Oscillon in Einstein-scalar system with double well potential and its properties.

Taishi Ikeda (Nagoya University in Japan)
with Chul-Moon Yoo, Vitor Cardoso

Introduction

- ✓ What is oscillon ?
- ✓ Critical behavior of the oscillon’s lifetime.

What we want to do.

- ✓ Self-gravitating oscillons

Method

- ✓ Our numerical code.

Result

- ✓ Critical behavior of self-gravitating oscillons
- ✓ New type of critical behavior ?

Introduction

- ✓ What is oscillon ?
- ✓ Critical behavior of the oscillon’s lifetime.

What we want to do.

- ✓ Self-gravitating oscillons

Method

- ✓ Our numerical code.

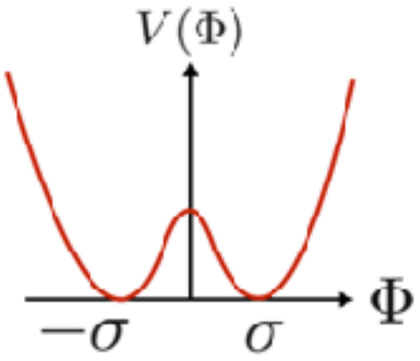
Result

- ✓ Critical behavior of self-gravitating oscillons
- ✓ New type of critical behavior ?

What is Oscillon ?

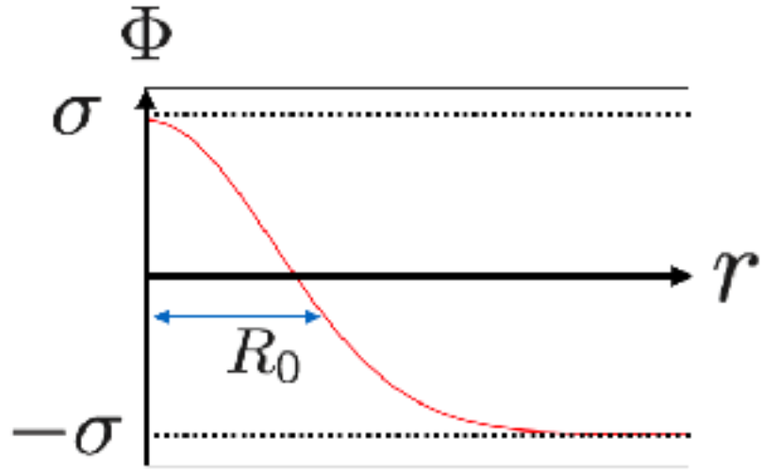
- Oscillon is a solution of a scalar field with double well potential.
  - EOM : $(-\frac{\partial^2}{\partial t^2} + \nabla^2)\Phi = V'(\Phi)$

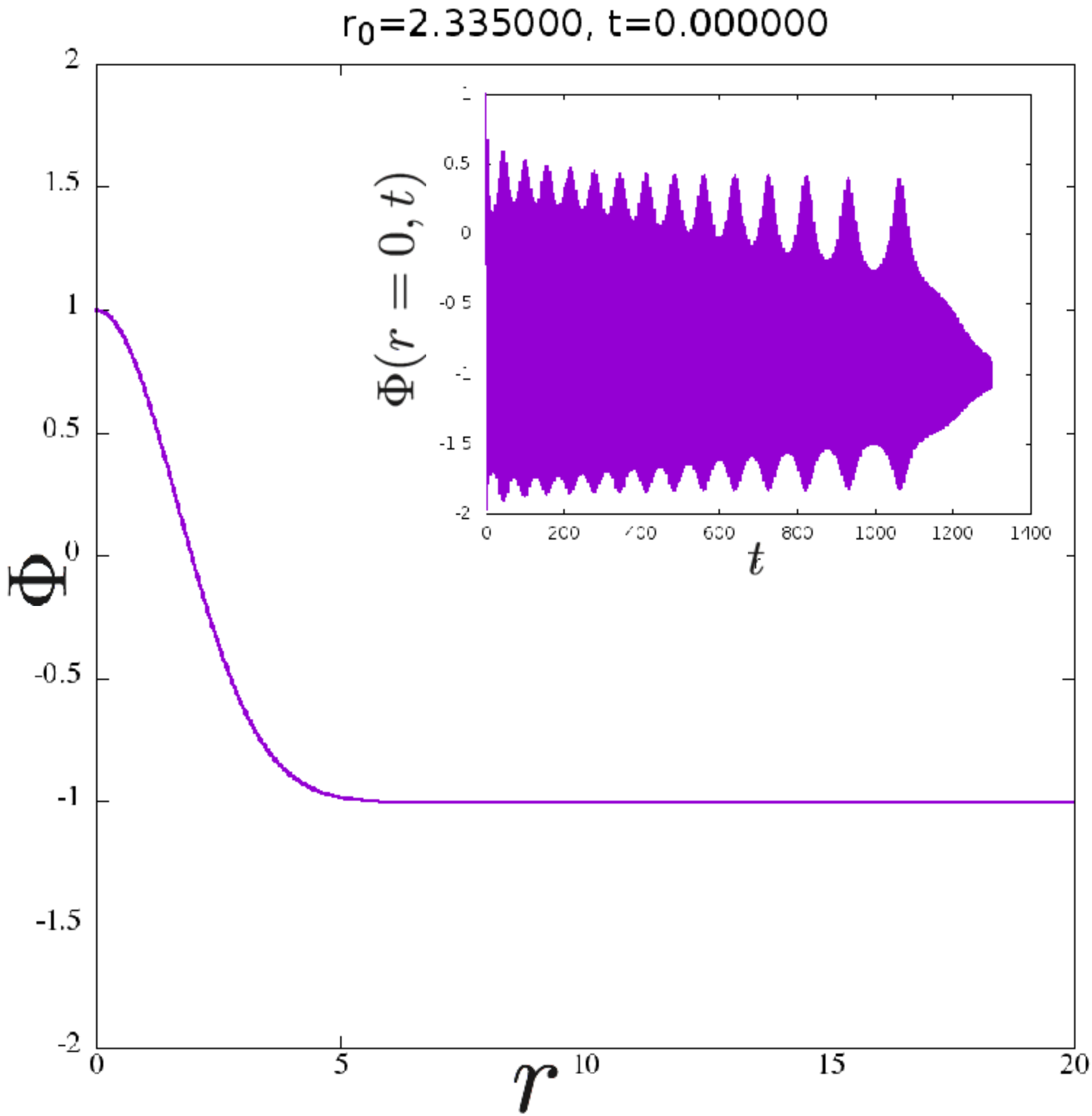
$$V(\Phi) = \frac{\lambda}{4}(\Phi^2 - \sigma^2)^2$$



- its fundamental properties
  - Oscillon is a longevity localized solution.
  - Its lifetime is very long, but finite.
  - Oscillon generally appears after bubble collapses.
  - The lifetime depends on the initial bubble radius.
- typical initial data (gaussian bubble)

$$\begin{cases} \Phi(t=0,r) &= 2\sigma e^{-(r/R_0)^2} - \sigma \\ \Pi(t=0,r) &= 0 \end{cases}$$

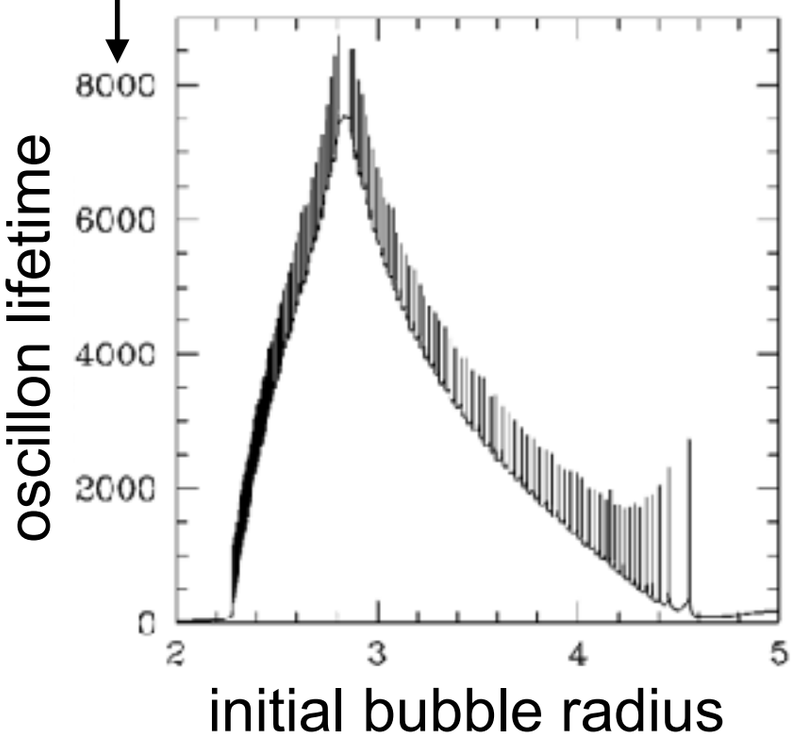


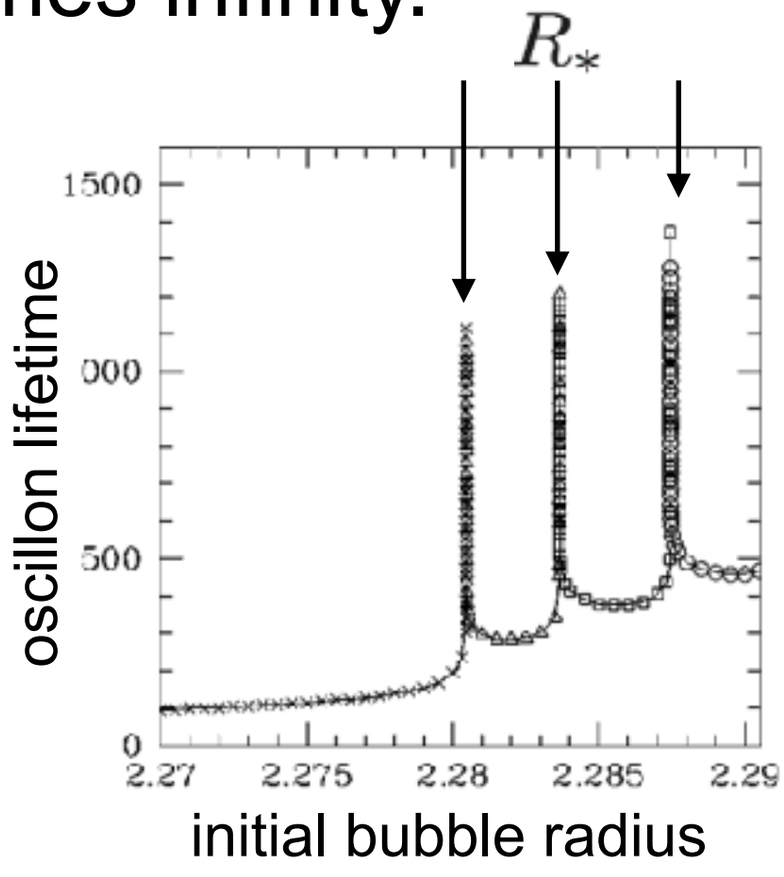

r_0=2.335000, t=0.000000
Φ
r
$\Phi(r=0,t)$
t

Critical behavior of oscillon's lifetime.

- Critical behavior of the oscillon. (Honda et al 2002) $\lambda = \sigma = 1$
  - If the initial parameter is fine-tuned, the lifetime of the oscillon becomes infinity.

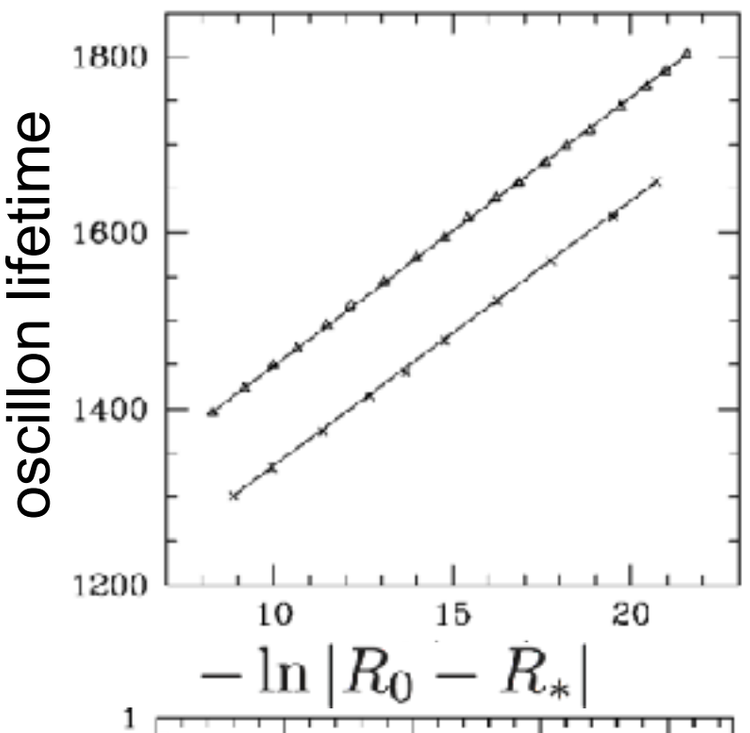
$\mathcal{O}(10^3 \sim 10^4)$

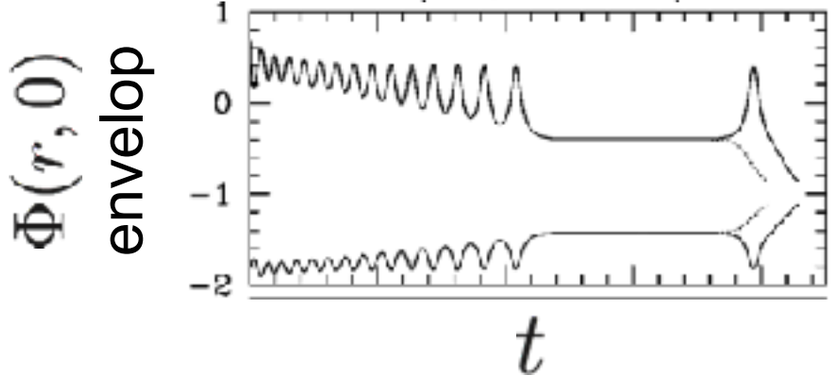




$\tau = -\gamma \ln |R_0 - R_*| + C$

type I critical behavior





Introduction

- ✓ What is oscillon ?
- ✓ Critical behavior of the oscillon's lifetime.

What we want to do.

- ✓ Self-gravitating oscillons

Method

- ✓ Our numerical code.

Result

- ✓ Critical behavior of self-gravitating oscillons
- ✓ New type of critical behavior ?

Self-gravitating oscillons

- Our Question :

  If oscillons feel gravity, how do the properties change ?
- We consider self-gravitating oscillons.

$$\begin{cases} G_{\mu\nu} = 8\pi G \left\{ -\frac{1}{2} g_{\mu\nu} (\nabla\Phi)^2 + \nabla_\mu \Phi \nabla_\nu \Phi - g_{\mu\nu} V(\Phi) \right\} \\ \nabla^2 \Phi = V'(\Phi) \end{cases}$$

DW potential $V(\Phi) = \frac{\lambda}{4}(\Phi^2 - \sigma^2)^2$

What we want to do.

We examine the lifetime of the self-gravitating oscillon in spherically symmetric spacetime.

In particular, we focus on the critical behavior in week gravity case.

Introduction

- ✓ What is oscillon ?
- ✓ Critical behavior of the oscillon’s lifetime.

What we want to do.

- ✓ Self-gravitating oscillons

Method

- ✓ Our numerical code.

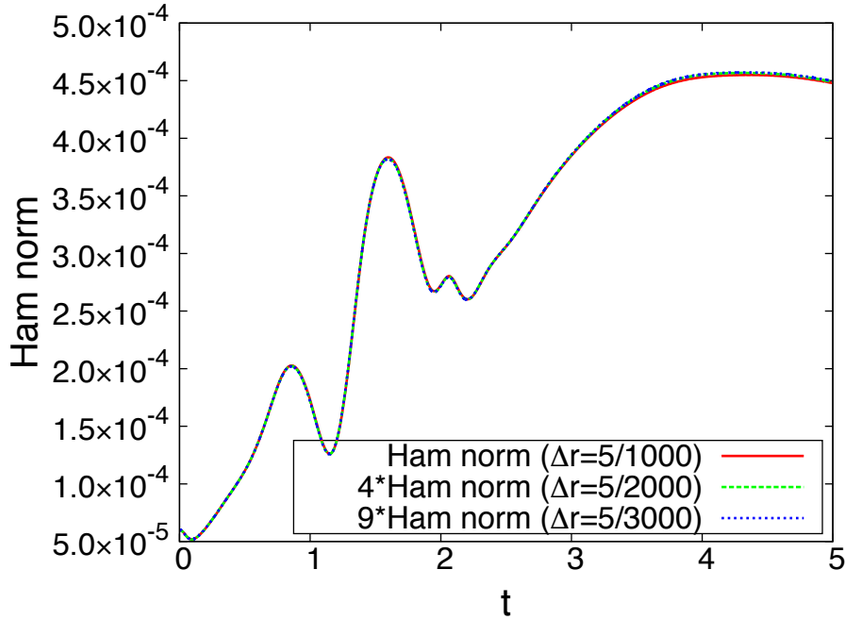
Result

- ✓ Critical behavior of self-gravitating oscillons
- ✓ New type of critical behavior ?

Our numerical code

- In order to solve the nonlinear equation, we must use the numerical calculation.
- Our numerical code
  - GBSSN formulation
  - free evolution
  - time integration : iterative Crank Nicholson scheme
  - spatial derivative : central difference

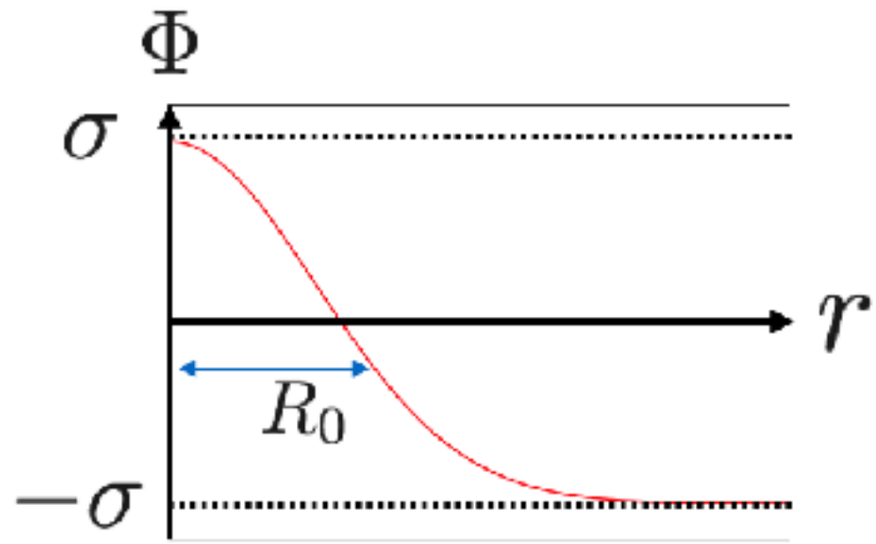
convergence check



Our numerical code

- initial data : Gaussian bubble

$$\begin{cases} \Phi(t=0,r) &= 2\sigma e^{-(r/R_0)^2} - \sigma \\ \Pi(t=0,r) &= 0 \end{cases}$$



- initial parameter : R_0

- model parameter
  - $\sigma^2 G$: coupling between gravity and scalar field
- definition of the lifetime
  - $M(t, r_0)$: Kodama mass inside the sphere with radius r_0
  - lifetime : τ

$$\frac{M(\tau, r_0)}{M(0, r_0)} < \epsilon = 0.01$$

Introduction

- ✓ What is oscillon ?
- ✓ Critical behavior of the oscillon’s lifetime.

What we want to do.

- ✓ Self-gravitating oscillons

Method

- ✓ Our numerical code.

Result

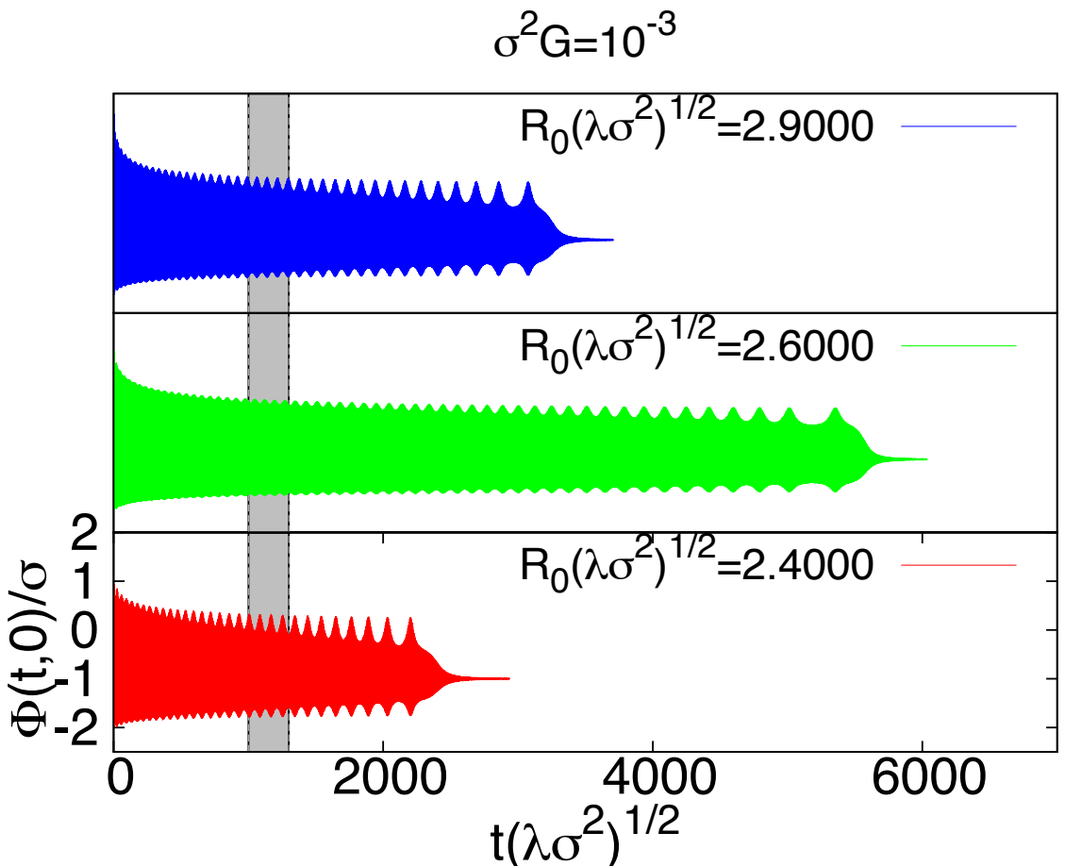
- ✓ Typical behavior of the oscillon.
- ✓ New type of critical behavior ?

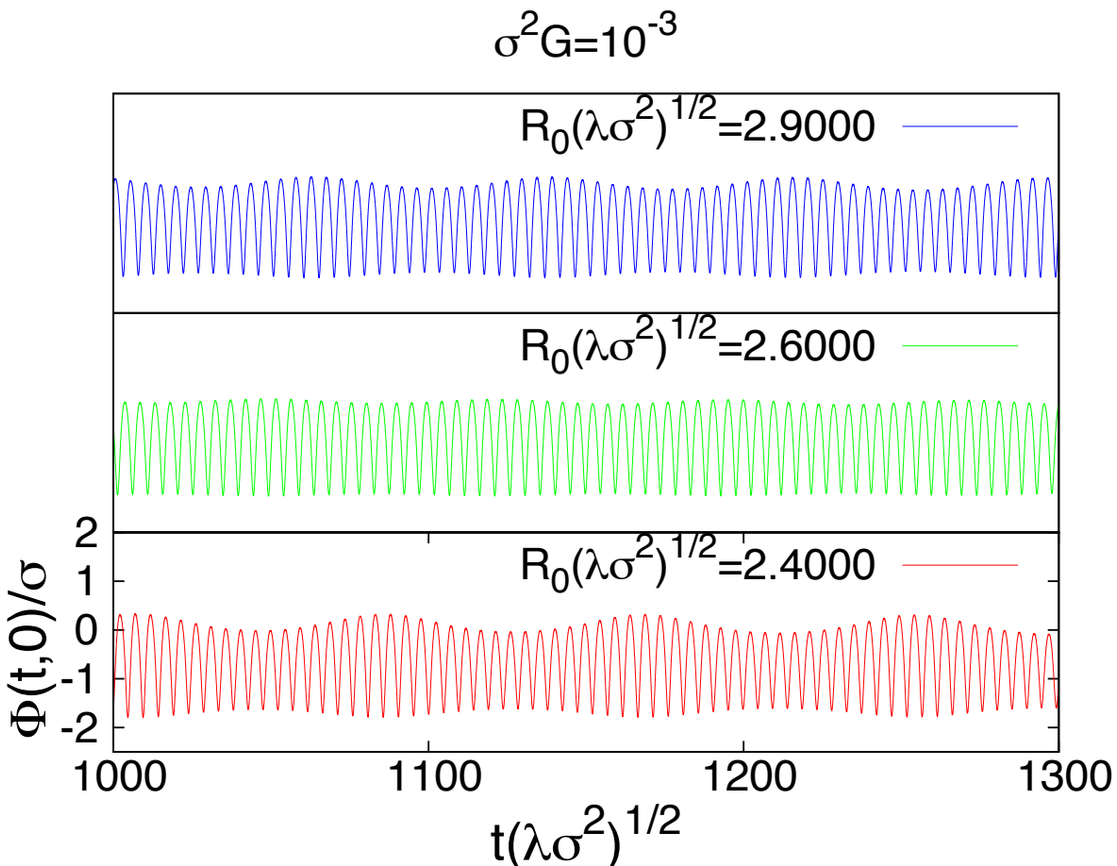
Typical behavior of the oscillon

- We examined the following parameter region.

$$\sigma^2 G = 10^{-4}, 5.0 \times 10^{-4}, 10^{-3}, 2.0 \times 10^{-3}$$

- time evolution of oscillon
  - The scalar field oscillates many times.
  - The envelop of the scalar field at the origin modulates.





r_0=2.600000 ,$\tilde{G}$=1.0e-03, t=0.010000

$\Pi(t,0)/(\sigma\alpha(t,0))$

2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0

-2 -1.5 -1 -0.5 0 0.5 1 1.5 2

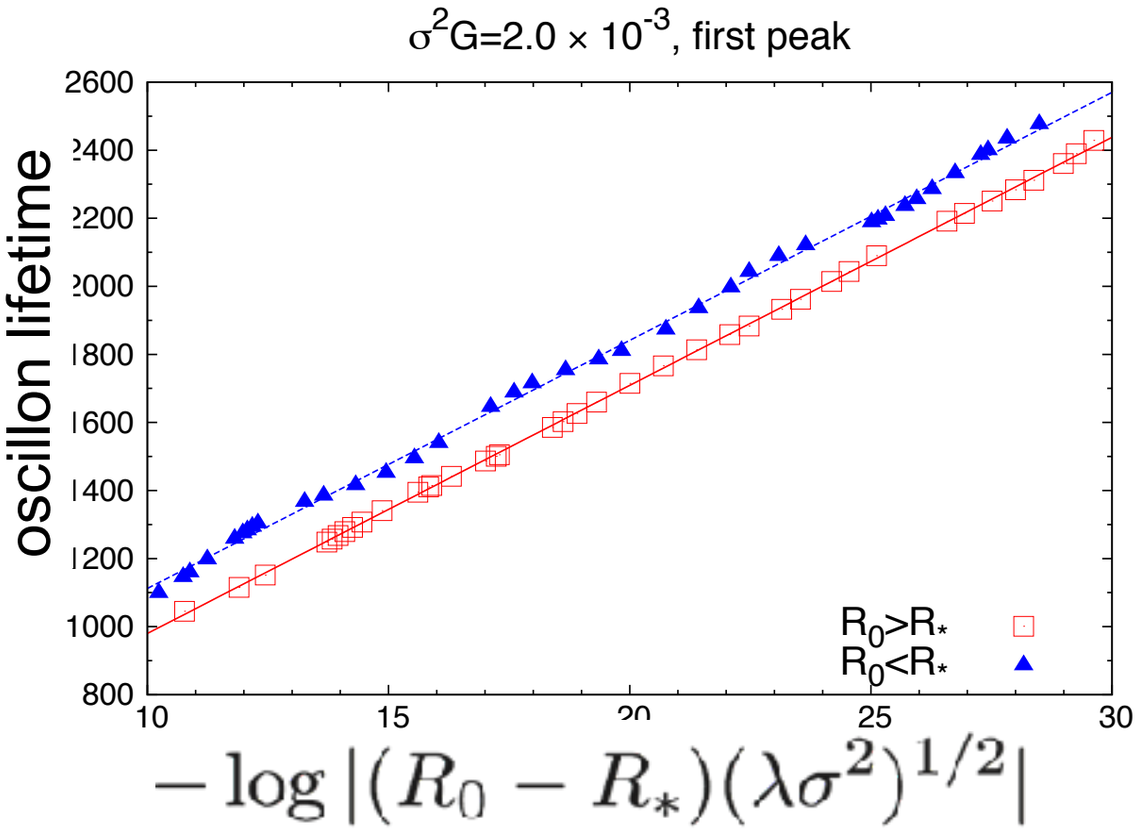
$\Phi(t,0)/\sigma$

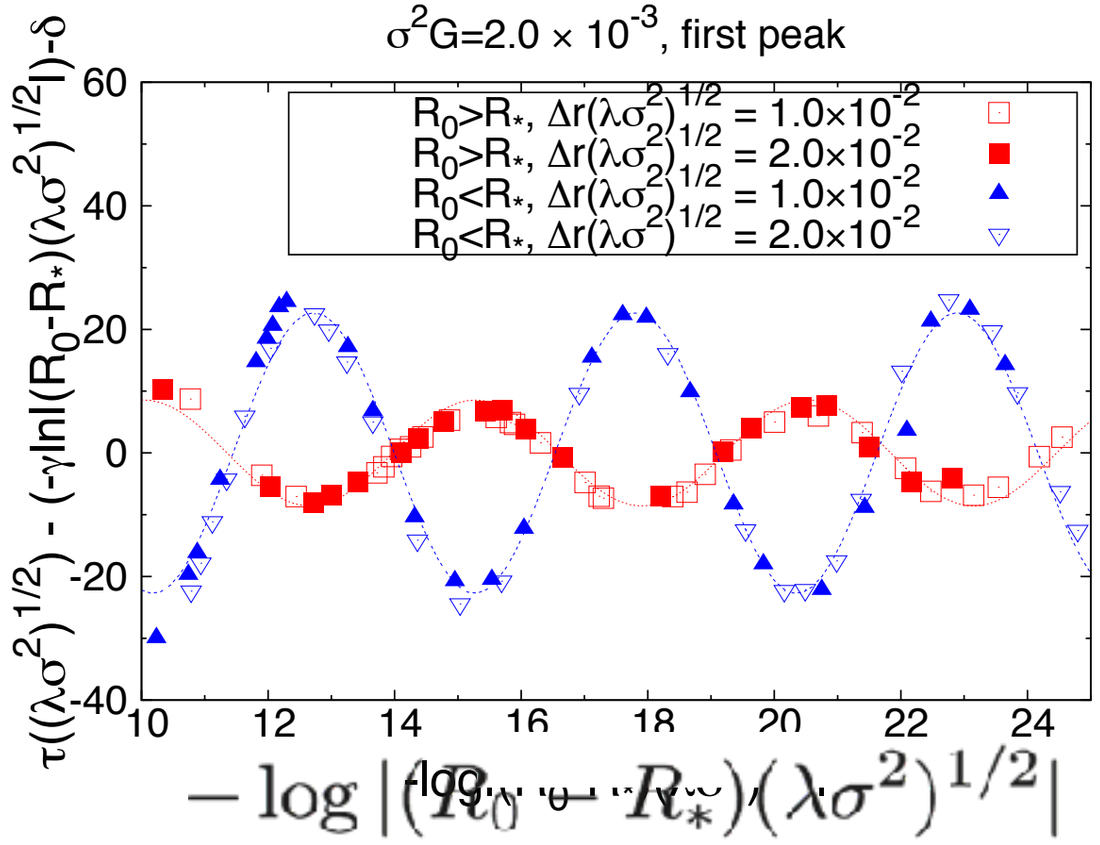
New type of critical behavior

- new type (?) of the scaling behavior
  - For $\sigma^2 G = 2.0 \times 10^{-3}$, new type critical behavior appears ?

$$\tau = -\gamma \log |R_0 - R_*| + f(-\log |R_0 - R_*|) + C$$

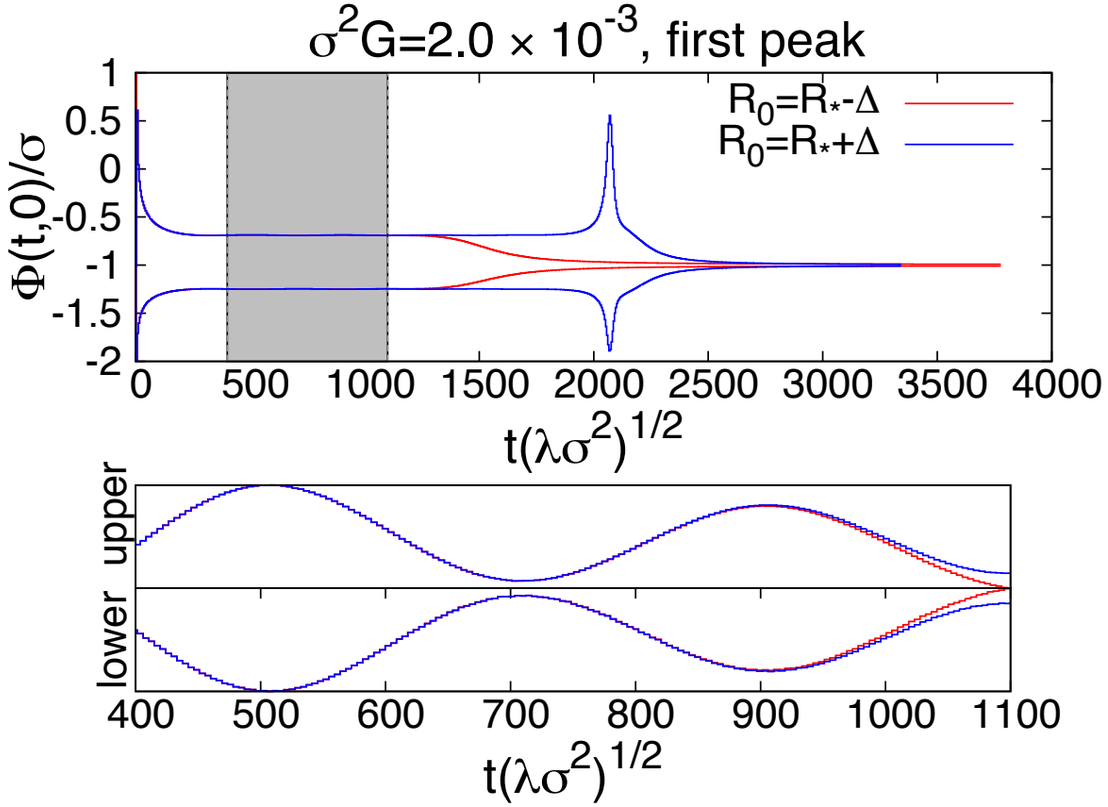
$$f(x) = f(x + \varpi)$$: periodic function

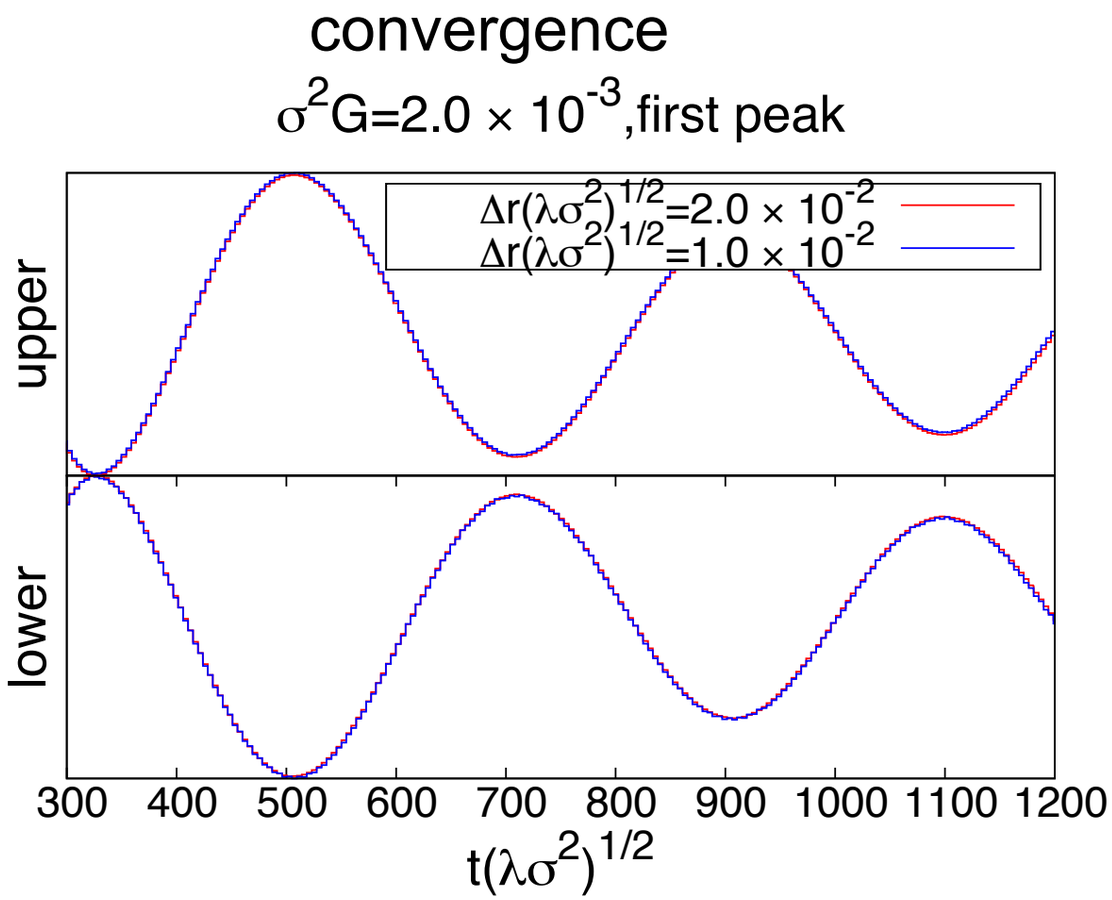




New type of critical behavior

- about its critical solution
  - The envelop of the critical solution oscillates.





Summary and discussion

- We examined the properties of self-gravitating oscillons.
  - typical behavior of the oscillon
  - critical behavior
  - When coupling between the gravity and matter is strong, new type (?) of type I critical behavior.
- Future work
  - How do the behavior change when the coupling is large ? $\sigma^2 G > 2.0 \times 10^{-3}$

Thank you !!

back up

Outline

1. Introduction
   - ★Critical behavior of gravitational collapse
   - ★Critical behavior of oscillon's lifetime
2. What we want to do. & Method
3. Result.
   - typical behavior of oscillon with gravity
   - ★"New" type I critical behavior ? (多分)

Message

gravity

potential

★Critical behavior of gravitational collapse

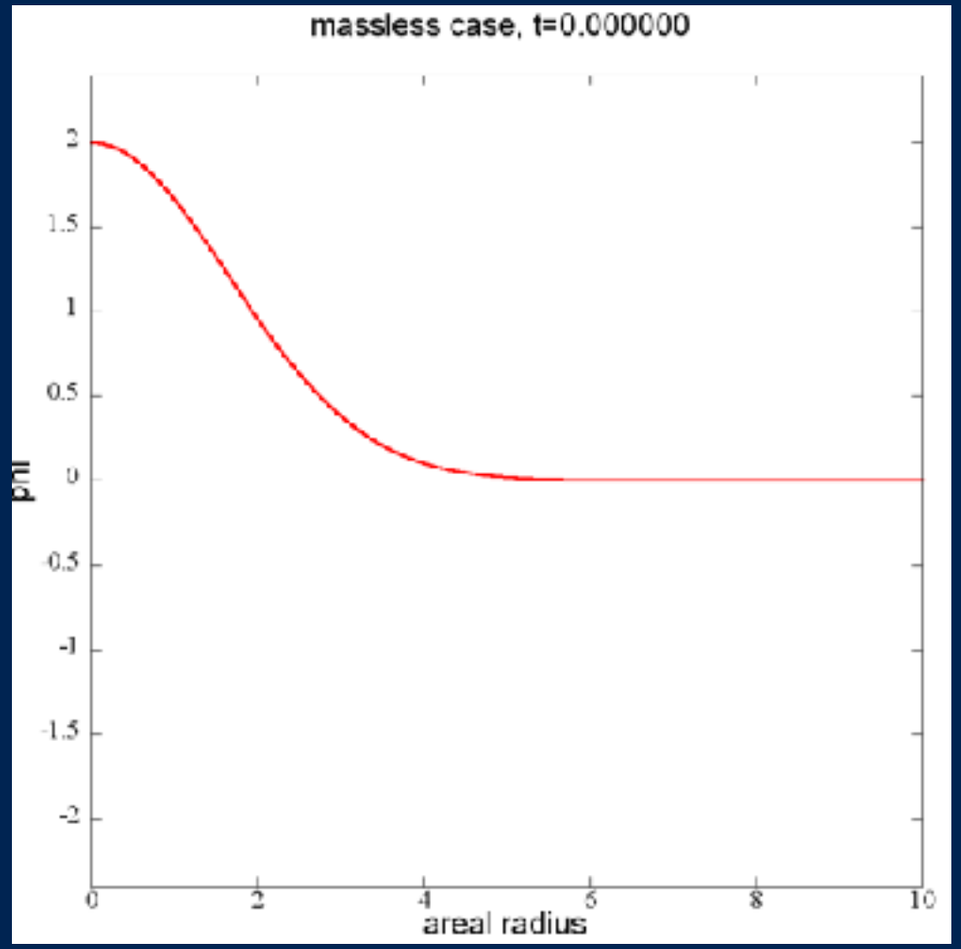
★"New" type I critical behavior ? (多分)

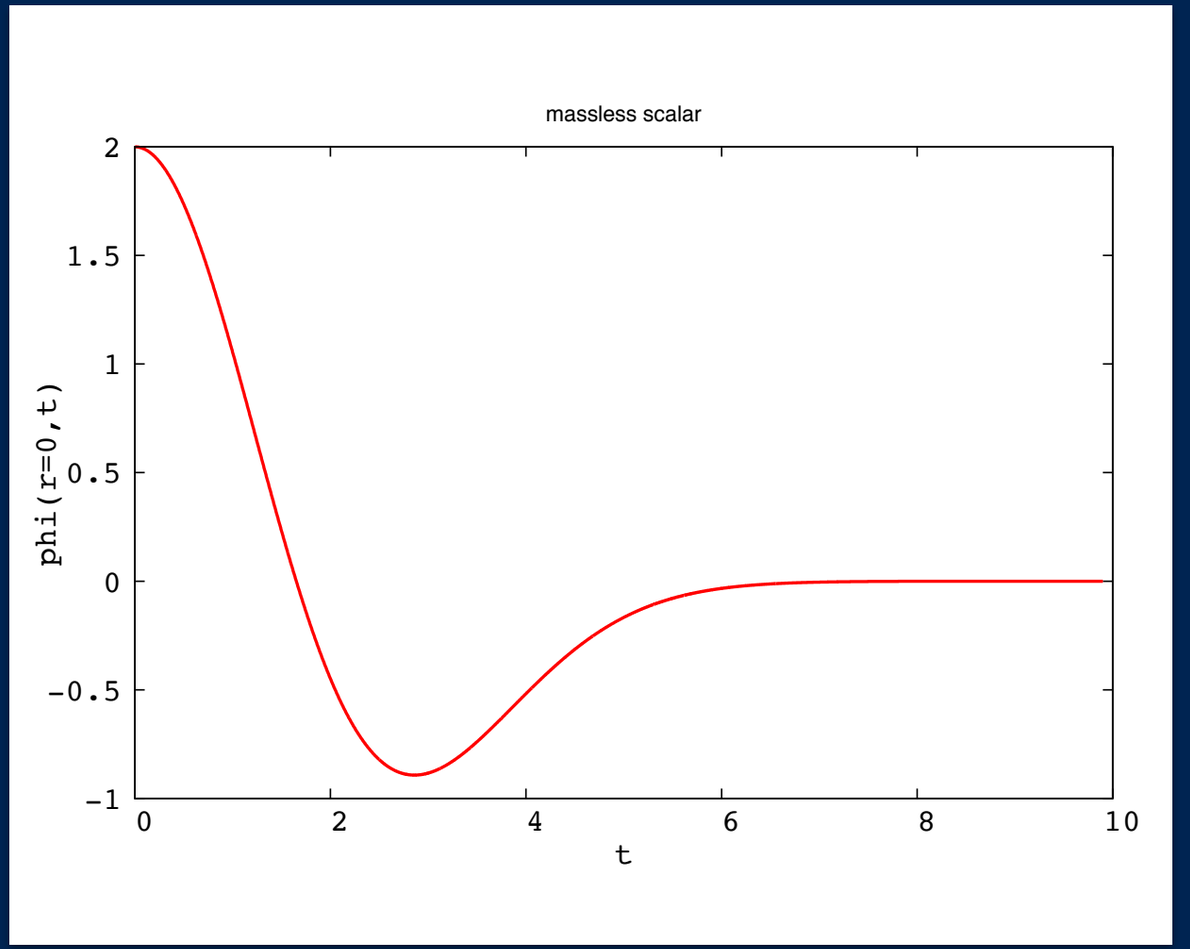
★Critical behavior of oscillon's lifetime

What is oscillon?

- Massless scalar field in Minkowski space time.
- EoM (spherically symmetry)

$$-\frac{\partial^2}{\partial t^2}\Phi + (\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r})\Phi = 0$$

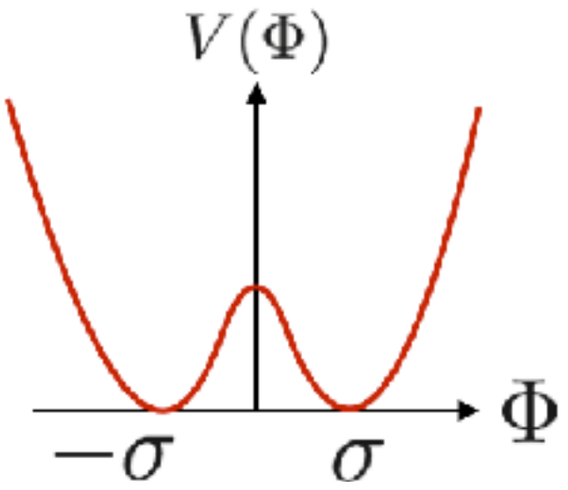




What is oscillon?

- Oscillon is a longevity localized dynamical solution of nonlinear Klein-Gordon eq.
  - Scalar field with double well potential in Minkowski spacetime.

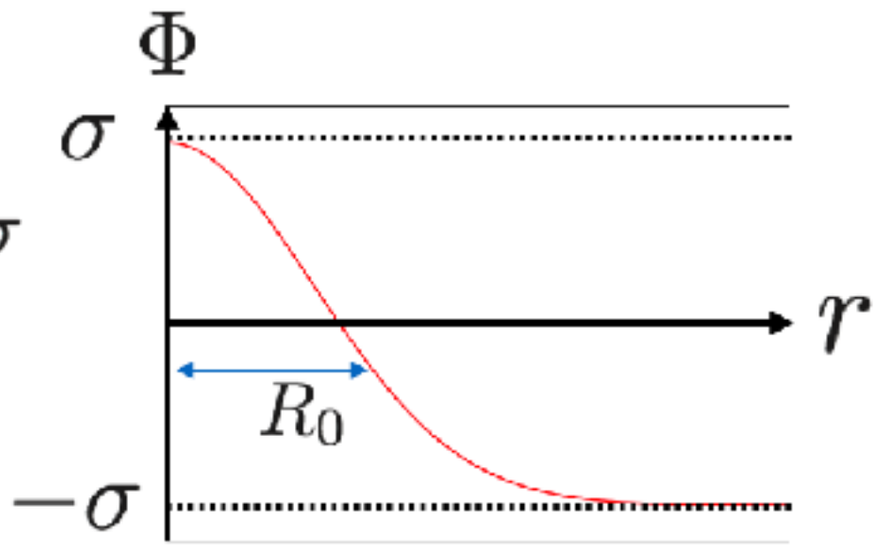
$$V(\Phi) = \frac{\lambda}{4}(\Phi^2 - \sigma^2)^2$$



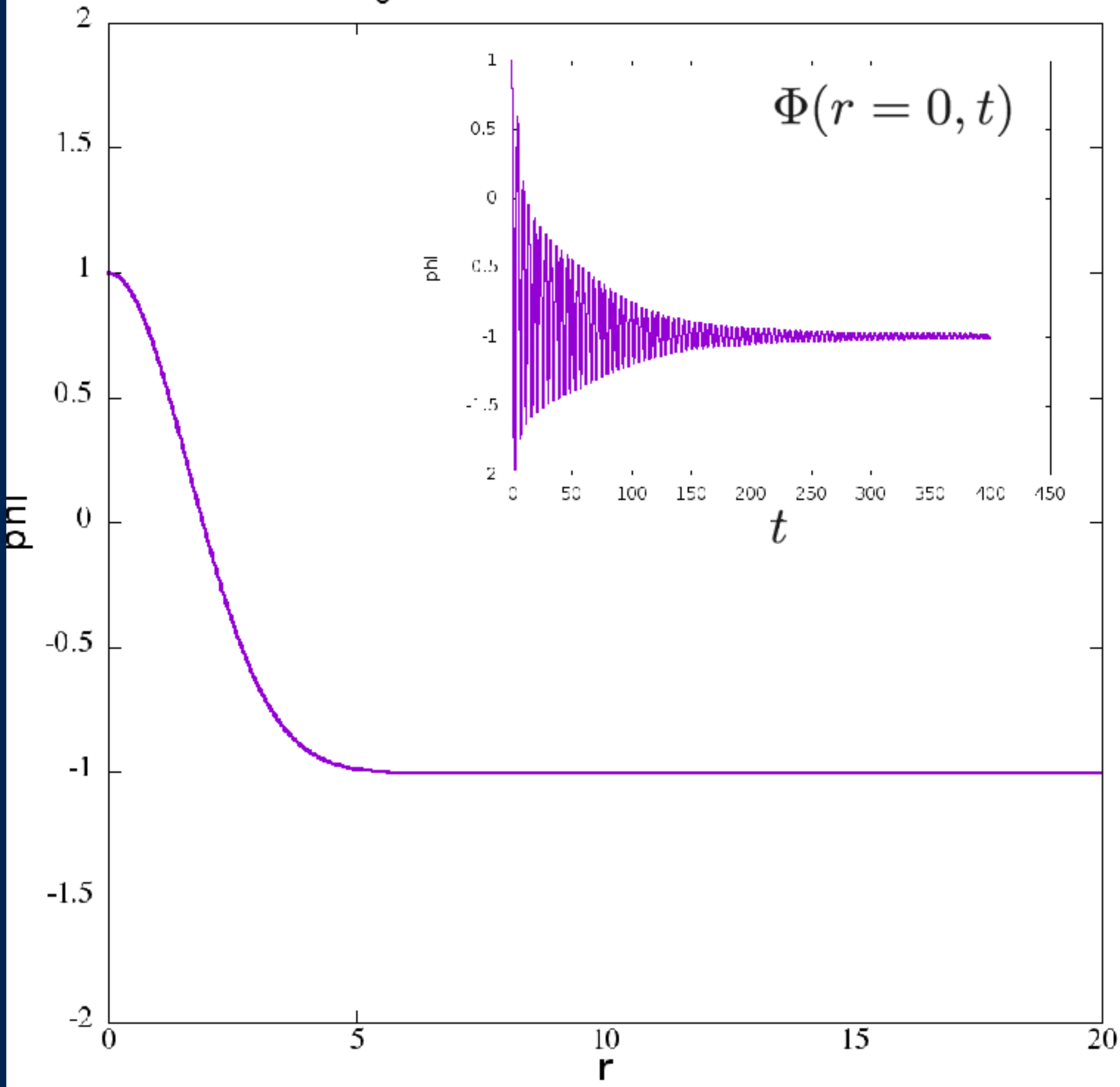
$$-\frac{\partial^2}{\partial t^2}\Phi + (\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r})\Phi = V'(\Phi)$$

  - initial data

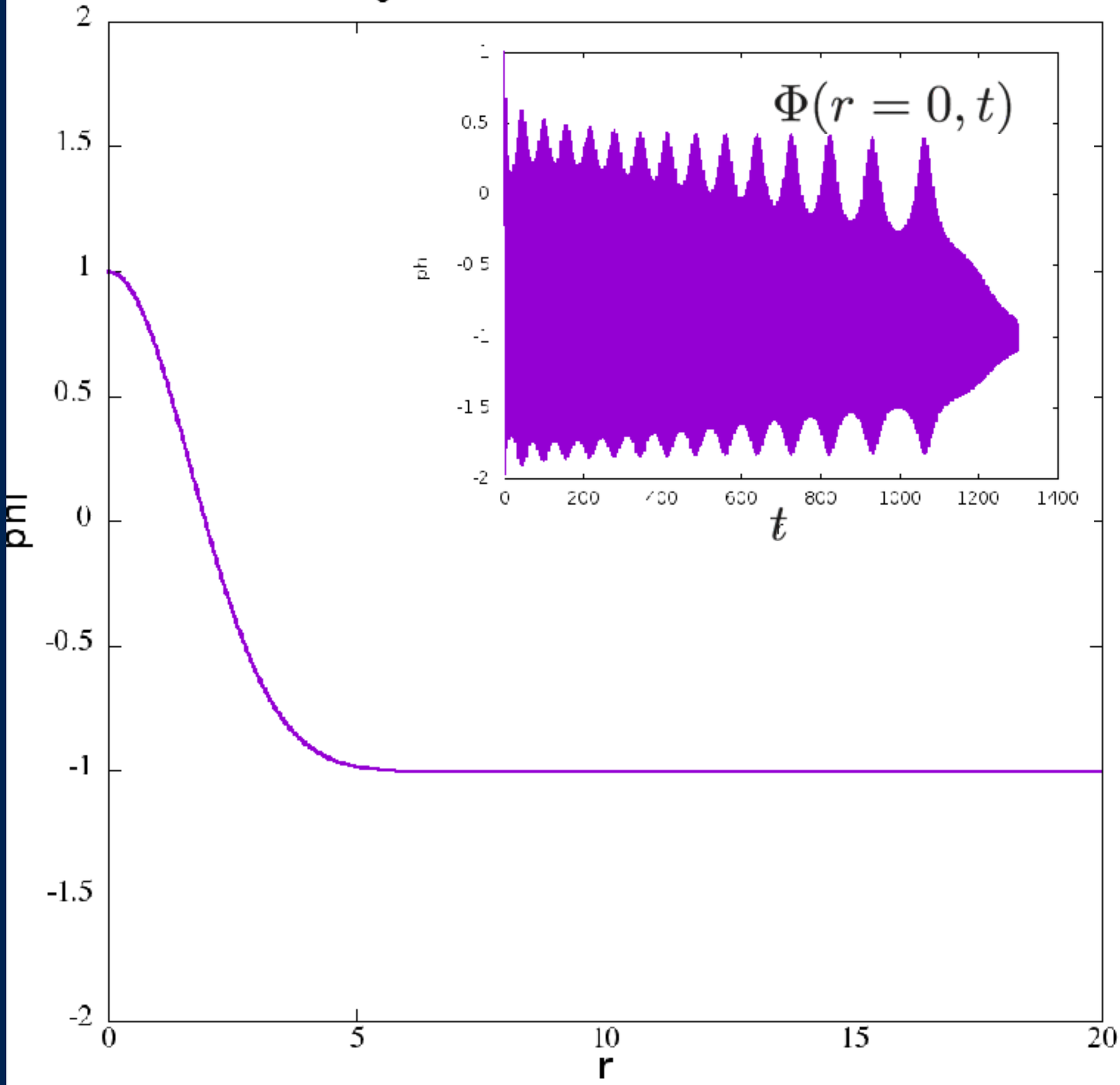
$$\begin{cases} \Phi(t=0,r) &= 2\sigma e^{-(r/R_0)^2} - \sigma \\ \Pi(t=0,r) &= 0 \end{cases}$$



r_0=2.275000, t=0.000000



r_0=2.335000, t=0.000000

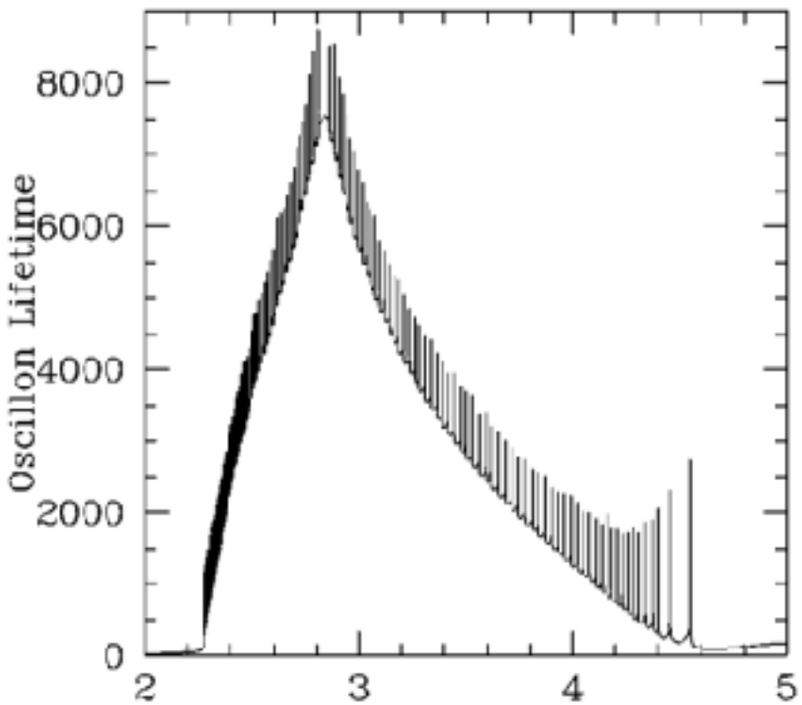


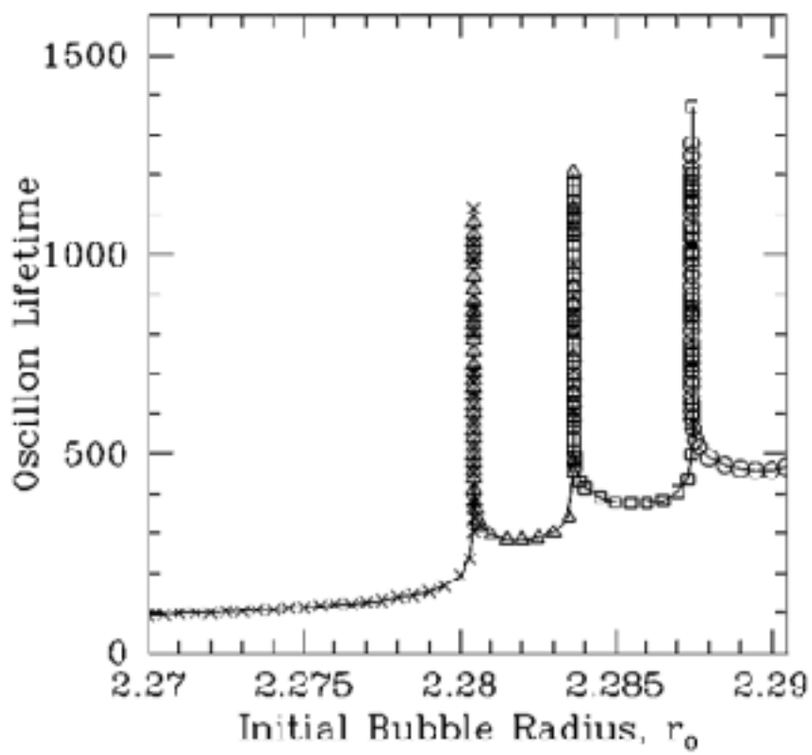
What is oscillon?

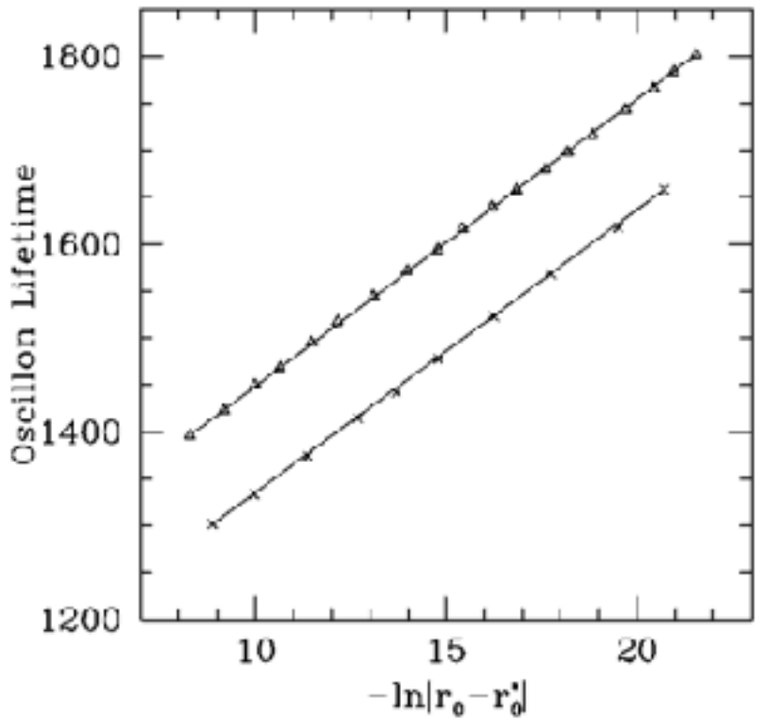
- Oscillon's lifetime depends on the initial parameter.

(Honda et al 2002)

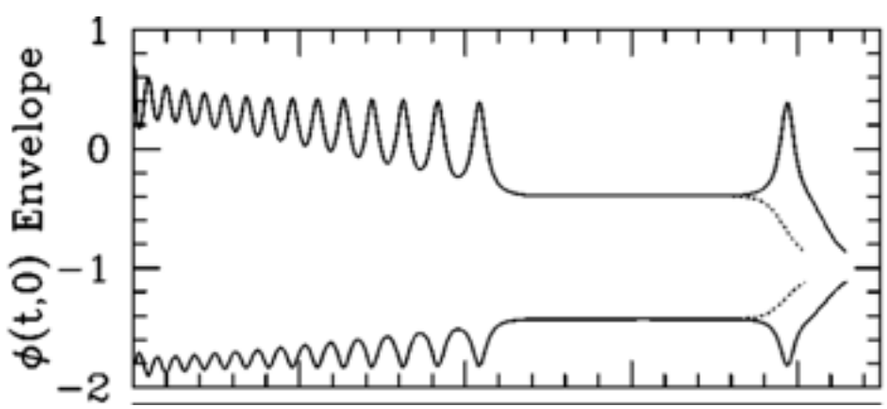
lifetime







initial radius of bubble



$$\tau = -\gamma \ln |R_0 - R_*| + C$$

critical solution = quasi-breather

(type I critical behavior)

Outline

1. Introduction
   - ★Critical behavior of gravitational collapse
   - ★Critical behavior of oscillon's lifetime
2. What we want to do. & Method
3. Result.
   - typical behavior of oscillon with gravity
   - ★"New" type I critical behavior ? (多分)

Message

gravity

potential

★Critical behavior of gravitational collapse

★"New" type I critical behavior ? (多分)

★Critical behavior of oscillon's lifetime

What we want to do.

Question

Do oscillons become the intermediate state of the gravitational collapse ?

How do the critical behavior changes ?

It may relate to interesting solution.

What we want to do.

What we want to do.

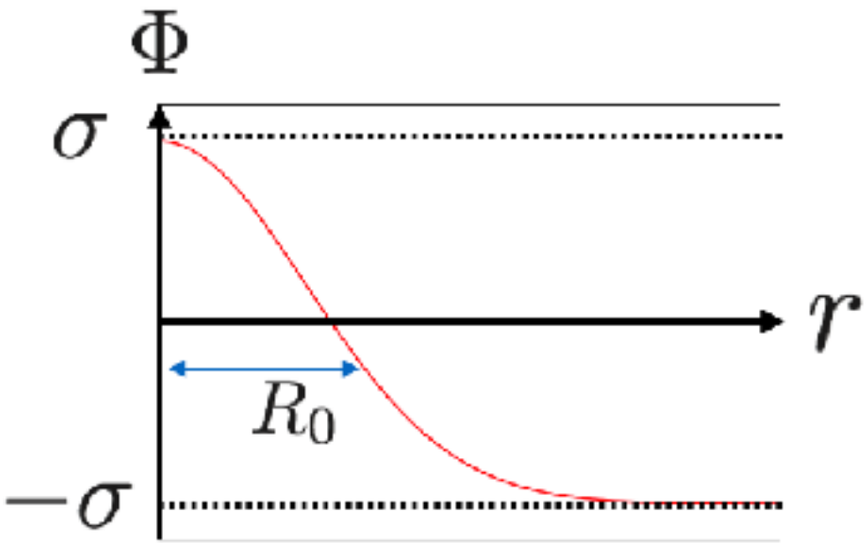
We examine the properties of the oscillon with gravity.

- EoM

$$\begin{cases} G_{\mu\nu} = & 8\pi G\left\{-\frac{1}{2}g_{\mu\nu}(\nabla\Phi)^2 + \nabla_\mu\Phi\nabla_\nu\Phi - g_{\mu\nu}V(\Phi)\right\} \\ \nabla^2\Phi = & V'(\Phi) \end{cases}$$

- Initial data

$$\begin{cases} \Phi(t=0,r) & = 2\sigma e^{-(r/R_0)^2} - \sigma \\ \Pi(t=0,r) & = 0 \end{cases}$$



- model parameter : $\sigma^2 G$

Numerical scheme

- Our numerical code
  - It is written in C++.
  - GBSSN formulation - spherically symmetric case
  - free evolution
  - time integration : iterative Crank Nicolson shceme
  - spatial derivative : central difference
  - totally second order accuracy
  - We add 2nd order numerical dissipation term in each time evolution equation.
  - We use inhomogeneous grid
  - parallel computation by using Open MP

Outline

1. Introduction
   - ★Critical behavior of gravitational collapse
   - ★Critical behavior of oscillon's lifetime
2. What we want to do. & Method
3. Result.
   - typical behavior of oscillon with gravity
   - ★"New" type I critical behavior ? (多分)

Message

gravity

potential

★Critical behavior of gravitational collapse

★"New" type I critical behavior ? (多分)

★Critical behavior of oscillon's lifetime

Result

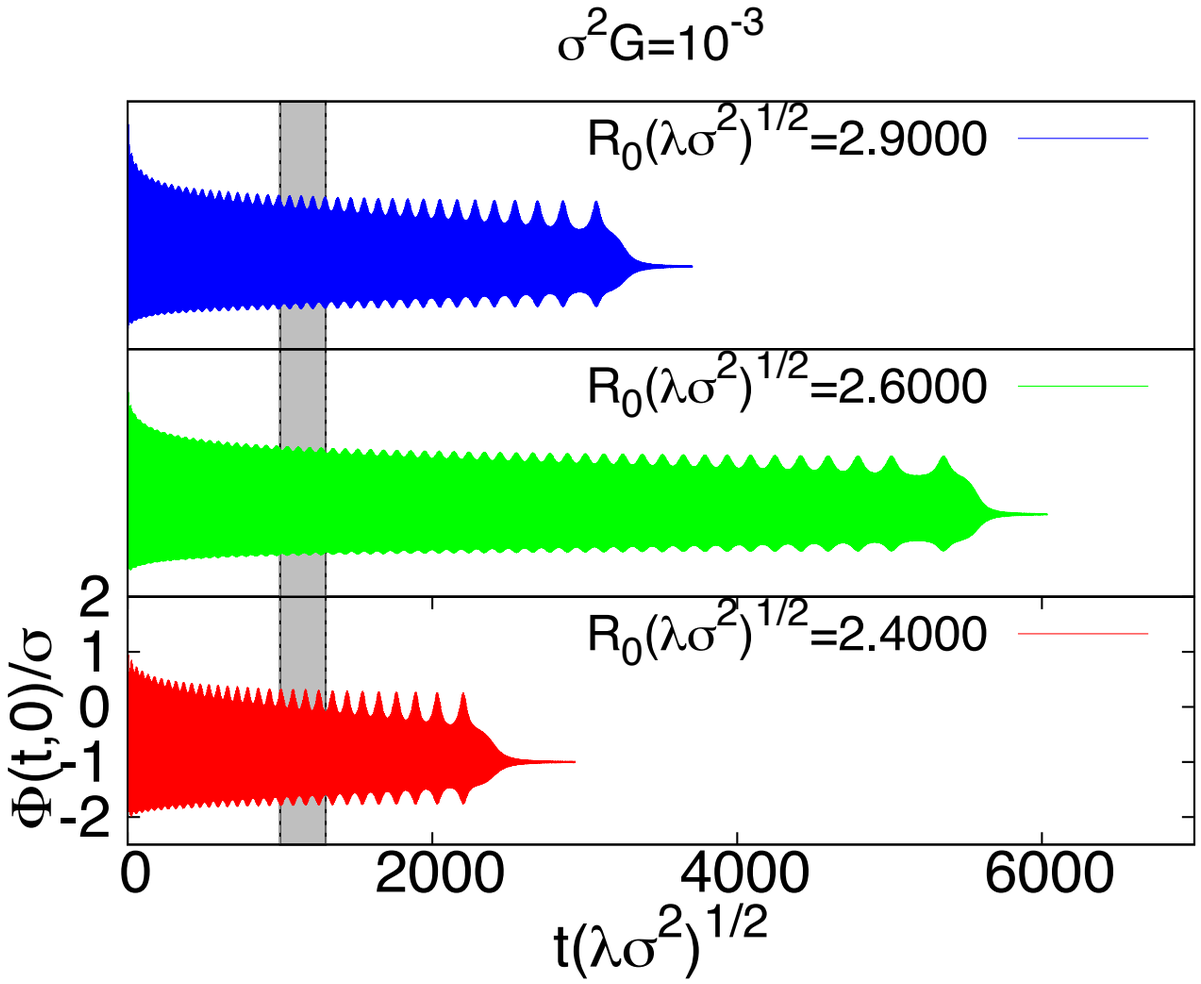
- We examined the following parameter region.

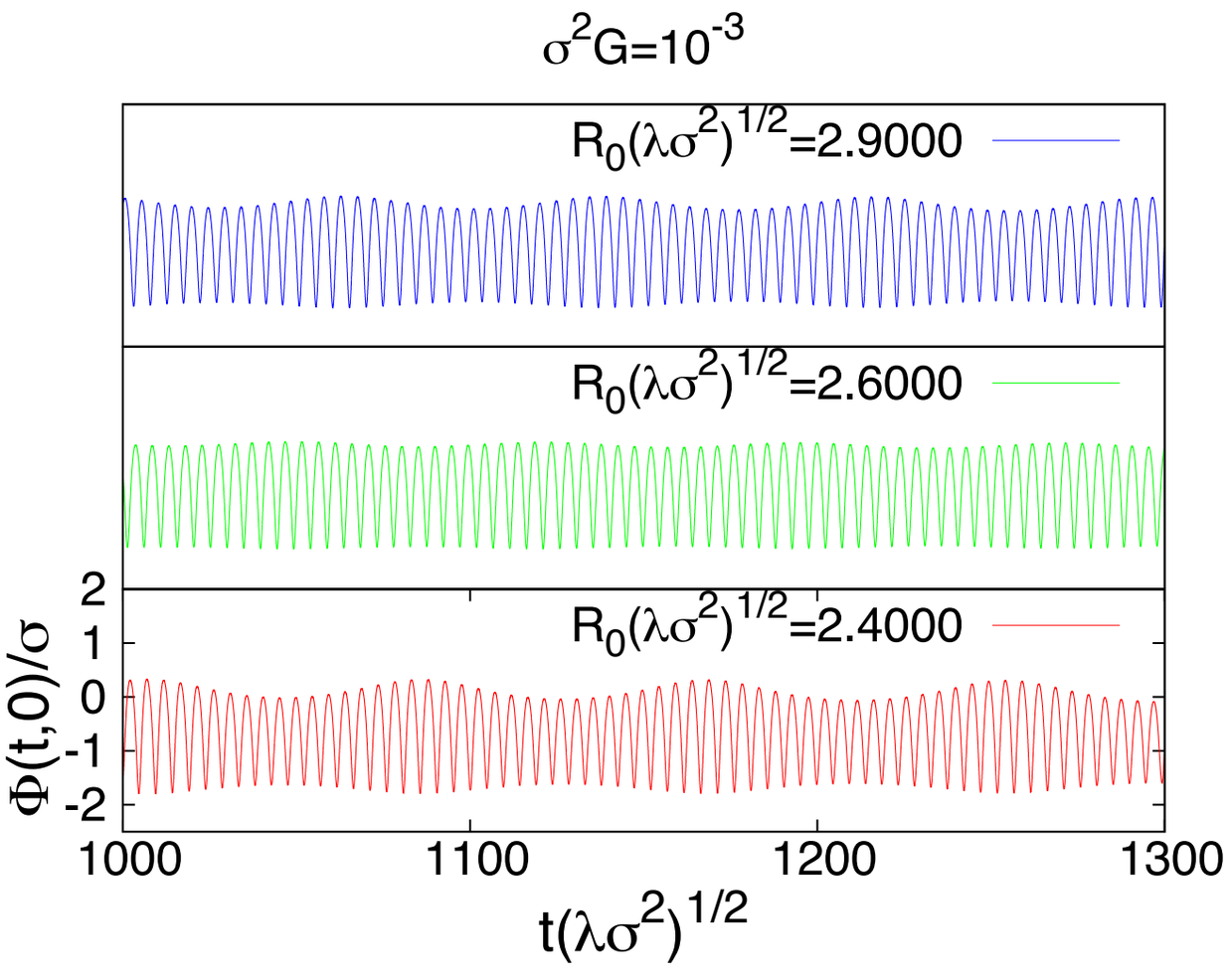
$$\begin{cases} \sigma^2 G = 10^{-4}, 5.0 \times 10^{-4}, 10^{-3}, 2.0 \times 10^{-3} \\ \text{first three peaks} \end{cases}$$

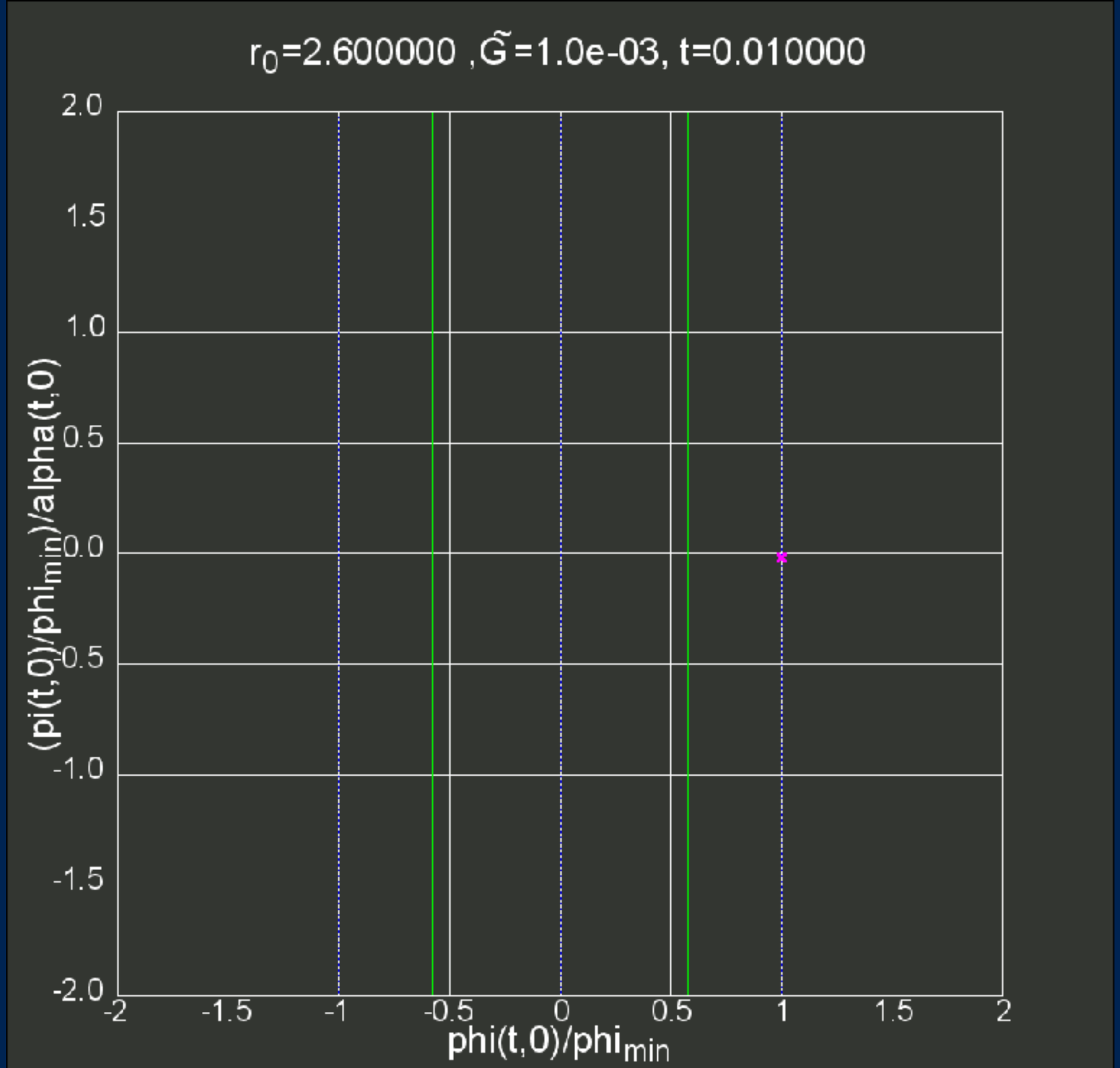
1. Typical behavior of oscillon
   - Time evolution of the oscillon
   - Time evolution of the energy
2. Properties of critical behavior
   - Time evolution around the critical point
   - Scaling behavior
   - New type (?) of the scaling behavior
   - About its critical solution

Result - 1.typical behavior of oscillon

- time evolution of oscillon
  - The scalar field oscillates many times.
  - The envelop of the scalar field at the origin modulates.



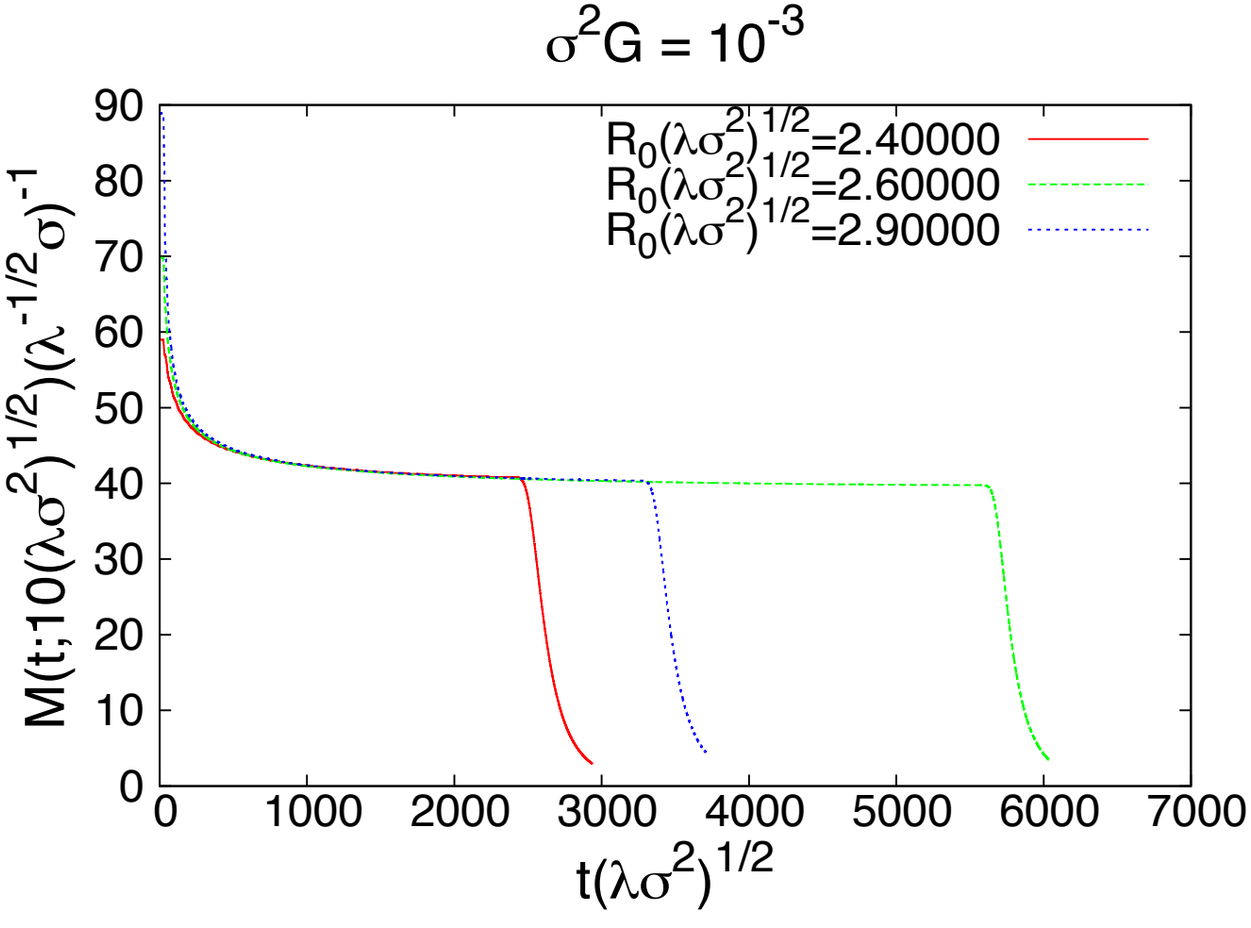



r0=2.600000 ,G̃=1.0e-03, t=0.010000
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
(pi(t,0)/phimin)/alpha(t,0)
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
phi(t,0)/phimin

Result - 1.typical behavior of oscillon

- time evolution of the energy
  - Typical oscillon's energy is universal.
  - It depends on $\sigma^2 G$.
  - Scenario
    1. Scalar wave is radiated.
    2. Oscillon phase
    3. Oscillon decays.

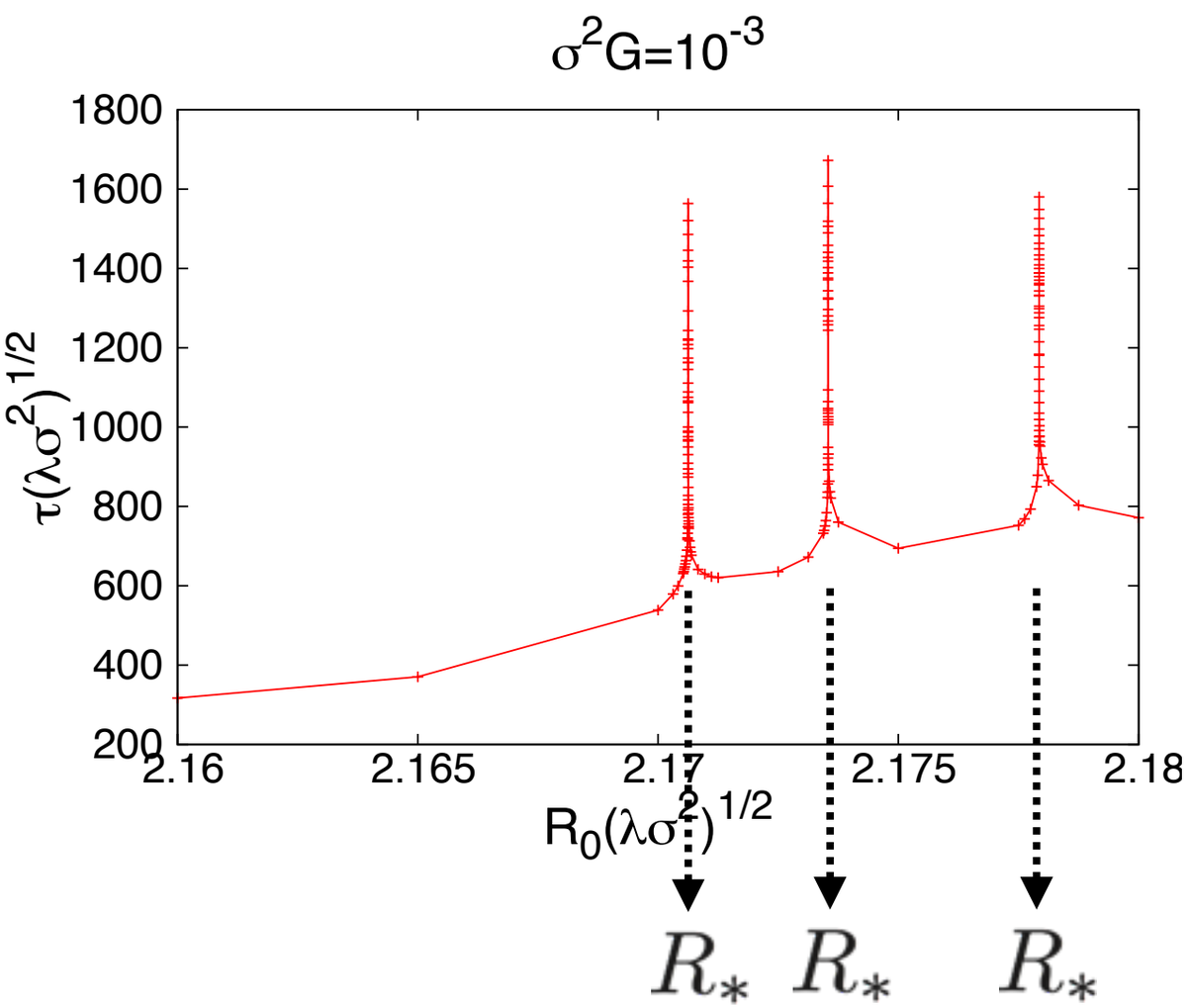
.

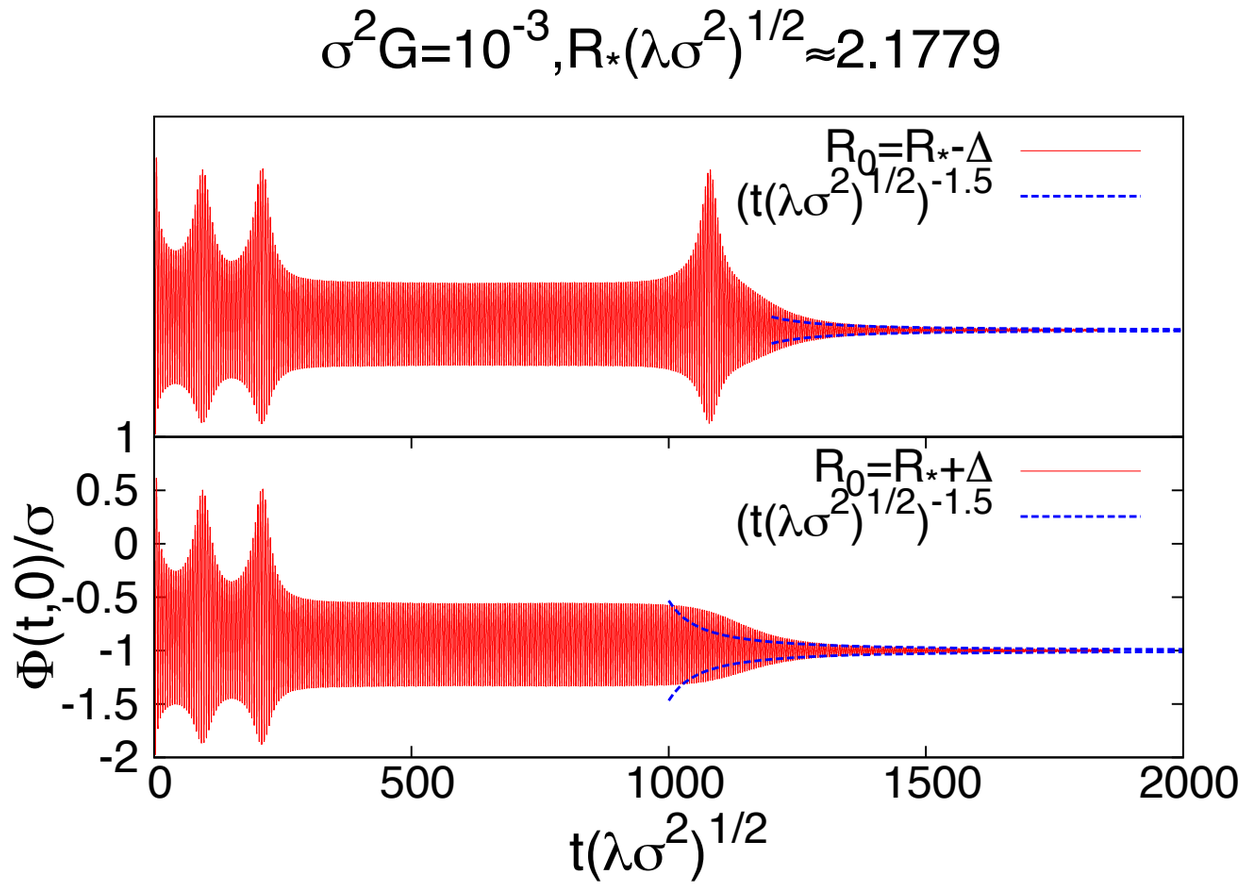


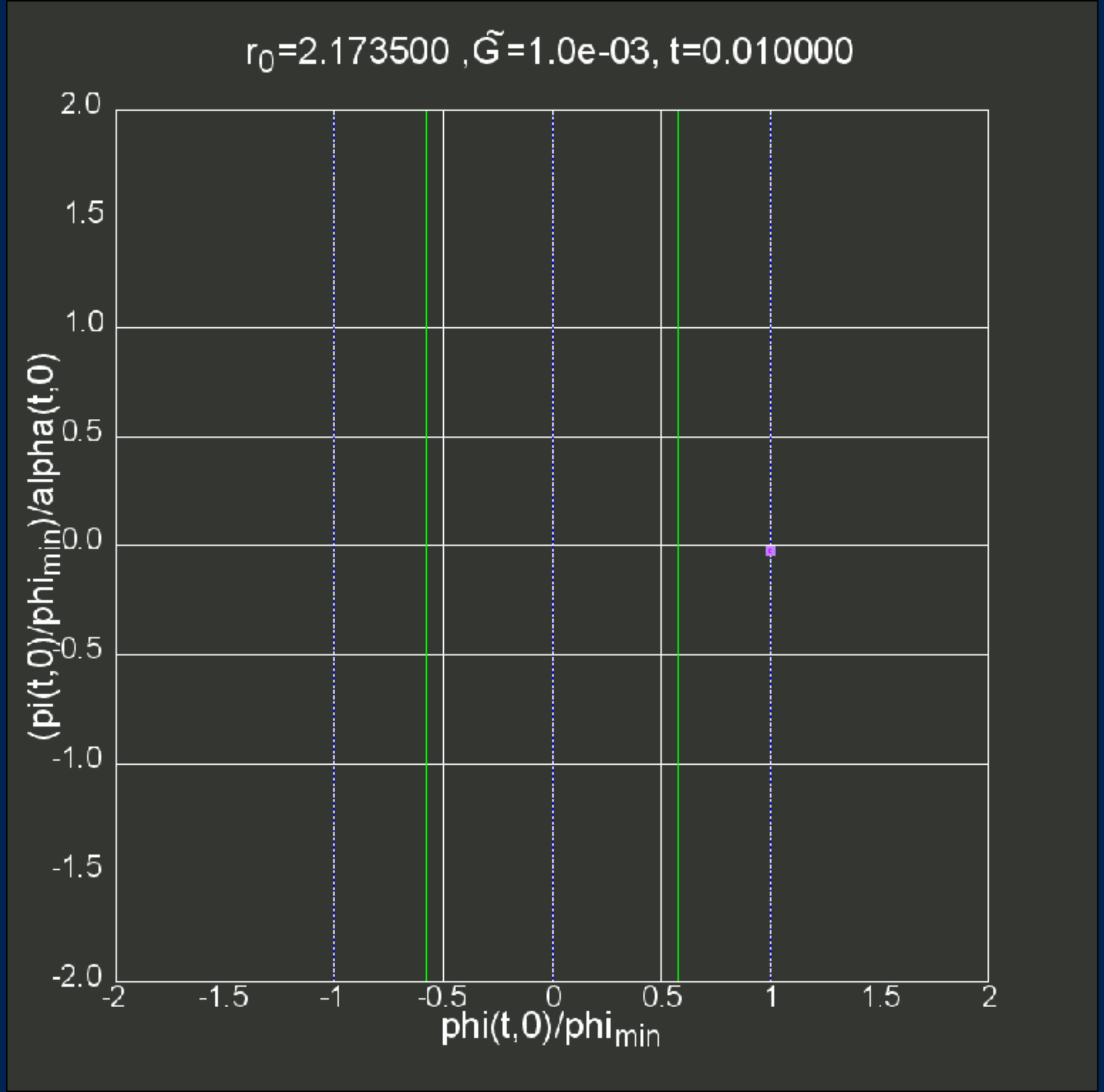
Result - 2.properties of critical behavior

- Time evolution around the critical point
  - When initial parameter is fine-tuned, the lifetime of oscillon becomes infinity. : $R_0 \to R_*$

.





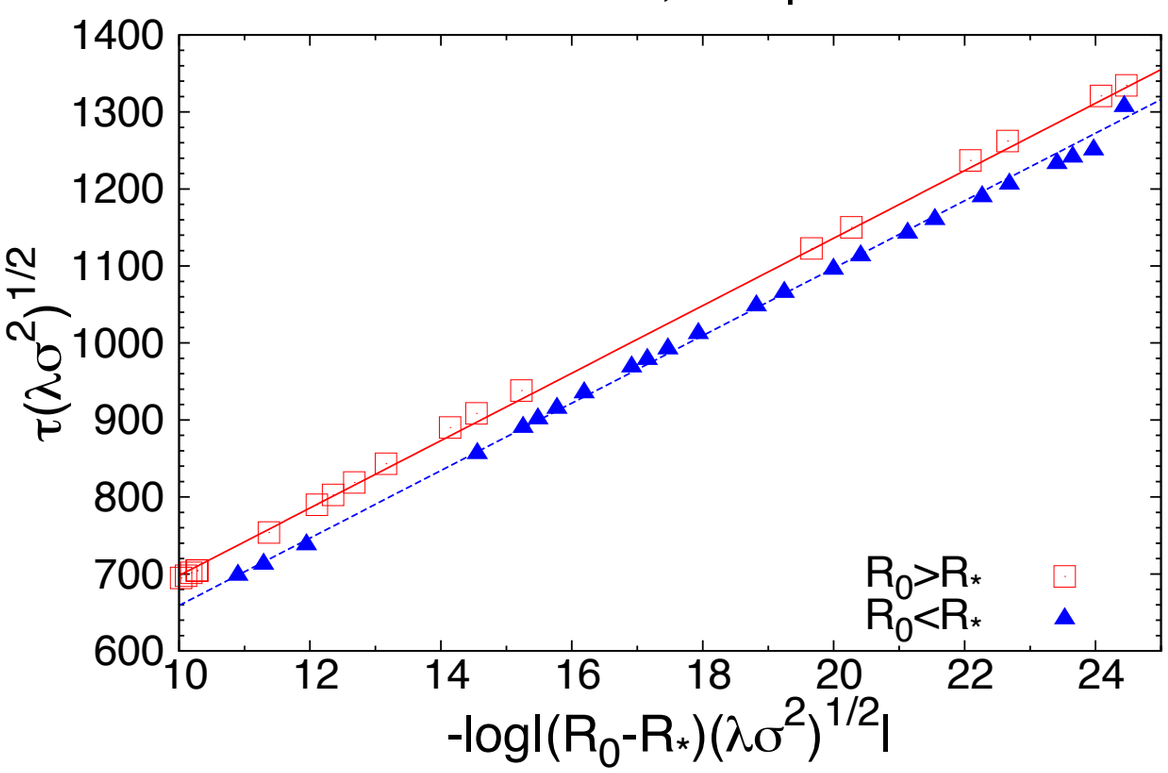

r0=2.173500 ,G̃=1.0e-03, t=0.010000
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
(pi(t,0)/phimin)/alpha(t,0)
phi(t,0)/phimin

Result - 2.properties of critical behavior

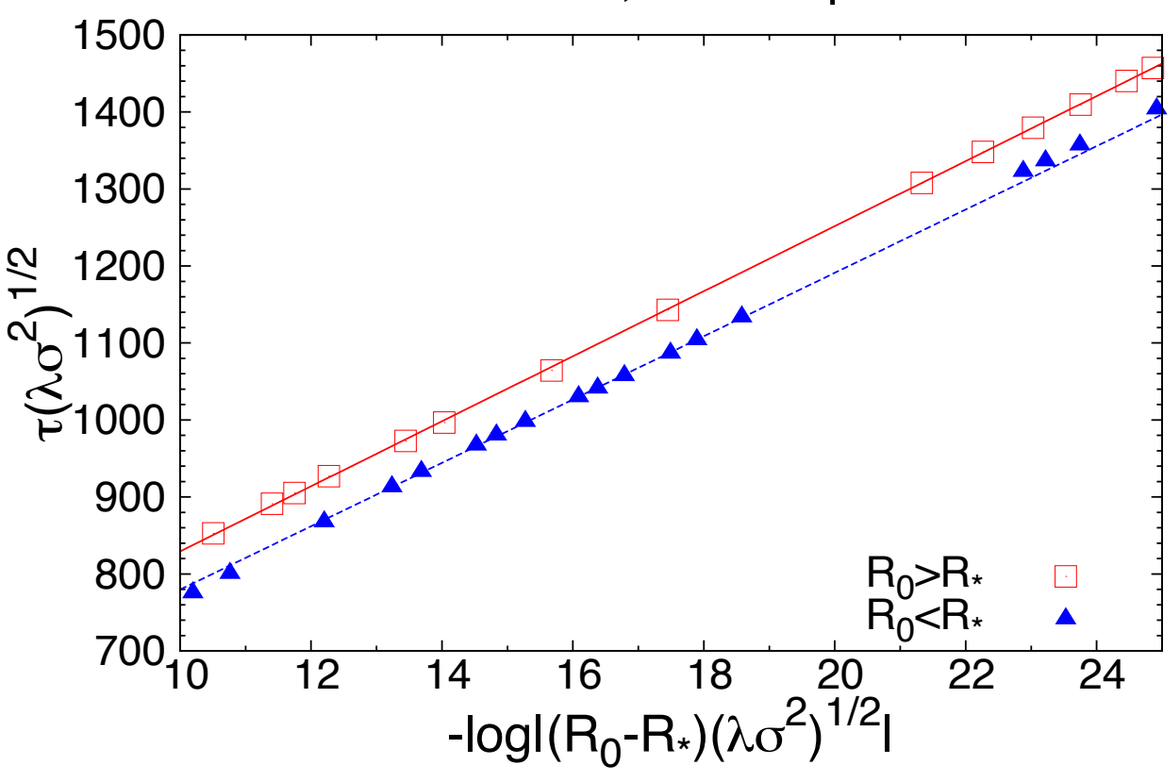
- scaling behavior
  - Around R_*, the lifetime of the oscillon obeys scaling low.

$$\tau = -\gamma \log |R_0 - R_*| + C$$

σ^2G=10^{-3}, first peak



σ^2G=10^{-3}, second peak

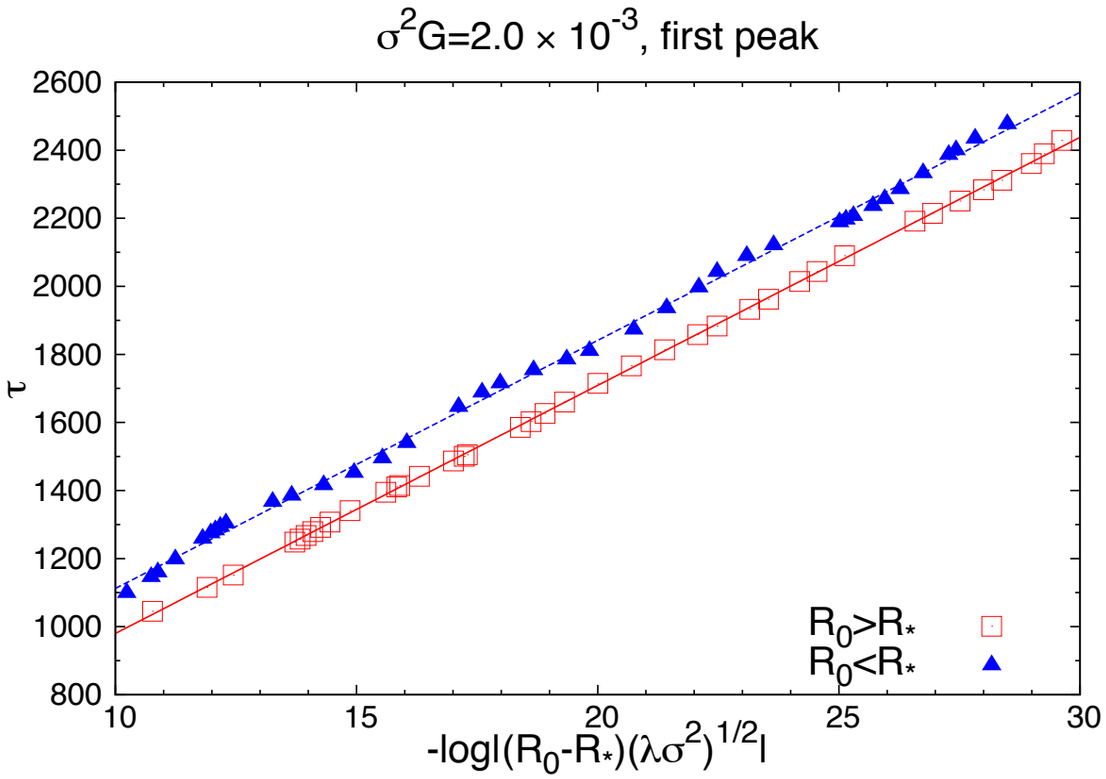


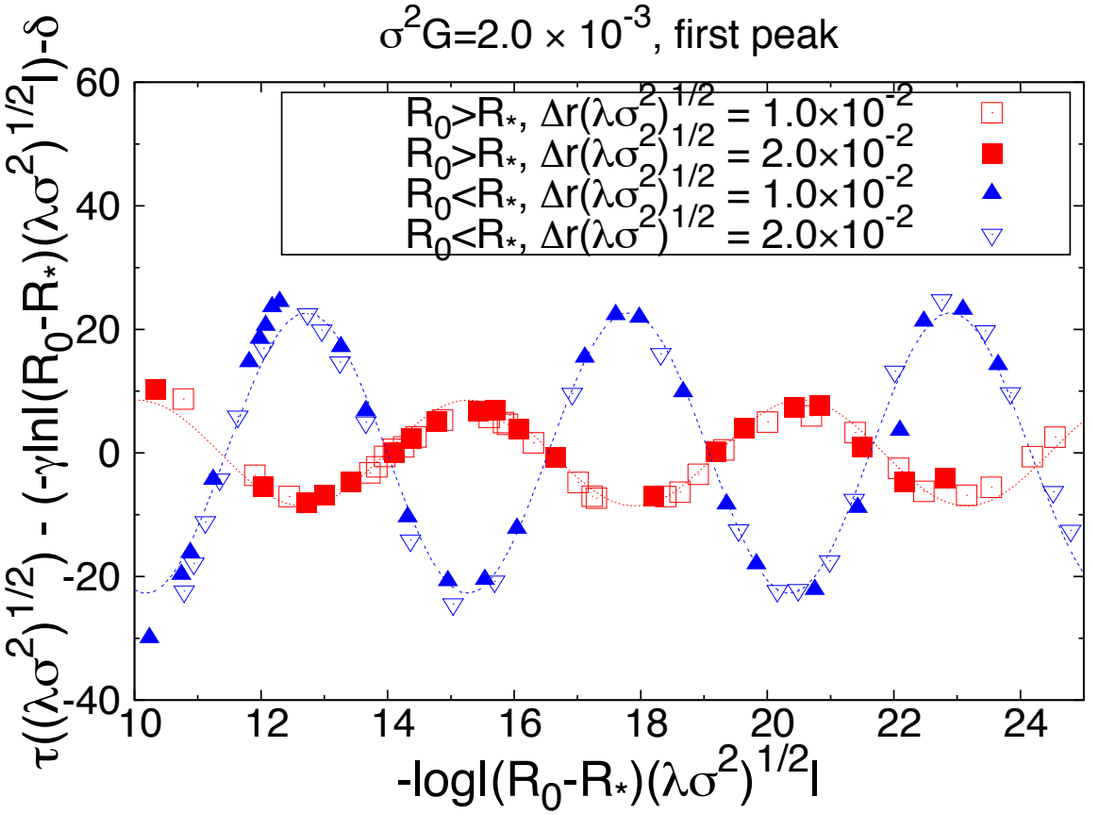
Result - 2.properties of critical behavior

- new type (?) of the scaling behavior
  - For $\sigma^2 G = 2.0 \times 10^{-3}$, new type critical behavior appears ?

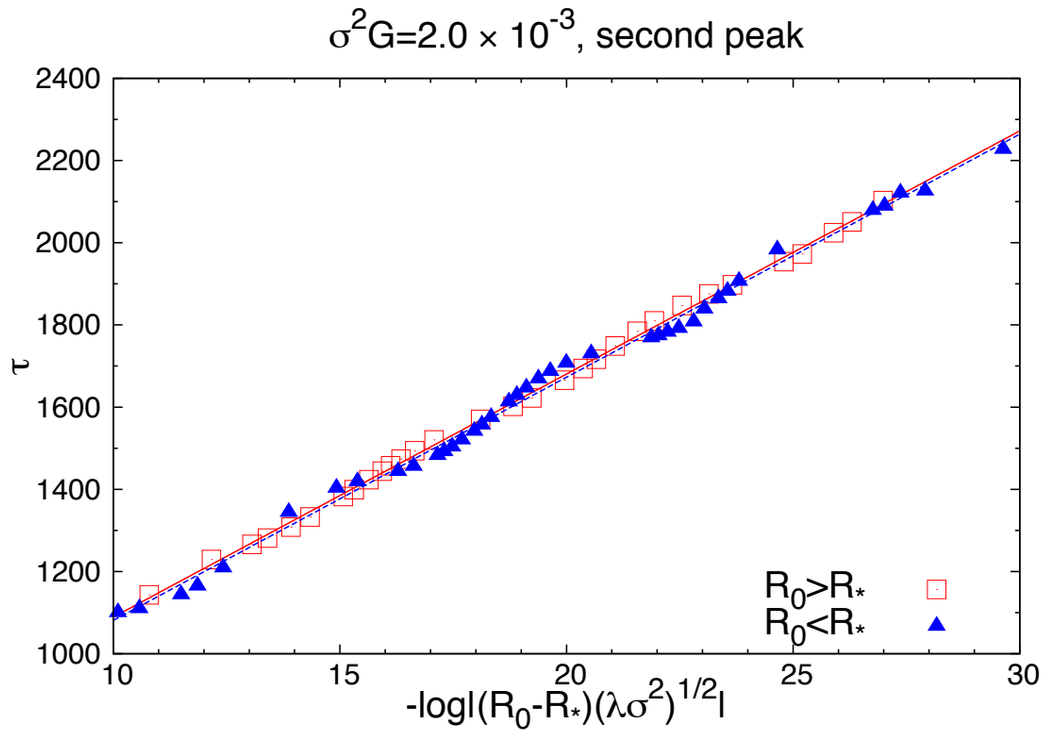
$$\tau = -\gamma \log |R_0 - R_*| + f(-\log |R_0 - R_*|) + C$$

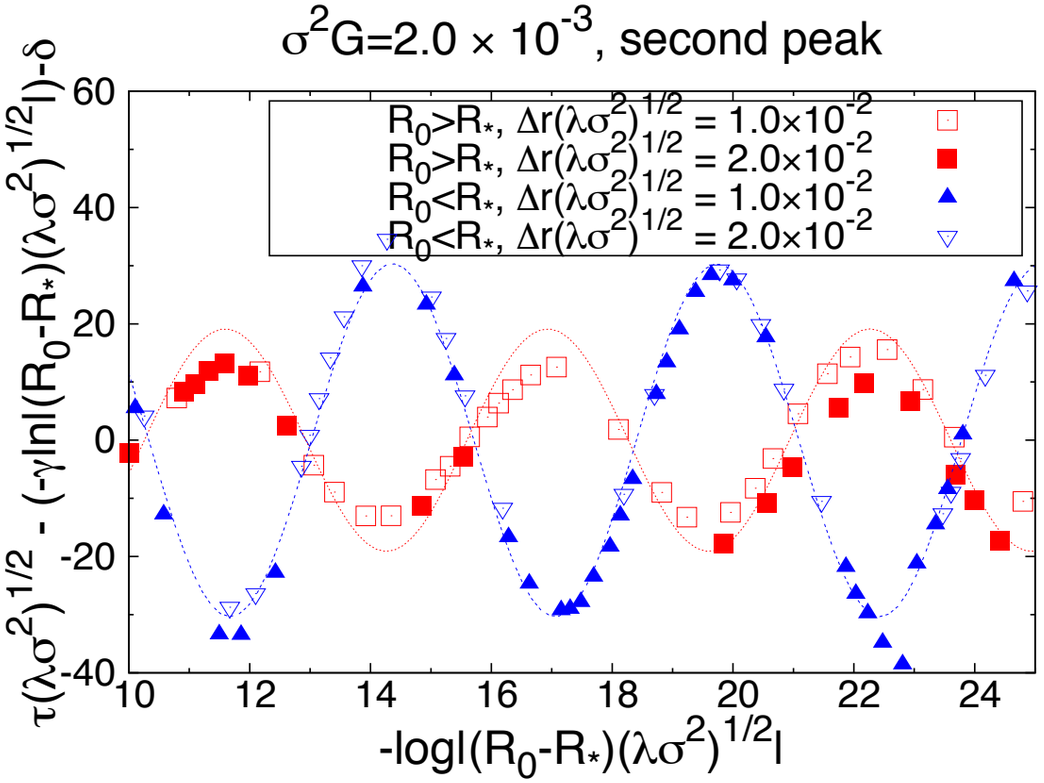
$f(x) = f(x + \varpi)$: periodic function

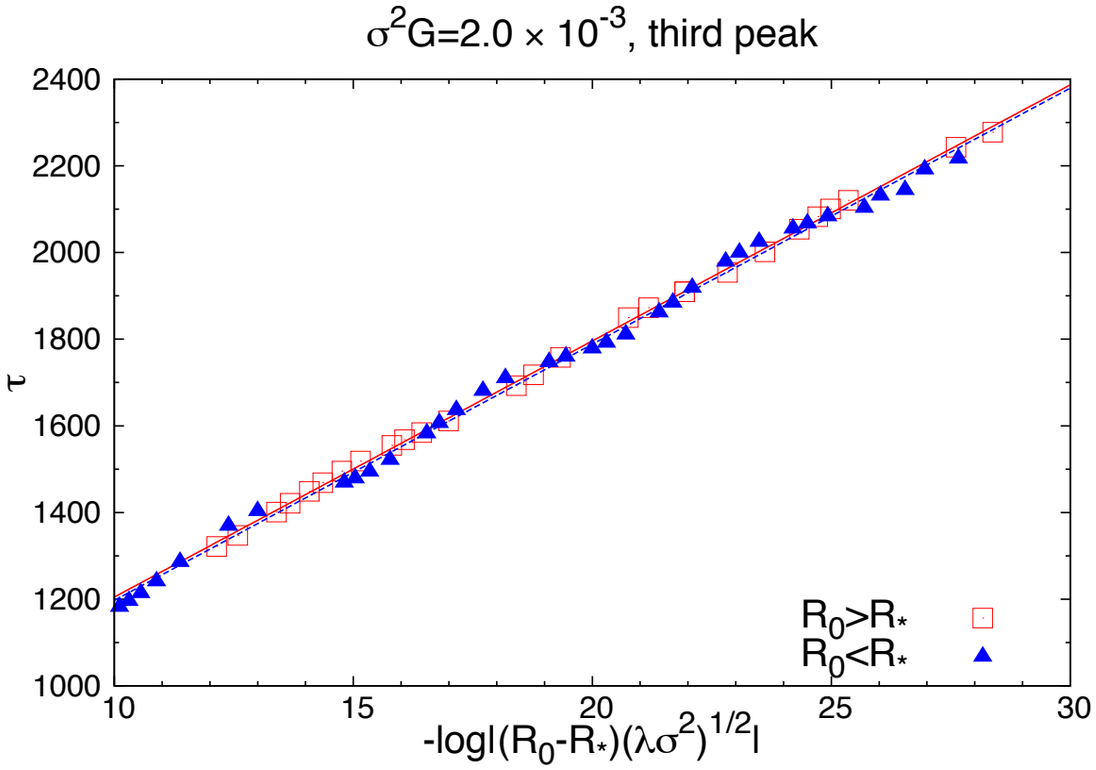


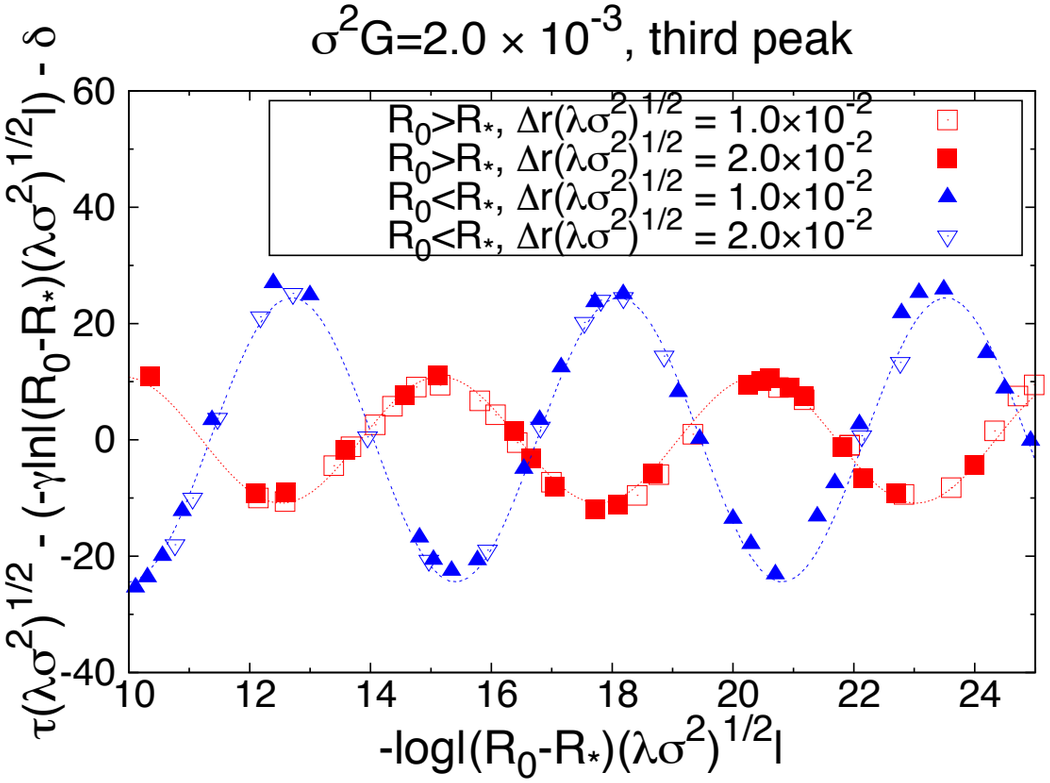


Result - 2.properties of critical behavior



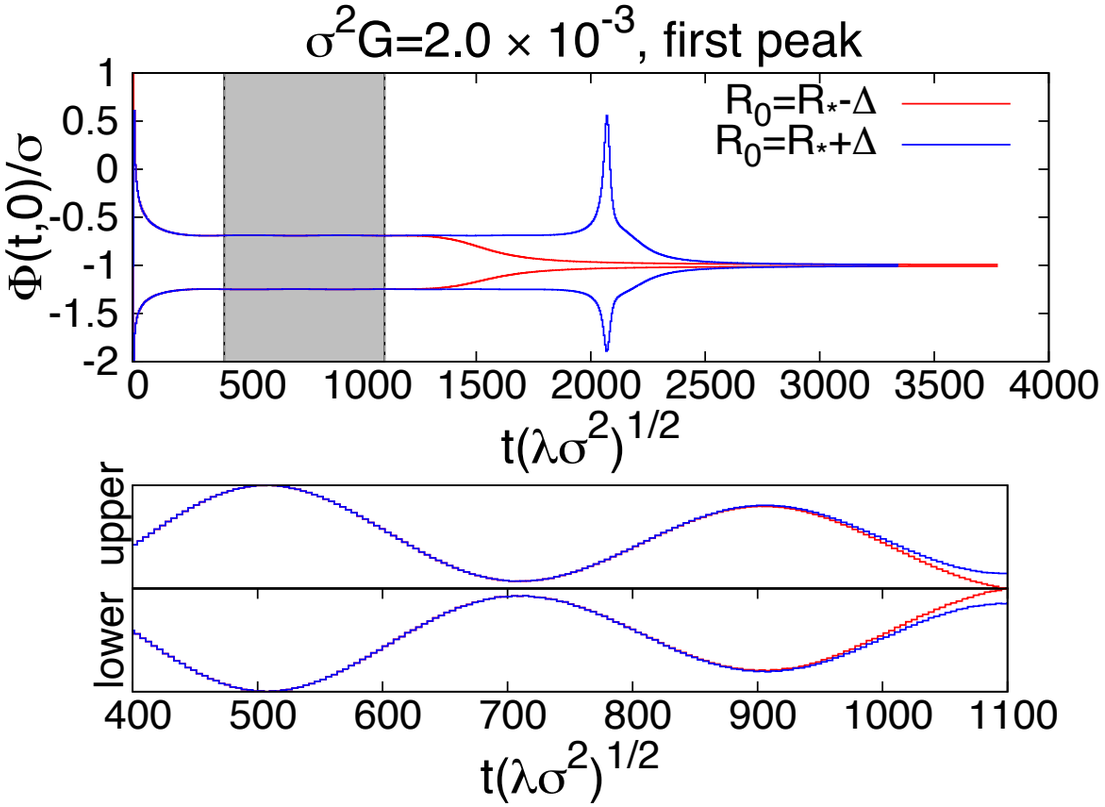


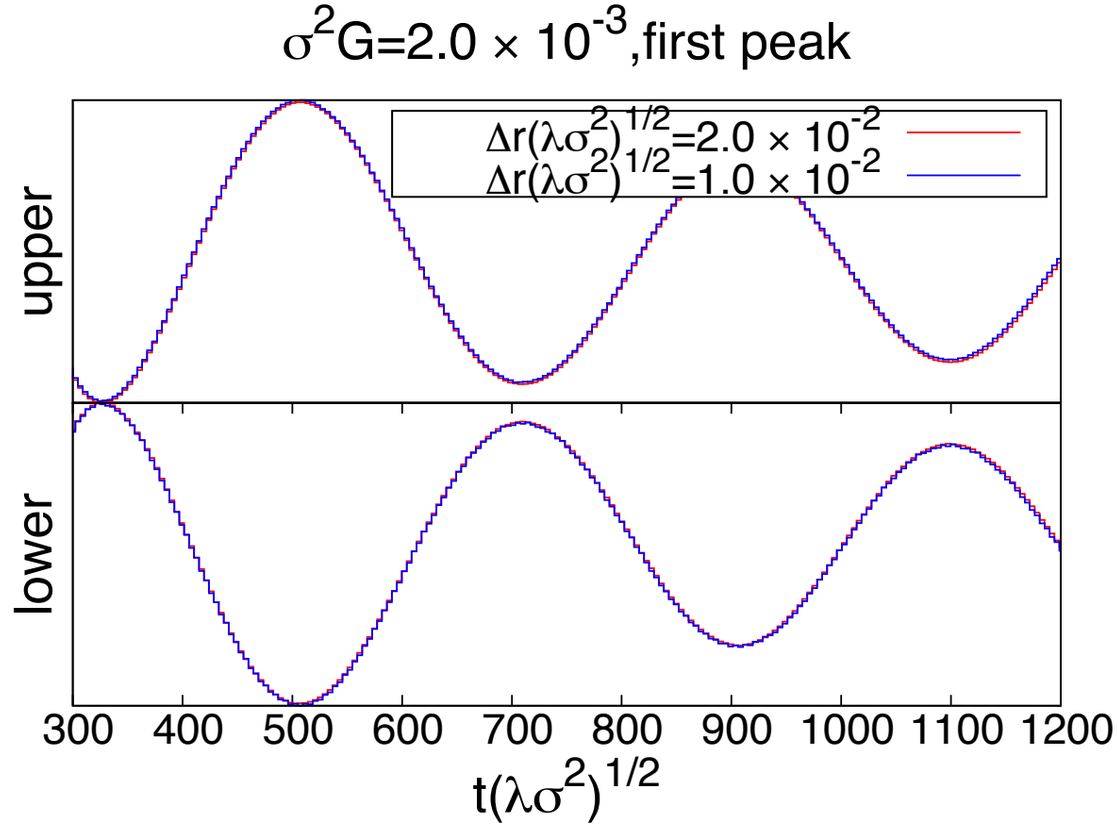




Result - 2.properties of critical behavior

- about its critical solution
  - For $\sigma^2 G = 2.0 \times 10^{-3}$, the envelop of the critical solution oscillates.

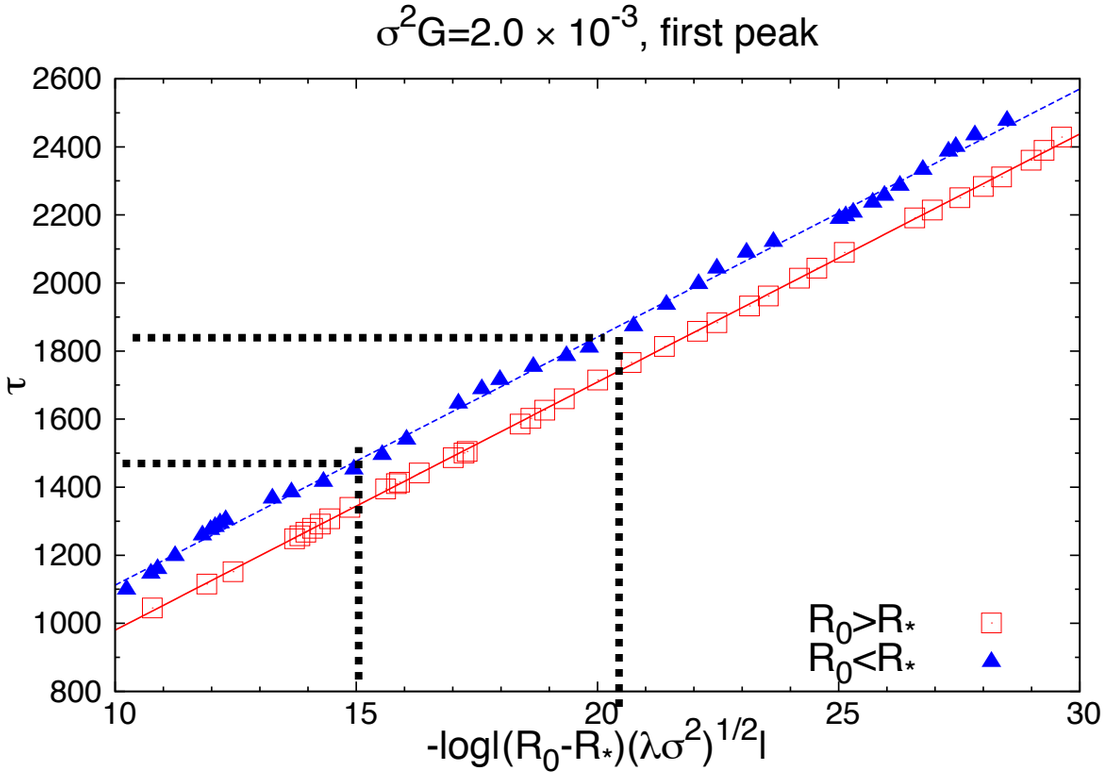


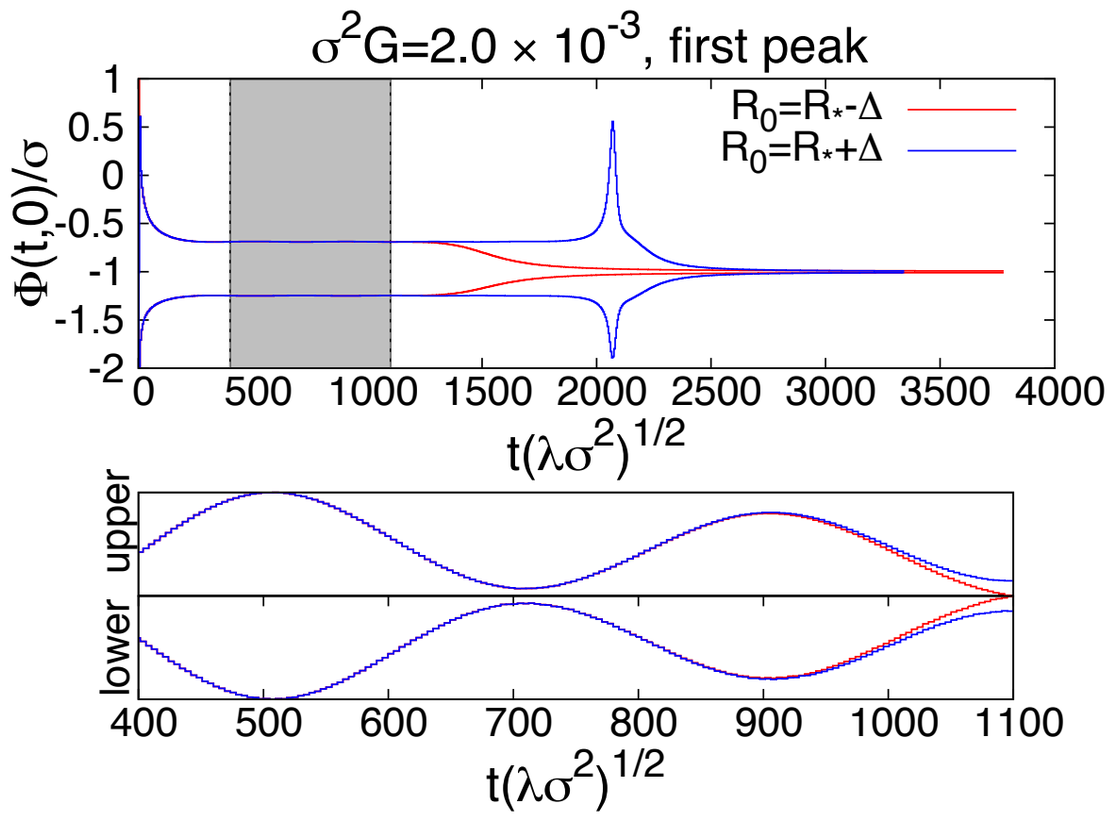


convergence

Result - 2.properties of critical behavior

- relation between two oscillations





- Oscillon's lifetime reflects to the feature of the critical solution.

Summary

- We focus on the GR + scalar field with double well potential.
- We examined the oscillon with gravity.
  - typical behavior of the oscillon with gravity.
  - critical behavior

Message

gravity

potential

★Critical behavior of gravitational collapse

★“New” type I critical behavior？(多分)

★Critical behavior of oscillon’s lifetime

Future work

- About oscillon
  - Can oscillons collapse to BH ?

- About critical behavior
  - Is new type (?) of critical behavior universal ?
  - How do the behavior change when the coupling is large ?

End

Back up