# Quintessential inflation: A unified scenario of inflation and dark energy

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## Plan:

- Cosmological dynamics of scalar field
- Quintessential inflation
  - Noncanonical scalar field
  - Canonical scalar field
- Summary

# Cosmic History

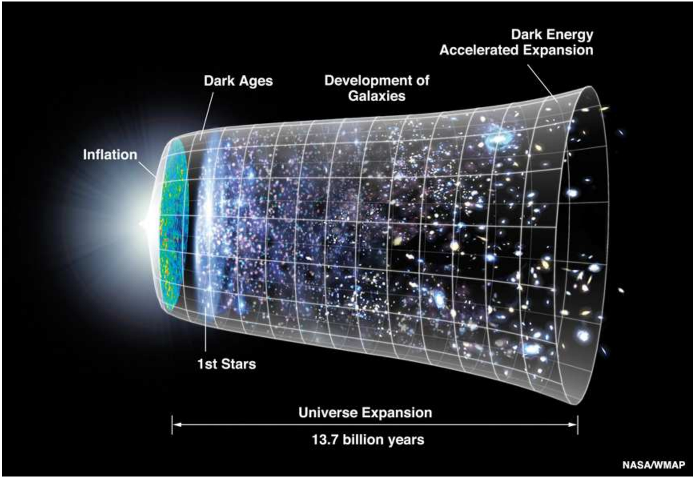


Figure: Cosmic history. Picture is taken from wfirst.gsfc.nasa.gov.

- Cosmic acceleration $\Longrightarrow$ Equation of state $\Longrightarrow$ $w =$ Pressure/Density $< -\frac{1}{3}$.
- Observationally $\Longrightarrow$ $w_{\rm inf} \approx -1$ and also currently $w_{\rm DE,0} \approx -1$.

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# Dark Energy:

Simplest candidate can be $\Lambda$.

Fine tuning problem

$$\implies \boxed{\frac{\rho_{\Lambda,\rm obs}}{\rho_{\Lambda,\rm theo}} = 10^{-120}}$$

Cosmic Coincidence

$$\implies \boxed{\rho_{\Lambda} \approx \rho_{\rm m0}}\ .$$

Alternatives $\implies$ Make DE dynamical $\implies$ Modification of gravity.

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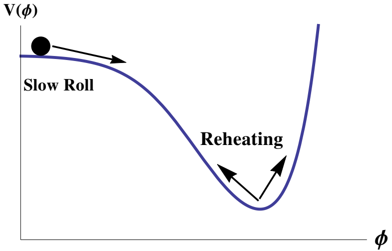
## Inflation:

$\mathcal{L} = \partial_\mu\phi\partial^\mu\phi + V(\phi)$. $\phi$ is a scalar field.



Scalar field tracks the background during the radiation and matter era and take over matter at recent past $\Longrightarrow$ Late time solution is an attractor for a wide range of initial conditions

P. J. Steinhardt, L. -M. Wang and I. Zlatev, PRD **59**, 123504 (1999)

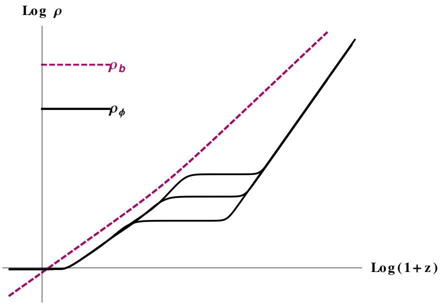



Figure: Schematic diagram of tracker behavior

- All paths are converging to a common evolutionary track.
- Not all potential can give rise to tracker behavior $\Longrightarrow$ A limitation.
- $\Gamma > 1$ where $\Gamma = \dfrac{V''V}{V'^2}$.
- Runaway potentials like $\dfrac{1}{\phi^n}$ or exponential potential $e^{M/\phi}$ can give rise to tracker solution.
- Field's EoS goes towards $-1$.

# Tracker

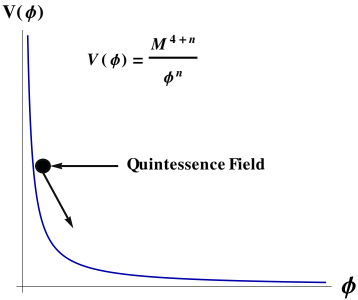



Figure: Schematic diagram of inverse power law potential, a runaway potential.

- Steep nature of the potential is needed. $\Longrightarrow$ Field's evolution freezes due to the large Hubble damping coming from the background and energy density becomes comparable with the background energy density $\Longrightarrow$ Field starts evolving and follow the background up to recent past.
- Some potentials which reduce to inverse power law and exponential nature asymptotically can also give tracker solution $\Longrightarrow$ Example: Double exponential or cos hyperbolic potential.

P. J. E. Peebles and A. Vilenkin, PRD **59**, 063505 (1999)

## What is it?

A unified description of inflation and late time cosmic acceleration:

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## What is it?

A unified description of inflation and late time cosmic acceleration:

- Scalar field behaves like inflaton at early epochs $\Longrightarrow$ Inflation.
- Same scalar field behaves like quintessence field at the late times $\Longrightarrow$ Late time cosmic acceleration.

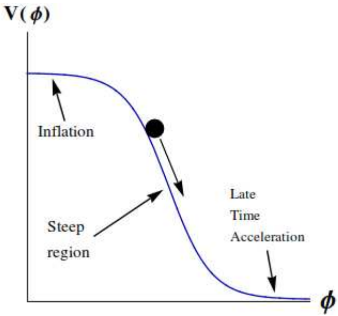
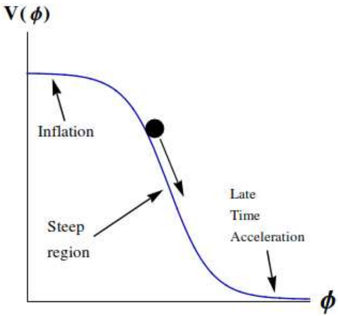



Figure: Schematic diagram of an effective potential which can give quintessential inflation.

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## Problems

1. Find out a suitable potential.
2. Scalar field survives until late times $\Longrightarrow$ potential is typically of a run-away type $\Longrightarrow$ One requires an alternative mechanism of reheating *e.g.*, instant preheating.
3. Long kinetic regime enhances the amplitude of relic gravitational waves $\Longrightarrow$ violates nucleosynthesis constraints at the commencement of radiative regime.

C. Wetterich, PRD **89**, 024005 (2014)

In the Einstien frame, let us consider:

- A noncanonical kinetic term of a scalar field.
- A nonminimal coupling between massive neutrinos and noncanonical scalar field.

Which lead to the following action,

C. Wetterich, PRD **89**, 024005 (2014)
MWH, R. Myrzakulov, M. Sami and E. N. Saridakis, PRD **90**, 023512 (2014)

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[-\frac{M_{\rm Pl}^2}{2}R + \boxed{\frac{k^2(\phi)}{2}\partial^\mu\phi\partial_\mu\phi} + V(\phi)\right] + \mathcal{S}_m + \mathcal{S}_r$$
$$\boxed{+\mathcal{S}_\nu(\mathcal{C}^2 g_{\alpha\beta};\Psi_\nu)},$$

$$k^2(\phi) = \left(\frac{\alpha^2-\tilde{\alpha}^2}{\tilde{\alpha}^2}\right)\frac{1}{1+\beta^2 {\rm e}^{\alpha\phi/{\rm M_{Pl}}}} + 1\,,$$
$$\mathcal{C}^2(\phi) = Ae^{2\tilde{\gamma}\alpha\phi/M_{\rm Pl}}, \qquad V(\phi) = M_{\rm Pl}^4{\rm e}^{-\alpha\phi/{\rm M_{Pl}}},$$

where $\beta \Longrightarrow$ can be fixed from inflation.

Let us consider the transformation,

$$\sigma = \mathbb{k}(\phi)\,,$$
$$k^2(\phi) = \left(\frac{\partial \mathbb{k}}{\partial \phi}\right)^2$$

The action becomes,

$$\mathcal{S}_E = \int d^4x\sqrt{g}\left[-\frac{M_{\rm Pl}^2}{2}R + \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma + V(\mathbb{k}^{-1}(\sigma))\right] + \mathcal{S}_m + \mathcal{S}_r + \mathcal{S}_\nu(\mathcal{C}^2 g_{\alpha\beta};\Psi_\nu)\,.$$

$\Longrightarrow$ $\sigma$ is the canonical scalar field.

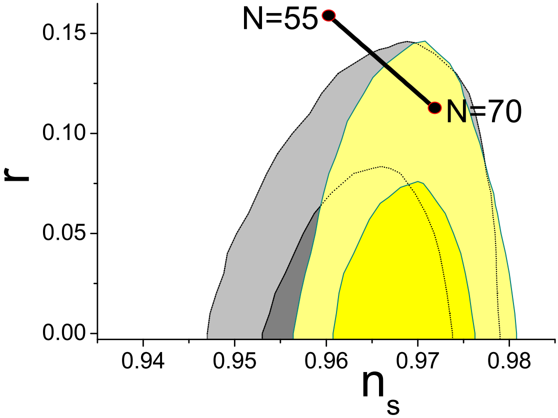


Figure: $1\sigma$ (yellow) and $2\sigma$ (light yellow) contours for Planck 2015 results, and $1\sigma$ (grey) and $2\sigma$ (light grey) contours for Planck 2013 results, on $n_s - r$ plane. Additionally, we depict the predictions of our scenario, for $\tilde{\alpha} \to 0$ and e-folding $\mathcal{N}$ varying between 55 and 70.

# CASE III: With canonical scalar field

C. Q. Geng, MWH, R. Myrzakulov, M. Sami and E. N. Saridakis, PRD **92**, no. 2, 023522 (2015)

Let us consider the following action,

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}\partial^{\mu}\phi\partial_{\mu}\phi - V(\phi)\right] + \mathcal{S}_{\rm m} + \mathcal{S}_{\rm r} + \mathcal{S}_{\nu}\left(\mathcal{C}^2 g_{\alpha\beta}, \Psi_{\nu}\right) \ ,$$

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$$\mathcal{C}^2(\phi) = Ae^{2\gamma\phi/M_{\rm Pl}}, \qquad V(\phi) = V_0{\rm e}^{-\lambda\phi^{\rm n}/{\rm M}_{\rm Pl}^{\rm n}},$$

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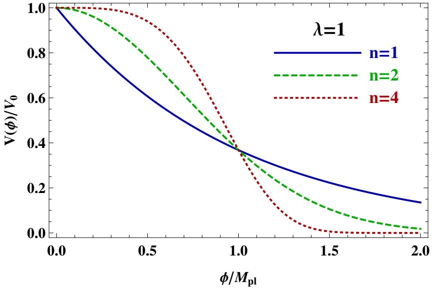

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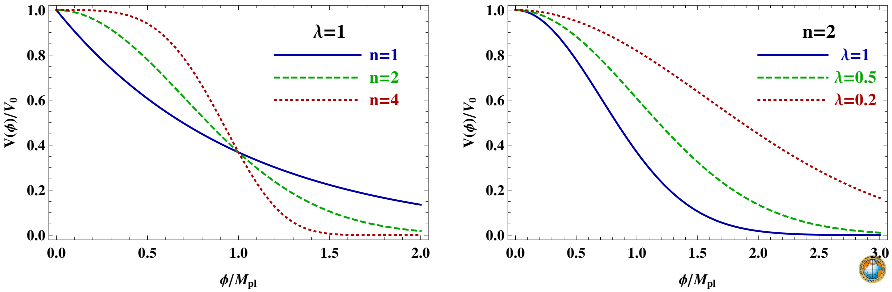

# Results

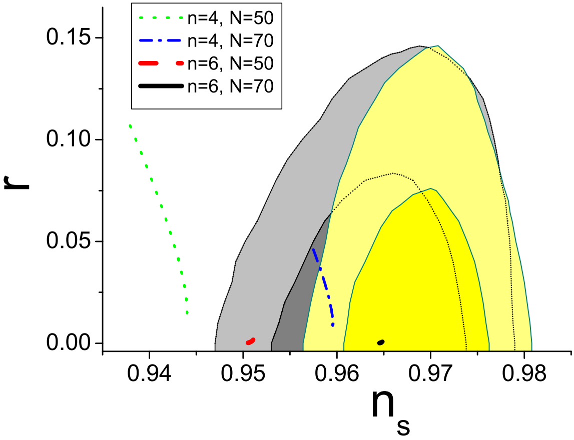


Figure: We depict the predictions of our scenario, for varying $\lambda$ (between $10^{-6}$ and $10^{-3}$), and $n$ being 4 or 6, with the e-folding value $\mathcal{N}$ being 50 or 70.

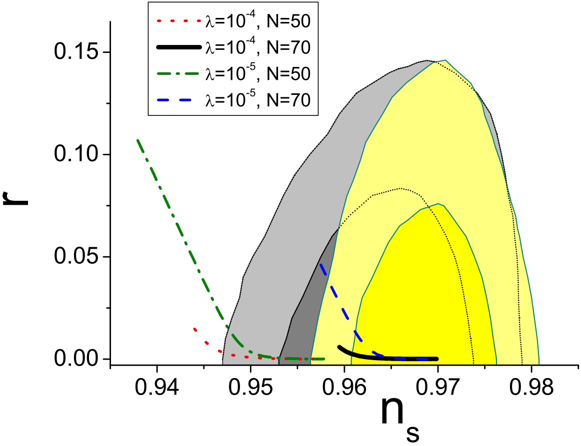


Figure: We depict the predictions of our scenario, for varying $n$ (between 4 and 20), and $\lambda$ being $10^{-4}$ or $10^{-5}$, with the e-folding value $\mathcal{N}$ being 50 or 70.

# Effect of Neutrinos Coupled with Scalar Field

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = f(\phi)$$

- Non-minimal coupling modifies the EoM of scalar field $\Longrightarrow \tilde{\gamma}\alpha\,(\rho_\nu - 3p_\nu)$.
- $p_\nu = \frac{1}{3}\rho_\nu \Longrightarrow$ Neutrinos behave like radiation $\Longrightarrow$ No modification.
- Modification comes into play only when neutrinos become nonrelativistic $\Longrightarrow p_\nu = 0$ $\Longrightarrow$ Effective potential forms $\Longrightarrow V_{\rm eff} = V(\phi) + f(\phi)$ where $f(\phi)$ is a growing function.

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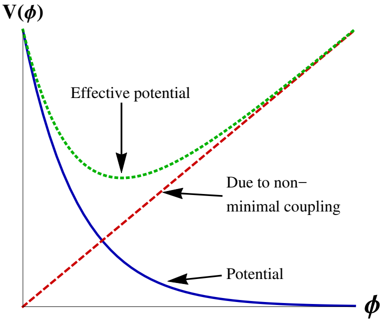


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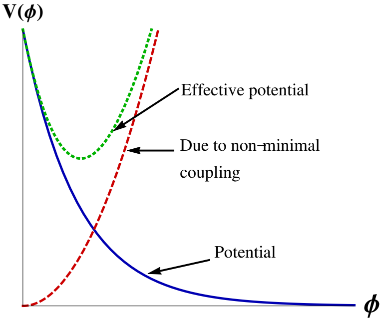


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# Dark Energy Scale

Effective potential for exponential potential

$$V_{\rm eff}(\phi) = V(\phi) + \hat{\rho}_\nu e^{(\tilde{\gamma}\alpha\phi/M_{\rm Pl})} \, .$$

where $\hat{\rho}_\nu = \rho_\nu e^{-(\tilde{\gamma}\alpha\phi/M_{\rm Pl})}$ is independent of $\phi$.

Effective potential at the minimum,

$$V_{\rm eff,min} = (1+\tilde{\gamma})\rho_\nu(\phi_{\rm min}) \, .$$

$\Longrightarrow \rho_{\rm DE} \approx V_{\rm eff,min} \sim \rho_\nu \Longrightarrow$ Sets dark energy scale through neutrino mass scale.

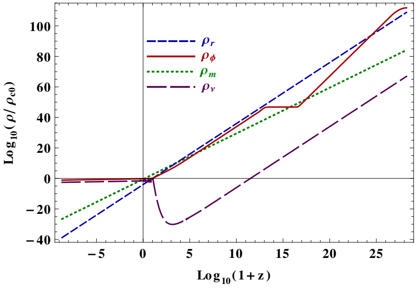
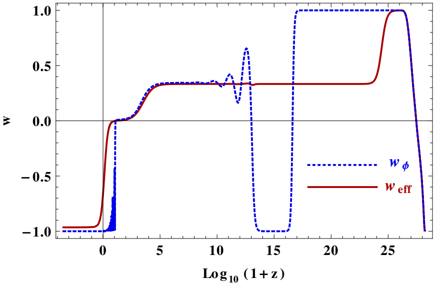



Figure:
$\gamma = 800$, $\lambda = 10^{-8}$ , $n = 6$ , $z_{\rm eq} = 2.55$ and $z_{\rm dur} = 3$.

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# Summary

- Quintessential inflation is a unified model of inflation and dark energy.
- Large tensor to scalar ratio can be achieved for noncanonical case.
- Small tensor to scalar ratio can be achieved for canonical case with potential steeper than exponential.
- Due the steep behavior of the potential tracker behavior is achieved during the post-inflationary era.
- Nonminimal coupling between massive neutrino and scalar field can give dark energy.

**THANK YOU**