TaylorF2 CBC waveform with eccentricity corrected phase

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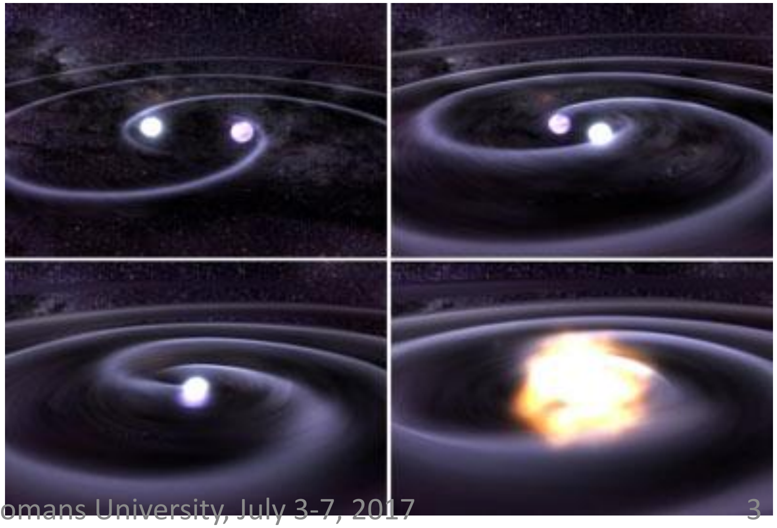
Motivation

- Compact binary inspirals are an important gravitational wave source for detectors (LIGO, Virgo and KAGRA).
- So far, circular binary orbit has been taken into account in GW data analysis.
- In this work, we perform GW parameter estimation for NS-NS binary inspirals in “eccentric” orbits.

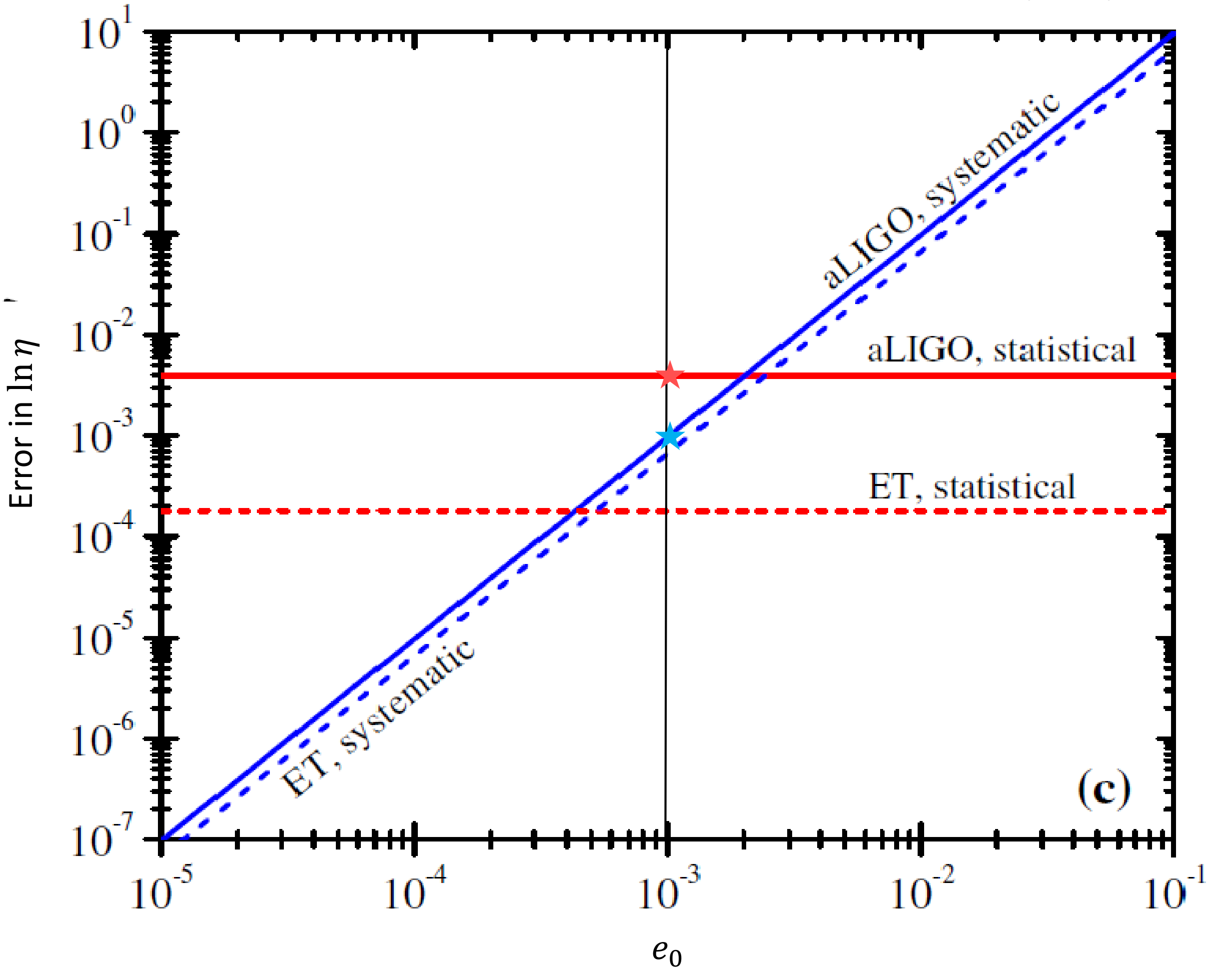


Motivation:

- Simplest next step beyond widely-used circular PN approximants.
- Produce waveforms that are structurally simple with nearly the same computational cost of circular, non-precessing waveforms.
- Fully PN-consistent in the small-eccentricity limit and accurate to 3PN order.
- Small (rather than large) eccentricity is (arguably) the astrophysically relevant regime to consider.
- Small eccentricities 0.001 to 0.01 can potentially bias mass parameters [MF PRL’14].
- Perform MCMC studies to determine eccentricity-induced bias; eventually add eccentricity as a template parameter.

If eccentricity = 0.001,
aLIGO statistical error is eta is 0.004,
aLIGO systematic error is expected to be 0.001

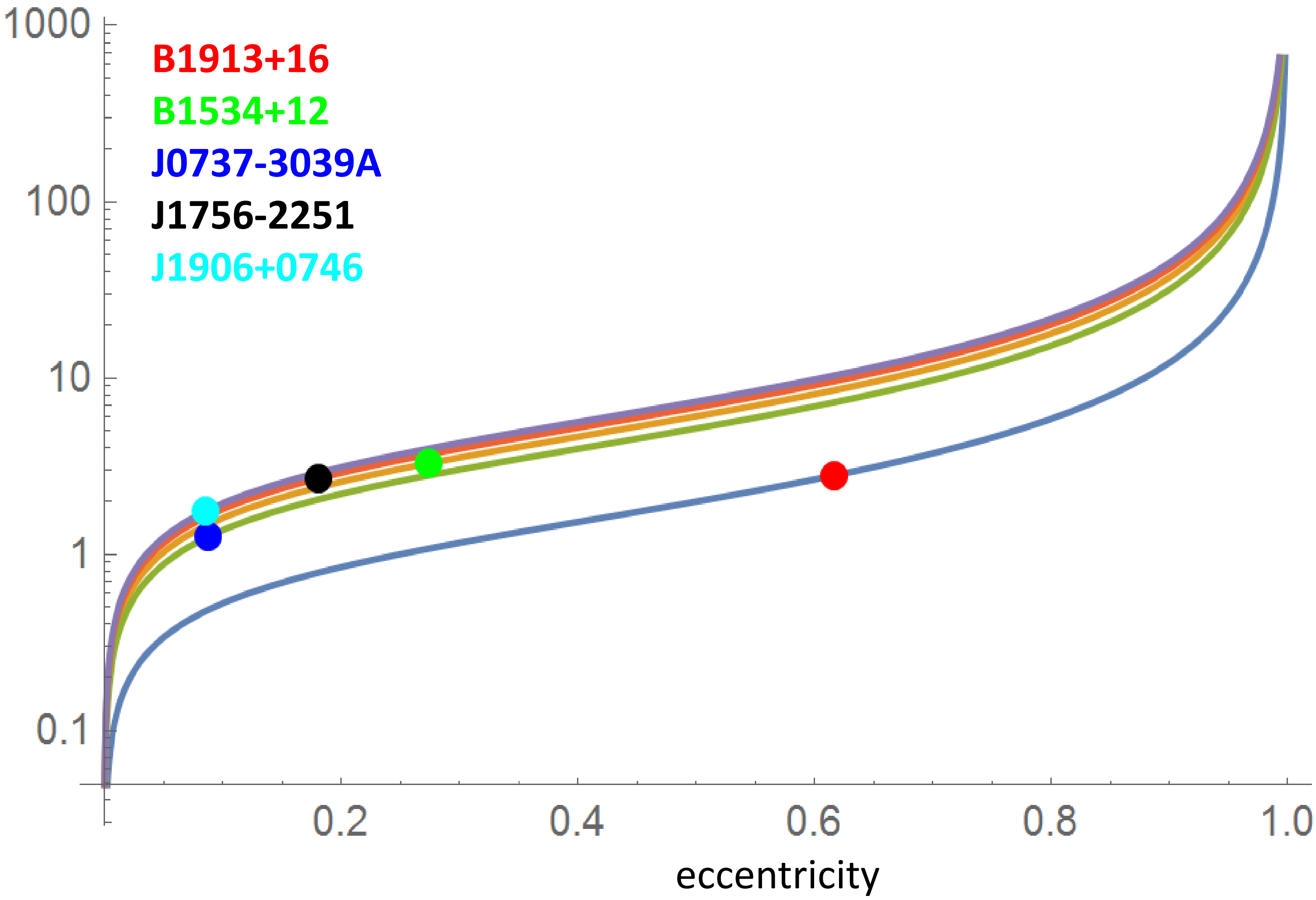
Marc Favata PRL(2014)

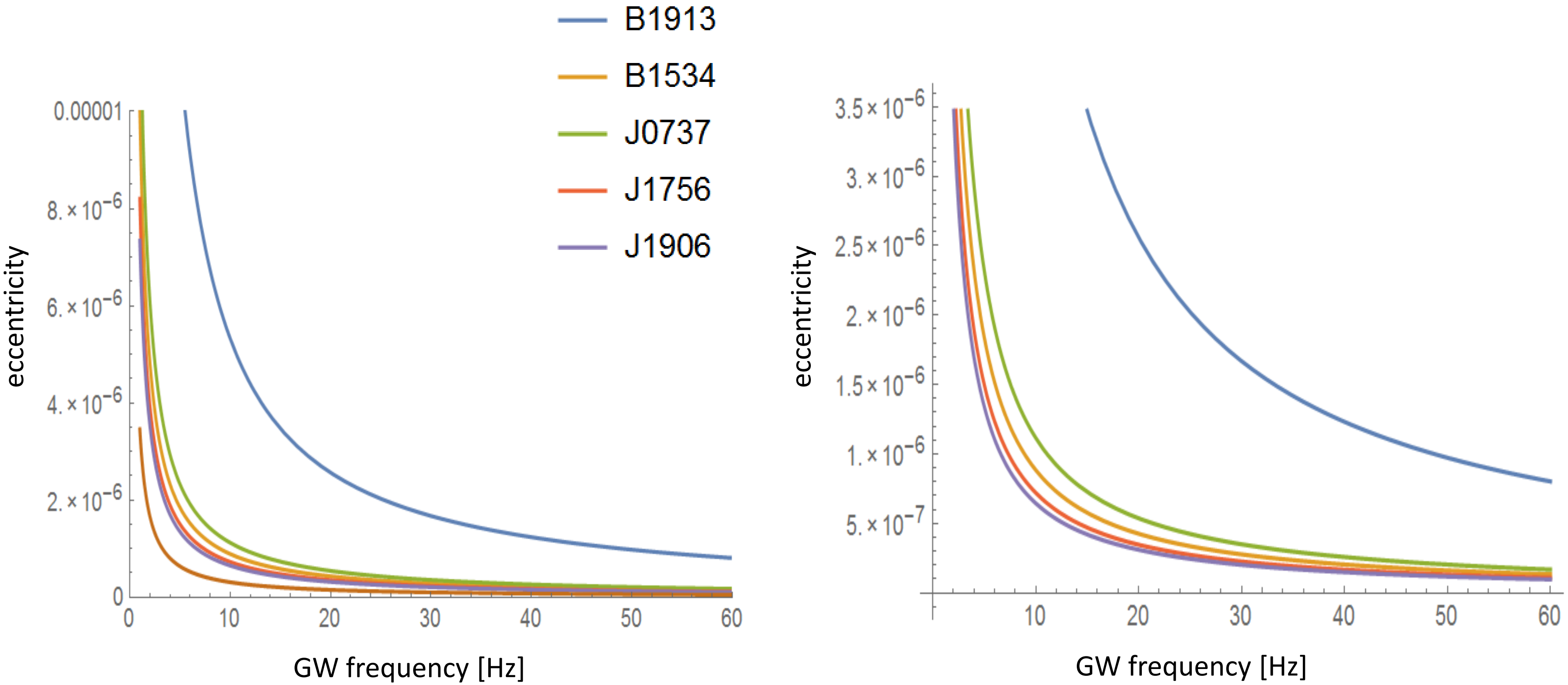


eta (η) : symmetric mass ratio

$$\eta = \frac{m_1 * m_2}{(m_1 + m_2)^2}$$

Orbital evolution of known NS-NS merger (following Peters 1964)




B1913
B1534
J0737
J1756
J1906
eccentricity
0.00001
$8. \times 10^{-6}$
$6. \times 10^{-6}$
$4. \times 10^{-6}$
$2. \times 10^{-6}$
0
0
10
20
30
40
50
60
GW frequency [Hz]
eccentricity
3.5×10^{-6}
$3. \times 10^{-6}$
2.5×10^{-6}
$2. \times 10^{-6}$
1.5×10^{-6}
$1. \times 10^{-6}$
$5. \times 10^{-7}$
10
20
30
40
50
60
GW frequency [Hz]

What we did:

- Computed leading-order eccentric correction [O(e_0^2)] to the *secular* phasing of PN approximants: TaylorT1, T2, T3, T4, F2/SPA.
- Evaluated relative importance of new PN terms.
- Evaluated role of periodic terms that also affect the phasing. (Not yet included in PN approximants; simple to do for time-domain approximants; working out details for frequency-domain.)
- Several other useful results for general eccentric waveforms.
- Modified codes to incorporate corrections to TaylorF2 [esp. LALSimInspiralTaylorF2, LALSimInspiralPNCoefficients]. Added parameters ecc, f_ecc, eccOrder.
- Modified LALInference routines to allow injected eccentric signal [e.g., LALInferenceReadData, LALInferenceTemplate]. Details at: https://dcc.ligo.org/LIGO-G1600512

Approximations:

- Derived from fully consistent PN formalism valid for any eccentricity (Damour, Gopakumar, & Iyer + Koningsdorferr & Gopakumar and Arun, Blanchet, Iyer, & Sinha.)

- Waveform amplitude is Newtonian-order and *circular*. (We derive and then ignore O(e_0) and higher corrections to amplitude; phase corrections are more important.) Only dominant harmonic is considered (this is appropriate for low-eccentricity).

$$h_+ = -\frac{\eta M v^2}{D}\Bigg((1+\cos^2\iota)\Bigg\{\left[2+3e_t\cos l+(4\cos 2l-1)e_t^2+(43\cos 3l-19\cos l)\frac{e_t^3}{8}\right]\cos 2\phi + \left[2e_t\sin l+3e_t^2\sin 2l+(17\sin 3l-7\sin l)\frac{e_t^3}{4}\right]\sin 2\phi\Bigg\} - \sin^2\iota\left[e_t\cos l+e_t^2\cos 2l+(9\cos 3l-\cos l)\frac{e_t^3}{8}\right]+O(e_t^4)\Bigg)+O(v^3)$$

$$\boxed{\begin{aligned} h_+^{\rm N} &= -2\frac{\eta M}{D}v^2(1+\cos^2\iota)\cos 2\phi(t)\ , \\ h_\times^{\rm N} &= -4\frac{\eta M}{D}v^2\cos\iota\sin 2\phi(t)\ . \end{aligned}}$$

- Phase corrections contain new terms to *O*(e_0^2). These are accurate for $e_0 \lesssim 0.1$ (validated against numerical phase evolution valid for large e_0).

- For comparable-mass binaries, a comparison w/ NR and self-force calculations [Le Tiec et al, PRL'11] suggests formalism is valid for:

$$f \lesssim 2585\text{Hz}\left(\frac{1M_\odot}{M}\right)$$

Example result: TaylorT2

circular terms (3.5PN)

$$\phi = \phi_c - \frac{1}{32\eta v^5}\left\{1 + \left(\frac{3715}{1008}+\frac{55}{12}\eta\right)v^2 + \cdots + O(v^7)\right.$$

$$\left. - \frac{785}{272}e_0^2\left(\frac{v_0}{v}\right)^{19/3}\underbrace{\left[1 + \left(\frac{6955261}{2215584}+\frac{436441}{79128}\eta\right)v^2 + \left(\frac{2833}{1008}-\frac{197}{36}\eta\right)v_0^2 + \cdots O(v^6)\right]}\right\}$$

$$v = (\pi M f)^{1/3}$$

$$v_0 = (\pi M f_0)^{1/3}$$

Relative PN corrections computed to 3PN order.

Eccentricity e_0 at reference frequency f_0 (e.g., 10 Hz).

Relative "0PN" order scaling.
Different expansion structure from circular terms.

Example result: TaylorF2/SPA

$$\tilde{h}(f) = A f^{-\frac{7}{6}} e^{i\psi(f)}$$

circular 3.5PN

$$\begin{aligned}\Psi = {} & \psi_0 + 2f\pi t_c + \frac{3}{128}\frac{1}{\eta v^5}\Bigg(1 + \left(\frac{3715}{756} + \frac{55}{9}\eta\right)v^2 - 16\pi v^3 + \left(\frac{15293365}{508032} + \frac{27145}{504}\eta + \frac{3085}{72}\eta^2\right)v^4 \\ & + \left\{\left[1 + \ln(v^3)\right]\left(\frac{38645}{756} - \frac{65}{9}\eta\right)\right\}\pi v^5 + \Bigg[\frac{11583231236531}{4694215680} - \frac{6848}{21}\gamma_E - \frac{640}{3}\pi^2 + \Bigg(-\frac{15737765635}{3048192} \\ & + \frac{2255}{12}\pi^2\Bigg)\eta + \frac{76055}{1728}\eta^2 - \frac{127825}{1296}\eta^3 - \frac{3424}{21}\ln(16v^2)\Bigg]v^6 + \left(\frac{77096675}{254016} + \frac{378515}{1512}\eta - \frac{74045}{756}\eta^2\right)\pi v^7\end{aligned}$$

eccentric 3PN

$$\begin{aligned} & - \frac{2355}{1462}e_0^2\left(\frac{v_0}{v}\right)^{19/3}\Bigg\{1 + \left(\frac{299076223}{81976608} + \frac{18766963}{2927736}\eta\right)v^2 + \left(\frac{2833}{1008} - \frac{197}{36}\eta\right)v_0^2 - \frac{2819123}{282600}\pi v^3 + \frac{377}{72}\pi v_0^3 \\ & + \left(\frac{16237683263}{3330429696} + \frac{24133060753}{971375328}\eta + \frac{1562608261}{69383952}\eta^2\right)v^4 + \left(\frac{847282939759}{82632420864} - \frac{718901219}{368894736}\eta - \frac{3697091711}{105398496}\eta^2\right)v^2 v_0^2 \\ & + \left(-\frac{1193251}{3048192} - \frac{66317}{9072}\eta + \frac{18155}{1296}\eta^2\right)v_0^4 - \left(\frac{2831492681}{118395270} + \frac{11552066831}{270617760}\eta\right)\pi v^5 + \Bigg(-\frac{7986575459}{284860800} \\ & + \frac{555367231}{10173600}\eta\Bigg)\pi v^3 v_0^2 + \left(\frac{112751736071}{5902315776} + \frac{7075145051}{210796992}\eta\right)\pi v^2 v_0^3 + \left(\frac{764881}{90720} - \frac{949457}{22680}\eta\right)\pi v_0^5 \\ & + \Bigg[-\frac{43603153867072577087}{132658535116800000} + \frac{536803271}{19782000}\gamma_E + \frac{15722503703}{325555200}\pi^2 + \left(\frac{299172861614477}{689135247360} - \frac{15075413}{1446912}\pi^2\right)\eta \\ & + \frac{3455209264991}{41019955200}\eta^2 + \frac{50612671711}{878999040}\eta^3 + \frac{3843505163}{59346000}\ln 2 - \frac{1121397129}{17584000}\ln 3 + \frac{536803271}{39564000}\ln(16v^2)\Bigg]v^6 \\ & + \left(\frac{46001356684079}{3357073133568} + \frac{253471410141755}{5874877983744}\eta - \frac{1693852244423}{23313007872}\eta^2 - \frac{307833827417}{2497822272}\eta^3\right)v^4 v_0^2 - \frac{1062809371}{20347200}\pi^2 v^3 v_0^3 \\ & + \left(-\frac{356873002170973}{249880440692736} - \frac{260399751935005}{8924301453312}\eta + \frac{150484695827}{35413894656}\eta^2 + \frac{340714213265}{3794345856}\eta^3\right)v^2 v_0^4 + \Bigg[\frac{26531900578691}{168991764480} \\ & - \frac{3317}{126}\gamma_E + \frac{122833}{10368}\pi^2 + \left(\frac{9155185261}{548674560} - \frac{3977}{1152}\pi^2\right)\eta - \frac{5732473}{1306368}\eta^2 - \frac{3090307}{139968}\eta^3 \\ & + \frac{87419}{1890}\ln 2 - \frac{26001}{560}\ln 3 - \frac{3317}{252}\ln(16v_0^2)\Bigg]v_0^6\Bigg\}\Bigg) \end{aligned} \tag{6.26}$$

Oscillatory corrections to the phase:

$$\phi = \phi_c - \frac{1}{32\eta v^5}\left\{1 + \left(\frac{3715}{1008} + \frac{55}{12}\eta\right) v^2 + \cdots + O(v^7) \right.$$

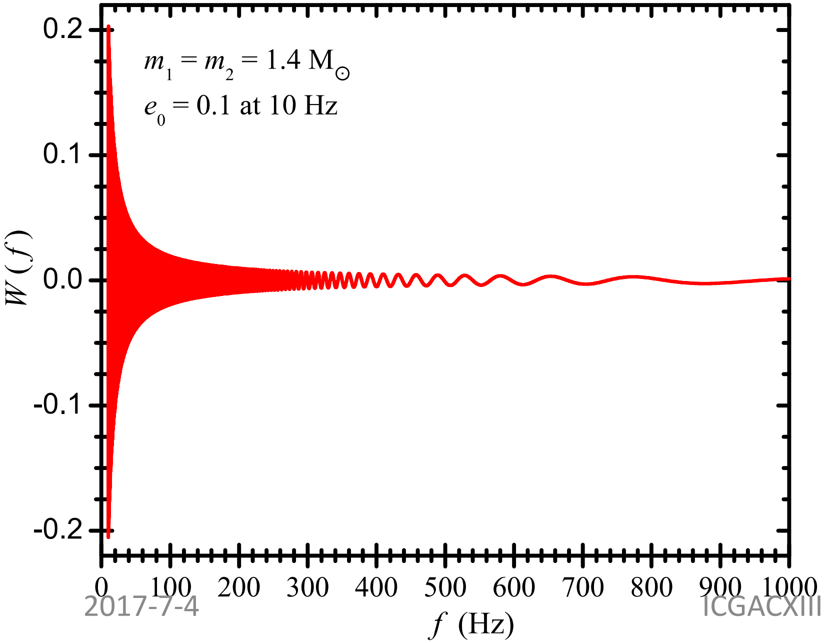
$$\left. - \frac{785}{272} e_0^2 \left(\frac{v_0}{v}\right)^{19/3} \left[1 + \left(\frac{6955261}{2215584} + \frac{436441}{79128}\eta\right) v^2 + \left(\frac{2833}{1008} - \frac{197}{36}\eta\right) v_0^2 + \cdots O(v^6)\right] \right\}$$

$$\phi(t) = \lambda(t) + W(t) + \tilde{\lambda}(t)$$

relative 5PN periodic correction; ignore.
(induced by rad. reaction force)

$$W(t) = 2e_0 \left(\frac{v_0}{v}\right)^{19/6} \sin l(t) \left[1 + O(v^2) + O(v^3) + \cdots\right]$$

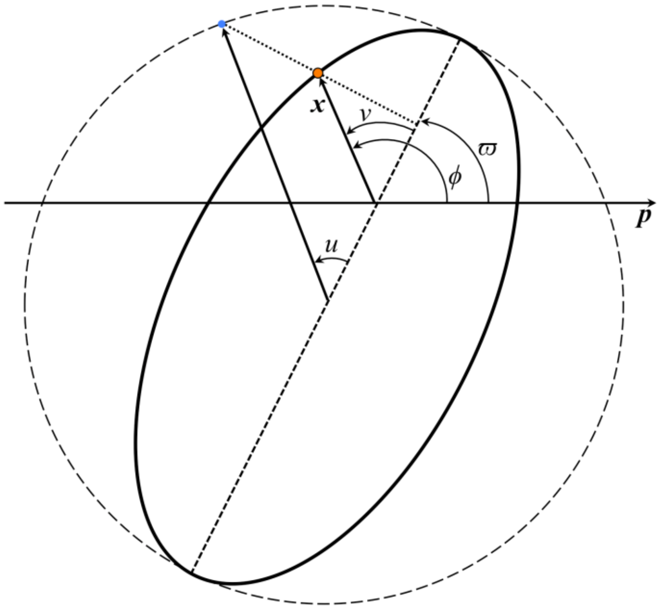
$$+ \frac{5}{4} e_0^2 \left(\frac{v_0}{v}\right)^{19/3} \sin 2l(t) \left[1 + O(v^2) + O(v^3) + \cdots\right] + O(e_0^3)$$



- *W* term is a Newtonian effect due to elliptical motion/variation of orbital speed.

- Produces phase error ~ *O*(0.1) GW cycles for e_0 ~ 0.1. Comparable to 2PN eccentric secular correction.

General considerations for eccentric waveforms:



- Circular PN waveforms evolve a single phase function: $\phi(t)$. Need to specify $\omega_\phi(t_0)$, $\phi(t_0)$, t_0
- Arbitrarily eccentric PN waveforms evolve two phase functions: $\phi(t)$ and $l(t)$.
- Must also specify $e_0(f_0)$, f_0, and $l(t_0)$.
- $l(t_0)$ is related to $\phi(t_0)$ and $\varpi(t_0)$ [argument of periastron].
- If ignoring periodic terms $W(l)$ and eccentric harmonics [$\cos(jl)$ and $\cos(jl \pm 2\phi)$], $l(t)$ and $\varpi(t_0)$ do not enter.

$$\frac{dl}{dt} = \omega_r(t)$$

$$\frac{d\lambda}{dt} = \omega_r(t)(1 + k(e(t), \omega_r(t))$$

$$\frac{d\omega_r}{dt} = F_{\omega_r}[\omega_r(t), e(t)]$$

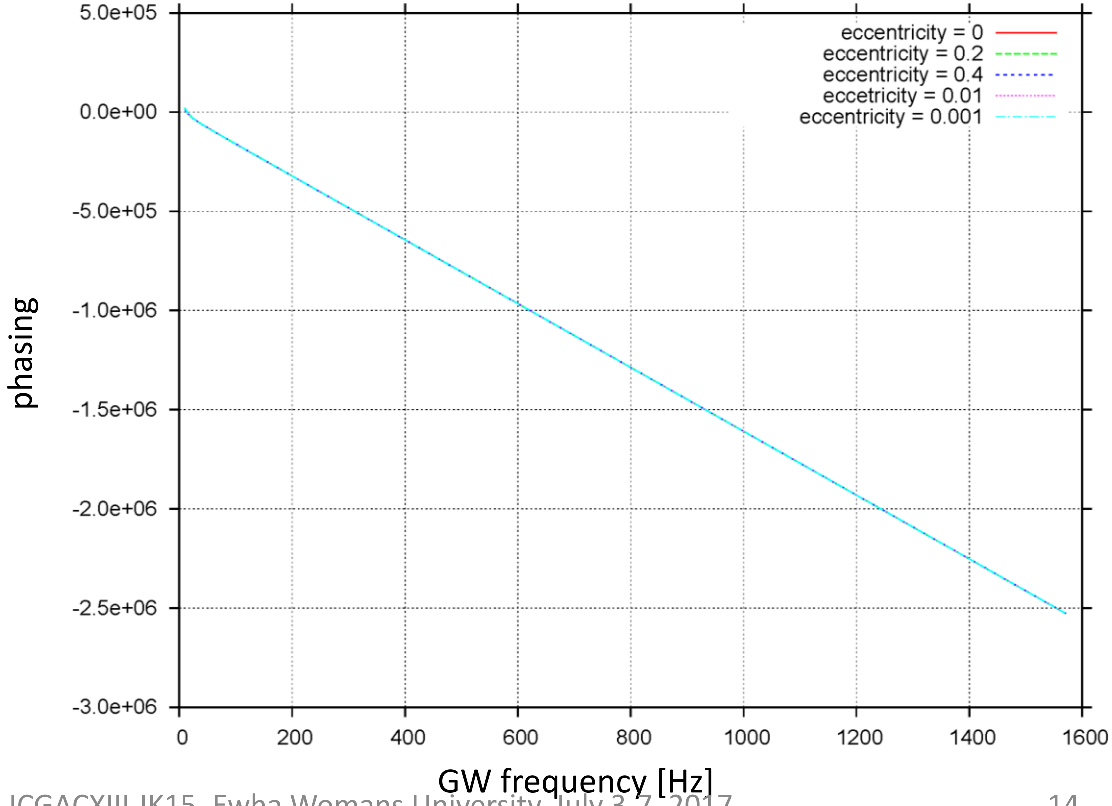
$$\frac{de}{dt} = F_e[\omega_r(t), e(t)]$$

$$h_+ = -\frac{\eta M v^2}{D}\Bigg((1+\cos^2\iota)\Bigg\{\Bigg[2 + 3e_t\cos l + (4\cos 2l - 1)e_t^2 + (43\cos 3l - 19\cos l)\frac{e_t^3}{8}\Bigg]\cos 2\phi + \Bigg[2e_t\sin l + 3e_t^2\sin 2l + (17\sin 3l - 7\sin l)\frac{e_t^3}{4}\Bigg]\sin 2\phi\Bigg\} - \sin^2\iota\Bigg[e_t\cos l + e_t^2\cos 2l + (9\cos 3l - \cos l)\frac{e_t^3}{8}\Bigg] + O(e_t^4)\Bigg) + O(v^3)$$

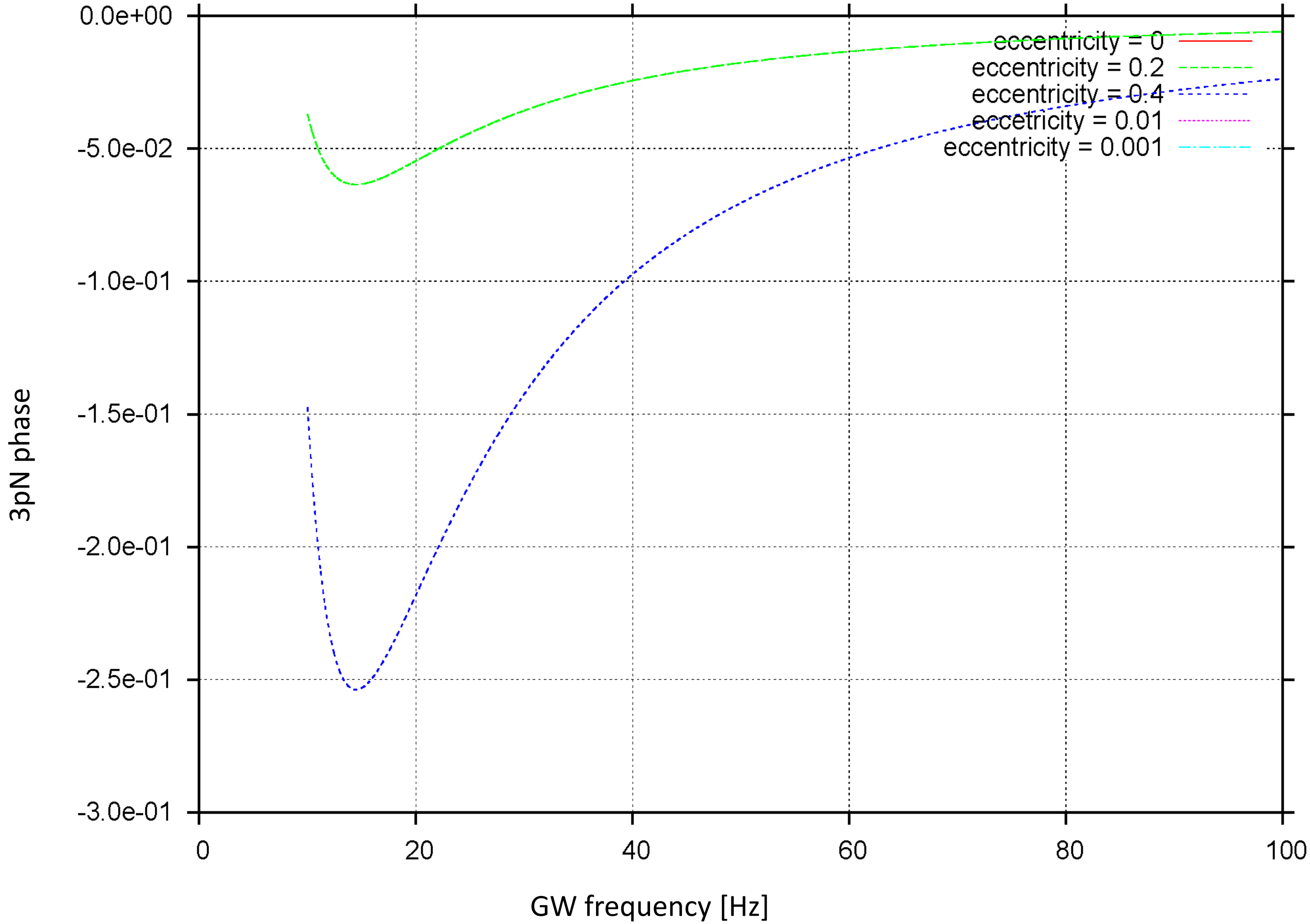
$$\phi(t) = \lambda(t) + W(t) + \tilde{\lambda}(t)$$

Phase term

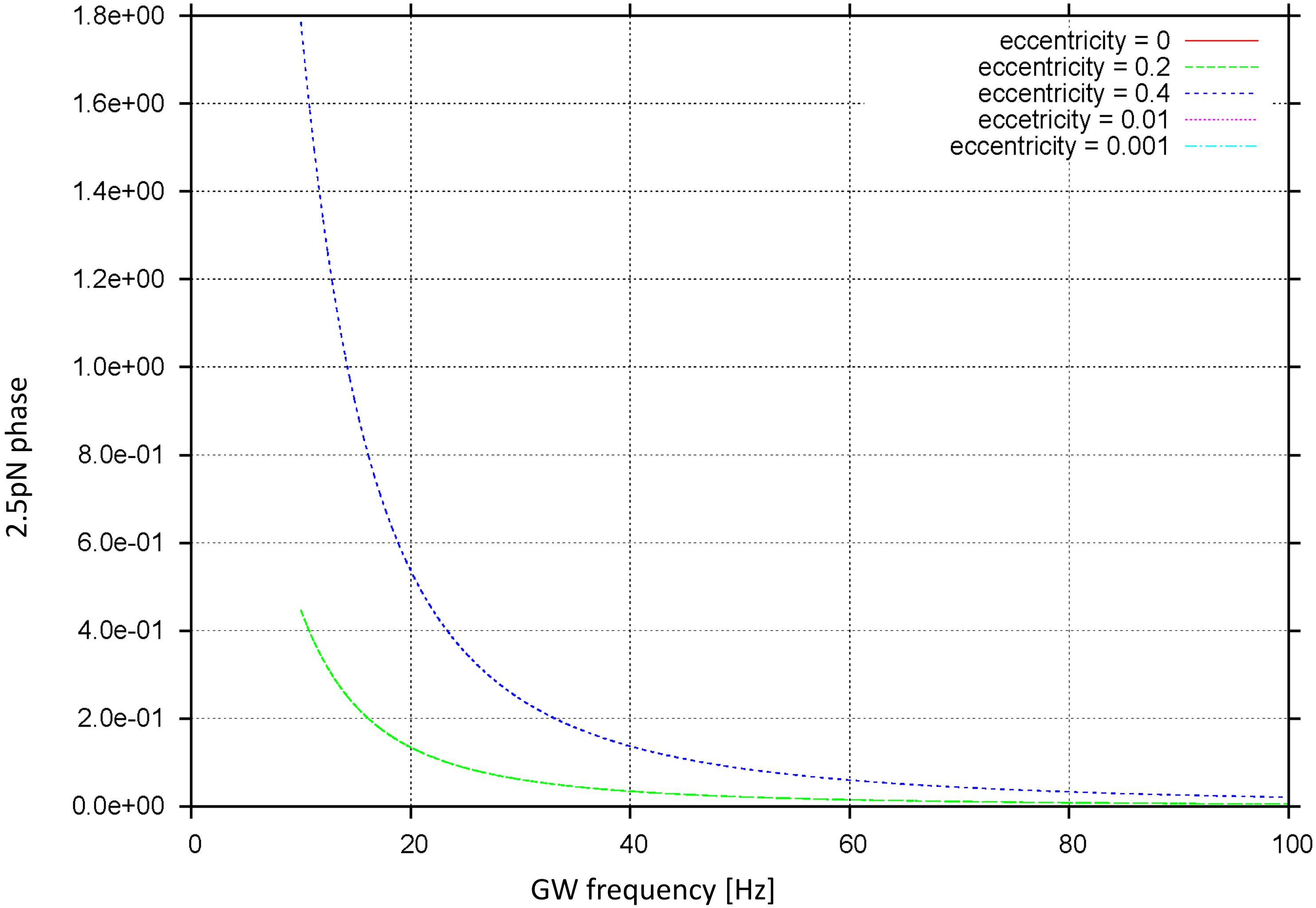
- Check the phase term of gravitational waveform for eccentricity.



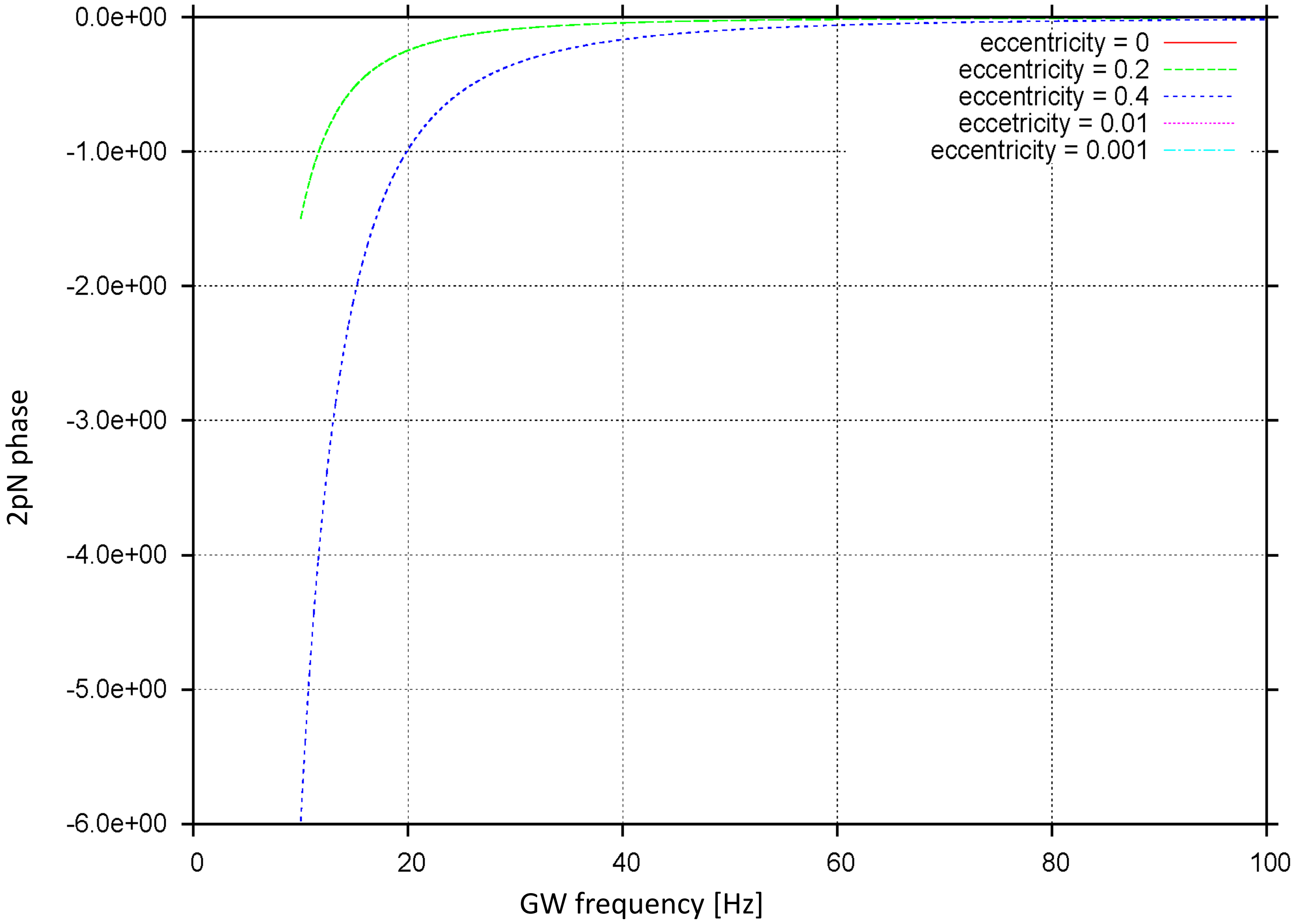
3 pN phase


0.0e+00
-5.0e-02
-1.0e-01
-1.5e-01
-2.0e-01
-2.5e-01
-3.0e-01
0
20
40
60
80
100
3pN phase
GW frequency [Hz]
eccentricity = 0
eccentricity = 0.2
eccentricity = 0.4
eccetricity = 0.01
eccentricity = 0.001

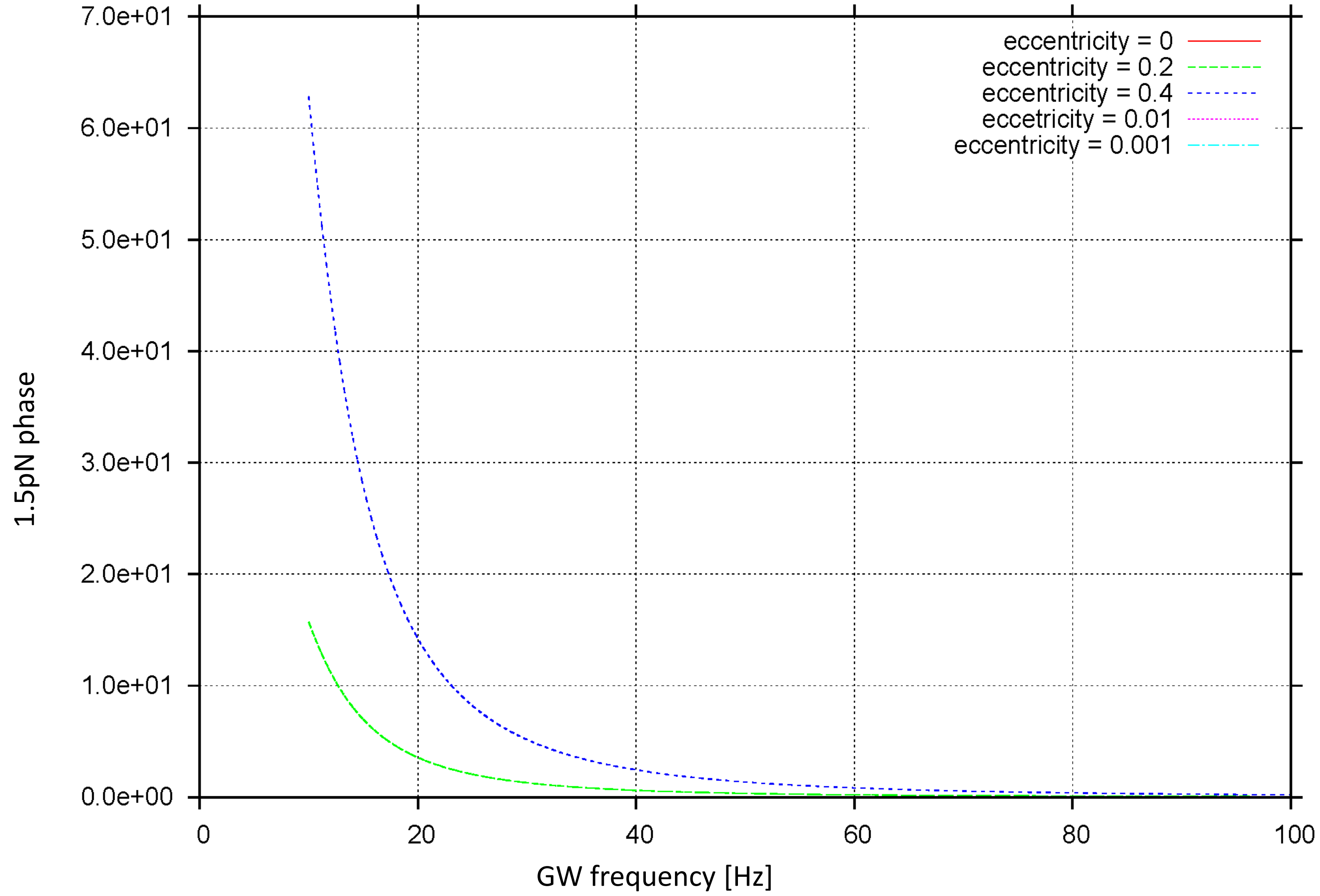
2.5pN phase


1.8e+00
1.6e+00
1.4e+00
1.2e+00
1.0e+00
8.0e-01
6.0e-01
4.0e-01
2.0e-01
0.0e+00
0
20
40
60
80
100
2.5pN phase
GW frequency [Hz]
eccentricity = 0
eccentricity = 0.2
eccentricity = 0.4
eccetricity = 0.01
eccentricity = 0.001

2pN phase


0.0e+00
-1.0e+00
-2.0e+00
-3.0e+00
-4.0e+00
-5.0e+00
-6.0e+00
2pN phase
0
20
40
60
80
100
GW frequency [Hz]
eccentricity = 0
eccentricity = 0.2
eccentricity = 0.4
eccetricity = 0.01
eccentricity = 0.001

1.5pN phase


7.0e+01
6.0e+01
5.0e+01
4.0e+01
3.0e+01
2.0e+01
1.0e+01
0.0e+00
0
20
40
60
80
100
1.5pN phase
GW frequency [Hz]
eccentricity = 0
eccentricity = 0.2
eccentricity = 0.4
eccetricity = 0.01
eccentricity = 0.001

PE for NS-NS inspirals

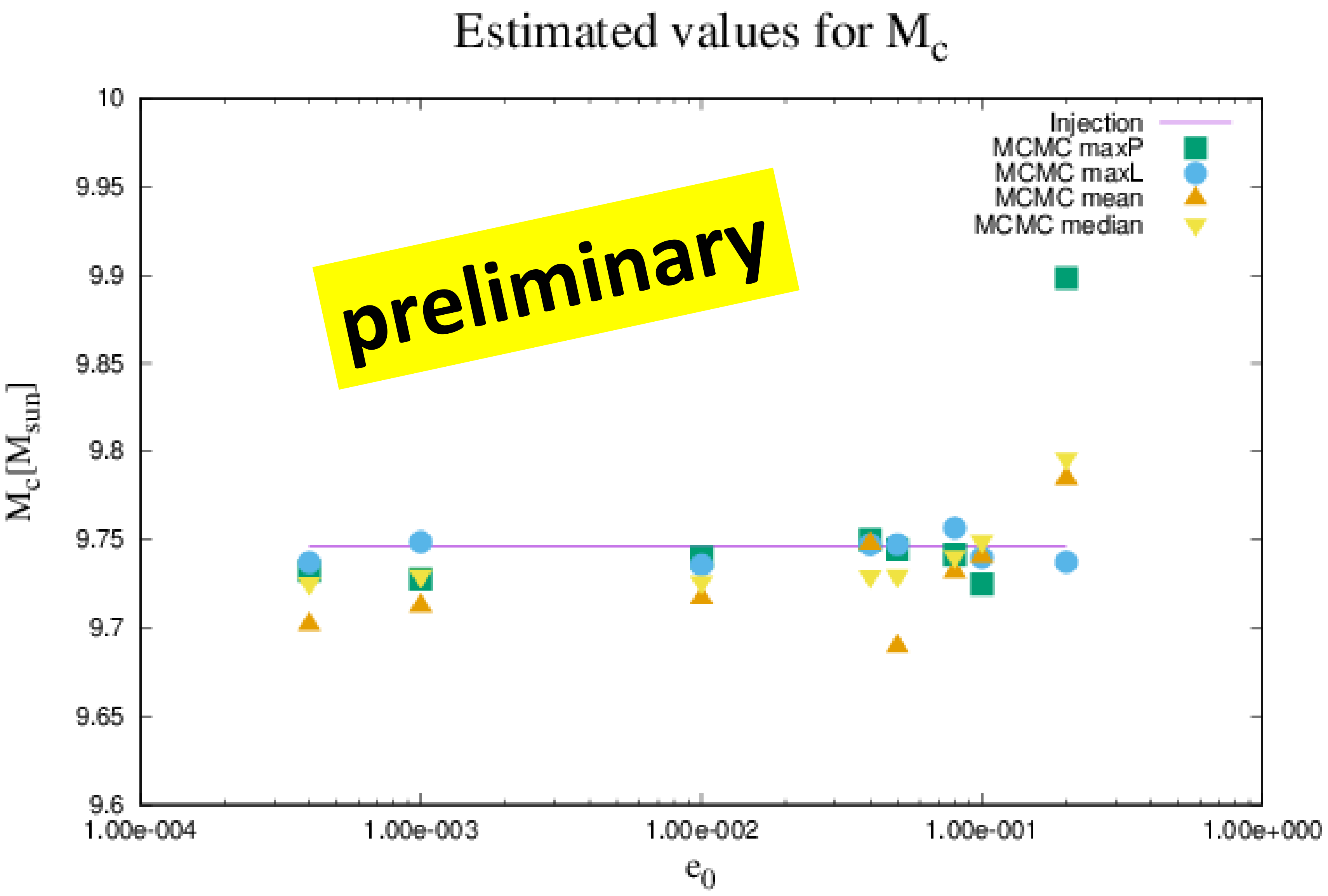
- non-spinning eccentric CBC inspirals (10 parameters)

$$\tilde{h}(f) = \tilde{h}\left(f;\ \begin{matrix}\text{chirpmass, symmetric mass ratio, distance, sky location (RA, dec),} \\ \text{eccentricity, inclination, orbital phase, polarization, coalescence time}\end{matrix}\right)$$

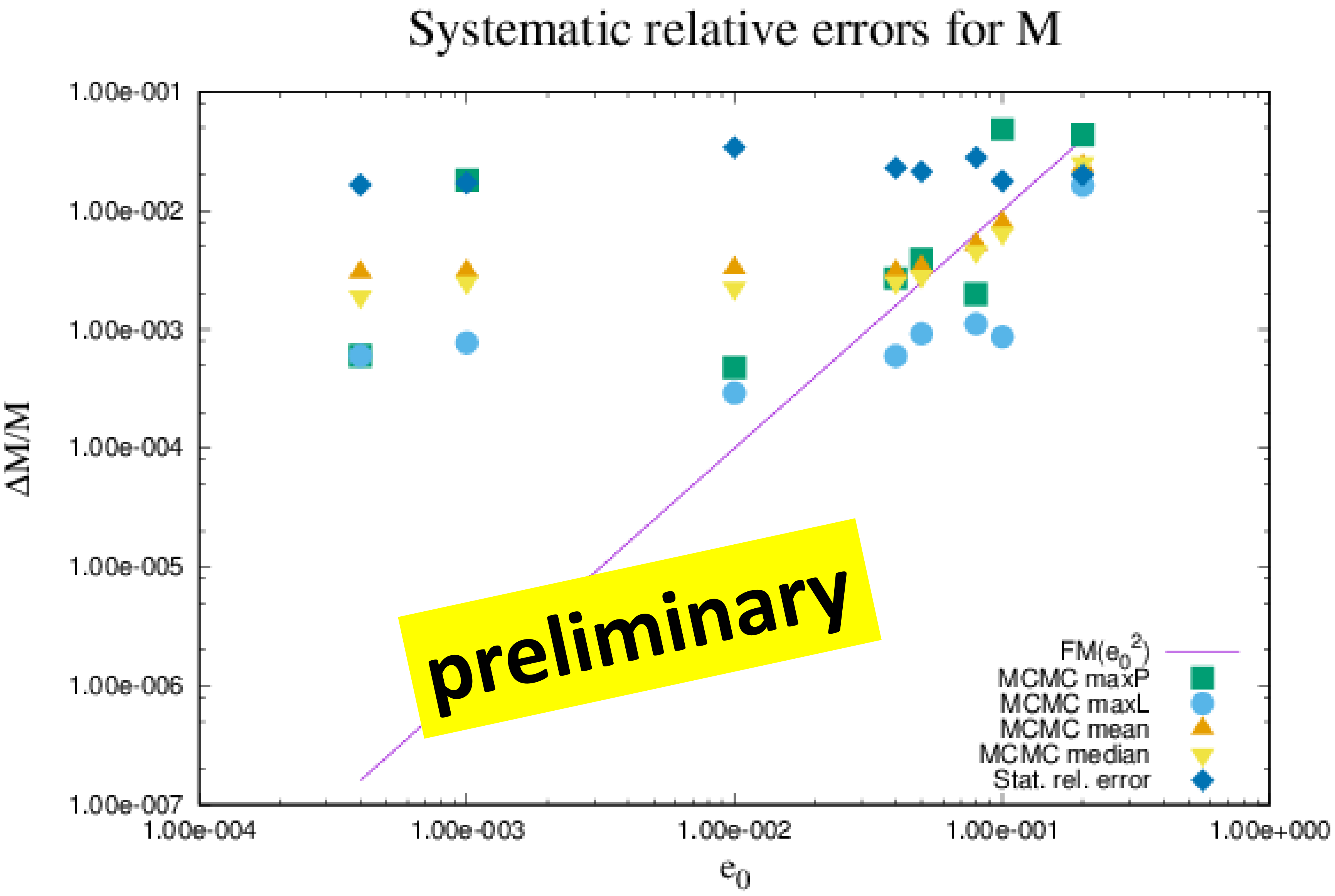
- Advanced LIGO-Virgo network (3 detectors), no noise, S/N ~ 20
- low cut frequency : 10Hz
- Injections waveform : eccentric, template waveform : circular

Binary	Chirp mass Mc (M_{sun})	Symmetric mass ratio η	distance (Mpc)	Inclination (rad)	Polarization (rad)	Orbital phase (rad)	Coalescence time (s)	RA (rad)	Dec (rad)
NS-NS	1.218733	0.249987	160.0	0.785	2.606	3.31	894383679	0.645	0.575

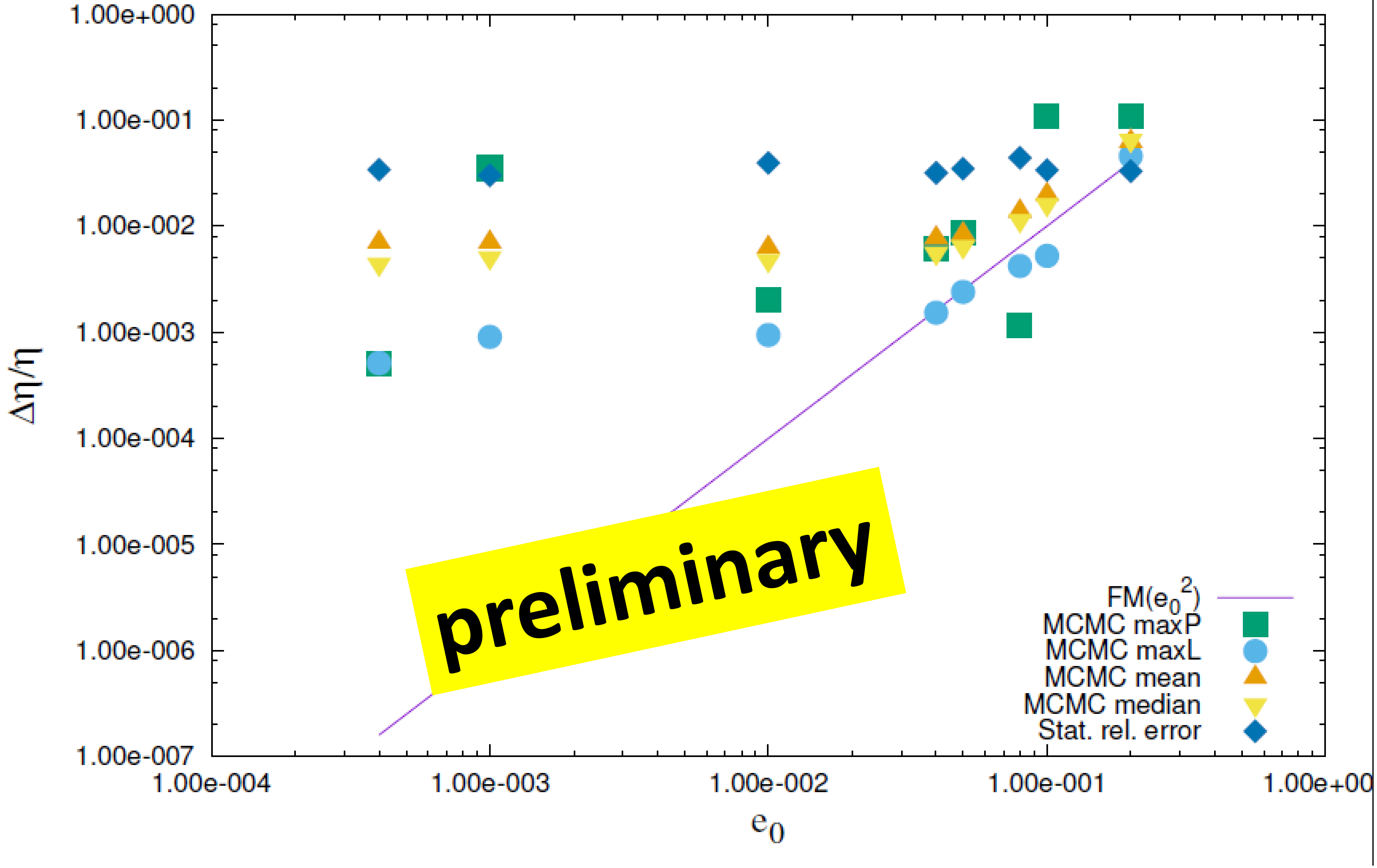
Eccentricity = 0, 0.0001, 0.001, 0.01, 0.1


Estimated values for M_c

preliminary

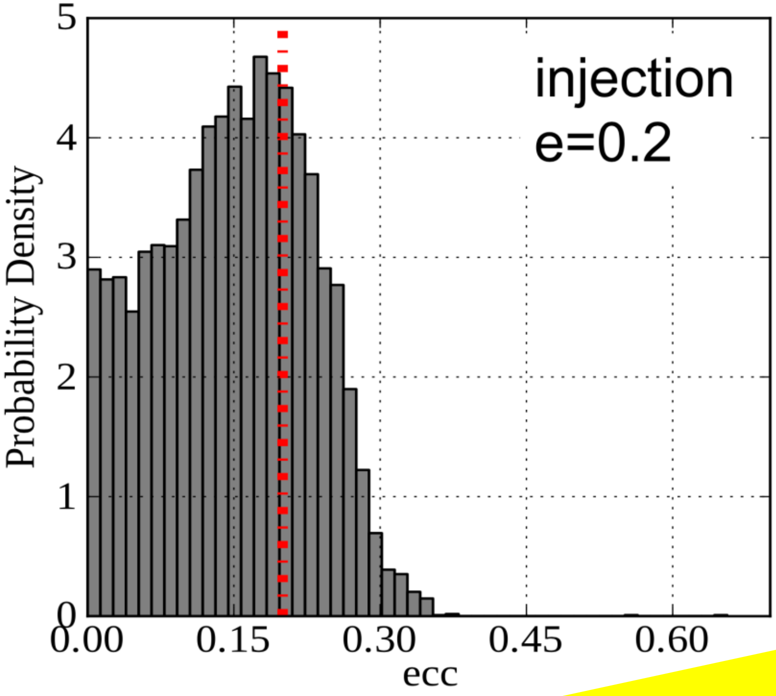

Systematic relative errors for M
ΔM/M
1.00e-001
1.00e-002
1.00e-003
1.00e-004
1.00e-005
1.00e-006
1.00e-007
1.00e-004
1.00e-003
1.00e-002
1.00e-001
1.00e+000
e_0
FM(e_0^2)
MCMC maxP
MCMC maxL
MCMC mean
MCMC median
Stat. rel. error
preliminary

Systematic relative errors for η

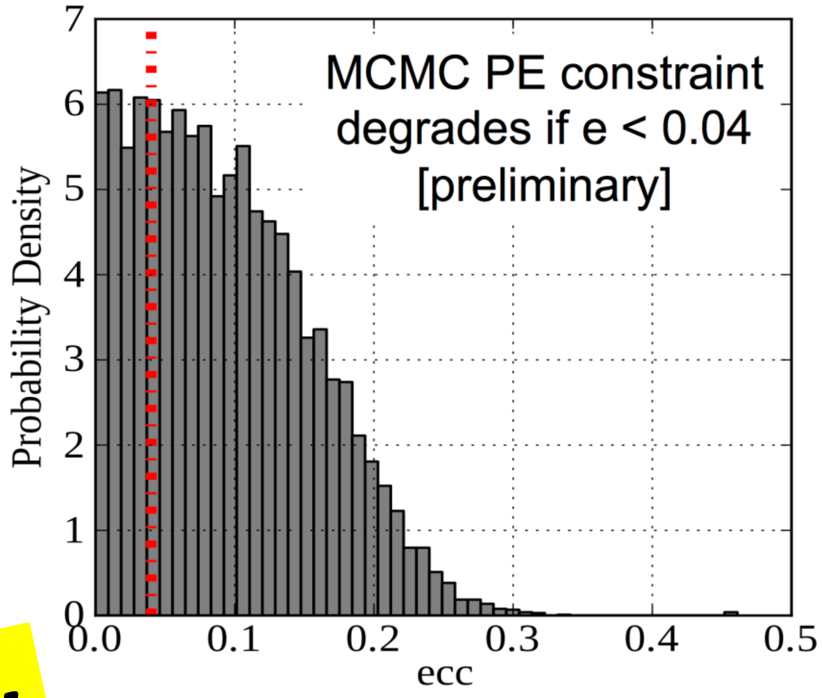


Individual probability density function of e0

[eccentric injection vs eccentric templates]



injection
e=0.2



MCMC PE constraint
degrades if e < 0.04
[preliminary]

preliminary

Summary and Future Work

- Circularization and in-spiral motion are consequences of GW emission from CBCs (Peters 1964).
- Non-negligible eccentricity changes the phase of GW waveform and this affects PE accuracy.
- Known NS-NS binary mergers found in the Galactic plane will be almost circular (e< 0.0001) when they enter the GW detection band ($\mathrm{f_{gw}} > 10\ \mathrm{Hz}$). CBCs with dynamical origins may have intrinsically eccentric orbits.
- Applying taylorF2 pN waveform model (up to 3.0 pN corrections for eccentricity) for NS-NS inspirals, we compare pN corrections quantitatively for different eccentricities (e=0.001,0.01,0.4,0.2 and 0) in GW phase.
- Neglecting orbital eccentricity does not affect to detection, but increases PE uncertainties for symmetric mass ratio.
- Favata (2014) showed that neglecting eccentricities (when e>0.002 @ 10Hz), systematic error due to eccentricity than statistical error of the advanced LIGO.
- Markov Chain PE using tayloF2 is underway, for NS-NS inspirals in eccentric orbit.

Thank you