac{\nabla F}{F}\cdot\nabla S + \mathrm{Im}(Q_{nn}) = 0$$

$$H_{nk}(h_a) := \langle \Phi_n(\phi, h_a) | \hat{H} | \Phi_k(\phi, h_a) \rangle \; ; \; \vec{A}_{nk}(h_a) := i\langle \Phi_n(\phi, h_a) | \nabla | \Phi_k(\phi, h_a) \rangle$$

$$Q_{nn}(h_a) := \frac{\nabla F}{F}\cdot\left(\frac{\nabla c_n}{c_n} - i\sum_k \vec{A}_{nk}\frac{c_k}{c_n}\right)$$

Semiclassical FRW Universe

- Extended superspace for a FRW with a minimal scalar and the cosmological time:

$$ds^2 = -ada^2 + a^3 d\phi^2$$

$$\frac{\partial}{\partial \tau} = -\frac{1}{Ma}\frac{\partial S(a)}{\partial a}\frac{\partial}{\partial a}, \quad \left(\frac{\partial a(\tau)}{\partial \tau} = -\frac{1}{Ma}\frac{\partial S(a)}{\partial a}\right)$$

- Heisenberg (matrix) equation for scalar field

$$i\hbar \frac{\partial c_n}{\partial \tau} = \sum_{k \neq n} H_{nk} c_k - \hbar \sum_k B_{nk} c_k - \frac{\hbar^2}{2Ma} \sum_{k \neq n} \Omega_{nk} c_k$$

$$H_{nk}(a(\tau)) := \langle \Phi_n | \hat{H} | \Phi_k \rangle \; ; \; B_{nk}(a(\tau)) := i \langle \Phi_n | \frac{\partial}{\partial \tau} | \Phi_k \rangle$$

$$\Omega_{nk}(a(\tau)) := -\frac{1}{\dot{a}^2}\left[\left(\frac{\partial^2}{\partial \tau^2} - \frac{\ddot{a}}{\dot{a}}\frac{\partial}{\partial \tau}\right)\delta_{nk} - 2iB_{nk}\frac{\partial}{\partial \tau} + \langle \Phi_n | \frac{\partial^2}{\partial \tau^2} - \frac{\ddot{a}}{\dot{a}}\frac{\partial}{\partial \tau} | \Phi_k \rangle\right]$$

Semiclassical Inflation Models

- Semiclassical chaotic models necessarily contain (higher) curvature terms

$$\left(\frac{\dot a}{a}\right)^2+\frac{k}{a^2}-\Lambda=\frac{8\pi}{3m_P^2a^3}\left[H_{nn}-\frac{4\pi\hbar^2}{3m_P^2a\dot a}U_{nn}\,\mathrm{Re}(R_{nn})+\frac{2\pi\hbar^2}{3m_P^2a}\left(U_{nn}^2+\frac{1}{\dot a}\dot U_{nn}\right)\right]$$

$$R_{nn}=\frac{\dot c_n}{c_n}-i\sum_k B_{nk}\frac{c_k}{c_n}$$

$$U_{nn}:=\frac{\partial F/\partial a}{F}=-\frac{1}{2}\frac{(a\dot a)^{\cdot}}{a\dot a^2+(4\pi\hbar/3m_P^2)\,\mathrm{Im}(R_{nn})}$$

- Effective energy density contains curvature terms

$$\rho_{nn}=H_{nn}-\frac{4\pi\hbar^2}{3m_P^2a\dot a}U_{nn}\,\mathrm{Re}(R_{nn})+\frac{2\pi\hbar^2}{3m_P^2a}\left(U_{nn}^2+\frac{1}{\dot a}\dot U_{nn}\right)$$

Semiclassical Massive Model

- Semiclassical chaotic model with a massive scalar at order of $\hbar/M$

$$\hat{H} = -\frac{\hbar^2}{2a^3}\frac{\partial^2}{\partial\phi^2} + \frac{m^2 a^3}{2}\phi^2$$

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} - \Lambda = \frac{8\pi}{3m_P^2 a^3}\left[H_{nn} + \frac{2\pi\hbar^2}{3m_P^2 a}\left(U_{nn}^{(0)2} + \frac{1}{\dot{a}}\dot{U}_{nn}^{(0)}\right)\right]$$

$$R_{nn}^{(0)} = 0 \;\; ; \;\; U_{nn}^{(0)} = -\frac{1}{2}\frac{(a\dot{a})^{\bullet}}{a\dot{a}^2}$$

$$H_{nn} = \hbar a^3\left(n + \frac{1}{2}\right)\left[\dot{\varphi}^*\dot{\varphi} + m^2\varphi^*\varphi\right] \;\; ; \;\; \ddot{\varphi} + 3\frac{\dot{a}}{a}\dot{\varphi} + m^2\varphi = 0$$

Gibbons-Hawking (GH) Temperature corrected by QC

Semiclassical dS with Symmetric Fluctuations

- Semiclassical equation for pure gravity model with spherically symmetric spacetime fluctuations (equivalent to massless fields) [SPK, PRD 55 ('97)]

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = H_\Lambda^2 - \frac{3\pi^2}{9m_P^2 a^4} V_Q$$

$$V_Q = -\frac{1}{2m_P^2}\frac{1}{F}\frac{\partial^2 F}{\partial a^2} = -\frac{1}{2m_P^2}\left[\left(\frac{(a\dot{a})^\bullet}{2a\dot{a}^2}\right)^2 - \frac{1}{2\dot{a}}\left(\frac{(a\dot{a})^\bullet}{2a\dot{a}^2}\right)^\bullet\right]$$

Semiclassical dS with Symmetric Fluctuations

- The dS-phase or Λ-dominated WKB approximation

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = H_\Lambda^2 + \frac{32\pi^2}{9m_P^4 a^6}$$

- Gibbons-Hawking (GH) temperature corrected by quantum cosmology [Gu, SPK, Shen, IJMPD 26 ('17)]

$$T_Q = \frac{H_\Lambda}{2\pi k_B}\left(1 + \bar{c}_6 \frac{H_\Lambda}{m_P}\right), \quad \bar{c}_6 = O(1)$$

Semiclassical dS with Inhomogeneous Perturbations

- The dS-phase or Λ-dominated WKB approximation for dS with inhomogeneous spacetime (or matter) perturbations

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = H_\Lambda^2 + \underbrace{\frac{c_4}{a_4}}_{\text{inhomogeneous perturbations}} + \underbrace{\frac{c_6}{a^6}}_{\text{spherically symmetric fluctuations}}$$

- The inverse GH temperature in a Euclidean time is independent of inhomogeneous perturbations

$$\frac{1}{k_B T_Q} = \frac{2}{H_\Lambda}\left(\int_{-a_1^*}^{-a_0^*} + \int_{a_0^*}^{a_1^*}\right) \frac{d\tilde{a}}{\sqrt{1 - \underbrace{\tilde{a}^2}_{\text{harmonic potential}} - \underbrace{(\tilde{c}_4/\tilde{a})^2}_{\text{angular momentum}} - (\tilde{c}_6/\tilde{a})^4}}$$

Semiclassical dS with Inhomogeneous Perturbations

- If $c_6 = 0$, the temperature is the same as GH temperature since $c_4$term plays a role of angular momentum and does not change the periodicity

$$\beta_0 = \frac{2}{H_\Lambda}\int_{x_-}^{x_+}\frac{dx}{\sqrt{x - x^2 - \tilde{c}_4^{\,2}}} = \frac{2\pi}{H_\Lambda}$$

- If $c_6 \neq 0$ (inhomogeneous perturbations), the leading term of the temperature is independent of $c_4$term:

$$\beta_2 = \frac{2}{H_\Lambda}\int_{x_-}^{x_+}\sqrt{\frac{x}{(x_+ - x)(x - x_-)(x - x_0)}}dx \approx \frac{2\pi}{H_\Lambda}$$

Gauge Invariant Quantum Cosmology

Mukhanov-Sasaki Hamiltonian

- Action for scalar perturbations of metric and field & Muhkanov-Sasaki variable up to quadratic order [Mena Marugan et al, JCAP ('15)] has the total Hamiltonian

$$H = \bar{N}_0 \left[H_0 + \sum_{\vec{n},\pm} \breve{H}_2^{\vec{n},\pm} \right] + \sum_{\vec{n},\pm} \breve{G}_{\vec{n},\pm} \breve{H}_1^{\vec{n},\pm} + \sum_{\vec{n},\pm} \breve{K}_{\vec{n},\pm} \tilde{H}_1^{\vec{n},\pm}$$

- Semiclassical cosmology from WDW equation provides the master equation for power spectrum of primordial scalar perturbations (vector and tensor perturbations)
- Semiclassical cosmology with (higher) curvatures via de Broglie-Bohm pilot theory and Born-Oppenheimer idea [SPK, Mena Maruga, work in progress].

Conclusion

- Semiclassical cosmology for FRW universe in dS-phase provides the Gibbons-Hawking (GH) temperature corrected by quantum cosmology (spherical fluctuations of spacetime or massless fields).
- Inhomogeneous fluctuations give the next leading order corrections to the GH temperature (but important in quantum gravity regime).
- Affect the BH thermodynamics for the dS space.