Exact holography of massive M2-brane theories and entanglement entropy

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Outline

- Mass-deformed ABJM theory and LLM geometry
- Construction of 4-dimensional LLM
- Exact holography and entanglement entropy
- Summary

Mass-deformed ABJM and LLM

- Duality properties of field theory and gravity theory

Quantum field theory	duality ⟺	Quantum gravity
d-dim		(d+1)-dim
Strongly coupled gauge theory	Large N ⟺	Weakly curved gravity (classical gravity)

➔ **Useful but difficult to check the duality!**

- Top-down and bottom-up approaches
- Most of works are focused on CFT and gravity on AdS
- We consider a non-conformal QFT in top-down approach with exact gauge/gravity duality

Mass-deformed ABJM and LLM

- N=6 Aharony-Bergman-Jafferis-Maldacena(ABJM) theory : low energy effective action of N coincident M2-branes on the C^4/Z_k orbifold fixed point

$\mathcal{N}$ =6 ABJM

dual gravity(large N) → **M-theory on** $\mathrm{AdS}_4 \times \mathrm{S}^7/\mathrm{Z}_k$

$k \ll N^{\frac{1}{5}}$

dual gravity(large N) → **IIA theory on** $\mathrm{AdS}_4 \times \mathbb{CP}^3$

$N^{\frac{1}{5}} \ll k \ll N$

↓ SUSY preserving mass deformation

$\mathcal{N}$ =6 mABJM

[Hosomichi et al 08]
[Gomis et al 08]

➔ Effective action of M2-branes in the presence of the transverse self-dual 4-form flux

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➔ Mass deformation ➔ breakdown of conformal symmetry

Mass-deformed ABJM and LLM

ABJM (CFT) + mass deformation = mABJM (non-CFT)

Vacua of mABJM ⟺ LLM geometry with $\mathbb{Z}_k$ orbifold

[Lin-Lunin-Maldacena, 2004]

$$\mathcal{M}_1^{(n)} = \begin{pmatrix} \sqrt{n} & 0 & & & & \\ & \sqrt{n-1} & 0 & & & \\ & & \ddots & \ddots & & \\ & & & \sqrt{2} & 0 & \\ & & & & 1 & 0 \end{pmatrix}$$

$$\mathcal{M}_2^{(n)} = \begin{pmatrix} 0 & 1 & & & & \\ & 0 & \sqrt{2} & & & \\ & & \ddots & \ddots & & \\ & & & 0 & \sqrt{n-1} & \\ & & & & 0 & \sqrt{n} \end{pmatrix}$$

Discrete Higgs vacua denoted by numerical values

➔Quantitative comparison is possible!!

[Gomis et al 08]

[Cheon-Kim-Kim 2011]

Mass-deformed ABJM and LLM

$n_i \times (n_i+1)$ **matrices**

$$Z^a = \sqrt{\frac{\mu k}{2\pi}} \begin{pmatrix} \mathcal{M}_a^{(n_1)} & & & & & \\ & \ddots & & & & \\ & & \mathcal{M}_a^{(n_i)} & & & \\ & & & \mathbf{0}_{(n_{i+1}+1)\times n_{i+1}} & & \\ & & & & \ddots & \\ & & & & & \mathbf{0}_{(n_f+1)\times n_f} \end{pmatrix}$$

$$W^{\dagger a} = \sqrt{\frac{\mu k}{2\pi}} \begin{pmatrix} \mathbf{0}_{n_1\times(n_1+1)} & & & & & \\ & \ddots & & & & \\ & & \mathbf{0}_{n_i\times(n_i+1)} & & & \\ & & & \bar{\mathcal{M}}_a^{(n_{i+1})} & & \\ & & & & \ddots & \\ & & & & & \bar{\mathcal{M}}_a^{(n_f)} \end{pmatrix}$$

$(n_i+1) \times n_i$ **matrices**

- Constraint: $\sum_{n=0}^{N-1} [nN_n + (n+1)N'_n] = N, \qquad \sum_{n=0}^{N-1} [(n+1)N_n + nN'_n] = N$
- Condition for supersymmetric vacua: $0 \le N_n \le k, \qquad 0 \le N'_n \le k$

[Kim-Kim 10]

Mass-deformed ABJM and LLM

Vacua are classified by occupation numbers

$$\{N_n, N'_n\}$$

Number of vacua for a given N (k=1)= partition of N

P(N)

For large N, P(N) $\sim e^{\pi\sqrt{\frac{2}{3}}\sqrt{N}}$

Mass-deformed ABJM and LLM

- Half-BPS solutions with SO(2,1)XSO(4)XSO(4) isometry in 11-dimensional supergravity [04, Lin-Lunin-Maldacena]

$$ds^2 = -G_{tt}\left(-dt^2 + dw_1^2 + dw_2^2\right) + G_{xx}(dx^2 + dy^2) + G_{\theta\theta} ds^2_{S^3/\mathbb{Z}_k} + G_{\tilde\theta\tilde\theta} ds^2_{\tilde S^3/\mathbb{Z}_k}$$

$$-G_{tt} = \left(\frac{4\mu_0^2 y\sqrt{\frac{1}{4} - z^2}}{f^2}\right)^{2/3},$$

$$G_{xx} = \left(\frac{f\sqrt{\frac{1}{4} - z^2}}{2\mu_0 y^2}\right)^{2/3},$$

$$G_{\theta\theta} = \left(\frac{fy\sqrt{\frac{1}{2} + z}}{2\mu_0\left(\frac{1}{2} - z\right)}\right)^{2/3},$$

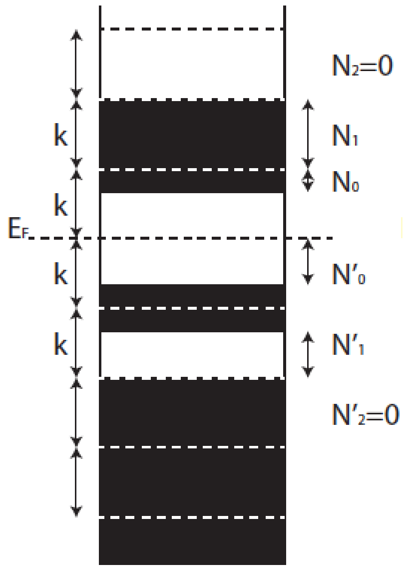
$$G_{\tilde\theta\tilde\theta} = \left(\frac{fy\sqrt{\frac{1}{2} - z}}{2\mu_0\left(\frac{1}{2} + z\right)}\right)^{2/3}.$$

$$f(x,y) = \sqrt{1 - 4z^2 - 4y^2V^2}.$$

Mass-deformed ABJM and LLM

- This solution is completely determined by two functions:

$$z(x,y) = \sum_{i=1}^{2m+1} \frac{(-1)^{i+1}(x-x_i)}{2\sqrt{(x-x_i)^2+y^2}}, \qquad V(x,y) = \sum_{i=1}^{2m+1} \frac{(-1)^{i+1}}{2\sqrt{(x-x_i)^2+y^2}}.$$



$y = 0$ case

$$z = \frac{1}{2} \text{ or } z = -\frac{1}{2}$$

white strip   black strip

Vacuum is identified by the occupation numbers :

$$\{N_n, N'_n\}$$

[Cheon-Kim-Kim 2011]

Mass-deformed ABJM and LLM

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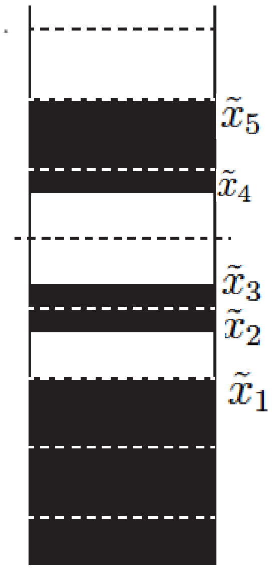
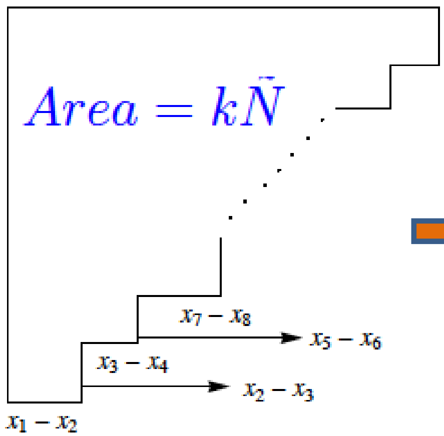
$$z(x,y)=\sum_{i=1}^{2m+1}\frac{(-1)^{i+1}(x-x_i)}{2\sqrt{(x-x_i)^2+y^2}},\qquad V(x,y)=\sum_{i=1}^{2m+1}\frac{(-1)^{i+1}}{2\sqrt{(x-x_i)^2+y^2}}$$



droplet



Young diagram

$$C_p=\sum_{i=1}^{2N_B+1}(-1)^{i+1}\left(\frac{\tilde{x}_i}{2\pi l_{\rm P}^3\mu_0\sqrt{k\tilde{N}}}\right)^p$$

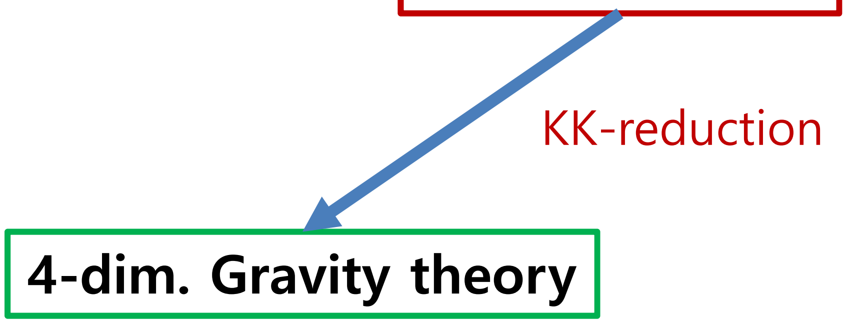
$$C_2-C_1^2=2$$

$\tilde{x}_i \rightarrow C_p$

new parametrization

Construction of 4-dimensional gravity

- To show the dual relation of two theories, correspondence between vacua of QFT and BPS solutions in gravity is not enough.
  - **mABJM theory: 3-dim. gauge theory**
  - **LLM geometry: BPS solution in 11-dim. SUGRA**



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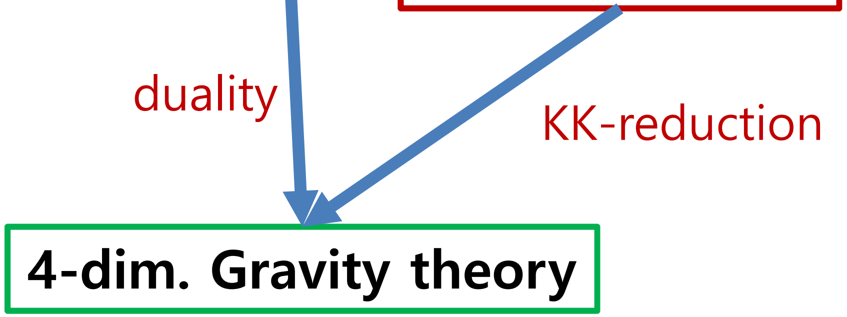


Construction of 4-dimensional gravity

- Equation of motion for g_{pq} and C_{pqr} :

$$R_{pq} - \frac{1}{2} g_{pq} R = \frac{1}{48}\Big(-\frac{1}{2} g_{pq} F_{rstu} F^{rstu} + 4 F_{pstu} F_{q}{}^{stu} \Big)$$

$$\partial_p (F^{pqrs}) + \frac{e}{2\cdot(4!)^2} \epsilon^{qrsu_1\cdots u_4 v_1 \cdots v_4} F_{u_1\cdots u_4} F_{v_1\cdots v_4} = 0 \qquad e = \sqrt{-g}$$

- Fluctuations around $\mathrm{AdS}_4 \times S^7$

$$g_{pq} = g^0_{pq} + h_{pq},$$

$$C_{pqr} = C^0_{pqr} + c_{pqr} \quad \Longleftrightarrow \quad F_{pqrs} = F^0_{pqrs} + f_{pqrs}$$

  - LLM geometry is asymptotically $\mathrm{AdS}_4 \times S^7$ + fluctuations
  - From the asymptotic expansion of the LLM geometry, we can read all h_{pq} and f_{pqrs}

Construction of 4-dimensional gravity

- Expansion in S^7 spherical harmonics:

$$h_{\mu\nu}(x,y) = h^{I_1}_{\mu\nu}(x)Y^{I_1}(y),$$
$$h_{\mu a}(x,y) = v^{I_7}_{\mu}(x)Y^{I_7}_{a}(y) + s^{I_1}_{\mu}(x)\nabla_a Y^{I_1}(y),$$
$$h_{(ab)}(x,y) = t^{I_{27}}(x)Y^{I_{27}}_{(ab)}(y) + v^{I_7}(x)\nabla_{(a}Y^{I_7}_{b)}(y) + s^{I_1}(x)\nabla_{(a}\nabla_{b)}Y^{I_1}(y),$$
$$h^{a}{}_{a}(x,y) = \phi^{I_1}(x)Y^{I_1}(y),$$

$$T_{(ab)} = \frac{1}{2}(T_{ab} + T_{ba}) - \frac{1}{7}g_{ab}T^{c}{}_{c}$$

$$f_{\mu\nu\rho\sigma}(x,y) = 4\nabla_{[\mu}s^{I_1}_{\nu\rho\sigma]}(x)Y^{I_1}(y),$$
$$f_{\mu\nu\rho a}(x,y) = 3\nabla_{[\mu}v^{I_7}_{\nu\rho]}(x)Y^{I_7}_{a}(y) - s^{I_1}_{\mu\nu\rho}(x)\nabla_a Y^{I_1}(y),$$
$$f_{\mu\nu ab}(x,y) = 2\nabla_{[\mu}t^{I_{21}}_{\nu]}(x)Y^{I_{21}}_{[ab]}(y) + 2v^{I_7}_{\mu\nu}(x)\nabla_{[a}Y^{I_7}_{b]}(y),$$
$$f_{\mu abc}(x,y) = \nabla_{\mu}t^{I_{35}}(x)Y^{I_{35}}_{[abc]}(y) - 3t^{I_{21}}_{\mu}(x)\nabla_{[a}Y^{I_{21}}_{bc]}(y),$$
$$f_{abcd}(x,y) = 4t^{I_{35}}(x)\nabla_{[a}Y^{I_{35}}_{bcd]}(y),$$

[H.J. Kim et al 1985, S.Lee et al 1998
Skenderis-Taylor 2006,…]

Construction of 4-dimensional gravity

- Near the UV fixed point where the ABJM theory lives on, (in the small mass expansion,) the 4-dimensional gravity action is given by

$$S_{4d} = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\,(R-2\Lambda) + A_T\int d^4x\sqrt{-g}\left(-\frac{1}{2}\nabla_\mu T\nabla^\mu T - \frac{1}{2}M_T^2T^2\right) + A_\psi\int d^4x\sqrt{-g}\left(-\frac{1}{2}\nabla_\mu\psi\nabla^\mu\psi - \frac{1}{2}M_\psi^2\psi^2\right)$$

Asymptotically AdS solution with relevant deformations

$$\psi = \frac{-162\hat{\phi}^{I_1=2} + 7\hat{\psi}^{I_1=2}}{70}, \qquad \hat{\psi}^{I_1=2} = 18h^{I_1=2} - U^{I_1=2}, \qquad \hat{\phi}^{I_1=2} = \phi^{I_1=2} + \frac{16}{L^2}S^{I_1=2}$$

$$T = T^{I_{35}=1}$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

$$\left(\Box - M_T^2\right)T = 0$$

$$\left(\Box - M_\psi^2\right)\psi = 0$$

Construction of 4-dimensional gravity

- Fluctuations around AdS space: $g_{\mu\nu} = g^{(0)}_{\mu\nu} + H_{\mu\nu}$

$$\frac{1}{2}\Big(- \Box H_{\mu\nu} + \nabla^\rho \nabla_\mu H_{\rho\nu} + \nabla^\rho \nabla_\nu H_{\rho\mu} - \nabla_\mu \nabla_\nu H \Big) + \frac{12}{L^2} H_{\mu\nu} - \frac{6}{L^2} g_{\mu\nu} H - \frac{1}{2} g_{\mu\nu} (\nabla^\rho \nabla^\sigma H_{\rho\sigma} - \Box H) = 8\pi G_N T_{\mu\nu}.$$

- To define 4-dimensional fields, we need field redefinition in terms of 11-dimensional modes with derivatives:

$$H_{\mu\nu} = h^0_{\mu\nu} - \frac{1}{4} g_{\mu\nu} \phi^0 + \frac{1}{24} g_{\mu\nu} \left(18 h^0 - U^0\right) + g_{\mu\nu} \left(C_1 \psi^2 + C_2 \nabla^\rho \psi \nabla_\rho \psi\right) + C_3 \nabla_\mu \psi \nabla_\nu \psi + g_{\mu\nu} C_4 \nabla^\rho \nabla^\sigma \psi \nabla_\rho \nabla_\sigma \psi + C_5 \nabla_\mu \nabla^\rho \psi \nabla_\nu \nabla_\rho \psi + g_{\mu\nu} C_T T^2$$

$$C_1 = -\frac{1}{40}\frac{1}{2^3 \times 3^3}, \quad C_2 = -\frac{1}{40}\frac{L^2}{2^8 \times 3^3}, \quad C_3 = -\frac{1}{40}\frac{7L^2}{2^8 \times 3^4}, \quad C_4 = -\frac{1}{40}\frac{L^4}{2^{11} \times 3^3},$$

$$C_5 = -\frac{1}{40}\frac{L^4}{2^{10} \times 3^4}, \quad C_T = -\frac{L^2}{2^5 \times 3}, \quad 8\pi G_N A_T = -\frac{L^2}{2^5 \times 3}, \quad 8\pi G_N A_\psi = -\frac{1}{2^8 \times 3^2}$$

Construction of 4-dimensional gravity

- From the asymptotic expansion of the 11d LLM solution, we read 11d modes and obtain the solutions of 4-dimensional fields, where **the whole information of the 11d LLM solutions (for all droplets) is included.**

$$H_{ij} = -\frac{L^2\mu_0^2}{180}\left(30+\beta_3^2\right)\eta_{ij} + \mathcal{O}\left(\mu_0^3\right), \quad (i,j=0,1,2) \qquad H_{zz} = -\frac{L^2\mu_0^2}{1440}\left(960+29\beta_3^2\right) + \mathcal{O}\left(\mu_0^3\right)$$

$$\psi = -24\beta_3\mu_0 z + \mathcal{O}\left(\mu_0^2\right), \qquad T = 16\sqrt{3}\mu_0 z + \mathcal{O}\left(\mu_0^2\right)$$

$$\beta_3 \equiv 2C_1^3 - 3C_1C_2 + C_3 \qquad C_p = \sum_{i=1}^{2N_B+1}(-1)^{i+1}\left(\frac{\tilde{x}_i}{2\pi l_{\rm P}^3\mu_0\sqrt{k\tilde{N}}}\right)^p$$

- One can examine the 4-dimensional gravity action:
  1. Vev of a chiral primary operator (CPO) with conformal dim. 1
  2. Holographic entanglement entropy

Exact holography

- Dictionary of the gauge/gravity duality (GKP-W relation)

$$\langle \mathcal{O}^{(\Delta)} \rangle = \mathbb{N}\phi_{(\Delta)}.$$ $\mathbb{N}$ is a normalization factor.

$$\langle \mathcal{O}^{(1)} \rangle = -24\mathbb{N}\mu_0\beta_3$$

- **Vevs of the CPO with conformal dimension 1 in mABJM**

$$\langle \mathcal{O}^{(\Delta=1)} \rangle = \frac{\mu k}{4\sqrt{2}\pi}\sum_{n=0}^{\infty}(N_n - N'_n)n(n+1)$$

→ Field theory calculation in the large N limit

$$\mathbb{N} = -\frac{N^{\frac{3}{2}}}{72\sqrt{2}\pi}$$

$$\{l_n, l'_n\} \Longleftrightarrow \{N_n, N'_n\}$$

$$\sum_{n=0}^{\infty}\left[n(n+1)(l_n - l'_n)\right] = \frac{N^{3/2}}{3}\beta_3,$$

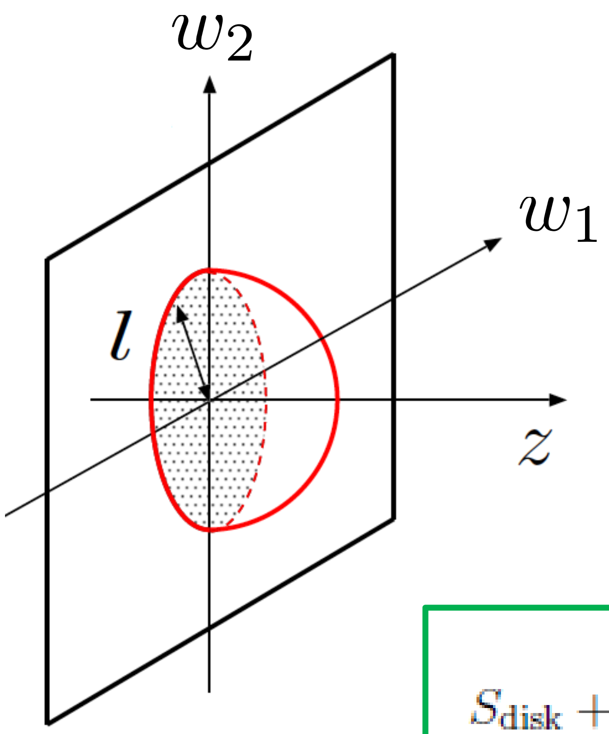
$$\langle \mathcal{O}^{(1)} \rangle = \frac{N^{\frac{3}{2}}\mu_0}{3\sqrt{2}\,\pi}\beta_3$$

→ GKP-W relation

→ Exact holography for all droplets

Holographic entanglement entropy

- RT conjecture for the holographic entanglement entropy



$$ds^2_{\rm AdS} = \frac{L^2_{\rm AdS}}{z^2}\left(-dt^2 + dw_1^2 + dw_2^2 + dz^2\right)$$
$$= \frac{L^2}{4z^2}\left(-dt^2 + dw_1^2 + dw_2^2 + dz^2\right),$$

$$t = \text{constant}, \quad w_1 = \rho\cos\sigma^1, \quad w_2 = \rho\sin\sigma^1, \quad z = z(\rho), \quad \rho = \sigma^2.$$

$$S_{\rm disk} = \frac{\pi L^2}{8G_N}\left(\frac{l}{\epsilon} - 1\right)$$

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + H_{\mu\nu}$$

$$S_{\rm disk} + \delta S = \frac{\pi L^2}{8G_N}\left[\frac{l}{\epsilon} - 1 - (\mu_0 l)^2\left(\frac{4}{3} + \frac{1}{24}\beta_3^2\right)\right]$$

This result exactly reproduces the HEE using PDE from the 11d LLM.

[Kim-Kim-OK 2016]

Conclusion

- Using the LLM geometry in 4-dim. gravity:
  1. $\delta S = \delta S(\langle \mathcal{O}_i^{(\Delta=1)} \rangle)$
  2. thermodynamic-like first law $\delta S = \delta E$
     for all droplet solution in non-conformal field theory
  3. $\delta S \leftrightarrow g_{\mu\nu}$ relation using exact holography
     and the entropic counterpart of the Einstein equation

- IR entanglement entropy for the mABJM and LLM