Induced gravitational waves

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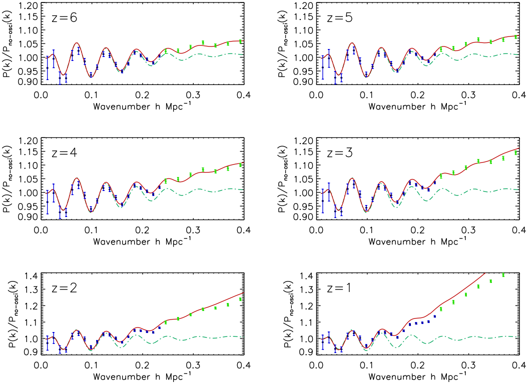
Based on

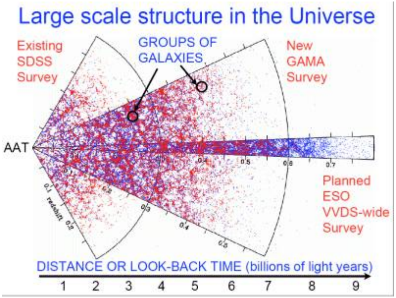
- JG, J.-c. Hwang, H. Noh and J. Yoo, arXiv:1706.07753 [gr-qc]
- JG, J.-c. Hwang, H. Noh and J. Yoo, to appear

Outline

Why non-linear GR perturbation theory?

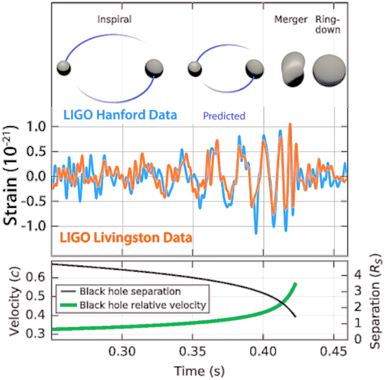
- All observable structure from perturbations
- Technical developments, e.g. search for nG in CMB
- Planned galaxy surveys cover huge volume
- GR (or any modification) becomes relevant





Why gravitational waves?

- A new window to the universe only through GR
- Detected by LIGO (GW150924): binary BHs with $\sim 30 M_\odot$
- Can affect structure formation

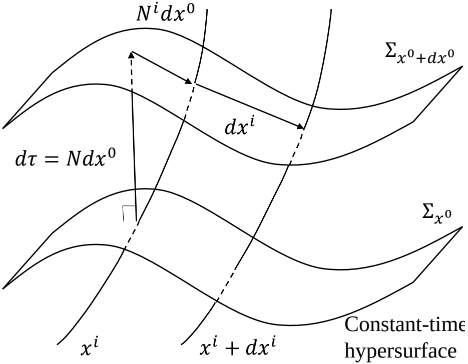


We need to understand GWs more exactly

ADM formulation and cosmological perturbations

$$ds^2 = -N^2\left(dx^0\right)^2 + h_{ij}\left(N^i dx^0 + dx^i\right)\left(N^j dx^0 + dx^j\right)$$

$$= -a^2(1+2\alpha)d\eta^2 - 2a\chi_i d\eta dx^i + a^2\left[(1+2\varphi)\delta_{ij} + 2\gamma_{,ij} + 2h^{(v)}_{(i,j)} + 2C_{ij}\right]dx^i dx^j$$



Finding the *exact* inverse metric h^{ij} with $h^{ik}h_{kj} = \delta^i{}_j$ is essential (e.g. $g^{0i} = h^{ij}N_j/N^2$)

Exact equations for tensor perturbations

"Exact" inverse spatial metric

$$h^{ij} = \frac{\delta^{ij} + H^{ij}}{a^2(1+2\varphi)} \quad \text{where} \quad H^{ij} \equiv -2\frac{(1+2\varphi)C^{ij} - 2C^{ik}C^j{}_k}{(1+2\varphi)^2 - 2C^{kl}C_{kl}}$$

Equation of tensor perturbations (and other traceless components) are contained in the trace-free part of the equation of h_{ij}

2nd order equation for tensor perturbations

Expanding up to 2nd order [n_{ij}: 2nd order terms (Hwang & Noh 2007)]

$$\frac{1}{a^2}\left(\partial_i\partial_j - \frac{\delta_{ij}}{3}\Delta\right)\left[\frac{1}{a}\frac{d}{dt}(a\chi) - \alpha - \varphi - 8\pi G\Pi\right] + \frac{1}{a}\left[\frac{1}{a^2}\frac{d}{dt}\left(a\chi^{(v)}_{(i,j)}\right) - 8\pi G\Pi^{(v)}_{(i,j)}\right]$$
$$+ \ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{\Delta}{a^2}h_{ij} - 8\pi G\Pi^{(t)}_{ij} = n_{ij}$$

Applying the projection operator ${P_{ij}}^{kl}$ singles out only tensor parts:

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} + \frac{\Delta}{a^2}h_{ij} - 8\pi G\Pi^{(t)}_{ij} = s_{ij}$$
$$s_{ij} \equiv {P_{ij}}^{kl} n_{kl}$$
$$= n_{ij} + \frac{1}{2}\left(\partial_i\partial_j - \delta_{ij}\right){n^k}_k - 2\Delta^{-1}\partial_{(i}{n^k}_{j),k} + \frac{1}{2}\Delta^{-2}\left(\partial_i\partial_j + \delta_{ij}\Delta\right){n^{kl}}_{,kl}$$

Secondary GWs from scalar perturbations

Induced GWs from scalar perturbations in previous studies

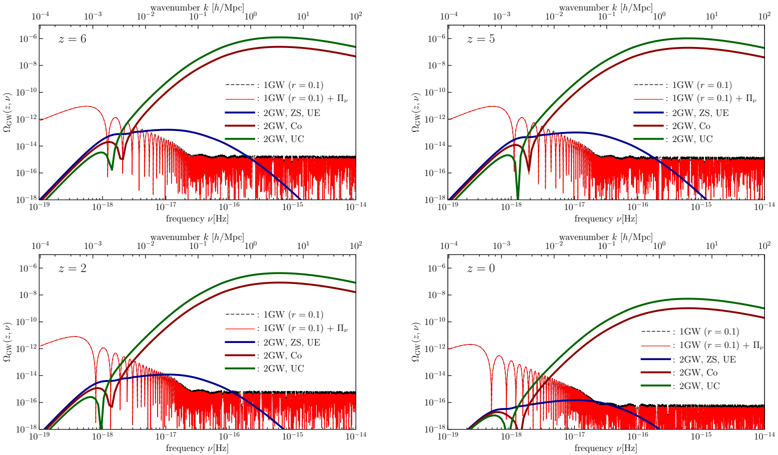
$$
\begin{aligned}
n_{ij} = & \frac{1}{a^3}\frac{d}{dt}\left[a\left(2\varphi\chi_{,ij}+2\varphi_{,(i}\chi_{,j)}\right)\right] + \frac{1}{a^2}\left(\kappa\chi_{,ij}-4\varphi\varphi_{,ij}-3\varphi_{,i}\varphi_{,j}\right) + \frac{1}{a^4}\chi^{,k}{}_{,i}\chi_{,jk} \\
& + \frac{1}{a^2}\left[2\dot{\chi}_{,ij}\alpha - H\chi_{,ij}\alpha + \chi_{,ij}\dot{\alpha} - 2(\alpha+\varphi)\alpha_{,ij} - \alpha_{,i}\alpha_{,j} - 2\alpha_{,(i}\varphi_{,j)}\right] + 8\pi G(\mu+p)v_{,i}v_{,j} \\
& - \frac{\delta_{ij}}{3}\left\{\frac{1}{a^3}\frac{d}{dt}\left[a\left(2\varphi\Delta\chi+2\varphi^{,k}\chi_{,k}\right)\right] + \frac{1}{a^2}\left(\kappa\Delta\chi-4\varphi\Delta\varphi-3\varphi^{,k}\varphi_{,k}\right) + \frac{1}{a^4}\chi^{,kl}\chi_{,kl}\right. \\
& \left. + \frac{1}{a^2}\left[2\alpha\Delta\dot{\chi} - H\alpha\Delta\chi + \dot{\alpha}\Delta\chi - 2(\alpha+\varphi)\Delta\alpha - \alpha^{,k}\alpha_{,k} - 2\alpha^{,k}\varphi_{,k}\right] + 8\pi G(\mu+p)v^{,k}v_{,k}\right\}
\end{aligned}
$$

- Generation in matter-dominated epoch
- Newtonian gauge results (Mollerach et al. 2004, Baumann et al. 2007, Ananda et al. 2007...)
- Extended to gauge-ready form only recently (Hwang, Jeong & Noh 2017)

Secondary GWs on small scales

Induced GWs are not negligible but even dominant on small scales

(Hwang, Jeong & Noh 2017)



Contributions to secondary GWs

Scalar perturbations are not the only source for the induced GWs

1. Scalar-scalar: it's done
2. Scalar-vector: $n_{ij}^{(sv)} = \dfrac{1}{a^3}\dfrac{d}{dt}\left[a\left(\chi_{(i,j)}\alpha + 2\varphi\chi_{(i,j)} + \varphi_{,j}\chi_i + \varphi_{,i}\chi_j - \varphi_{,k}\chi^k\delta_{ij}\right)\right] + \cdots$
3. Scalar-tensor:
   $n_{ij}^{(st)} = \dfrac{1}{a^3}\dfrac{d}{dt}\left\{a^3\left[\dot{C}_{ij}\alpha + 2\left(\left(\varphi C_{ij}\right)^{\cdot} + \dfrac{1}{a^2}C_i^k\chi_{,jk}\right) + \dfrac{\chi^{,k}}{a^2}\left(C_{ik,j} + C_{jk,i} - C_{ij,k}\right)\right]\right\} + \cdots$
4. Vector-vector: $n_{ij}^{(vv)} = -\dfrac{1}{a^4}\chi^k\chi_{(i,j)k} + \dfrac{1}{a^4}\left(\chi^k\chi_{k,ij} + \chi^k{}_{,i}\chi_{k,j}\right) + \cdots$
5. Vector-tensor: $n_{ij}^{(vt)} = \dfrac{1}{a^3}\dfrac{d}{dt}\left\{a\left[2h_i{}^k\chi_{(j,k)} + \chi^k\left(h_{ik,j} + h_{jk,i} - h_{ij,k}\right)\right]\right\} + \cdots$
6. Tensor-tensor: $n_{ij}^{(tt)} = \dfrac{2}{a^3}\dfrac{d}{dt}\left[a^3\left(h_i{}^k\dot{h}_{jk}\right)\right] - \dfrac{\delta_{ij}}{3}\dfrac{2}{a^3}\dfrac{d}{dt}\left[a^3\left(h^{kl}\dot{h}_{kl}\right)\right] + \cdots$

Simplifications for secondary GWs

There are several simplifications

- Without (vector type) anisotropic stress, $\chi_i = 0$: no vector
- In MD, scalar all i.t.o. initial curvature perturbation
- Evolution of (linear) GWs during MD is well-known

Induced GWs from (tt)-(tt), (ss)-(tt) and (st)-(st)

All the necessary pieces are already there!

(and we are working hard...)

Conclusions

- Need to understand cosmological perturbations more exactly
- Non-linear regime / GR context
  - Structure formation involves non-linear evolution
  - Galaxy surveys probing huge volume
- Exact non-linear equations are presented
  - From exact inverse spatial metric & ADM eqs
  - Can be applied to extreme situations
- Induced gravitational waves
  - Sourced not only by scalar-scalar, but also by tensor-tensor, scalar-tensor...
  - May dominant on small scales – report under progress