# Dark Matter and Dark Energy from Emergent Spacetime Picture

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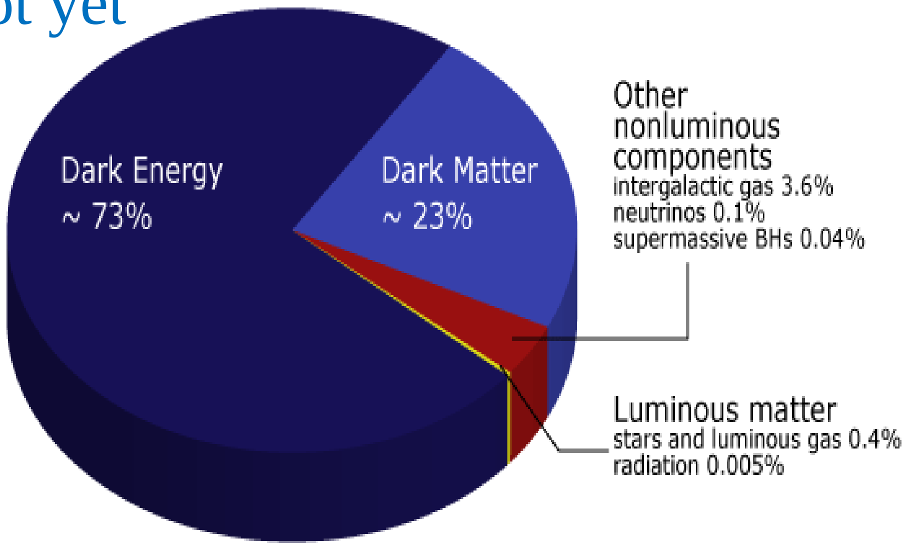
☞ Dark matter and dark energy are a great mystery of the 20th century physics, which is not yet resolved.




☞ In retrospect, the resolution of great puzzles requires the upheaval of a radical new physics.

☞ For example, the luminiferous ether $\Longrightarrow$ special relativity, blackbody radiation, photoelectric effect $\Longrightarrow$ quantum mechanics.

☞ To resolve the puzzle of dark matter and dark energy, is it also necessary to introduce a novel concept of spacetime?

Cf. E. Verlinde, "Emergent gravity and the dark universe" (arXiv:1611.02269)

# In this talk

☞ I emphasize that noncommutative (NC) spacetime necessarily implies emergent spacetime if spacetime at microscopic scales should be viewed as NC.
(HSY, arXiv:0808.4728, arXiv:1312.0580)

☞ The emergent gravity from NC $U(1)$ gauge theory is the large $N$ duality and the emergent spacetime picture admits a background-independent formulation of quantum gravity.
(HSY, arXiv:1503.00712, arXiv:1610.00011)

☞ Dark matter and dark energy arise as a holographic (UV-IR) manifestation of the coherent vacuum of Planck energy condensate.
(HSY, arXiv:0808.4728, arXiv:1111.0015)

☞ Emergent gravity distinguishes dark matter and dark energy according to the causal structure of emergent spacetime.
(HSY, arXiv:1503.00712, arXiv:1610.00011)

# Emergent gravity (spacetime) as a large $N$ duality

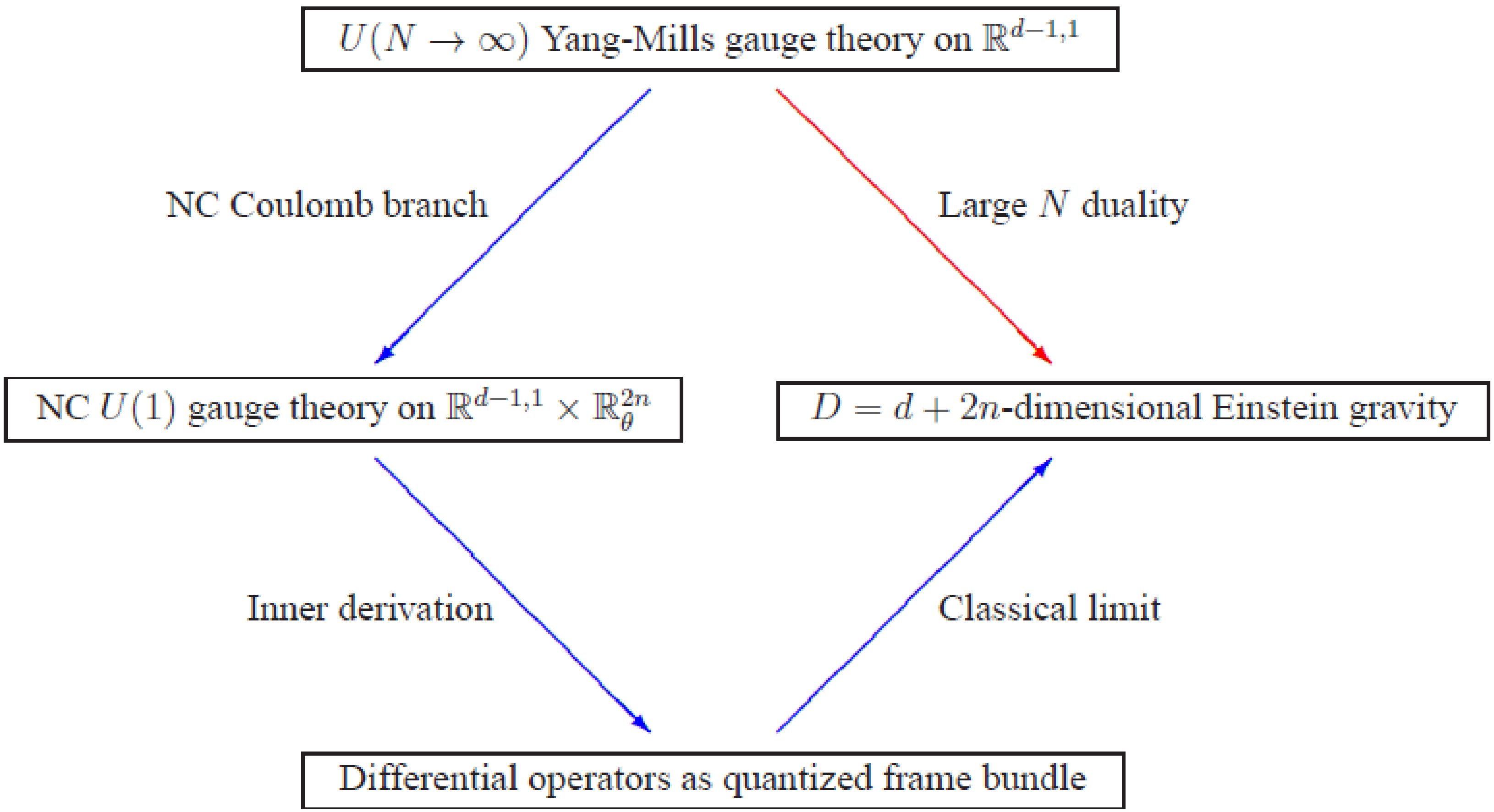



Figure 2: Flowchart for large $N$ duality

# NC Electromagnetism from Matrix Model

Let us start with a zero-dimensional matrix model with a bunch of $N \times N$ Hermitian matrices, $\{\phi_a \in \mathcal{A}_N | a = 1, \cdots, 2n\}$, whose action is given by

$$S = \frac{1}{4} \sum_{a,b=1}^{n} [\phi_a, \phi_b]^2.$$

We require that the matrix algebra $\mathcal{A}_N$ is associative, which leads to the Jacobi identity

$$[\phi_a, [\phi_b, \phi_c]] + [\phi_b, [\phi_c, \phi_a]] + [\phi_c, [\phi_a, \phi_b]] = 0.$$

We also assume the action principle, from which we yield the equation of motion

$$\sum_{b=1}^{2n} [\phi_b, [\phi_a, \phi_b]] = 0.$$

First suppose that the vacuum configuration of $\mathcal{A}_N$ is given by

$$\langle \phi_a \rangle_{vac} = p_a \in \mathcal{A}_N.$$

An obvious solution in the limit $N \to \infty$ is given by the Moyal-Heisenberg algebra

$$[p_a, p_b] = -iB_{ab},$$

where $(B)_{ab} = -L_P^{-2}(\mathrm{I}_n \otimes i\sigma^2)$ is a $2n \times 2n$ constant symplectic matrix.

A general solution is generated by considering all possible deformations of the Moyal-Heisenberg algebra. They take the form

$$\phi_a = p_a + \hat{A}_a \in \mathcal{A}_N,$$

obeying the deformed algebra

$$[\phi_a, \phi_b] = -i\big(B_{ab} - \hat{F}_{ab}\big),$$

where $\hat{F}_{ab} = \partial_a \hat{A}_b - \partial_b \hat{A}_a - i[\hat{A}_a, \hat{A}_b]$ with the definition $\partial_a \equiv ad_{p_a} = -\, i[p_a, \ \cdot\ ]$.

For a general matrix $\phi_a \in \mathcal{A}_N$ to be a solution, the set of matrices $\hat{F}_{ab} \in \mathcal{A}_N$ must satisfy the following equations

$$\widehat{D}_a \hat{F}_{bc} + \widehat{D}_b \hat{F}_{ca} + \widehat{D}_c \hat{F}_{ab} = 0,$$

$$\sum_{b=1}^{2n} \widehat{D}_b \hat{F}_{ab} = 0,$$

where $\widehat{D}_a \hat{F}_{bc} \equiv -i \ ad_{\phi_a} \hat{F}_{bc} = -i\left[\phi_a, \hat{F}_{bc}\right] = -[\phi_a, [\phi_b, \phi_c]].$

Let us apply the Lie algebra homomorphism $\rho: \mathcal{A}_\theta \to \mathfrak{D}$ to yield

$$X_{\widehat{D}_a \hat{F}_{bc}} \xrightarrow{|\theta| \to 0} [V_a, [V_b, V_c]] \in \Gamma(TM).$$

It is then straightforward to get the following correspondence

$$\sum_{b=1}^{2n} \widehat{D}_b \hat{F}_{ab} = 0 \overset{|\theta| \to 0}{\Longleftrightarrow} \sum_{b=1}^{2n} [V^b, [V_a, V_b]] = 0\,,$$

$$\widehat{D}_a \hat{F}_{bc} + \text{cyclic} = 0 \overset{|\theta| \to 0}{\Longleftrightarrow} [V_a, [V_b, V_c]] + \text{cyclic} = 0.$$

Note that the torsion $T$ and curvature $R$ are multi-linear differential operators

$$T(X,Y) = \nabla_X Y - \nabla_Y X - [X,Y],$$
$$R(X,Y)Z = [\nabla_X, \nabla_Y]Z - \nabla_{[X,Y]}Z,$$

where $X, Y$ and $Z$ are vector fields on $M$. It is easy to derive the relation

$$T(V_a, V_b) = \lambda^2 T(E_a, E_b),$$
$$R(V_a, V_b)V_c = \lambda^3 T(E_a, E_b)E_c,$$

when $V_a = \lambda E_a$.

After imposing the torsion free condition $T(E_a, E_b) = 0$, it is easy to derive the identity below

$$R(E_a, E_b)E_c + \text{cyclic} = \lambda^{-3}([V_a,\ [V_b, V_c]] + \text{cyclic}) .$$

Therefore we see that the Bianchi identity for NC $U(1)$ gauge fields is equivalent to the first Bianchi identity for the Riemann curvature tensors, i.e.,

$$\hat{D}_a \hat{F}_{bc} + \text{cyclic} = 0 \quad \overset{|\theta|\to 0}{\Longleftrightarrow} \quad R(E_a, E_b)E_c + \text{cyclic} = 0.$$

The mission for the equations of motion is more involved. So let us focus on four dimensions, i.e., $n = 2$.

Basically we are expecting the Einstein equations

$$\sum_{b=1}^{2n} \hat{D}_b \hat{F}_{ab} = 0 \quad \Leftrightarrow \quad R_{ab} = 8\pi G \left( T_{ab} - \frac{1}{2} \delta_{ab} T \right).$$

After a straightforward but tedious calculation, we get a remarkably simple but cryptic result

$$R_{ab} = -\frac{1}{\lambda^2} [g_d^{(+)i} g_d^{(-)j} (\eta^i_{ac} \bar{\eta}^j_{bc} + \eta^i_{bc} \bar{\eta}^j_{ac}) - g_c^{(+)i} g_d^{(-)j} (\eta^i_{ac} \bar{\eta}^j_{bd} + \eta^i_{bc} \bar{\eta}^j_{ad})].$$

To get the above result, we have introduced the structure equation of vector fields $V_a \in \Gamma(TM)$,

$$[V_a, V_b] = -g_{ab}{}^c \, V_c$$

and the canonical decomposition

$$g_{abc} = g_c^{(+)i} \, \eta^i_{ab} + g_c^{(-)i} \bar{\eta}^i_{ab}.$$

First it is convenient to decompose the energy-momentum tensor into two parts

$$8\pi \mathrm{G}\, T_{ab}^{(M)} = -\frac{1}{\lambda^2}\left(g_{acd}g_{bcd} - \frac{1}{4}\delta_{ab}\, g_{cde}\, g_{cde}\right),$$

$$8\pi G\, T_{ab}^{(L)} = \frac{1}{2\,\lambda^2}\left(\rho_a\rho_b - \Psi_a\Psi_b - \frac{1}{2}\delta_{ab}(\rho_c\rho_c - \Psi_c\Psi_c)\right),$$

where $\rho_a \equiv g_{bab}$ and $\Psi_a \equiv -\frac{1}{2}\varepsilon^{abcd}g_{bcd}$.

A close inspection reveals that the first one is the Maxwell energy-momentum tensor given by

$$T_{ab}^{(em)} = \frac{\hbar^2 c^2}{g_{YM}^2}\left(F_{ac}F_{bc} - \frac{1}{4}\delta_{ab}F_{cd}F_{cd}\right),$$

but the second one seems to be very mysterious.

In order to descry closer aspects of the second energy-momentum tensor, let us consider the following decomposition

$$\rho_a\rho_b = \frac{1}{4}\delta_{ab}\rho_c\rho_c + \left(\rho_a\rho_b - \frac{1}{4}\delta_{ab}\rho_c\rho_c\right),$$

$$\Psi_a\Psi_b = \frac{1}{4}\delta_{ab}\Psi_c\Psi_c + \left(\Psi_a\Psi_b - \frac{1}{4}\delta_{ab}\Psi_c\Psi_c\right)$$

In a long wavelength limit, the quadruple modes can be ignored and then it behaves like a cosmological constant

$$T_{ab}^{(L)} = -\frac{c^4}{32\pi G} \, R \, \delta_{ab},$$

where $R = \frac{1}{2\lambda^2}(\rho_a\rho_b - \Psi_a\Psi_b)\delta^{ab}$. So it may be related to dark energy/dark matter.

In order to get a corresponding result in (3+1)-dimensional Lorentzian spacetime,
let us take the analytic continuation defined by $x^4 = ix^0$. .
Under this Wick rotation,

$$\delta_{ab} \quad \longrightarrow \quad \delta_{ab}, \qquad \Psi_a \quad \longrightarrow \quad i\Psi_a,$$

the so-called Poisson-Liouville energy-momentum (PLEM) tensor is given by

$$T_{\mu\nu}^{(L)} = \frac{1}{16\pi G\,\lambda^2}\Big(\rho_\mu\rho_\nu + \Psi_\mu\Psi_\nu - \frac{1}{2}\, g_{\mu\nu}\big(\rho_\lambda^2 + \Psi_\lambda^2\big)\Big)$$
$$\approx -\frac{1}{32\pi G} \, R \, g_{\mu\nu}.$$

Note that $\rho_\mu$ and $\Psi_\mu$ are four vectors and random fluctuations in nature.
So they are classified into two classes:

$(\rho_\mu, \Psi_\mu)$: spacelike vectors, i.e., $\rho_\mu\rho_\nu \, g^{\mu\nu} > 0$, etc.
$(\rho_\mu, \Psi_\mu)$: timelike vectors, i.e., $\rho_\mu\rho_\nu \, g^{\mu\nu} < 0$, etc.

Given a timelike unit vector $u^\mu$, i.e., $u^\mu u_\mu = -1$, the Raychaudhuri equation in four dimensions is given by

$$\dot{\Theta} - \dot{u}^\mu_{;\mu} + \Sigma_{\mu\nu}\Sigma^{\mu\nu} - \Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{3}\Theta^2 = -R_{\mu\nu}u^\mu u^\nu,$$

where

$$-R_{\mu\nu}u^\mu u^\nu = -\frac{1}{2\lambda^2}u^\mu u^\nu(\rho_\mu\rho_\nu + \Psi_\mu\Psi_\nu).$$

Suppose that all the terms except the expansion evolution, $\dot{\Theta}$, in the Raychaudhuri equation vanish or become negligible. In this case the Raychaudhuri equation reduces to

$$\dot{\Theta} = -\frac{1}{2\lambda^2}u^\mu u^\nu(\rho_\mu\rho_\nu + \Psi_\mu\Psi_\nu).$$

Note that $\dot{\Theta} \approx \frac{R}{4}$ and $R < 0$ when $\rho_\mu$ and $\Psi_\mu$ are timelike while $R > 0$ when $\rho_\mu$ and $\Psi_\mu$ are spacelike:

$\dot{\Theta} \approx \frac{R}{4} < 0$ for timelike fluctuations;

$\dot{\Theta} \approx \frac{R}{4} > 0$ for spacelike fluctuations.

Note that curvature tensors in GR are decomposed as

$$R = \begin{pmatrix} W^+ + \frac{1}{12}s & B \\ B^T & W^- + \frac{1}{12}s \end{pmatrix}$$

where $s$ is the scalar curvature, $B$ is the traceless Ricci tensor, and $W_\pm$ are the Weyl tensors.
A strange thing in our case is that $R_{ab} \in su(2)_L \times su(2)_R$ like $B$, nevertheless it has a non-vanishing Ricci scalar, which is impossible in GR.
Hence $T^{(L)}_{\mu\nu}$ cannot be realized in the context of GR.

It turns out that the weird energy $T^{(L)}_{\mu\nu}$ is originated from "vacuum fluctuations" with the largest possible wavelength $L_H$ due to the UV/IR mixing triggered by the noncommutativity of spacetime:

$$\rho = \rho_{vac} + \delta\rho = \frac{1}{4g_{YM}^2}\left(B_{\mu\nu} - \hat{F}_{\mu\nu}\right)^2 = \frac{1}{4g_{YM}^2}\left|B_{\mu\nu}\right|^2\left(1 + \theta\hat{F}\right)^2$$
$$\sim M_P^4\left(1 + \frac{L_P^2}{L_H^2}\right)^2 = M_P^4 + \frac{1}{L_P^2 L_H^2},$$

where a simple dimensional analysis implies that $|\hat{F}(x)| = \frac{1}{L_H^2}$.

Then $R \sim \frac{1}{L_H^2}$, so the PLEM tensor is given by

$$T_{\mu\nu} \approx -\frac{1}{L_P^2 L_H^2} g_{\mu\nu} \approx (10^{-3} eV)^4$$

if we identify with the size of cosmic horizon of our observable universe, $L_H \sim 1.3 \times 10^{26}$m. This extended (nonlocal) energy is in good agreement with the observed value of current dark energy.

In the Lorentzian signature, the sign of the Ricci scalar $R$ depends on whether fluctuations are spacelike $(R > 0)$ or timelike $(R < 0)$.
As a result, the spacelike perturbations act as a repulsive force whereas the timelike ones act as an attractive force.
Thus remarkably emergent gravity predicts the existence of dark matter as well as dark energy.

When considering the fact that the fluctuations are random in nature and we are living in (3+1) (macroscopic) dimensions, the ratio of the repulsive and attractive components will end in $\frac{3}{4} : \frac{1}{4} = 75 : 25$ and this ratio curiously coincides with the dark composition of our universe.

# Conclusions

☞ The emergent gravity from NC $U(1)$ gauge theory is the large $N$ duality and the emergent spacetime is an inevitable consequence of the background-independent formulation of quantum gravity.

☞ NC spacetime brings about UV/IR mixing, so UV fluctuations are necessarily paired with IR fluctuations, which means that these UV/IR fluctuations are extended to large scales $L_H$.

☞ Dark matter and dark energy arise as a holographic (UV-IR) manifestation of the coherent vacuum of Planck energy condensate.

☞ A surprising feature is that emergent gravity distinguishes dark matter and dark energy according to the causal structure of emergent spacetime.