Bell Inequality From Holographic Gravity

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arXiv 1612.09513: Holographic Bell Inequality

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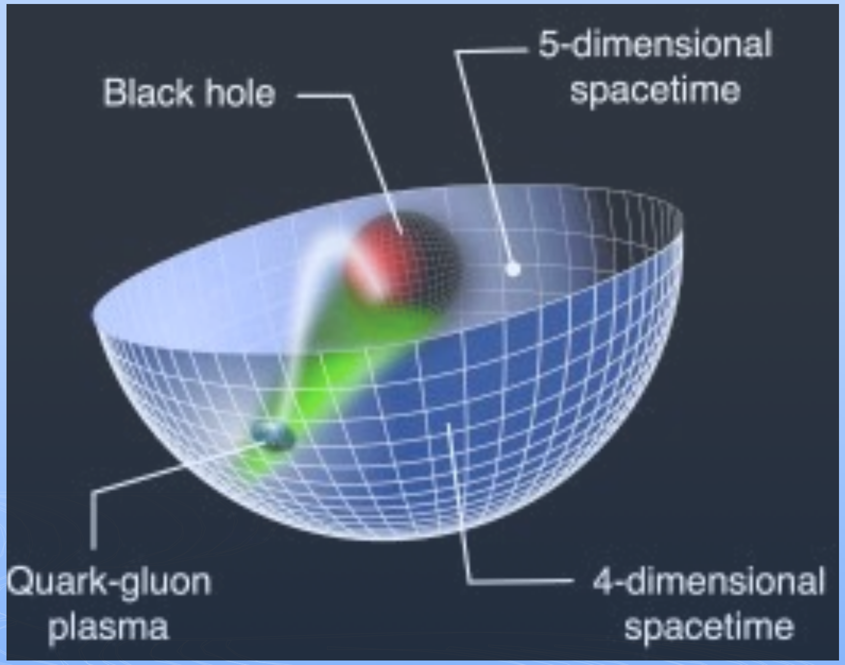
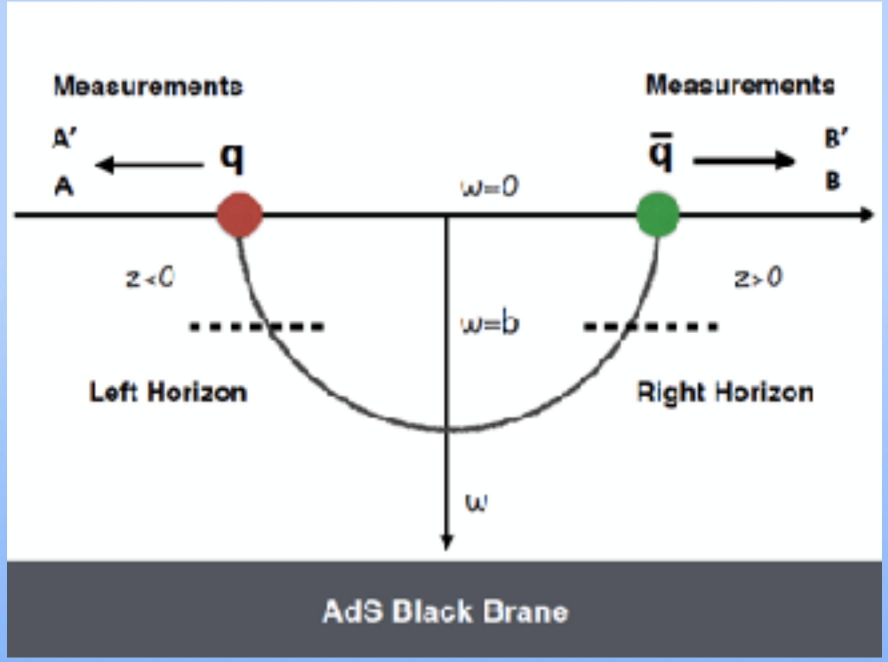
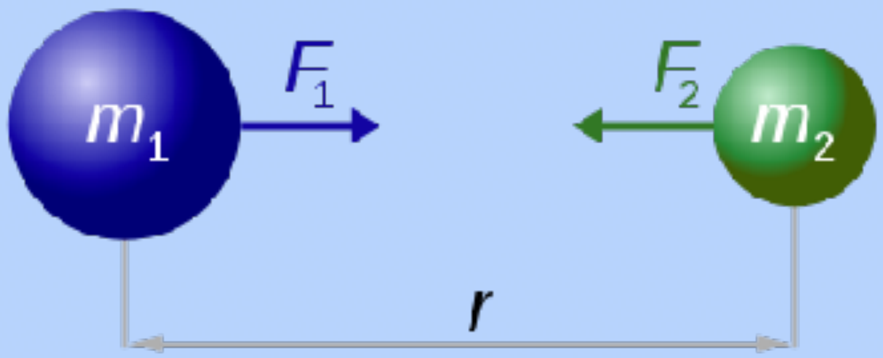
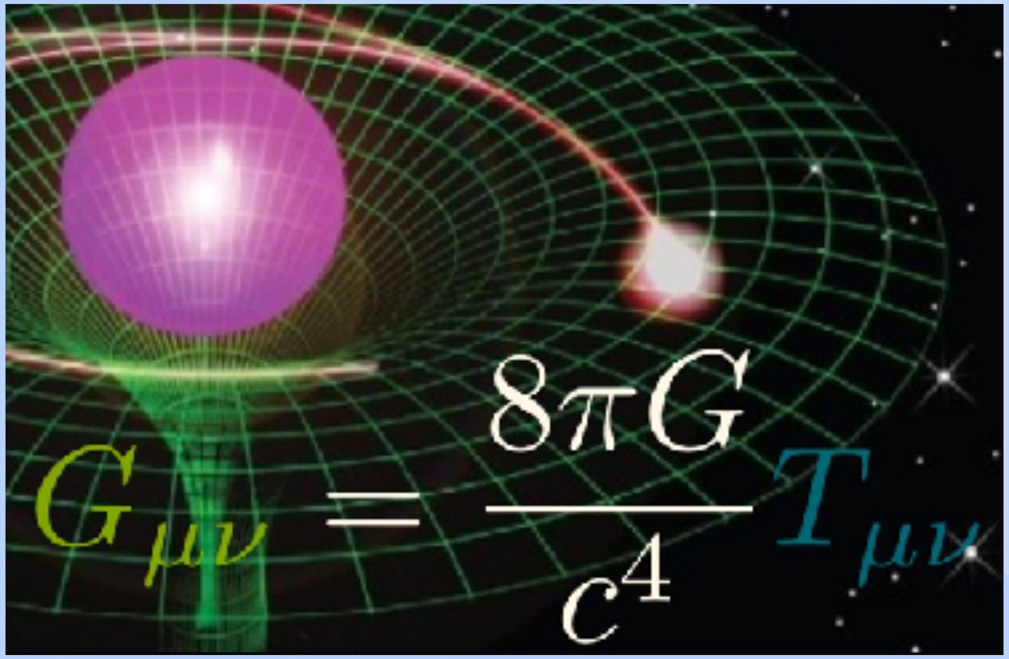




From Newton Gravity to Einstein Gravity

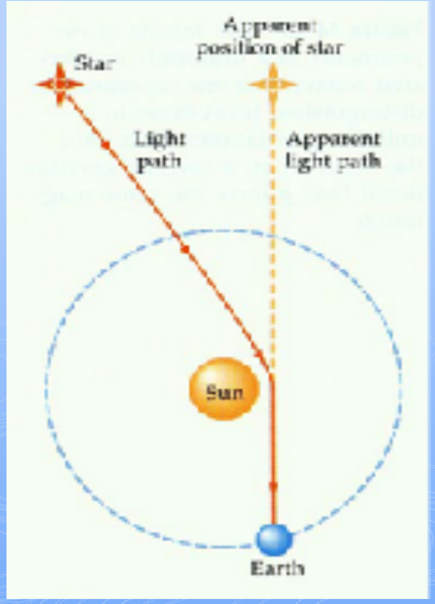


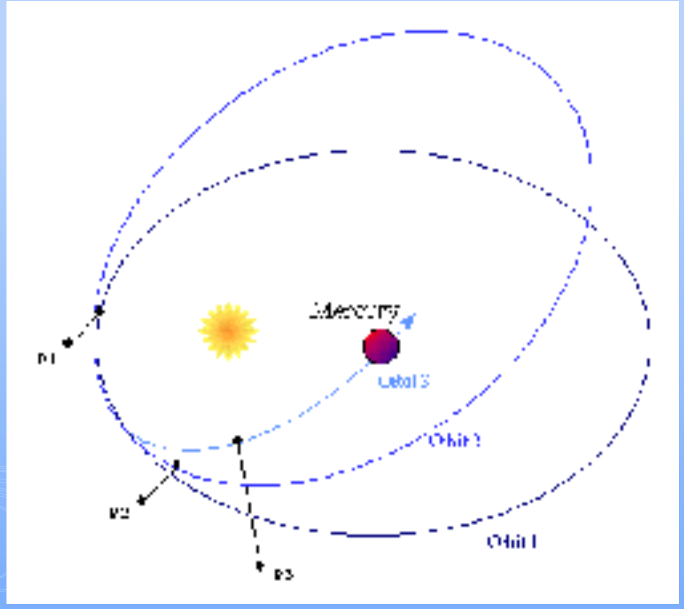
$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$

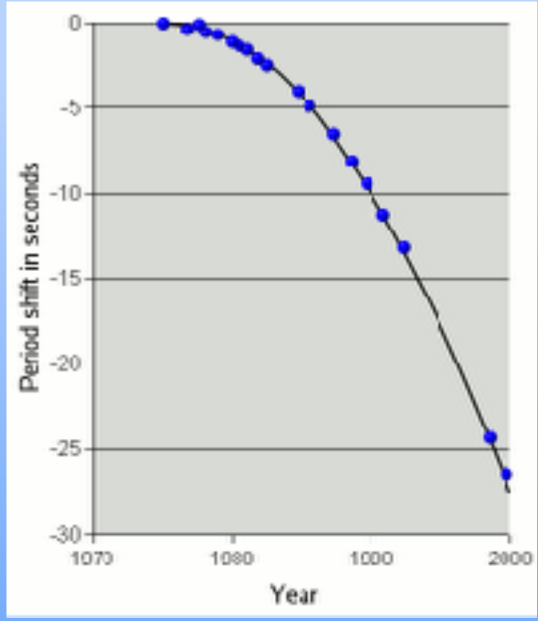


Newton's Gravity

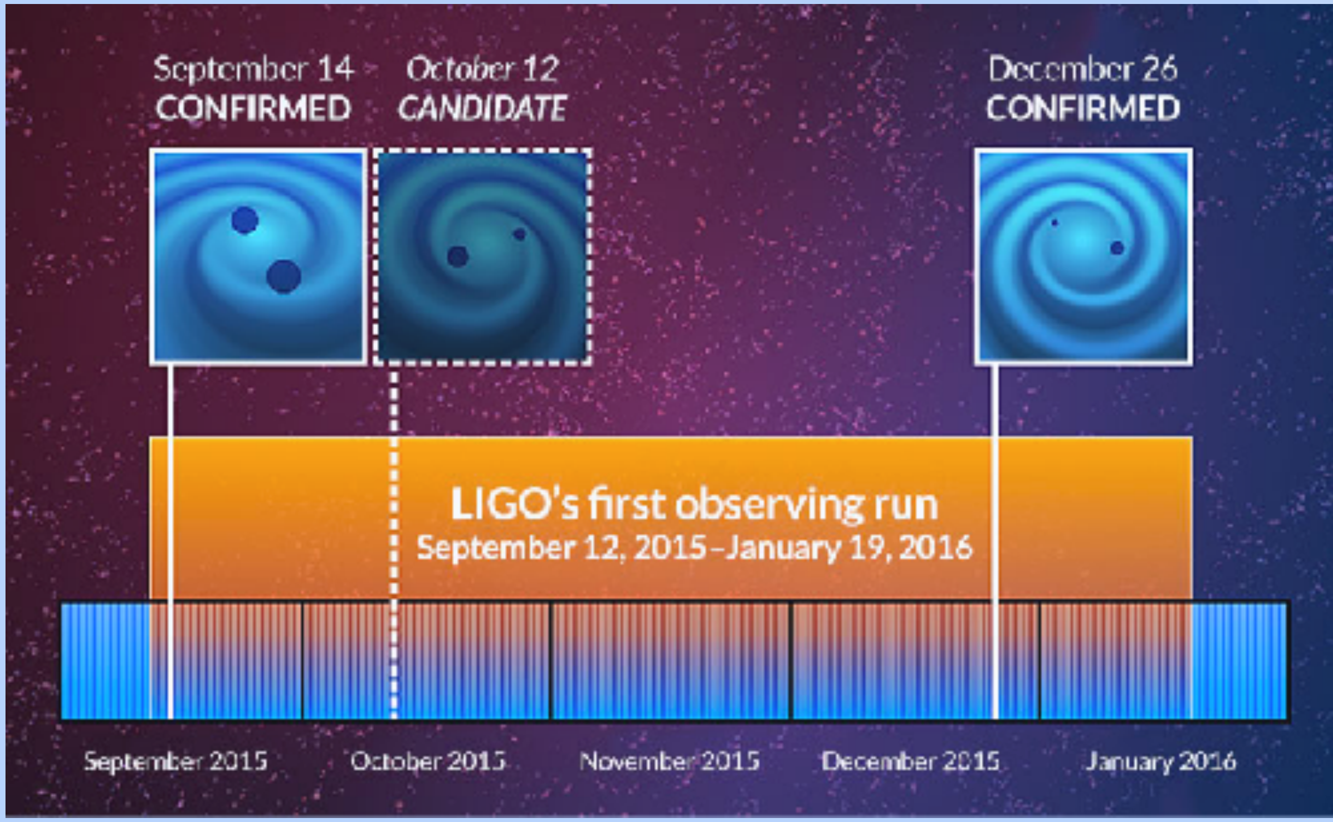
Einstein's Gravity

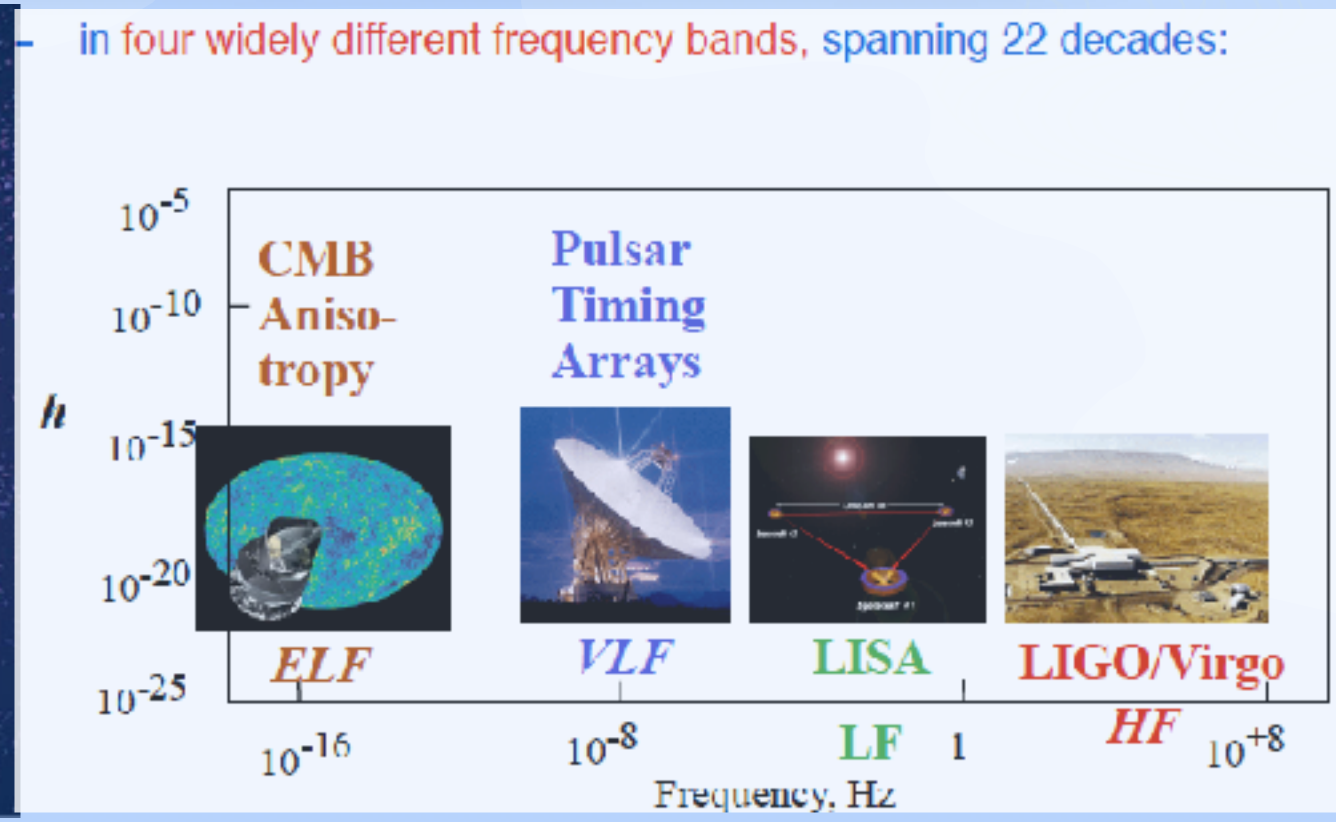






Black Hole Dynamics & Gravitational waves





Schwarzschild Solution(1916)

$$c^2 d\tau^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right),$$

Black Holes(1960s): Golden age of general relativity

Kerr-Newman Solution

Figures credit: LIGO

Holographic Properties of Gravity

(2010s) Gravity/Entanglement: Effective Metric

(2000s) AdS/CFT Duality: Black Hole in a Box

Astrophysical Black Holes

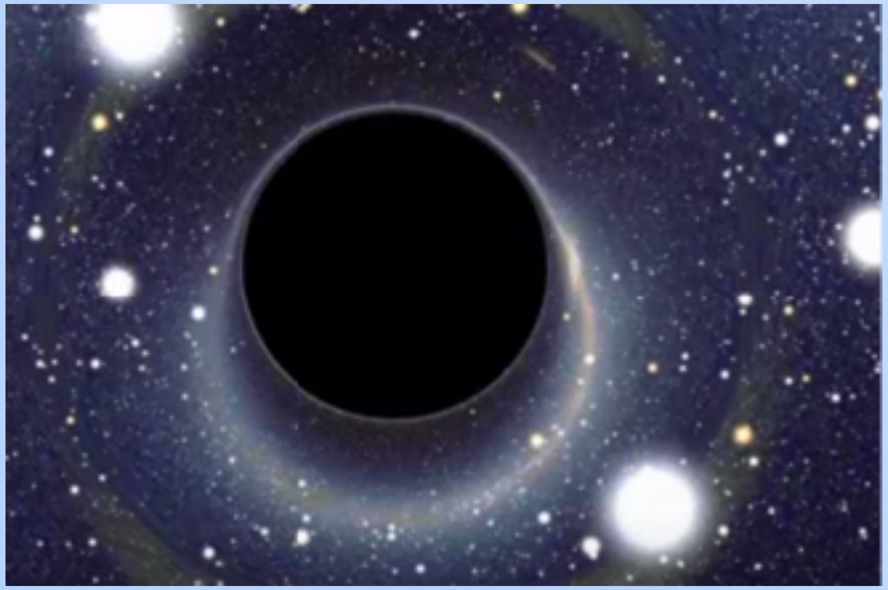
(1990s) Holographic Principle: Horizon Encoding

(1980s) Membrane Paradigm: Effective Fluid

(1970s) Hawking Radiation: Thermodynamics

Thermodynamics (1970s): Hawking Radiation

Bekenstein & Hawking, …



Hawking Temperature $T_H = \frac{\hbar c^3}{8\pi G M k_B} = \frac{\kappa}{2\pi},$

Bekenstein-Hawking Entropy $S_{\mathrm{BH}} = \frac{kA}{4\ell_{\mathrm{P}}^2}$

0th Law: Constant surface gravity

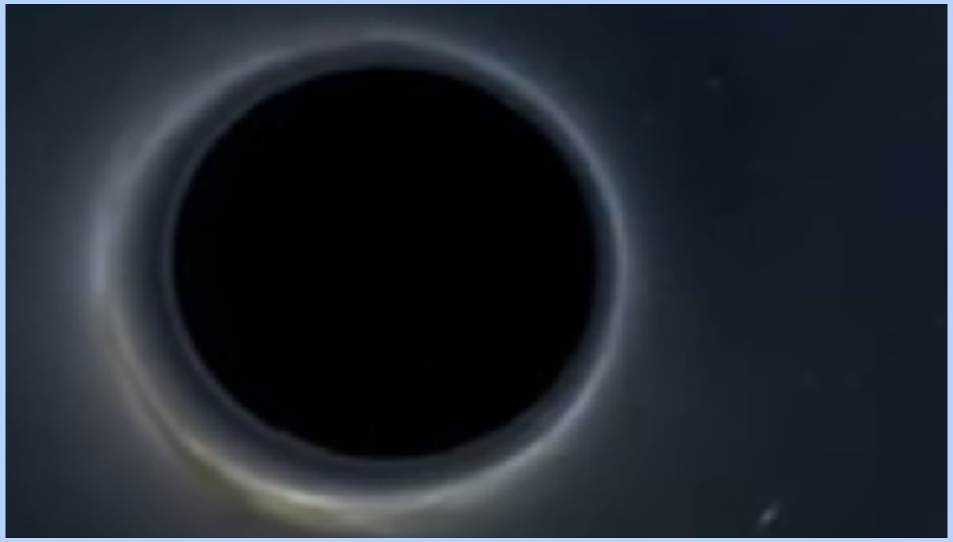
1st Law: $dE = \frac{\kappa}{8\pi}\, dA + \Omega\, dJ + \Phi\, dQ,$

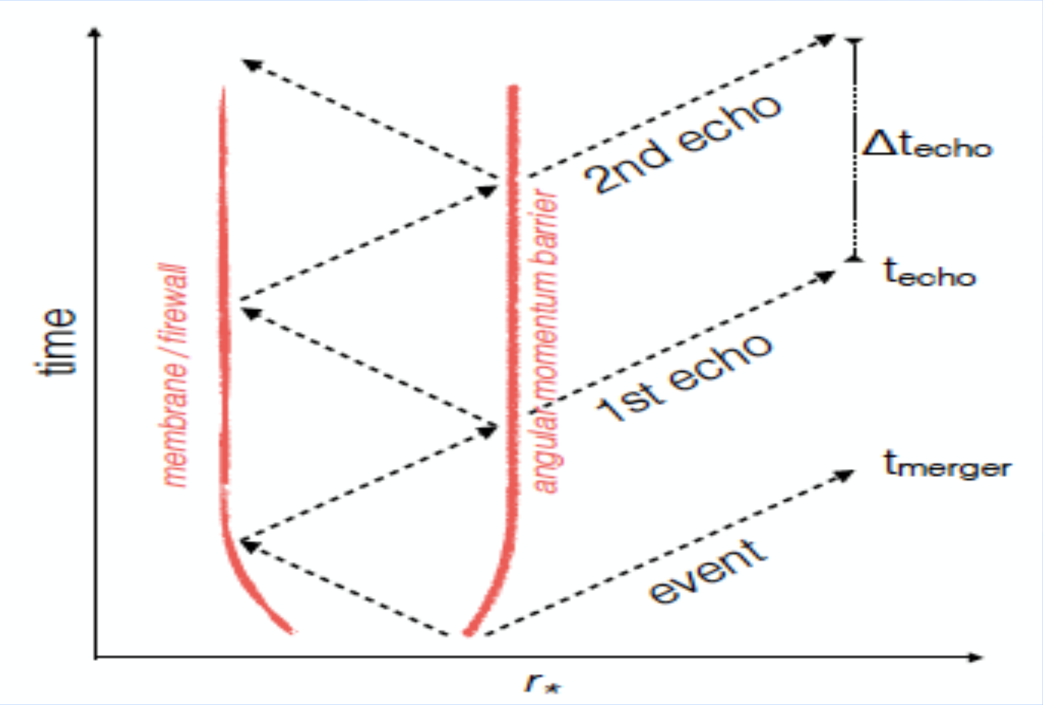
2nd Law: Non-decreasing of entropy

3rd Law: Approaching extremal black hole

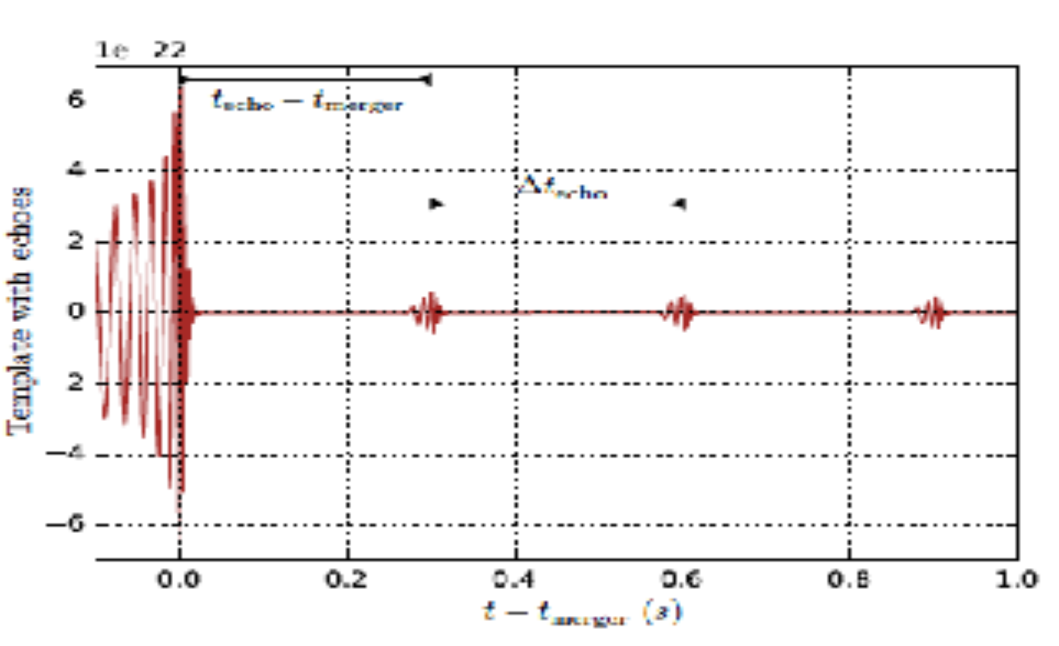
Membrane paradigm(1980s): Effective Fluid

T. Doumer & K. Thorne, …





Effective Description



Membrane on Stretched horizon

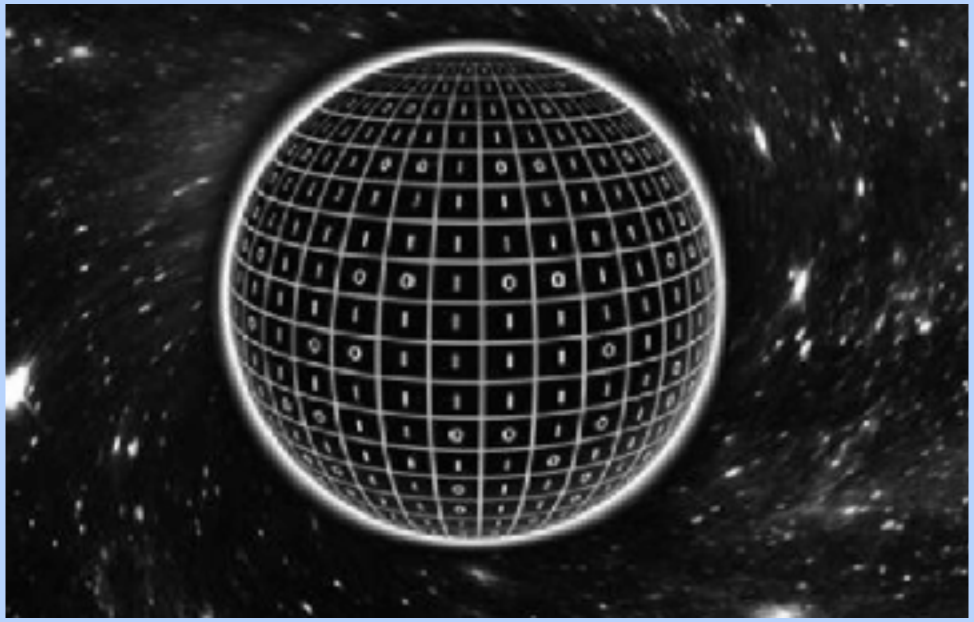
Conductivity & Viscosity

aiXiv: 1612.00266 Echoes from the Abyss

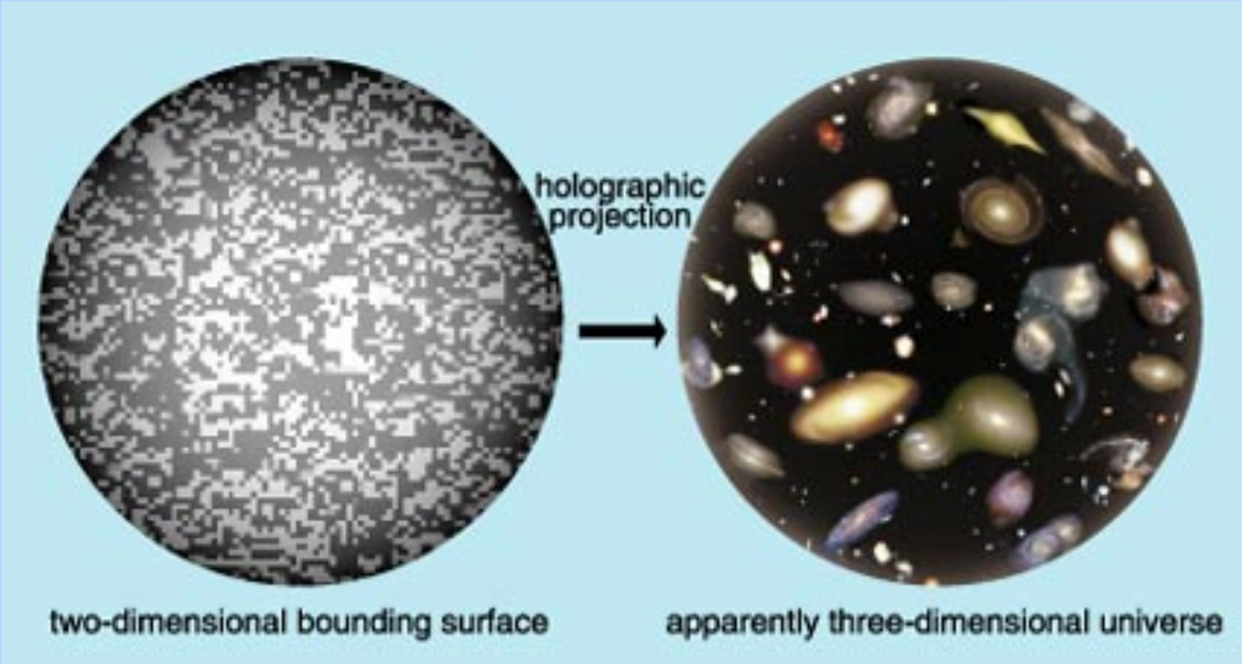
Holographic Principle (1990s): Horizon encoding

Susskind & ’t Hooft, …

Gravity in the Bulk= Theory on the light-like boundary



Black hole Horizon



Cosmological Horizon

AdS/CFT Duality (2000s): Maldacena & Gubser & Witten,et al

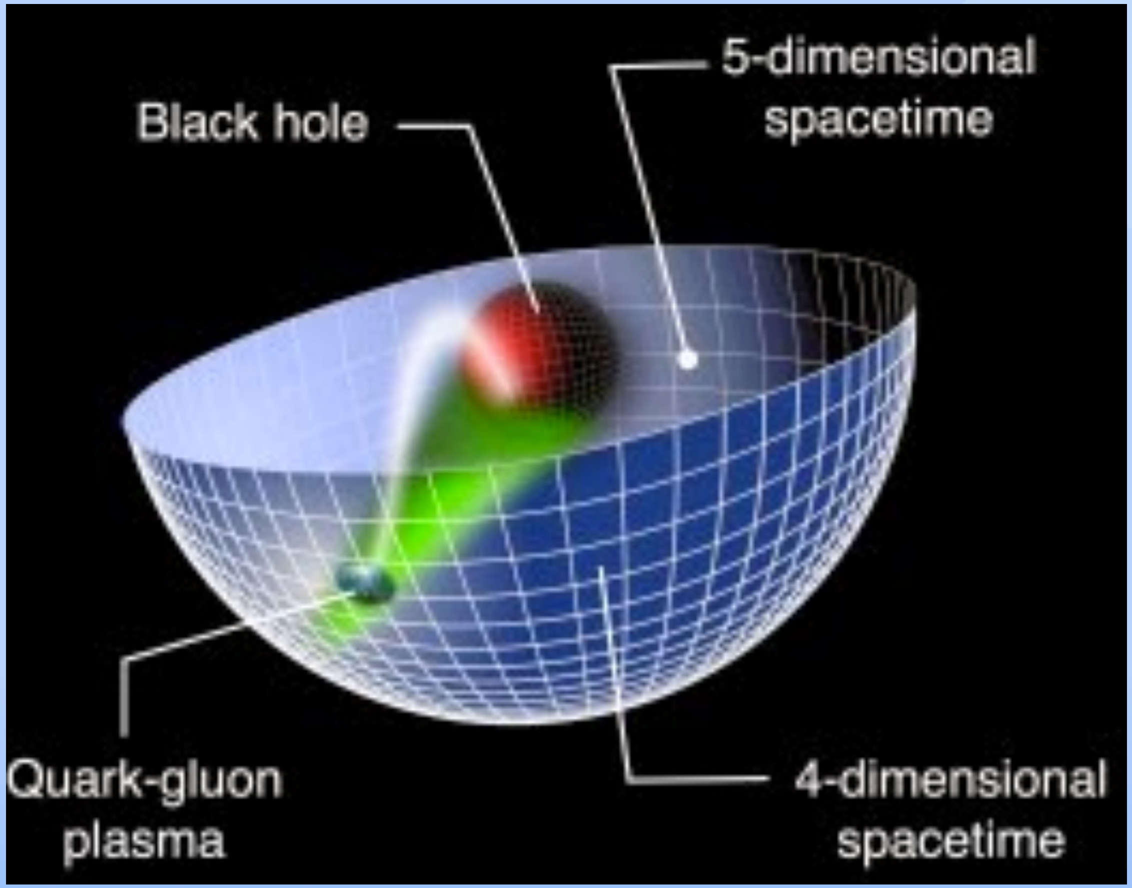
AdS/CMT Correspondence Black Hole in a natural Box

Shear Viscosity $\frac{\eta}{s} \approx \frac{\hbar}{4\pi k}$

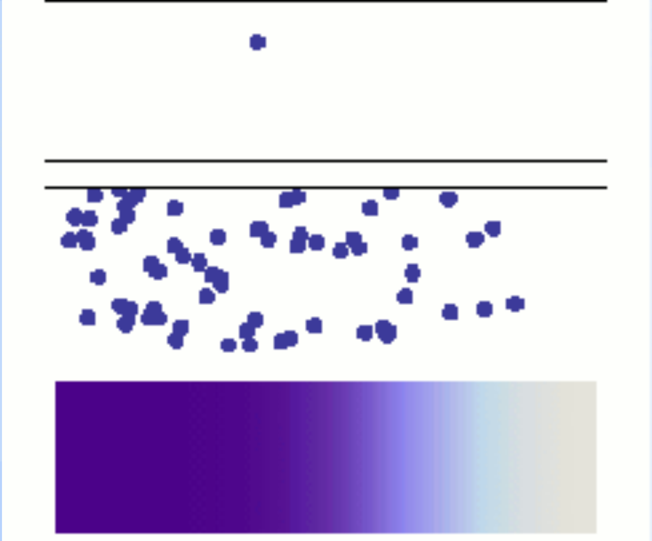
Conductivity

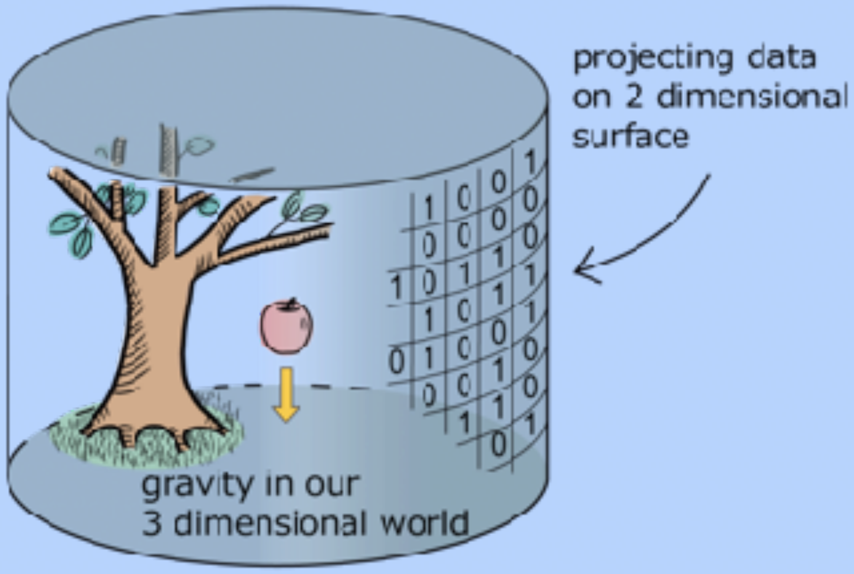
Holographic Superconductor

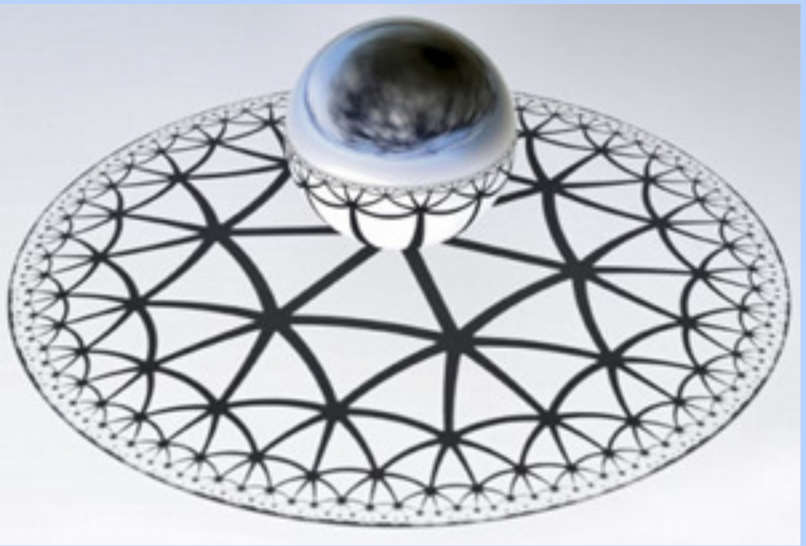
Holographic Non-Fermi Liquid



Gravity and Entanglement (2010s)





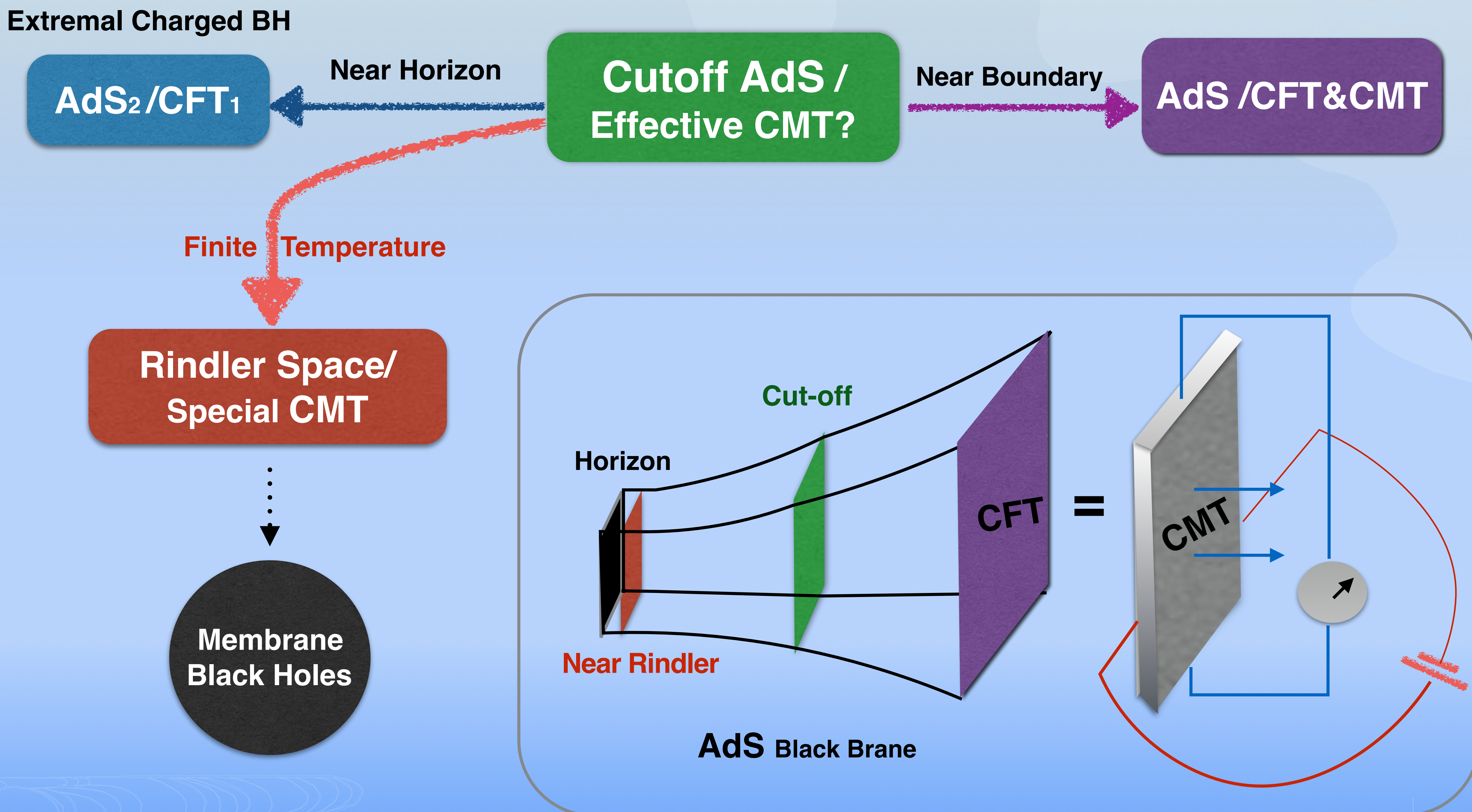


Emergent Gravity & Dark Matter from Entropic (E. Verlinde)

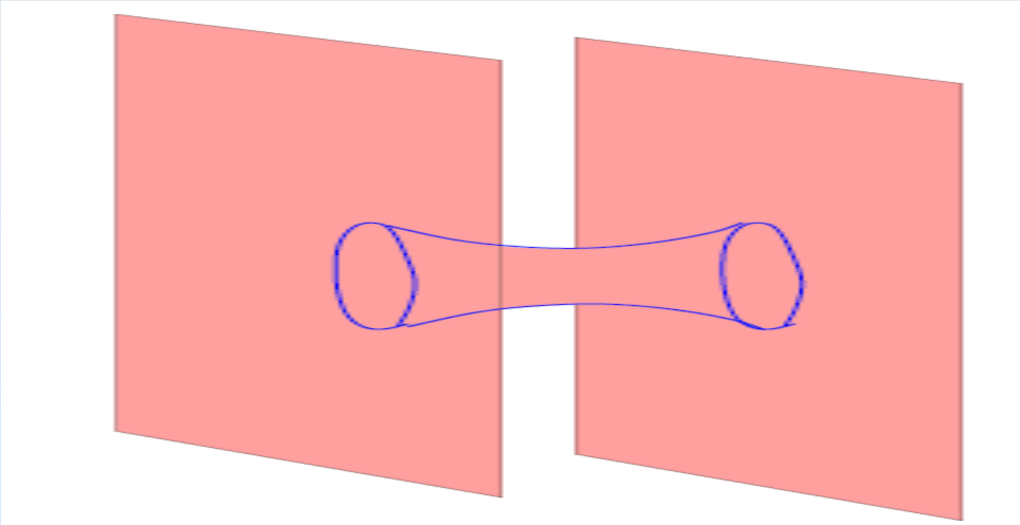
Holographic Geometry from Tensor Network (S.Ryu &T. Takayanagi)

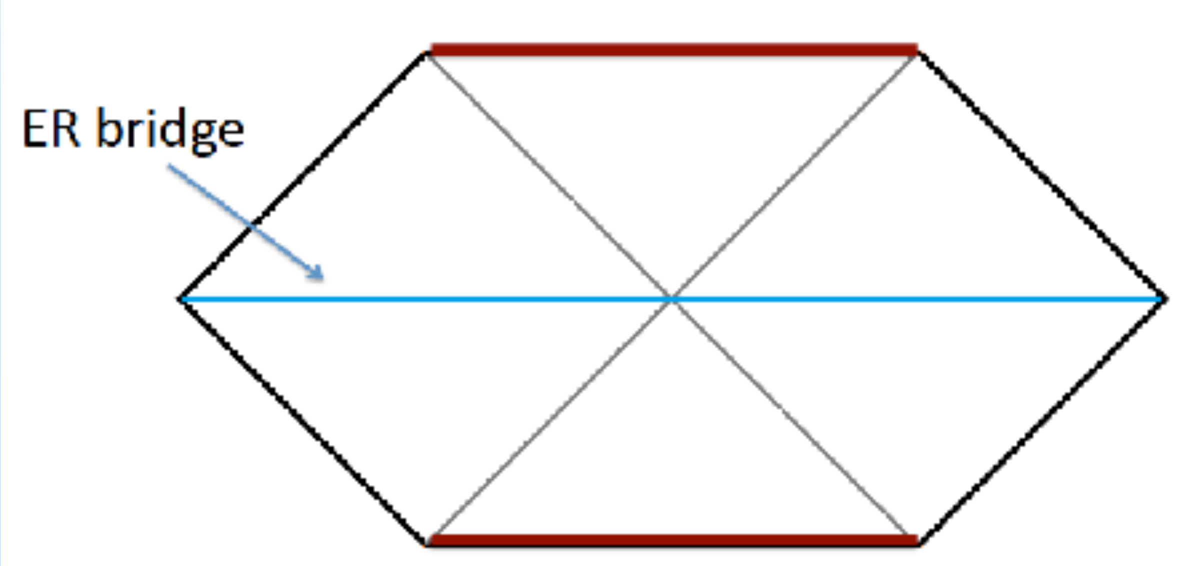
Emergent Spacetime from Quantum States (H. Ooguri)

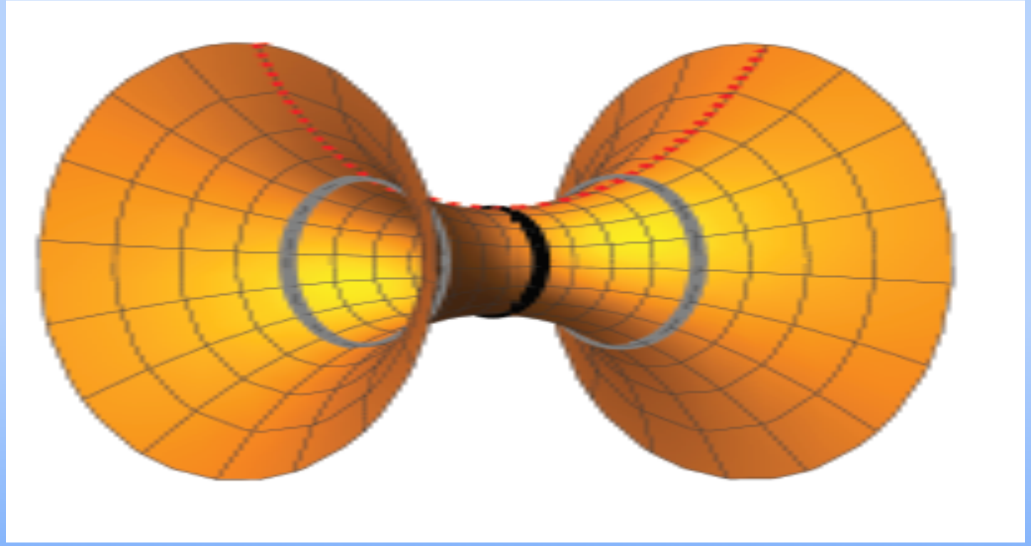
on a finite Cutoff surface

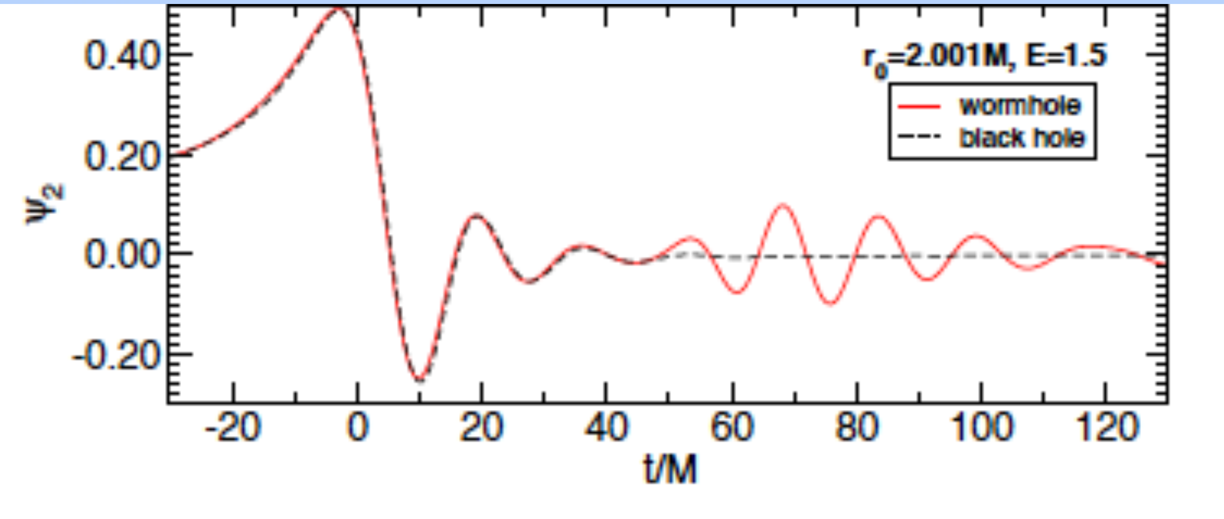


Wormhole and Black Holes



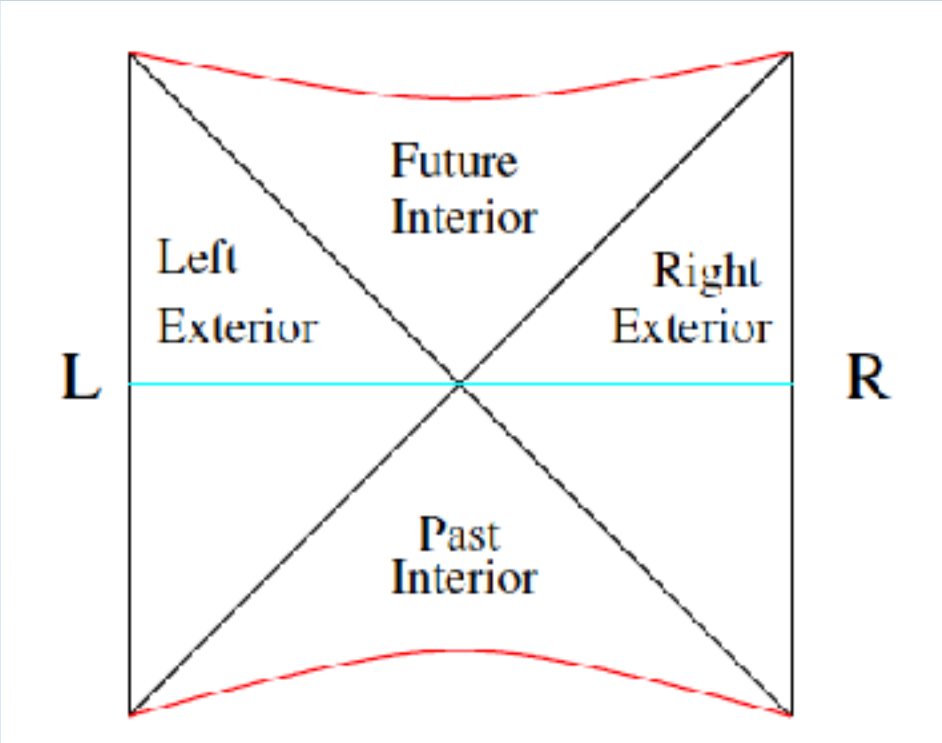


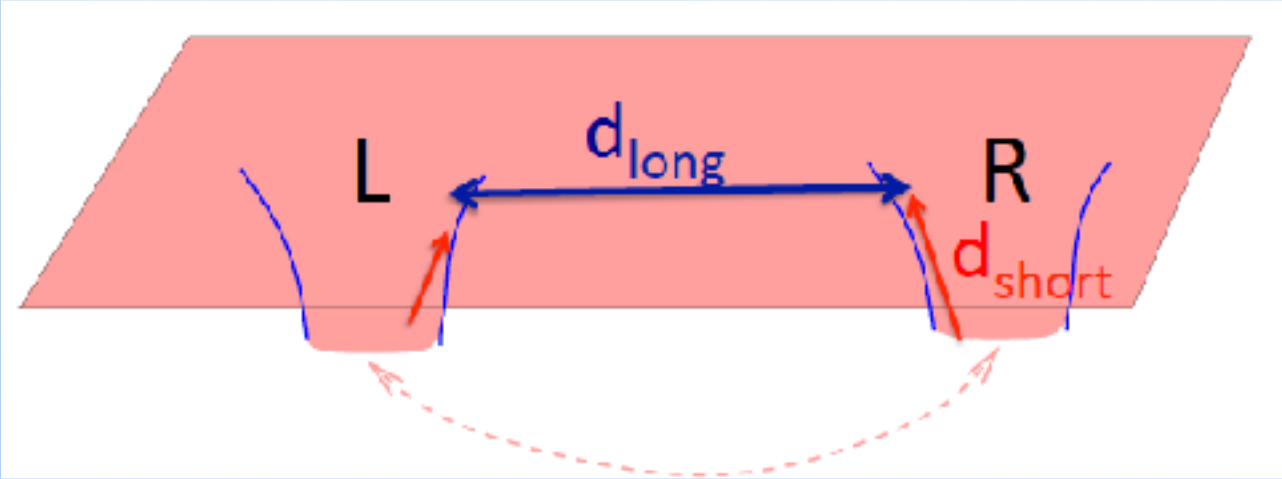




Is the Gravitational-Wave Ringdown a Probe of the Event Horizon? PRL. 116, 171101 (2016)

Wormhole=Entangled Pair (ER=EPR) ?





Wormhole =

EPR pair of two black holes in a particular entangled state.

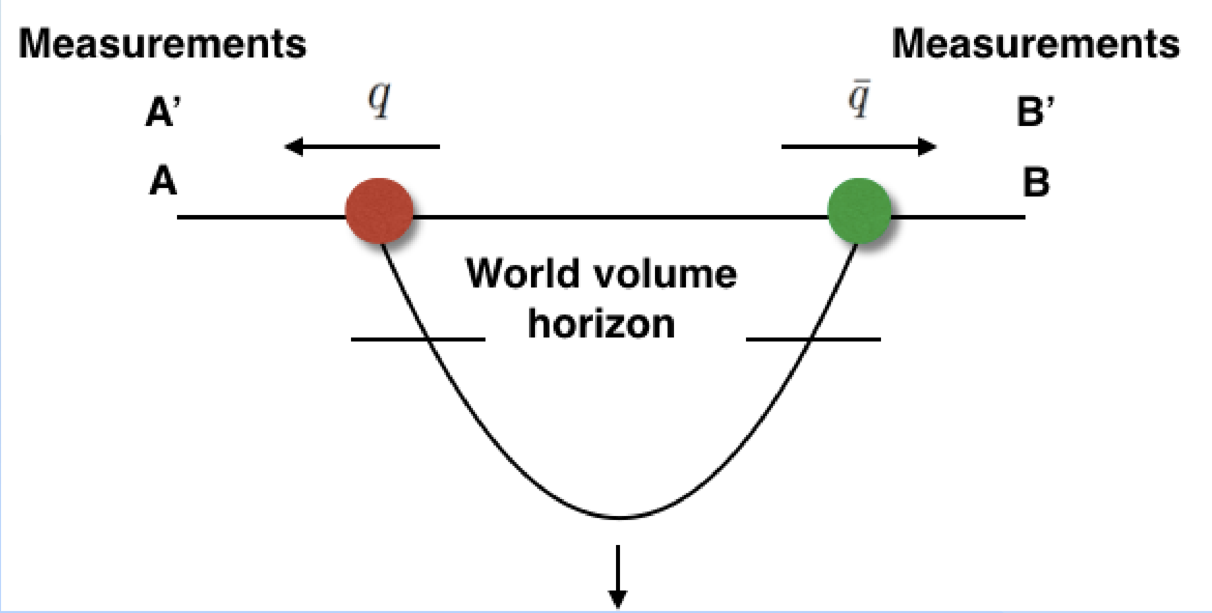
$$|\Psi\rangle = \sum_n e^{-\beta E_n/2} |E_n\rangle_L^{CPT} \times |E_n\rangle_R$$

Maldacena(2013)

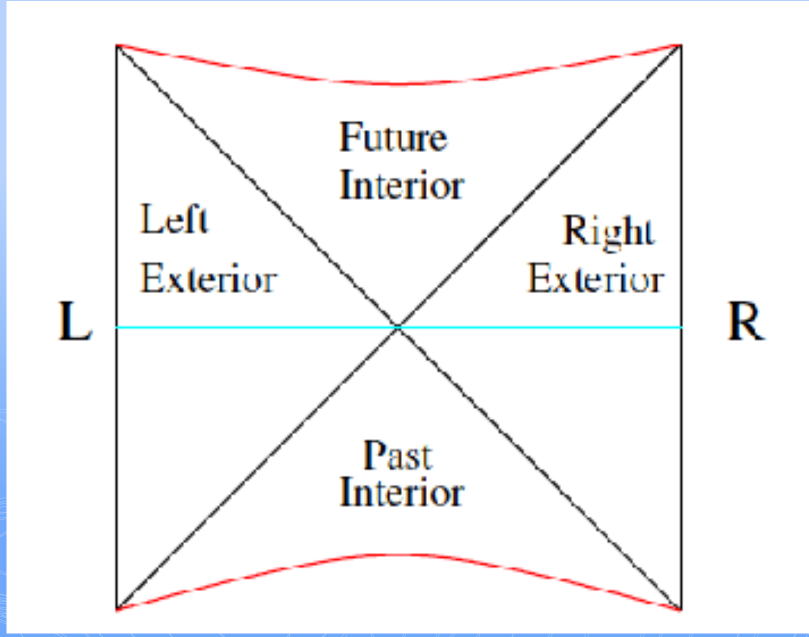
Holographic EPR Pair

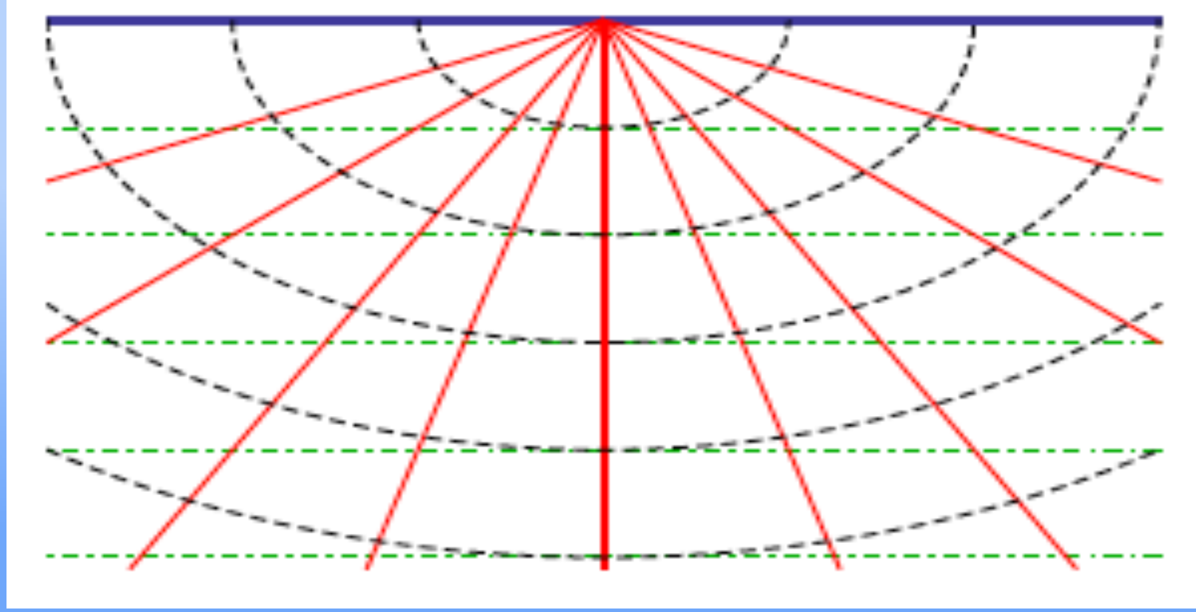
$$ds^2 = \frac{R^2}{w^2}\left[-dt^2 + dw^2 + (dx^2 + dy^2 + dz^2)\right],$$

$$|z| = b\sqrt{1-\tilde{r}e^{\tilde{z}}}\cosh\tilde{\tau},$$
$$t = b\sqrt{1-\tilde{r}e^{\tilde{z}}}\sinh\tilde{\tau},$$
$$w = b\sqrt{\tilde{r}}e^{\tilde{z}}.$$



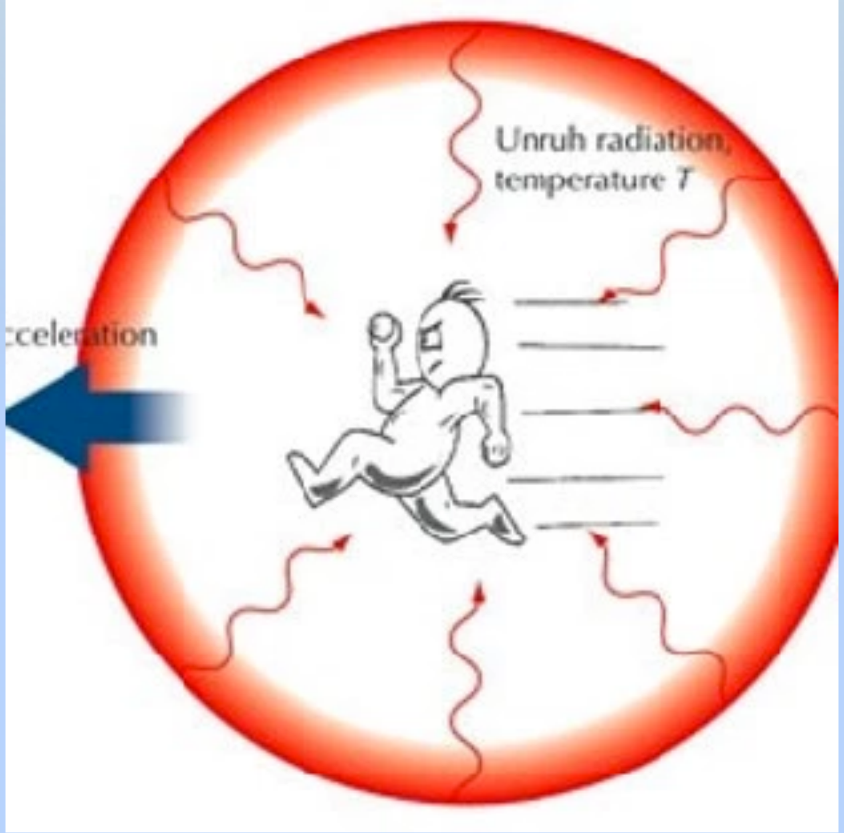
$$ds^2 = \frac{R^2}{b^2\tilde{u}}\left[-f(\tilde{u})\,d\tau^2 + \frac{b^2}{4\tilde{u}}\frac{d\tilde{u}^2}{f(\tilde{u})} + d\tilde{z}^2 + \left(dx^2 + dy^2\right)\exp\left(-2\tilde{z}/b\right)\right],$$

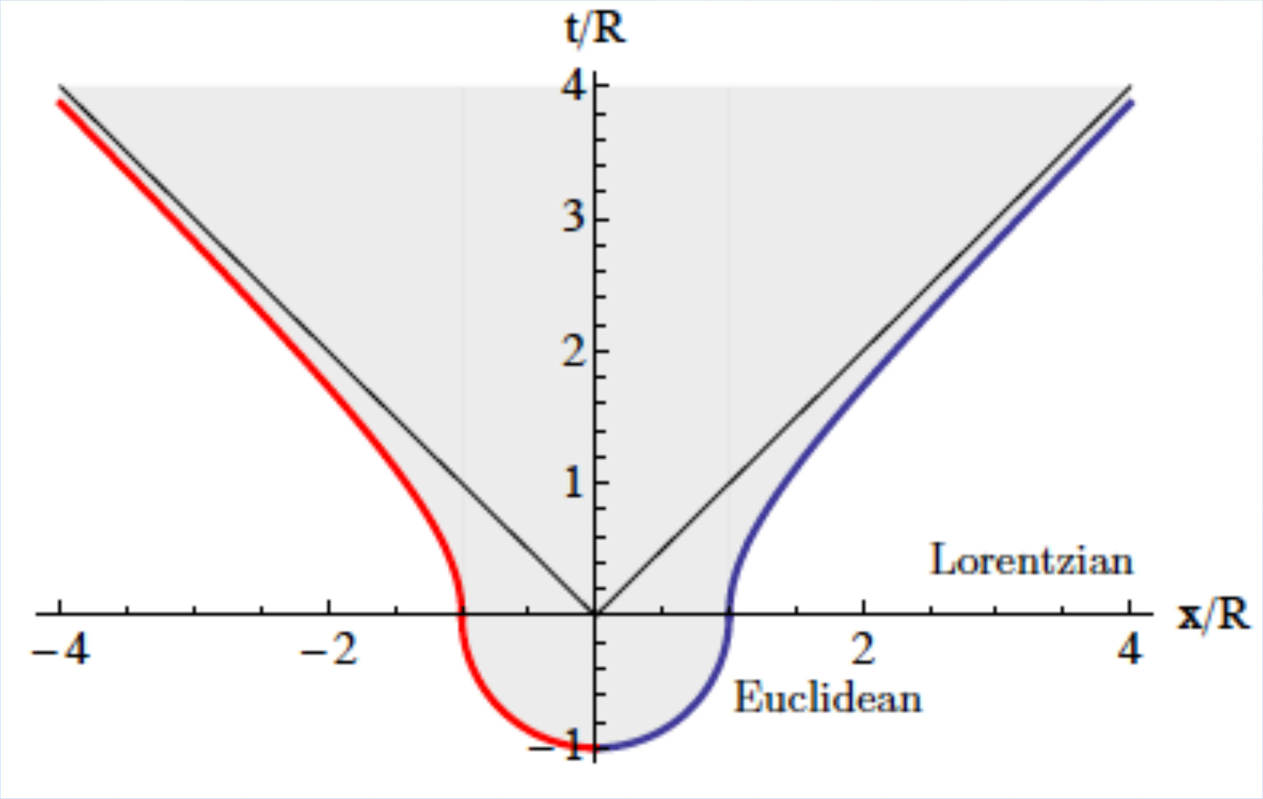




by Karch and Jensen (2013) PRL 111.211602

No causal connection



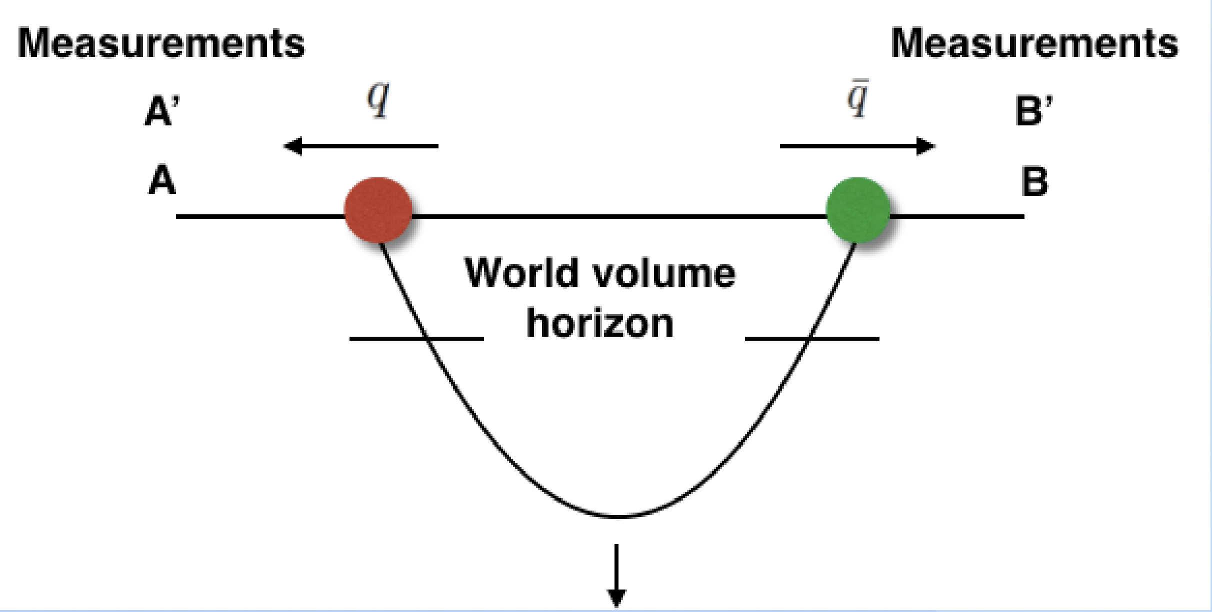


No causal connection Worldsheet = ER bridge (finite distance)

Holographic Schwinger effect J.Sonner(2013), PRL111.211603

Holographic Correlations

$$S \simeq -T_0 L^2 \int \frac{d\tilde{\tau}\, d\tilde{r}}{2\tilde{r}^{3/2}} \left(1 + 2\tilde{r} f \sum_i \tilde{\delta}_i'^2 - \frac{1}{2f} \sum_i \dot{\tilde{\delta}}_i^2\right)$$



$$\tilde{\delta}_i(\tilde{r}, \tilde{\tau}) = \int \frac{d\omega}{2\pi} e^{-i\omega\tilde{\tau}} \tilde{\delta}_i(\omega) Y_\omega(\tilde{r}),$$

$$\begin{aligned} G_R^{ij}(\omega) &= -\frac{2T_0 L^2}{b^2 \tilde{r}^{1/2}} f(\tilde{r}) Y_{-\omega}(\tilde{r}) \partial_{\tilde{r}} Y_\omega(\tilde{r}) \delta^{ij} \big|_{\tilde{r}\to 0} \\ &= -\frac{a^2\sqrt{\lambda}}{2\pi} \mathrm{i}\omega\delta^{ij} + O(\omega^2), \end{aligned}$$

$$\mathrm{i}G_R^{ij}(\tau) = \theta(\tau)[\mathcal{F}^i(\tau), \mathcal{F}^j(0)]\rangle$$

$$G_{AB}^{ij}(\omega) = \frac{2\mathrm{i}e^{-\omega/2T_U}}{1 - e^{-\omega/T_U}} \mathrm{Im}G_R^{ij}(\omega)$$

$$\mathrm{i}G_{AB}^{ij}(\tau, x) = \langle \mathcal{F}_A^i(\tau, x) \mathcal{F}_B^j(0) \rangle$$

$$\mathrm{i}G_{AB}^{xx} = \mathrm{i}G_{AB}^{yy} = \frac{\sqrt{\lambda} a^3}{2\pi^2}, \quad \mathrm{i}G_{AB}^{xy} = \mathrm{i}G_{AB}^{yx} = 0.$$

$$G_{AB}^{ij} \propto \delta^{ij}$$

Bell's Theorem(CHSH formula)

For classical system

$$\langle C \rangle = \langle AB \rangle + \langle AB' \rangle + \langle A'B \rangle - \langle A'B' \rangle,$$

$$|\langle C \rangle| \leq 2$$

For Quantum system

$$|\psi_s\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle \otimes |\downarrow\rangle - |\downarrow\rangle \otimes |\uparrow\rangle),$$

$$|\langle C \rangle| \leq 2\sqrt{2}$$

$$A = \vec{n}_A \cdot \vec{\sigma}, \;\; A' = \vec{n}_{A'} \cdot \vec{\sigma},$$
$$B = \vec{n}_B \cdot \vec{\sigma}, \;\; B' = \vec{n}_{B'} \cdot \vec{\sigma}.$$

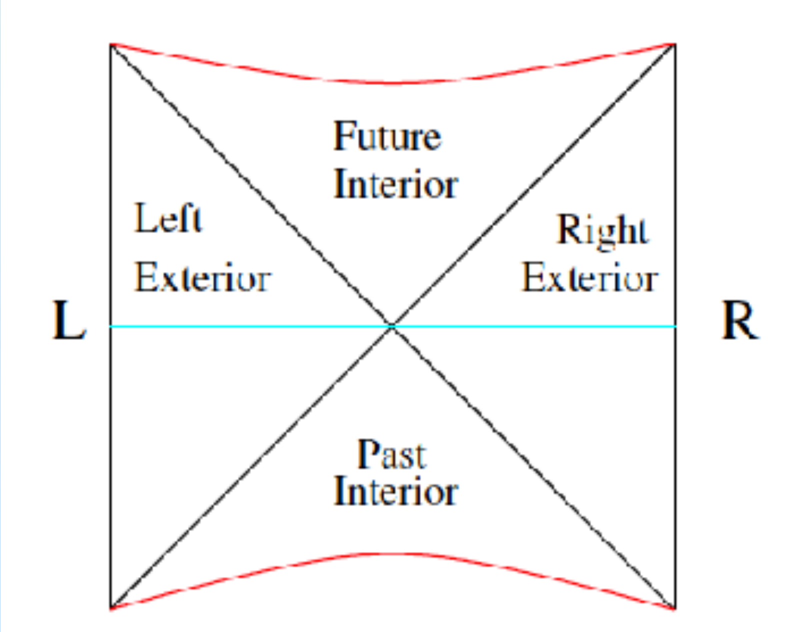
Local Hidden Variables & Quantum Mechanics

Constructing Bell inequality for Holographic EPR

$$G^{ij}_{AB}(\omega) = \frac{2\mathrm{i}e^{-\omega/2T_U}}{1-e^{-\omega/T_U}}\mathrm{Im}G^{ij}_R(\omega)$$

$$\mathrm{i}G^{ij}_{AB}(\tau,x) = \langle \mathcal{F}^i_A(\tau,x)\mathcal{F}^j_B(0)\rangle.$$

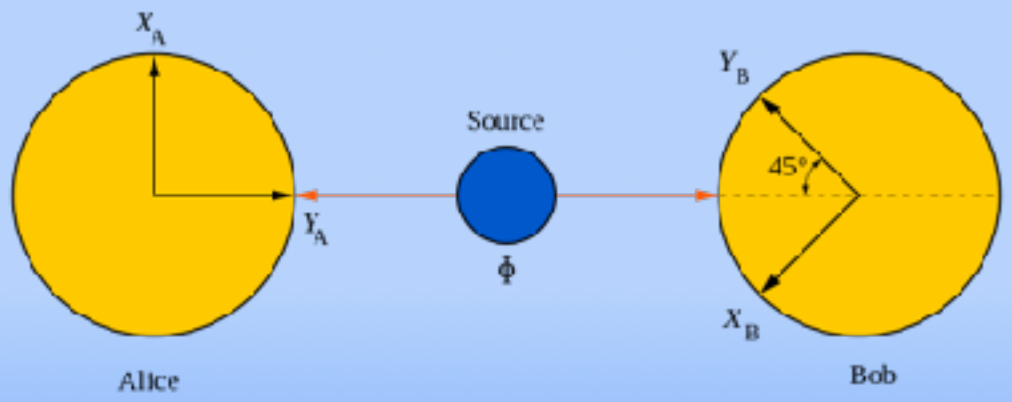
$$G^{ij}_{AB} \propto \delta^{ij}$$



$$A_{\mathcal{F}} = (\cos\theta_A \mathcal{F}^x_A + \sin\theta_A \mathcal{F}^y_A)/\langle \mathcal{F}^x_A \mathcal{F}^x_B\rangle^{1/2},$$
$$B_{\mathcal{F}} = (\cos\theta_B \mathcal{F}^x_B + \sin\theta_B \mathcal{F}^y_B)/\langle \mathcal{F}^x_A \mathcal{F}^x_B\rangle^{1/2},$$

$$\langle A_{\mathcal{F}} B_{\mathcal{F}}\rangle = \cos(\theta_A - \theta_B) \equiv \cos\theta_{AB}.$$

$$\langle C_{\mathcal{F}}\rangle = \langle A_{\mathcal{F}} B_{\mathcal{F}}\rangle + \langle A_{\mathcal{F}} B'_{\mathcal{F}}\rangle + \langle A'_{\mathcal{F}} B_{\mathcal{F}}\rangle - \langle A'_{\mathcal{F}} B'_{\mathcal{F}}\rangle$$
$$= \cos\theta_{AB} + \cos\theta_{AB'} + \cos\theta_{A'B} - \cos\theta_{A'B'}.$$



$$\theta_{AB} = \theta_{AB'} = \theta_{A'B} = \pi/4, \quad \theta_{A'B'} = 3\pi/4$$

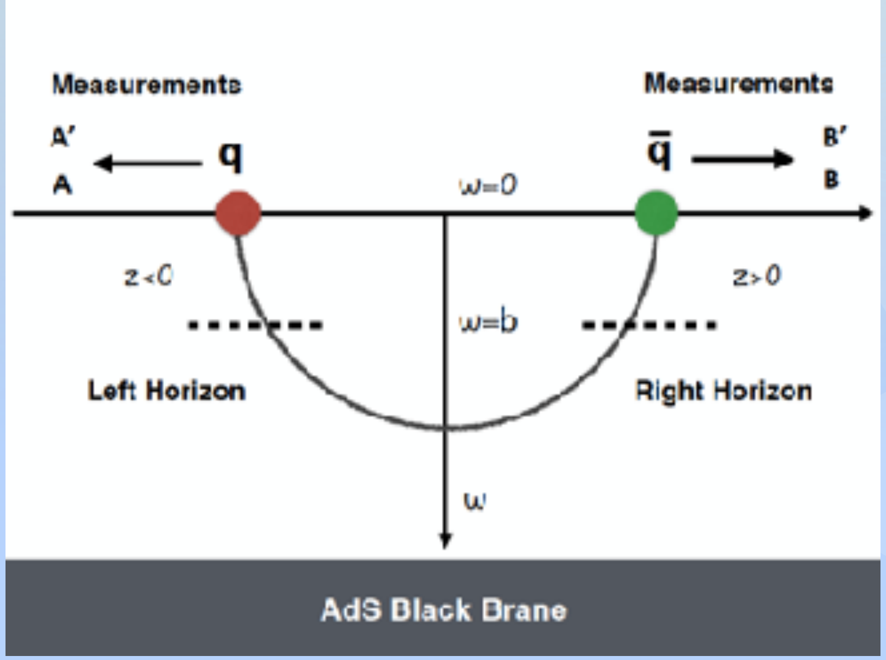
$$\langle C_{\mathcal{F}}\rangle = 2\sqrt{2}.$$

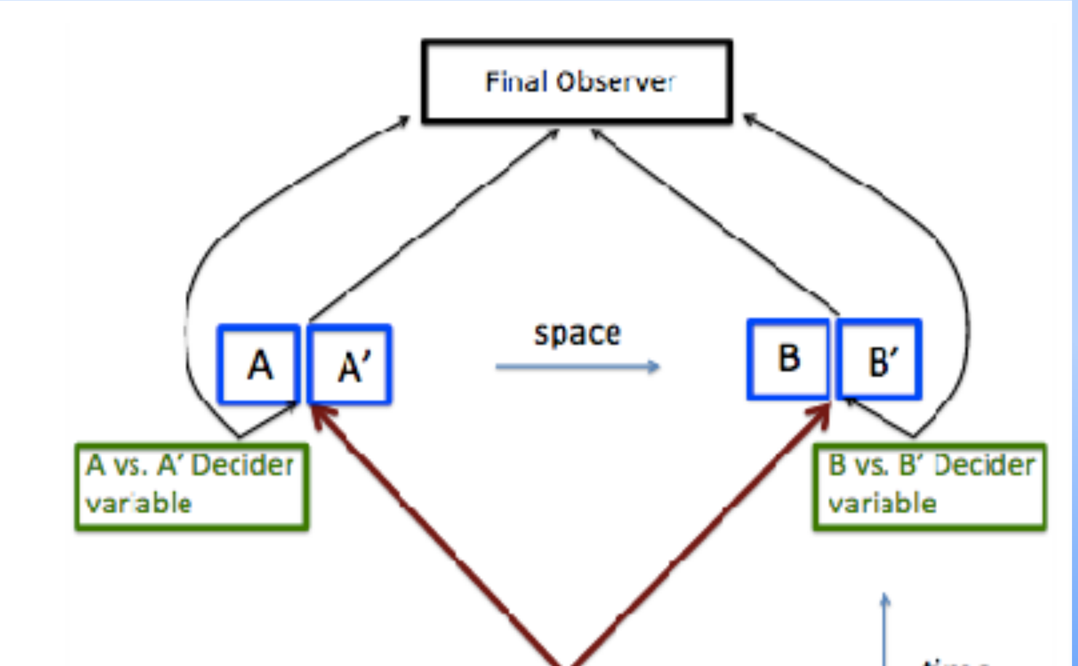
Discussions and Outlook

Holographic EPR

Bell Inequality

Measurement?

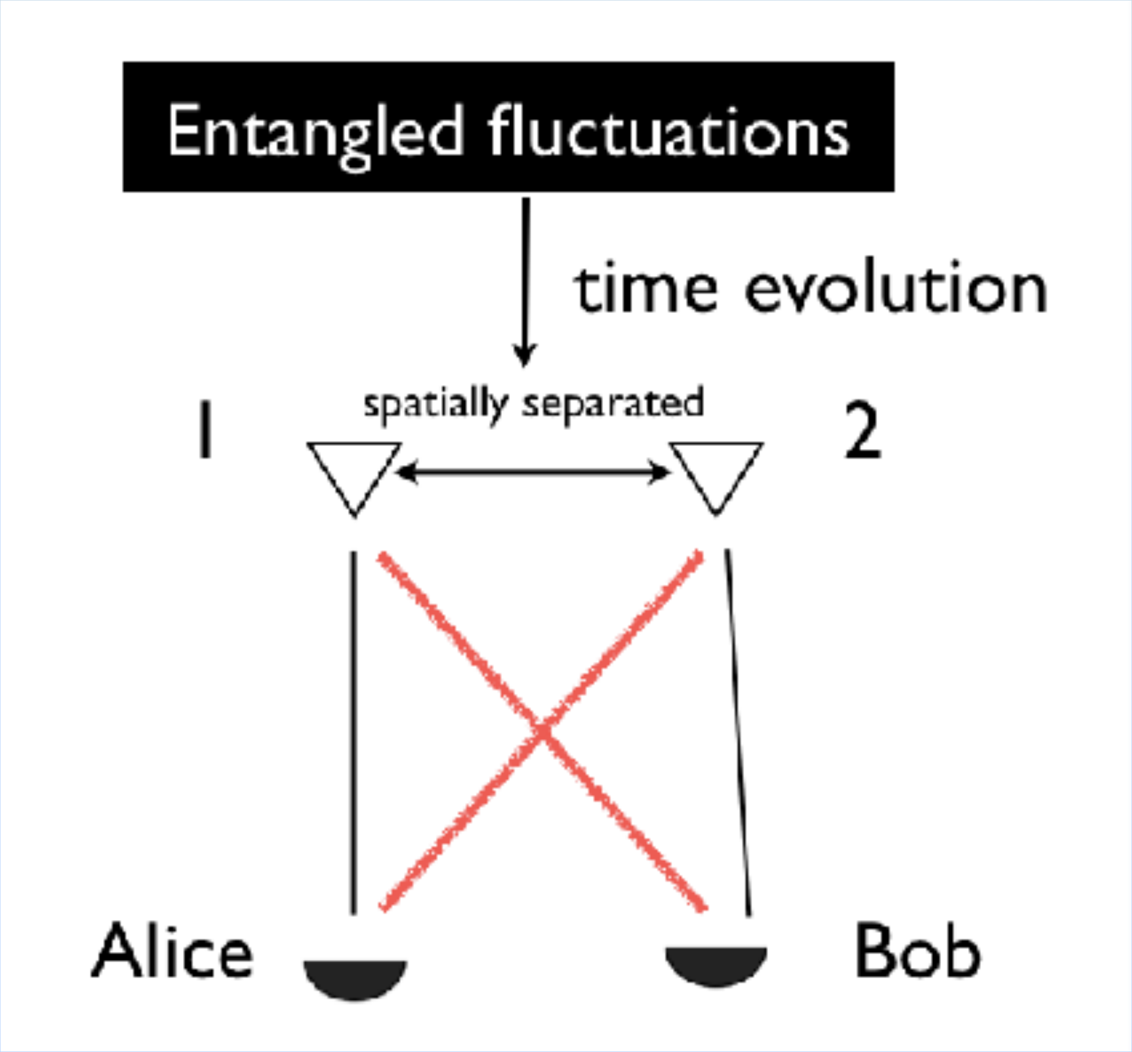


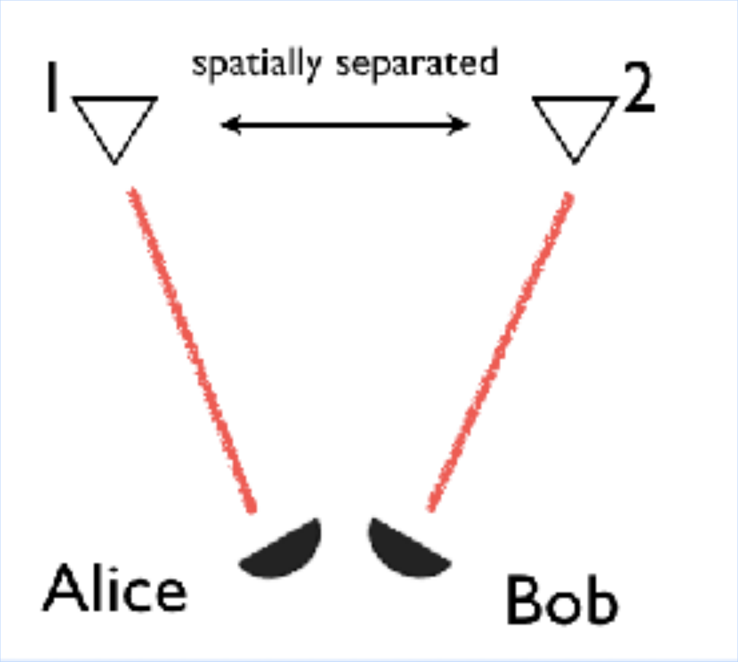


1508.01082
J. Maldecena

Thanks for your attention!

Towards Searching for Entangled Photons in the CMB Sky





arXiv: 1701.03437, by J.W.Chen, S.H.Dai, D. Maity, S. Sun and **Y. L. Zhang**

Holographic SK Correlators

$$S \simeq -T_s L^2 \int \frac{d\bar\tau\, d\bar r}{2\bar r^{3/2}} \left\{1+\left[2\bar r f(\bar r)\bar y_i'\bar y_j' - \frac{1}{2f(\bar r)}\dot{\bar y}_i\dot{\bar y}_j\right]h^{ij}\right\},$$

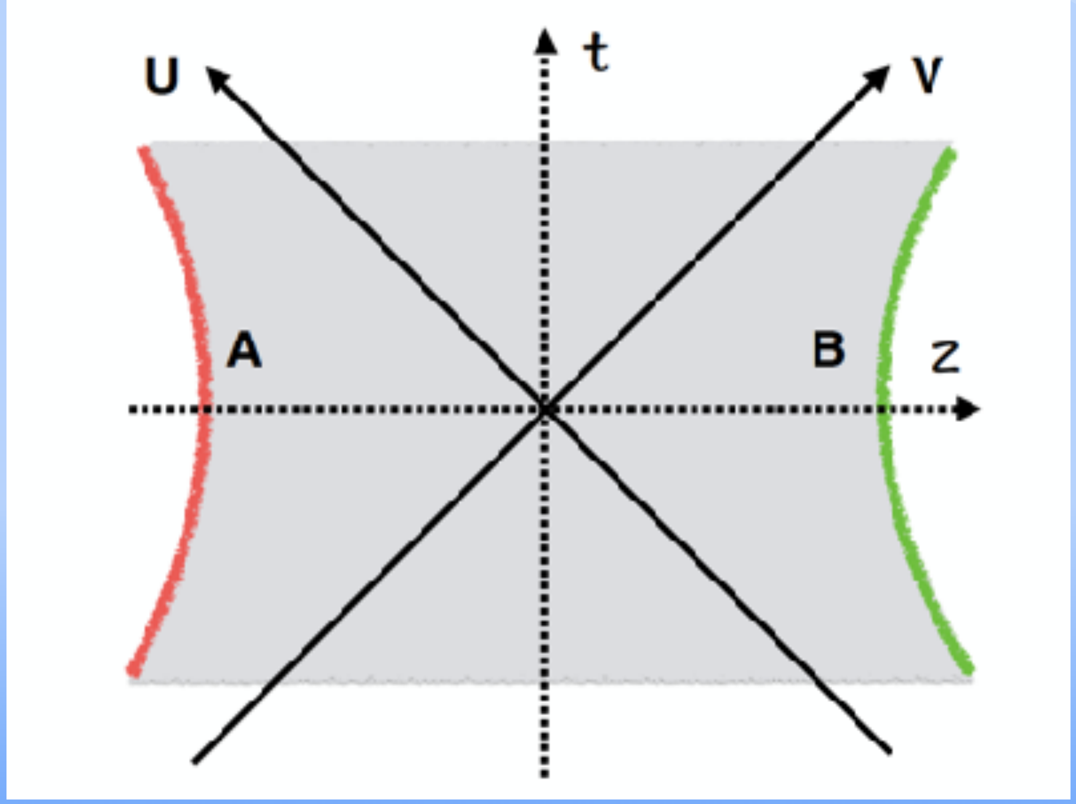
$$Y_\omega''(\bar r) - \frac{f(\bar r)-2\bar r f'(\bar r)}{2\bar r f(\bar r)}Y_\omega'(\bar r) + \frac{\omega^2 Y_\omega(\bar r)}{2f(\bar r)\bar r^{3/2}} = 0.$$

$$Y_\omega''(\bar r) - \frac{f(\bar r)-2\bar r f'(\bar r)}{2\bar r f(\bar r)}Y_\omega'(\bar r) + \frac{\omega^2 Y_\omega(\bar r)}{2f(\bar r)\bar r^{3/2}} = 0.$$

$$\bar y_-^B = e^{-i\omega\bar\tau}Y_\omega(\bar r) \sim e^{-i(\omega/2)\ln(V)},$$
$$\bar y_+^B = e^{-i\omega\bar\tau}Y_\omega^*(\bar r) \sim e^{i(\omega/2)\ln(-U)}.$$

$$\bar y_+(\omega) = \bar y_+^B + e^{+\pi\omega/2}\bar y_+^A,$$
$$\bar y_-(\omega) = \bar y_-^B + e^{-\pi\omega/2}\bar y_-^A,$$

$$\begin{aligned} S_\theta = -\frac{1}{2}\int\frac{d\omega}{2\pi}\Big\{ & \left[\bar y_i^A(-\omega)\bar y_j^B(\omega) + \bar y_i^B(-\omega)\bar y_j^A(\omega)\right] \\ & \times\sqrt{n_\omega(1+n_\omega)}\left[G_A^{ij}(\omega) - G_R^{ij}(\omega)\right] \\ & + \bar y_i^A(-\omega)\bar y_j^A(\omega)\left[(1+n)G_R^{ij}(\omega) - nG_A^{ij}(\omega)\right] \\ & + \bar y_i^B(-\omega)\bar y_j^B(\omega)\left[nG_R^{ij}(\omega) - (1+n)G_A^{ij}(\omega)\right]\Big\}, \end{aligned}$$



1508.01082 by J.Maldecena