Walls of massive Kähler sigma models on SO(2N)/U(N) in three dimensions

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Contents

- moduli matrices of walls on the Grassmann manifold [Y.Isozumi& M.Nitta & K.Ohashi & N.Sakai (2004)]
- moduli matrices of walls on $SO(2N)/U(N)$ $(N \leq 3)$ [M.Arai & SS (2011)]
  moduli matrices of magnetic monopoles on $SO(2N)/U(N)$ [M.Eto& T.Fujimori & S.B.Gudnason & Y.Jiang & K.Konishi & M.Nitta & K.Ohashi (2011)]
- moduli matrices of walls on $SO(2N)/U(N)$ $(N > 3)$ [B-H. Lee & C. Park & SS (2017)]
  $\rightarrow$ penetrable walls

What we know $\cdots$

- The strong gauge coupling limit of $U(N_C)$ gauge theory $(N_F > N_C)$ becomes the complex Grassmann manifold G_{N_F,N_C}.
- It is shown that the vector multiplet part of BPS eq. does not produce additional moduli parameters. It is proven in the case of compact Kähler base space and domain walls in $U(1)$ and non-Abelian gauge theories, etc. [Mundet I Riera(2000)], [Cieliebak, Rita Gaio, Salamon(2000)], [Sakai,Yang(2005)], [Sakai,Tong(2005),K.S.M.Lee(2003)]

Lagrangian

•4D Lagrangian [K.Higashijima & M.Nitta (1999)] $\rightarrow$ massless
•mass-deformed 3D Lagrangian

$$
\begin{aligned}
&\mathcal{L}_{\mathrm{bos3D}} = \\
&-(\overline{D_\mu\phi})_i{}^a(D^\mu\phi)_a{}^i - |i\phi_a{}^j M_j{}^i - i\Sigma_a{}^b\phi_b{}^i|^2 + |F_a{}^i|^2 + \frac{1}{2}(D_a{}^b\phi_b{}^i\bar{\phi}_i{}^a - D_a{}^a) \\
&+\Big[(F_0)^{ab}\phi_b{}^i J_{ij}\phi^{Tj}{}_a + (\phi_0)^{ab}F_b{}^i J_{ij}\phi^{Tj}{}_a + (\phi_0)^{ab}\phi_b{}^i J_{ij}F^{Tj}{}_a + c.c.\Big]
\end{aligned}
$$

$$
\begin{aligned}
\phi_a{}^i\bar{\phi}_i{}^b - \delta_a{}^b &= 0 \\
\phi_a{}^i J_{ij}\phi^{Tj}{}_b &= 0
\end{aligned}
$$

•J: invariant tensor of $O(2N)$

$$
J_{2N} = \sigma^1 \otimes I_N,
$$

•potential

$$
V = |i\phi_a{}^j M_j{}^i - i\Sigma_a{}^b\phi_b{}^i|^2 + 4|(\phi_0)^{ab}\phi_b{}^i|^2
$$

Lagrangian cont'd

•mass matrix

In this basis, Cartan generators are

$$H_n = \left(\begin{array}{c|c} h_n & \\ \hline & -h_n \end{array}\right), \quad (n = 1, \cdots, N)$$

with $N \times N$ matrix h_n which has only component 1 in (n, n) element.

$$\underline{m} := (m_1, m_2 \cdots, m_N)$$
$$m_1 > m_2 > \cdots > m_N \quad \text{w/o loss of generality}$$

$$\underline{H} := (H_1, H_2, \cdots, H_N)$$

$$M = \underline{m} \cdot \underline{H}$$

Lagrangian cont'd

•vacuum

$$\phi_a^i M_j{}^i - i\Sigma_a{}^b \phi_b{}^i = 0$$
$$(\phi_0)^{ab} = 0$$

Σ can be diagonalized by $U(N)$ transformation

$$\Sigma = \mathrm{diag}(\Sigma_1, \Sigma_2, \cdots, \Sigma_N)$$

therefore the vacua are labelled by

$$(\Sigma_1, \Sigma_2, \cdots, \Sigma_N) = (\pm m_1, \pm m_2, \cdots, \pm m_N)$$

$\rightarrow \#$ of vauca$= 2^{N-1}$
Euler's characteristic [S.B. Gudnason, Y. Jiang, K. Konishi (2010)]

BPS equation

The BPS equation for wall solutions is derived from the Bogomol'nyi completion of the Hamiltonian. It is assumed that fields are static and all the fields depend only on the $x_1 \equiv x$ coordinate. It is also assumed that there is Poincare invariance on the two-dimensional world volume of walls to set $A_0 = A_2 = 0$. The energy is saturated when

$$(D\phi)_a{}^i \mp (\phi_a{}^j M_j{}^i - \Sigma_a{}^b \phi_b{}^i) = 0.$$

We choose the upper sign for the BPS equation without loss of generality.

moduli matrices

•BPS equation

$$(D\phi)_a{}^i - (\phi_a{}^j M_j{}^i - \Sigma_a{}^b \phi_b{}^i) = 0$$

By introducing complex matrix functions $S_a{}^b(x)$ and $f_a{}^i(x)$ defined by

$$\Sigma_a{}^b - iA_a{}^b \equiv (S^{-1}\partial S)_a{}^b,\ \phi_a{}^i \equiv (S^{-1})_a{}^b f_b{}^i,$$

the BPS eq. is solved as

$$\phi_a{}^i = (S^{-1})_a{}^b H_{0b}^j (e^{Mx})_j{}^i.$$

H_0 : moduli matrix
All the quantities are invariant under the transformation

$$S_a'{}^b = V_a{}^c S_c{}^b, \quad H_{0a}'{}^i = V_a{}^c H_{0c}^i, V \in GL(N, \mathbf{C}).$$

The V defines an equivalent class of (S, H_0). $\rightarrow$world-volume symmetry
[Y.Isozumi & M.Nitta & K.Ohashi & N.Sakai(2004)].

moduli matrices cont'd

•constraints

$$\begin{array}{c} \phi_a{}^i \bar{\phi}_i{}^b - \delta_a{}^b = 0 \\ \phi_a{}^i J_{ij} \phi^{Tj}{}_b = 0 \end{array} \quad \rightarrow \quad \begin{array}{c} H_{0a}^{\ i} (e^{2Mx})_i{}^j H_{0j}^{\dagger\ b} = (S\bar{S})_a{}^b \equiv \Omega_a{}^b \\ H_{0a}^{\ i} J_{ij} H_b^{Tj} = 0 \end{array}$$

•moduli space

$$\begin{array}{l} H'_{0a}{}^{i} = V_a{}^c H_{0c}^{\ i}, V \in GL(N, \mathbf{C}) \\ H_{0a}^{\ i} J_{ij} H_b^{Tj} = 0 \end{array}$$

$\rightarrow$ Moduli space is $SO(2N)/U(N)$.

$SO(2N)/U(N)$

$$SO(4)/U(2) \simeq CP^1, \quad SO(6)/U(3) \simeq CP^3$$

$SO(2N)/U(N)$, $N \leq 3 \rightarrow$ Abelian gauge theory
$SO(2N)/U(N)$, $N > 3 \rightarrow$ non-Abelian gauge theory

In non-Abelian gauge theory, there are penerable walls.

elementary walls in Gr_{N_F,N_C}

In [Isozumi& Nitta & Ohashi & Sakai (2004)], walls are algebraically constructed from elementary walls. On the Grassmann manifold, an elementary wall connects two nearest vacua of the same color index changing the flavor by one unit. An elementary wall interpolating two vacua $\langle A\rangle$ and $\langle B\rangle$ in the flavor i and $i+1$ in the same color is $H_{0\langle A\leftarrow B\rangle} = H_{0\langle A\rangle} e^{E_i(r)}$ where $E_i(r) \equiv e^r E_i (r \in \mathbf{C})$. The E_i of an elementary wall carrying tension $T_{\langle A\leftarrow B\rangle}$ is defined by

$$[cM, E_i] = c(m_i - m_{i+1})E_i = T_{\langle i\leftarrow i+1\rangle},$$

where c is a constant, M is the mass matrix and E_i is an $N_f \times N_f$ square matrix generating an elementary wall. The E_i has an nonzero component only in the $(i, i+1)$-th element.

$\Longrightarrow$ This definition is not compatible with $SO(2N)/U(N)$.

elementary walls in $SO(2N)/U(N)$

We can generalize the formula as

$$[cM, E_i] = c(\underline{m} \cdot \underline{\alpha})E_i = T_{\langle i \leftarrow i+1 \rangle},$$

$$\underline{\alpha} := (\alpha_1, \cdots, \alpha_N), \quad \underline{m} := (m_1, \cdots, m_N),$$

where α_i are simple roots of E_i, which are positive step operators of $SO(2N)$. We can restrict ourselves to the case where $m_1 > m_2 > \cdots > m_N$ then the vector $\underline{m}$ is a vector in the interior of the positive Weyl chamber,

$$\underline{m} \cdot \underline{\alpha} > 0.$$

positive step operators & roots

$$E_i = \begin{array}{cc} & \begin{array}{cc} j & i+N \end{array} \\ \begin{array}{c} i \\ j+N \end{array} & \left(\begin{array}{c|c} 1 & \\ \hline & -1 \end{array}\right) \end{array}, \quad E_N = \begin{array}{cc} & \begin{array}{cc} i+N & j+N \end{array} \\ \begin{array}{c} i \\ j \end{array} & \left(\begin{array}{c|cc} & & 1 \\ & -1 & \\ \hline & & \end{array}\right) \end{array}$$

$(i = 1, \cdots, N-1)$

$$\begin{aligned}
&\alpha_1 = (1, -1, 0, \cdots, 0, 0, 0) \\
&\alpha_2 = (0, 1, -1, \cdots, 0, 0, 0) \\
&\cdots \\
&\alpha_{N-1} = (0, 0, 0, \cdots, 0, 1, -1) \\
&\alpha_N = (0, 0, 0, \cdots, 0, 1, 1)
\end{aligned}$$

walls

- elementary walls

$$H_{0\langle a\leftarrow b\rangle} = H_{0\langle a\rangle} e^{E_{(a\leftarrow b)}(r)}$$

- compressed wall with a level n

$$H_{0\langle a\leftarrow b\rangle} = H_{0\langle a\rangle} e^{[E_{a_1},[E_{a_2},[E_{a_3},\cdots,[E_{a_n},E_{a_{n+1}}]\cdots](r)}$$

corresponding root $g_{n+1} = g_1 + g_2 + \cdots + g_n$

- multiwalls

$$H_{0\langle a\leftarrow b\rangle} = H_{0\langle a\rangle} e^{E_{a_1}(r_1)} e^{E_{a_2}(r_2)} \dots e^{E_{a_n}(r_n)}$$

- Walls are penetrable if

$$[E_{a_i}, E_{a_j}] = 0.$$

corresponding roots $g_i \cdot g_j = 0$

walls (cont'd)

In $SO(2N)$, there are

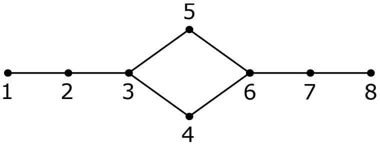
${}_{2N}C_2$ generators

N Cartan generators

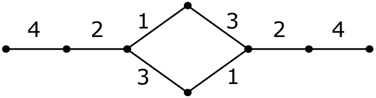
$2N^2 - 2N$ root generators

We are only interested in positive roots. So there are $N^2 - N$ root generators, which generate walls.

vacua & roots

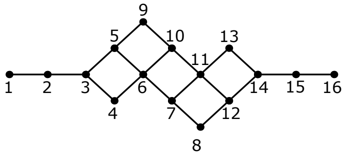


vacua on $SO(8)/U(4)$

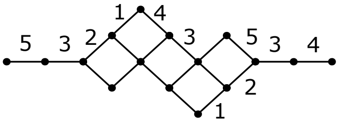


roots on $SO(8)/U(4)$

vacua & roots cont'd



vacua on $SO(10)/U(5)$



roots on $SO(10)/U(5)$

Summary

- SO(2N) constraint is imposed to the moduli matrices of walls.
- Operators which generate elementary walls are defined accordingly.
- Penetrable walls are observed in nonlinear sigma models on $SO(2N)/U(N)$ with $N > 3$.

Summary

- SO(2N) constraint is impose to the projective moduli matrices.
- Operators which generate elementary walls are defined accordingly.
- Penetrable walls are observed in nonlinear sigma models on $SO(2N)/U(N)$ with $N > 3$.

Thank you !