



Third Quantization of Taub Universe

 Quantum Field Theory of Taub Cosmology

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[Why "Third" Quantization ?]

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physical system → matter + gravity in which :
matter = dynamical field (to be second quantized)
gravity = background geometry~ arena (to be third quantized)
That is, the gravity is an arena (play ground) on which the (quantum) matter propagates!
  Quantum Gravity versus Quantum Cosmology
  Quantum Gravity
the degree of freedom to be quantized → metric tensor
the quantization scheme \rightarrow covariant quantization in which:
            metric tensor = background field + quantum fluctuation [perturbation expansion]
                  Lorentz invariance → preserved
                 Renormalization \rightarrow Einstein's general theory of relativity is just
                 1-loop renormalizable [ 'tHooft - Veltman ]
  Quantum Cosmology
the degree of freedom to be quantized → state vector (universe wave function
which is a solution to the Wheeler-DeWitt eq.)
the quantization scheme → canonical(Hamiltonian) quantization in which:
                  universe wave function = creation/annihilation op. *mode functions [mode expansion]
              Lorentz invariance → broken [ADM's space-plus-time split formalism]
          Issue to be addressed \rightarrow (time) evolution of the universe state(wave function)
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[from "Quantum Mechanics" to "Quantum Field Theory" of cosmology]

Quantum Mechanics (1st Quantization) of the universe wave function(=state): due to Wheeler-DeWitt

Classical Hamiltonian constraint H=0. [time-re parametrization invariance in the Einstein-Hilbert action]

 \rightarrow [off-shell promotion] the Wheeler-DeWitt (WDW) equation $^{\text{h}}_{\text{op}} = 0$.

Quantum Field Theory (2nd Quantization) of the universe field operator : due to Hosoya-Morikawa

- a Lagrangian (action)
- →[on-shell condition] the Euler-Lagrange's field equation ~Wheeler-DeWitt (WDW) equation

 $^{\text{H}_{\text{op}}}=0$

-> this is the starting point of :Third Quantization of Taub Universe

Quantum Field Theory of Taub Cosmology

[Bogoliubov transformation] which is responsible for the

creation/production of quanta (universes in the Quantum Gravity/Cosmology case) in the 2nd Quantization(i.e., Quantum Field Theory)

In the context of the canonical(Hamiltonian) quantization in ordinary particle field theory, or [ADM's space-plus-time split formalism] for the case of quantum cosmology,

the choice of mode functions to span the (universe) field operator is NOT unique as each set of mode functions is separately complete, orthonormal (basis in the Hilbert space).

This is because the physical state of the system (subject to the mode expansions) keeps changing during its (time) evolution.

As a result, the out-state ($t \rightarrow =+infty$) mode functions can be expanded in terms of the the in-state ($t \rightarrow =-infty$) mode functions and vice versa

→ This (linear) Bogoliubov transformations result in non-trivial mode(or, frequency) mixing, Which eventually leads to the creation/production of quanta (universes for the case of quantum cosmology)!

[Bogoliubov transformation] in terms of mode functions

the Bogoliubov coefficient C2 is responsible for "frequency-mixing" that associates the negative-frequency in/out modes with the positive-frequency in/out modes.

[Bogoliubov transformation] in terms of ladder operators

the Bogoliubov coefficient C2 is associated with "mode-mixing" that associates in/out creation operator with out/in annihilation operator.

à Therefore, this Bogoliubov coefficient C2 is of our major interest and concern as It is the one that is responsible for the frequency/ mode-mixing!

[What is the Taub Cosmology model ?]

A.H. Taub, Am. Math. <u>53</u>, 472 (1951)

Bianchi – type IX (Mixmaster) Cosmology

: the most general context of homogeneous but anisotropic cosmology model

→ take a special limit

Taub (universe) Cosmology model

: the simplest homogeneous but anisotropic cosmology model



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Bianchi - type IX (Mixmaster)

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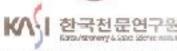


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Taub Universe limit

Branchi-type IX cosmology





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[Concluding Remarks-after applying to Taub Universe]

- [1] Creation of largely-anisotropic universes with (lambda \neq 0) from "nothing"
- * Probably, we are living in one of those universes in which the cosmological constant (lambda) was non-zero from the beginning and had large anisotropy and at the same time slowly-varying anisotropy trend since they would have been produced in large numbers at the time of creation!
- * In addition, these (Taub) universes with large anisotropy but with slowly-varying anisotropy trend Would have gone through an exponential expansion (i.e., inflation) period after the creation due to the non-zero cosmological constant (lambda \neq 0).
- Therefore, any initial large anisotropy would have been isotropized rapidly.
- * Certainly, this is consistent with the homogeneous, isotropic LSS of the present universe!
- [2] Creation of nearly-isotropic universes with (lambda = 0) from "nothing"
- * Probably, we are living in one of those universes in which the cosmological constant (lambda) was zero from the beginning and had small anisotropy and at the same time slowly-varying anisotropy trend since they would have been produced in large numbers at the time of creation!
- *Certainly, once again, this is consistent with the homogeneous, isotropic LSS of the present universe!





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Chapter I

Creation

largely-anisotropic with (140)



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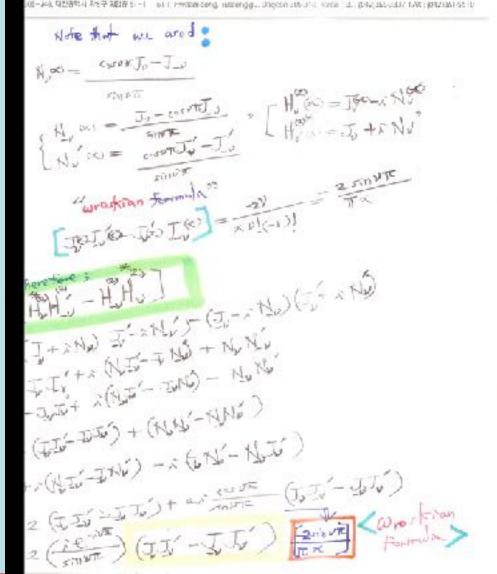


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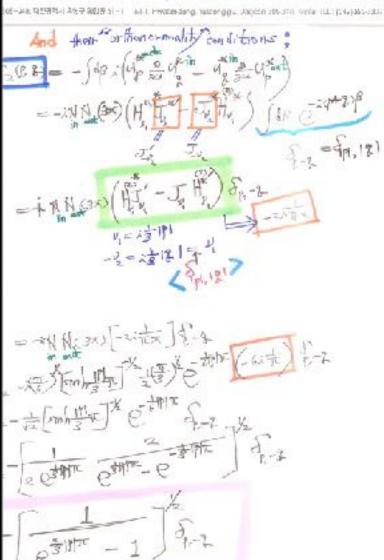


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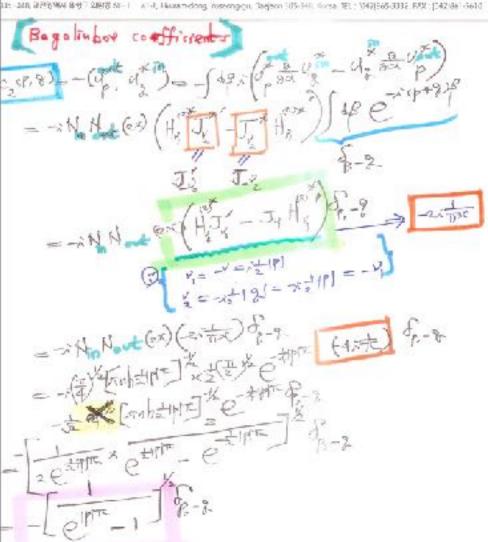
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To concludes [page Tand universes with small animotropy an at the rame time Cuten commological roomant A is when obably we are living in one of those ivenses in which the cosmological constant ias zero from the beginning and d small anisotropy and at the same fine lowly-varying anisotropy frend since by would have been produced in rge numbers at the time of crentron

