Charged Cosmological Black Hole

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Outline

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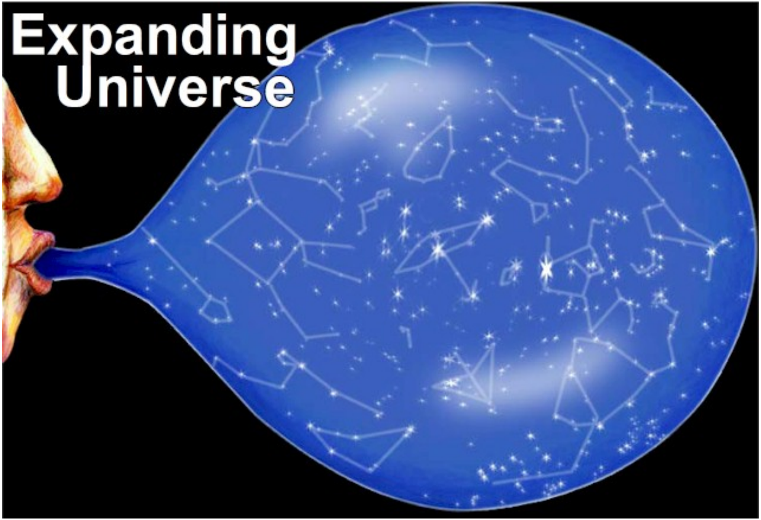
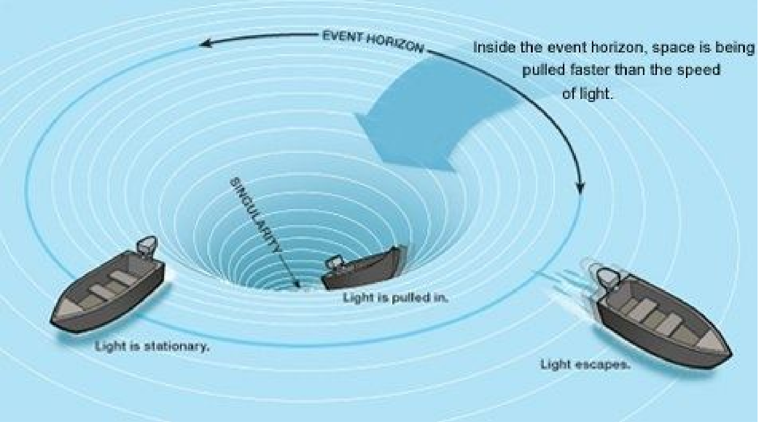
Collapsing structures and Expanding Universe

The cosmological black holes are black holes living not in an asymptotically flat universe but in an expanding spacetime. They have a rich dynamics in particular for their mass and horizon

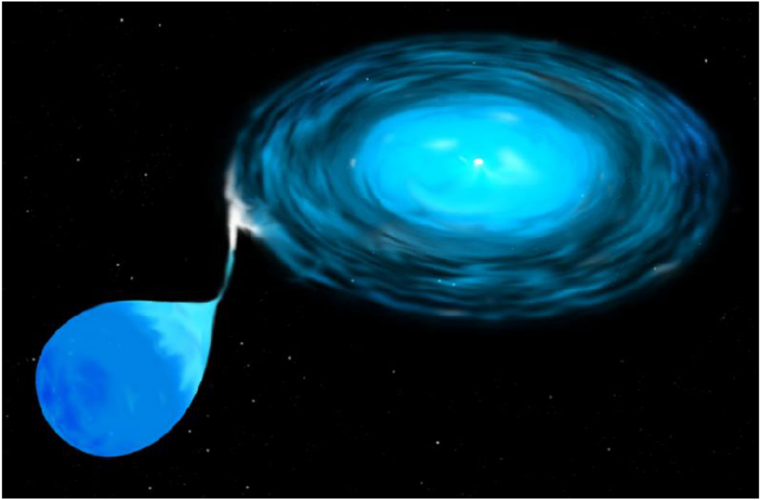


What happens when there is no asymptotically flat universe?

The usual definition of black hole event horizon turns out to be pretty much useless for practical purposes in highly dynamical situations.

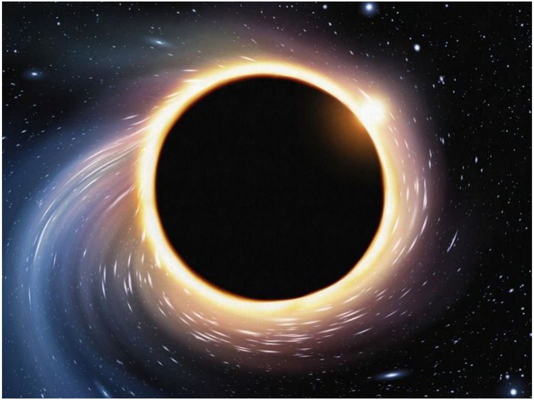


What happens when boundary of black hole is evolving?



why charged evolving black hole?

Completing the literature on cosmological black holes by making a natural step forward: in the same way the Schwarzschild solution generalizes to the ReissnerNordstrom solution by considering the coupled Einstein-Maxwell system, we will generalize the neutral CBH to its charged counterpart: **Charged Cosmological Black Hole (CCBH)**



charged black hole and nature

it is believed that black holes with large charge-mass ratio don't exist in nature. For instance, Wald has shown that the charge-mass ratio for a Kerr black hole rotating in the small uniform magnetic field of a galaxy ($10^{-4} - 10^{-5}$ Gauss) is $\simeq 10^{-24}$. However, if a highly magnetized plasma accretes onto the black hole, the charge-to-mass ratio can be much larger. In particular, in the merging of a binary system of neutron stars, it is expected at the final steps of a gravitational collapse to a black hole to obtain electromagnetic fields larger than the critical value for vacuum polarization. In this case, the charge-to-mass ratio could be near to 1. This would produce the most energetic known objects in the universe: the gamma-ray bursts (GRB) with an energy around 10^{54} ergs ($\simeq 1 M_{\odot} c^2$) released in few seconds.

Metric

Consider a general inhomogeneous spherically symmetric spacetime constructed with a charged perfect fluid and a metric expressed in $x^\mu = (t,\ r,\ \theta,\ \phi)$:

$$ds^2 = -e^{2\sigma}\, dt^2 + e^\lambda\, dr^2 + R^2\, d\Omega^2\ , \tag{1}$$

where $\sigma = \sigma(t, r)$, $\lambda = \lambda(t, r)$ are functions to be determined, $R = R(t, r)$ is the physical radius, and $d\Omega^2 = d\theta^2 + \sin^2\theta\, d\phi^2$ is the metric of the unit 2-sphere. The energy momentum tensor of the perfect fluid is

$$T_M^{\mu\nu} = (\rho + p)\, u^\mu\, u^\nu + g^{\mu\nu}\, p\ , \tag{2}$$

and the electromagnetic tensor is

$$T_{EM}^{\mu\nu} = \frac{1}{4\pi}\left(F^{\mu\alpha}F^{\nu}{}_{\alpha} - \frac{1}{4}g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right), \tag{3}$$

where $\rho = \rho(t, r)$ is the mass-energy density, $p = p(t, r)$ is the pressure, and $u^\mu = (e^{-\sigma}, 0, 0, 0)$ is the charged perfect fluid four-velocity. Choosing a perfect fluid implies that there is no heat flow, radiation, or viscosity.

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Electromagnetic field

The electromagnetic field $F^{\mu\nu}$ satisfies Maxwell's equation:

$$\nabla_\mu F^{\mu\nu} = 4\pi J^\nu \tag{4}$$

and

$$F_{[\alpha\beta,\gamma]} = 0, \tag{5}$$

where J^ν is the 4-current. To describe the charged black hole, we consider an electric charge at rest in the comoving coordinate of fluid ; in this case, the potential and the current are given by:

$$A_\mu(t,r) = A(t,r)\delta^0_\mu \qquad J^\nu = \rho_{EM}(t,r)u^\nu. \tag{6}$$

Electromagnetic field

Because of the spherical symmetry, the only non-vanishing component of the electromagnetic field is $F^{01} = -F^{10}$ so from Eq. (4) we have

$$F^{01} = e^{-\left(\sigma+\frac{\lambda}{2}\right)}\frac{Q}{R^2}. \tag{7}$$

Here we consider that the only non-vanishing current density is J^0, so Q is not an explicit function of time

$$Q(r) = \int_0^R 4\pi e^{-\left(\sigma+\frac{\lambda}{2}\right)} R^2 J^0 dr. \tag{8}$$

Therefore, the electromagnetic invariants are

$$\begin{aligned}
\mathcal{F} &\equiv \frac{1}{2}F_{\mu\nu}F^{\mu\nu} = (\mathbf{B}^2 - \mathbf{E}^2) \\
&= -\mathbf{E}^2 = -\left[\frac{Q(t,r)}{R(t,r)^2}\right]^2, \\
\mathcal{G} &\equiv \frac{1}{4}F_{\mu\nu}{}^*F^{\mu\nu} = \mathbf{E}\cdot\mathbf{B} = 0,
\end{aligned} \tag{9}$$

where ${}^*F^{\mu\nu}$ is the dual of the electromagnetic tensor $F^{\mu\nu}$.

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Einstein equations

The Einstein equations $G^{\mu\nu} = \kappa\, T^{\mu\nu} - g^{\mu\nu}\Lambda$ can be reduced to

$$e^{2\sigma}\, G^{tt} = -\left(\frac{2R''}{R} + \frac{R'^2}{R^2} - \frac{R'}{R}\,\lambda'\right) e^{-\lambda} + \left(\frac{\dot{R}^2}{R^2} + \frac{\dot{R}}{R}\,\dot{\lambda}\right) e^{-2\sigma} + \frac{1}{R^2} = \kappa\rho + \frac{Q^2}{R^4} + \Lambda\ , \tag{10}$$

$$e^{\lambda}\, G^{tr} = \left(\frac{2\dot{R}'}{R} - \frac{2\dot{R}}{R}\,\sigma' - \frac{R'}{R}\,\dot{\lambda}\right) e^{-2\sigma} = 0\ , \tag{11}$$

Einstein equations

$$
\begin{aligned}
e^{\lambda}\, G^{rr} \;=\;& \left(\frac{R'^2}{R^2} + \frac{2R'}{R}\,\sigma' \right) e^{-\lambda} \\
& - \left(\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} - \frac{2\dot{R}}{R}\,\dot{\sigma} \right) e^{-2\sigma} - \frac{1}{R^2} = \kappa p - \frac{Q^2}{R^4} - \Lambda \,, \qquad (12)
\end{aligned}
$$

$$
\begin{aligned}
R^2\, G^{\theta\theta} =& \left(\frac{R''}{R} + \frac{R'}{R}\,\sigma' + \sigma'' + \sigma'^2 - \frac{R'}{2R}\,\lambda' - \frac{1}{2}\,\sigma'\,\lambda' \right) e^{-\lambda} \\
+ & \left(\frac{\dot{R}}{R}\,\dot{\sigma} - \frac{\ddot{R}}{R} - \frac{1}{2}\,\ddot{\lambda} + \frac{1}{2}\dot{\lambda}\,\dot{\sigma} - \frac{\dot{R}}{2R}\dot{\lambda} - \frac{1}{4}\,\dot{\lambda}^2 \right) e^{-2\sigma} = \kappa p + \frac{Q^2}{R^4} - \Lambda \;. \qquad (1
\end{aligned}
$$

The conservation laws

$$\frac{2e^{2\sigma}}{(\rho+p)} \nabla_\mu T^{t\mu} = \dot{\lambda} + \frac{2\dot{\rho}}{(\rho+p)} + \frac{4\dot{R}}{R} = 0, \tag{14}$$

$$\frac{e^{\lambda}}{(\rho+p)} \nabla_\mu T^{r\mu} = \sigma' + \frac{p'}{p+\rho} - \frac{QQ'}{4\pi(\rho+p)R^4} = 0, \tag{15}$$

where the dot denotes a partial derivative with respect to t, and the prime denotes a partial derivative with respect to r.

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Partial Differential Equations

Using Misner-Sharp mass definition

$$\frac{2M}{R} = \dot{R}^2 e^{-2\sigma} - R'^2 e^{-\lambda} + 1 + \frac{Q^2}{R^2} - \frac{1}{3}\Lambda R^2 \,. \tag{16}$$

and some simplifications the five coupled partial differential equations governing the evolution of the CCBH are:

Partial Differential Equations

$$\dot{R} = \pm e^{\sigma}\sqrt{\frac{2M}{R} - \frac{Q^2}{R^2} + 2E + \frac{\Lambda R^2}{3}} \ , \tag{17}$$

$$\dot{M} = \frac{-\kappa p\,\dot{R}R^2}{2} \ , \tag{18}$$

$$\dot{\rho} = -p'\,\frac{\dot{R}}{R'} + \frac{2\dot{R}QQ'}{4\pi R^4} - (\rho + p)\left[\frac{\dot{R}'}{R'} + \frac{2\dot{R}}{R}\right] \ , \tag{19}$$

$$\dot{p} = \frac{dp}{d\rho}\,\dot{\rho} \ , \tag{20}$$

$$\dot{\lambda} = \frac{2}{R'}\left(\frac{p'\dot{R}}{(\rho + p)} + \dot{R}' - \frac{2\dot{R}QQ'}{4\pi(\rho + p)R^4}\right) \ , \tag{21}$$

where

$$2E(t,r) = R'^2 e^{-\lambda} - 1 \tag{22}$$

is the curvature term, analogous to $E(r)$ in the LTB model.

Analytical solutions

Assuming $\Lambda = 0$, $p = 0$ and $Q =$ cst the metric (1) reduces to the charged LTB metric:

$$ds^2 = -dt^2 + \frac{R'^2(t,r)}{1+2E(r)}dr^2 + R^2(t,r)d\Omega^2. \quad (23)$$

For our purpose of describing charged black holes, the system to solve becomes:

$$\dot{R}^2(t,r) = 2E(r) + \frac{2M(r)}{R} - \frac{Q^2}{R^2(t,r)}, \quad (24)$$

$$M'(r) = \frac{1}{2}\kappa\rho R^2(t,r)R'(t,r), \quad (25)$$

since $\dot{M} = 0$, M is only a function of r, *i.e.* $M = M(r)$.
The explicit solutions of Eq. (24) involve elliptic function which in the case of $Q = 0$ were discussed by Lemaitre and Omer.

LTB solution

In mathematical physics, the Lemaitre-Tolman metric is the spherically symmetric dust solution of Einstein's field equations. It was first found by Lematre in 1933 and Tolman in 1934 and later investigated by Bondi in 1947. This solution describes a spherical cloud of dust (finite or infinite) that is expanding or collapsing under gravity. It is also known as the Lemaitre-Tolman-Bondi metric and the Tolman metric.

LTB solution

The metric is:

$$ds^2 = dt^2 - \frac{(R')^2}{1+2E}dr^2 - R^2\,d\Omega^2$$

where:

$$d\Omega^2 = d\theta^2 + \sin^2\theta\,d\phi^2$$

$$R = R(t,r)\,, \qquad R' = \partial R/\partial r\,, \qquad E = E(r) > -\frac{1}{2}$$

The matter is comoving, which means its 4-velocity is:

$$u^a = \delta^a_0 = (1,0,0,0)$$

so the spatial coordinates (r,θ,ϕ) are attached to the particles of dust.

The pressure is zero (hence *dust*), the density is

$$8\pi\rho = \frac{2M'}{R^2\,R'}$$

and the evolution equation is

$$\dot{R}^2 = \frac{2M}{R} + 2E$$

where

$$\dot{R} = \partial R/\partial t$$

The evolution equation has three solutions, depending on the sign of E,

$$E>0: \qquad R = \frac{M}{2E}(\cosh\eta - 1)\,, \qquad (\sinh\eta - \eta) = \frac{(2E)^{3/2}(t-t_B)}{M}\;;$$

$$E=0: \qquad R = \left(\frac{9M(t-t_B)^2}{2}\right)^{1/3}\;;$$

$$E<0: \qquad R = \frac{M}{2E}(1-\cos\eta)\,, \qquad (\eta - \sin\eta) = \frac{(-2E)^{3/2}(t-t_B)}{M}\;;$$

Analytical solutions

When $Q \neq 0$ we obtain the explicit solutions as follow

- $E(r) < 0$:

$$R(t,r) = \frac{M(r)}{2E(r)}\left(\cosh\eta - 1 - \frac{E(r)e^{\eta}Q^2}{M(r)^2}\right),$$
$$\eta - \sinh\eta = \frac{(-2E(r))^{3/2}}{M}\left[t - t_B(r)\right] - \frac{E(r)e^{\eta}Q^2}{M(r)^2}.$$

- $E(r) = 0$:

$$R(t,r) = \frac{1}{6}\left[\frac{5Q^2}{M(r)} + \frac{Q^4}{M(r)^2 L(t,r)} + L(t,r)\right],$$
$$L(t,r) = \left(486\left[t - t_B(r)\right]^2 M(r) - \frac{Q^6}{M(r)^3} - \frac{18\sqrt{3}\sqrt{243\left[t - t_B(r)\right]^4 M(r)^4 - \left[t - t_B(r)\right]^2 Q^6}}{M(r)}\right)^{1/3}.$$

- $E(r) > 0$:

$$R(t,r) = \frac{M(r)}{2E(r)}\left(\cosh\eta - 1 + \frac{E(r)e^{\eta}Q^2}{M(r)^2}\right),$$
$$\eta - \sinh\eta = -\frac{(2E(r))^{3/2}}{M(r)}\left[t - t_B(r)\right] + \frac{E(r)e^{\eta}Q^2}{M(r)^2}.$$

These solutions are valid even for $Q = Q(r)$ with the mass of Eq. (27):

$$M'(r) = \frac{1}{2}\kappa\rho R^2(t,r)R'(t,r) + \frac{Q(r)Q'(r)R'(t,r)}{2R(t,r)}.$$

Characterization of the horizons

The expansion for ingoing and outgoing null geodesics is:

$$\theta_{(\ell)} \propto \left(1 - \frac{\sqrt{\frac{2M}{R}+2E-\frac{Q^2}{R^2}}}{\sqrt{1+2E}}\right),\ \theta_{(n)} \propto \left(-1 - \frac{\sqrt{\frac{2M}{R}+2E-\frac{Q^2}{R^2}}}{\sqrt{1+2E}}\right) < 0.$$

$\theta_{(\ell)} = 0 \to 1 = \frac{2M}{R} - \frac{Q^2}{R^2}$. Its roots are

$$R_{\pm} = M \pm \sqrt{M^2 - Q^2}. \tag{26}$$

To study the horizon of the CCBH, we consider $R = R_+$ where the expansion for null outgoing geodesic changes its sign.

Slowly evolving horizon

We define the evolution parameter C such that the tangent vector to the dynamical horizon, V , is given by

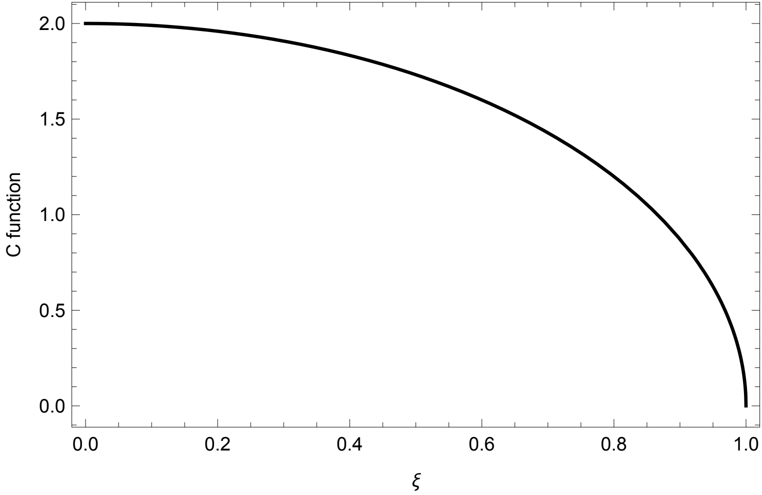
$$V^{\mu} = \ell^{\mu} - C n^{\mu} \tag{27}$$

$$C = \frac{\rho}{1/8\pi R^2 - \frac{\rho}{2}}|_{AH} = \frac{2M'}{R' - M'}|_{AH}, \tag{28}$$

where $A = 4\pi R_+^2$ is the area of the black hole. For $Q = cst$

$$C = 2\sqrt{1 - \frac{Q^2}{M^2}}\bigg|_{AH}. \tag{29}$$

Slowly evolving horizon



C function for different values of $\xi = \frac{Q_{BH}}{M_{BH}}$. From this behavior for the C-function, one can conclude that the charge helps to reach the quasi-event (isolated) horizon in a shorter time.

Initial conditions

In order to fulfill the FLRW limit and to have a structure with a void, we choose the the initial density as follows

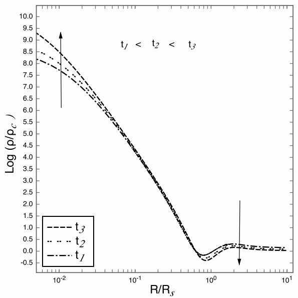
$$\rho(t_0, r) = \rho_c + \rho_s - r^2 \rho_G(r), \tag{30}$$

where ρ_c is the background density, ρ_s is the density of the collapsing object and $\rho_G(r) = a \exp(-r^2)$ is a Gaussian term that controls the location of the void. a is a dimensionless normalization constant. The initial conditions for the curvature term $E(t, r)$ and for the physical radius $R(t, r)$ are:

$$\begin{aligned} E(t_0, r) &= -b_0 r^2 e^{-b_1 r} \\ R(t_0, r) &= r, \end{aligned} \tag{31}$$

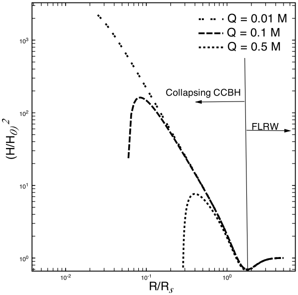
b_0 and b_1 are constants. FLRW limit is fullfiled for $r \to \infty$

Analytical solutions for density evolution



Evolution of the density of the CCBH with our initial density. with $\rho_s = \frac{\rho_c}{r^2}$, $a = \rho_c$. In the FLRW limit, the density is decreasing while it is increasing inside the structure density. R_s is the radius of the collapsing structure, ρ_c is the background density. For decreasing $R(t, r)$, one can see first the decreasing FLRW density corresponding to the expanding universe, second a void and third an increasing density for the gravitational collapse.

Numerical solutions



The effect of the electric charge (Q) on the collapse rate ($H = \frac{\dot{R}}{R}$) at a constant time for three different values of Q. When Q is large, the collapse rate tends to zero for large radius R. This illustrates the repulsive nature of the electromagnetic energy. In the case $Q = M$, this repulsive force halts the collapse and the BH does not form. Note also that the equation of state for three cases is the same

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Thank you!