Neutron Stars from the Skyrme Model Perspective

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    $\rightarrow$ beyond mean field
    $\rightarrow$ inverse TOV
  - **beyond** the BPS limit

    $\rightarrow$ crust
    model independent $M(R)$
    crust in $\mathcal{L}_{24}$ Skyrme model

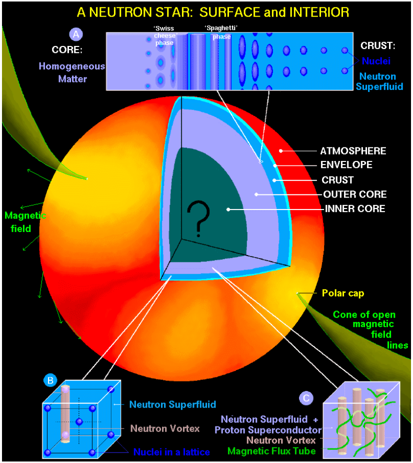
Neutron Stars

structure and observables

structure and observables

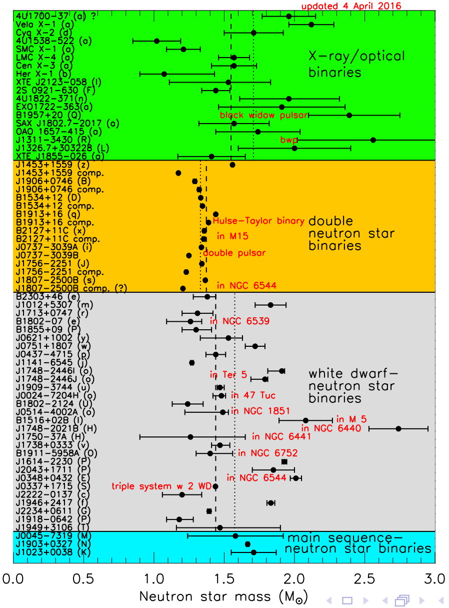
- neutron star structure from nuclear physics

  http://www.astroscu.unam.mx/neutrones/NS-Picture/NStar/



- liquid core: 99% of mass
- core/crust with inner structure (phases): paste, lasagne, meat ball → Coulomb force

structure and observables



TOV approach

(self) gravitating (nuclear) matter

neutron stars in TOV approach

- the canonical approach to neutron stars
  - Einstein eqs

$$G^{\mu\nu} = \frac{\kappa^2}{2} T^{\mu\nu}$$

  - prescribed energy-momentum tensor - **perfect fluid**

$$T^{\rho\sigma} = (p+\rho)u^\rho u^\sigma - p g^{\rho\sigma}$$

- spherically symmetric metric

$$ds^2 = \mathbf{A}(r)dt^2 - \mathbf{B}(r)dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

TOV equations

$$\text{TOV1}: \quad M' = 4\pi r^2 \rho, \quad \mathbf{B}(r) \equiv \left(1 - \frac{\kappa^2}{8\pi}\frac{M(r)}{r}\right)^{-1}$$

$$\text{TOV2}: \quad rp' = (\rho + p)\left(\frac{1}{2}(1-\mathbf{B}) - \frac{\kappa^2}{4} r^2 \mathbf{B} p\right)$$

$$\left(\frac{\mathbf{A}'}{\mathbf{A}} = \frac{1}{r}(\mathbf{B}-1) + \frac{\kappa^2}{2} r\mathbf{B}p\right)$$

- to close the system: **equation of state** (EoS)

$$p = p(\rho, ...)$$

neutron stars in TOV approach

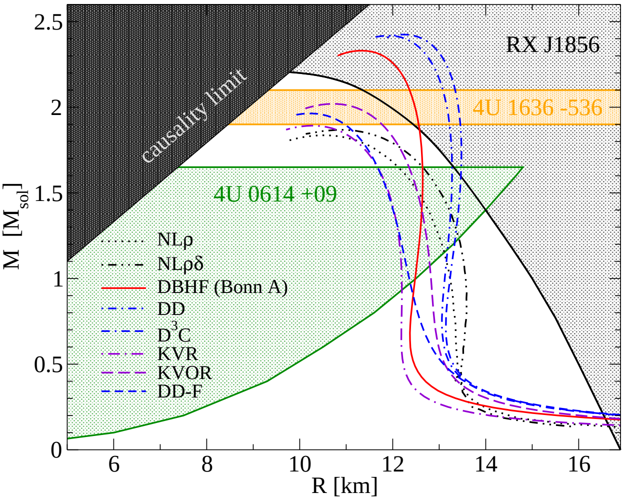
- EoS $\rightarrow$ input from nuclear physics
  EFTs for nuclear matter - no perfect fluid
  $\rightarrow$ **mean-field** approximation
  examples
  - Walecka model
  - NJL model
  - Skyrme model
- mean-field EoS
  - algebraic EoS $p = \rho(p)$
  - constant densities $\rho = const.$
- typically combined EoS
  $\rightarrow$ for different phases (densities)

How non-mean-field affects TOV?
Can we do gravitating nuclear matter in a full FT+GR?
$\rightarrow$ we need an effective model (action) of nuclear matter
$\rightarrow$ couple it to gravity
$\rightarrow$ find nuclear stars
$\rightarrow$ verify the universality of EoS

solvable nuclear matter action in a **thermodynamical limit (perf. fluid)**
$\rightarrow$ **the BPS Skyrme model**: low B.E. + perfect fluid

neutron stars in TOV approach

T. Klähn et al, Phys. Rev. C**74** (2006) 035802



the solitonic Skyrme model

beyond the mean-field limit

solitonic Skyrme model

- **the Skyrme framework** Skyrme (61)

  pionic EFT of

  - baryons and nuclei $\rightarrow$ emergent objects: **solitons**

    extended, non-perturbative
  - nuclear matter
  - with applications to neutron stars
    $\rightarrow$ complementary to lattice

- support form $N_c \rightarrow \infty$ limit t'Hooft (83), Witten (84)
  - chiral effective meson/baryon theory
  - primary d.o.f. are mesons
  - baryons (nuclei) are realized as solitons

  - simplest case (two flavors): $U(x) = e^{i\vec{\pi}\vec{\sigma}} \in SU(2)$
  - $\vec{\pi}$ - pions
  - topological charge = baryon number

$$U : \mathbb{R}^3 \cup \{\infty\} \cong \mathbb{S}^3 \ni \vec{x} \rightarrow U(\vec{x}) \in SU(2) \cong \mathbb{S}^3$$

$$\pi_3(\mathbb{S}^3) = \mathbb{Z}$$

- **what is the proper action?**

solitonic Skyrme model

- Lorentz inv., standard Hamiltonian, max. first time derivative squared

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_0 + \mathcal{L}_6$$

- **sextic term**

$$\mathcal{L}_6 = -\mathbb{B}_\mu \mathbb{B}^\mu, \quad \mathbb{B}^\mu = \frac{1}{24\pi^2} \mathrm{Tr}\left(\epsilon^{\mu\nu\rho\sigma} L_\nu L_\rho L_\sigma\right)$$

- topological i.e., the baryon current squared
- coherent i.e., "many"-body interaction
- short range repulsive interaction
- leading at **high density/pressure** Adam, Haberichter, Wereszczynski (15)
- **perfect fluid** Adam, Naya, Sanchez-Guillen, Speight, Wereszczynski (14)

- ideal for **nuclear star liquid core**

higher order terms? Gudnason (17)

the Skyrme model in the BPS limit

- **the BPS Skyrme model**: $\lambda_\pi, \lambda_2, \lambda_4 \to 0$ limit

$$\mathcal{L}_{BPS} = \lambda_6 \mathcal{L}_6 + \mathcal{L}_0$$

  - **BPS** zero (classical) BE
  - **perfect fluid field theory** for any B

    Physically well motivated idealization of the nuclear matter

  - **solvablity**
    very simple solvable model which covers the main features of nuclear matter

  - **the leading part in high density regime**
    should capture the leading contribution to the bulk observables of NS
    liquid core of NS

- **BPS Skyrme model - classical aspects**
  - topological bound ASW(10) PLB, Speight (10)

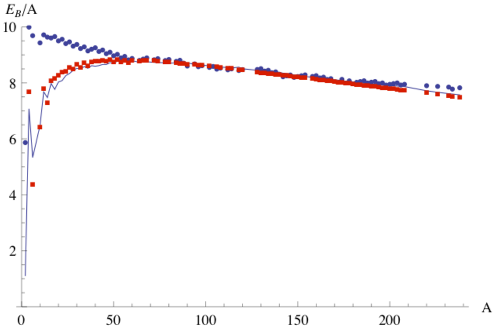
$$
\begin{aligned}
E_{06} &= \int d^3x \left(\lambda^2\pi^4\mathbb{B}_0^2 + \mu^2\mathcal{U}\right) \\
&= \int d^3x \left(\lambda\pi^2\mathbb{B}_0 \pm \mu\sqrt{\mathcal{U}}\right)^2 \mp \int d^3x \lambda\mu\pi^2\sqrt{\mathcal{U}}\mathbb{B}_0 \\
&\geq \int d^3x \lambda\mu\pi^2\sqrt{\mathcal{U}}\mathbb{B}_0 = 2\pi^2\lambda\mu < \sqrt{\mathcal{U}} >_{S^3} |B|
\end{aligned}
$$

- the bound is saturated $\Rightarrow$ BPS equation

$$\lambda\pi^2\mathbb{B}_0 \pm \mu\sqrt{\mathcal{U}} = 0$$

- **exact, analytical solutions for any topological charge**
- classically **zero binding energy** $E = \lambda\mu C|B|$

- **BPS Skyrme model - quantum aspects**



Binding energy per nucleon: BPS model (blue), Weizsäcker's formula (red), experimental values (solid line)

- axially symmetric solutions ⇒ **exact result**
- weakly depend on the potential

Heavy atomic nuclei (binding energies) can be described by a solitonic model Adam, Naya, Ssanchez, Wereszczynski (2013) PRL

the Skyrme model in a BPS limit perfect fluid

- **SDiff symmetries**
- **energy-momentum tensor of a perfect fluid**

$$T^{00} = \lambda^2 \pi^2 \mathbb{B}_0^2 + \nu^2 \mathcal{U} \equiv \varepsilon$$

$$T^{ij} = \delta^{ij} \left(\lambda^2 \pi^2 \mathbb{B}_0^2 - \nu^2 \mathcal{U} \right) \equiv \delta^{ij} P$$

  - *local* thermodynamical quantities

  - BPS eq. = zero pressure condition
  - e-m. conservation: $\partial_\mu T^{\mu\nu} = 0$
    static: $\partial_i T^{ij} = 0 \Rightarrow \partial_j P = 0 \Rightarrow P = const.$

  - constant pressure equation is a first integral of static EL eq.

$$\lambda^2 \pi^4 \mathcal{B}_0^2 - \nu^2 \mathcal{U} = P > 0$$

$$\lambda \pi^2 \mathcal{B}_0 = \pm \nu \sqrt{\mathcal{U} + \tilde{P}}, \quad \tilde{P} \equiv (P/\nu^2)$$

    static non-BPS solutions with $P > 0$

perfect fluid - exact thermodynamics

- **energy density EoS**

$$\varepsilon - P = 2\nu^2 \mathcal{U}$$

  - **non-barotropic chiral fluid** $\varepsilon \neq \varepsilon(P)$

    the step-function potential $\varepsilon = P + 2\nu^2$

    no potential $\varepsilon = P$

    high pressure limit - potential independent

$$\varepsilon = P$$

  - *on-shell* EoS

$$\varepsilon = \varepsilon(P, \vec{x})$$

beyond mean-field thermodynamics:

$P = const.$ but $\varepsilon \neq const.$

Neutron Stars in the Skyrme model

beyond the mean-field limit

the BPS Skyrme model with gravity Adam, Naya, Sanchez, Vazquez, Wereszczynski (15)

$$S_{06} = \int d^4x |g|^{\frac{1}{2}} \left(-\lambda^2 \pi^4 |g|^{-1} g_{\rho\sigma} \mathbb{B}^\rho \mathbb{B}^\sigma - \mu^2 \mathcal{U} \right)$$

- energy-momentum tensor

$$T^{\rho\sigma} = -2|g|^{-\frac{1}{2}} \frac{\delta}{\delta g_{\rho\sigma}} S_{06}$$
$$= 2\lambda^2 \pi^4 |g|^{-1} \mathcal{B}^\rho \mathcal{B}^\sigma - \left(\lambda^2 \pi^4 |g|^{-1} g_{\pi\omega} \mathcal{B}^\pi \mathcal{B}^\omega - \mu^2 \mathcal{U} \right) g^{\rho\sigma}$$

- the energy-momentum tensor of **a perfect fluid**

$$T^{\rho\sigma} = (p + \rho) u^\rho u^\sigma - p g^{\rho\sigma}$$

  where the four-velocity $u^\rho = \mathcal{B}^\rho / \sqrt{g_{\sigma\pi} \mathcal{B}^\sigma \mathcal{B}^\pi}$ and

$$\rho = \lambda^2 \pi^4 |g|^{-1} g_{\rho\sigma} \mathcal{B}^\rho \mathcal{B}^\sigma + \mu^2 \mathcal{U}$$
$$p = \lambda^2 \pi^4 |g|^{-1} g_{\rho\sigma} \mathcal{B}^\rho \mathcal{B}^\sigma - \mu^2 \mathcal{U}$$

- for a static case with diagonal metric $u^\rho = (\sqrt{g^{00}}, 0, 0, 0)$

$$T^{00} = \rho g^{00} \,, \quad T^{ij} = -p g^{ij}.$$

  flat space case $\Rightarrow$ pressure must be constant

- In general, ρ and p arbitrary functions of the space-time coordinates, $\Rightarrow$ **no universal equation of state $p = p(\rho)$ valid for all solutions**

the BPS Skyrme model with gravity

- **Einstein equations**
  - static, spherically symmetric metric (Schwarzschild coordinates)

$$ds^2 = \mathbf{A}(r)dt^2 - \mathbf{B}(r)dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

  - axially symmetric ansatz for the Skyrme field with baryon number B

$$U = e^{i\xi\vec{n}\cdot\vec{\tau}}$$

$$\xi = \xi(r), \quad \vec{n} = (\sin\theta\cos B\phi, \sin\theta\sin B\phi, \cos\theta)$$

are compatible with the Einstein equations

$$G_{\rho\sigma} = \frac{\kappa^2}{2} T_{\rho\sigma}$$

FT + GR with *full backreaction* $\leftrightarrow$ TOV: fix EoS

Einstein equations

$$\frac{1}{r}\frac{\mathbf{B}'}{\mathbf{B}} = -\frac{1}{r^2}(\mathbf{B}-1) + \frac{\kappa^2}{2}\mathbf{B}\rho$$

$$r\,(\mathbf{B}p)' = \frac{1}{2}(1-\mathbf{B})\mathbf{B}(\rho+3p) + \frac{\kappa^2}{2}\mu^2 r^2 \mathbf{B}^2 \mathcal{U}(h)p$$

$$\frac{\mathbf{A}'}{\mathbf{A}} = \frac{1}{r}(\mathbf{B}-1) + \frac{\kappa^2}{2} r\mathbf{B}p$$

- **A**, **B** and ξ are functions of $r \Rightarrow p$ and ρ are functions of r
  $h = (1-\cos\xi)/2$

  $$\rho = \frac{4B^2\lambda^2}{\mathbf{B}r^4}h(1-h)h_r^2 + \mu^2\mathcal{U}(h), \quad p = \rho - 2\mu^2\mathcal{U}(h)$$

  eliminate $r \Rightarrow$ **on-shell EoS** $p = p(\rho)$
  except Θ-step-function potential $\Rightarrow$ **off-shell EoS** $\rho = p + 2\mu^2$

- axially symmetric ansatz **is the correct one** because gravity straightens out all deviations from spherical symmetry

- **parameters fitting**
  two parameters in the model λ and μ

  - chose a potential

    $$\mathcal{U}_\pi^2, \quad \mathcal{U}_\pi = 1 - \cos\xi$$

    $$\mathcal{U}_{step} = \begin{cases} \sin^4\xi & \xi \in \left[0, \frac{\pi}{2}\right] \\ 1 & \xi \in \left[\frac{\pi}{2}, \pi\right] \end{cases}$$

  - $\mathbf{m} \equiv \lambda\mu$ has the dimensions of mass (energy)
    fit to the binding energy of nucleon of infinite nuclear matter

    $$E_b = 16.3 \text{ MeV}$$

  - $\mathbf{l} \equiv (\lambda/\mu)^{1/3}$ has the dimensions of length
    fit to the nuclear saturation density

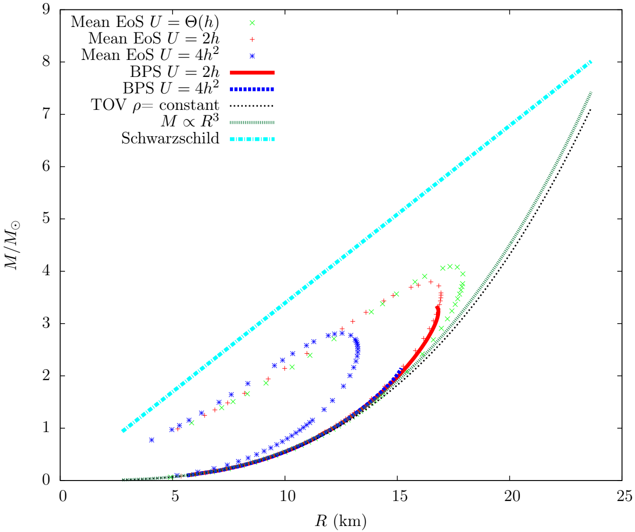
    $$n_0 = 0.153\text{fm}^{-3}$$

  particular potentials

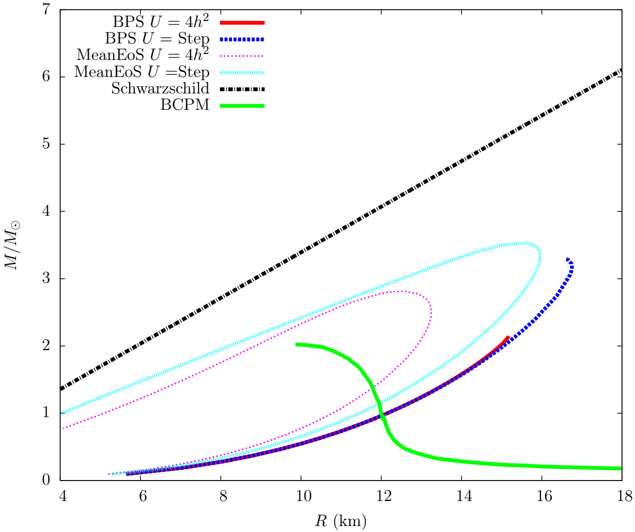
$$\mathcal{U}_\pi^2: \quad \lambda^2 = 15.49 \text{ MeVfm}^3, \quad \mu^2 = 141.22 \text{ MeVfm}^{-3}$$

$$\mathcal{U}_{step}: \quad \lambda^2 = 23.60 \text{ MeVfm}^3, \quad \mu^2 = 121.08 \text{ MeVfm}^{-3}$$

- **results: mass-radius relation I**



- **results: mass-radius relation II**



- maximal mass

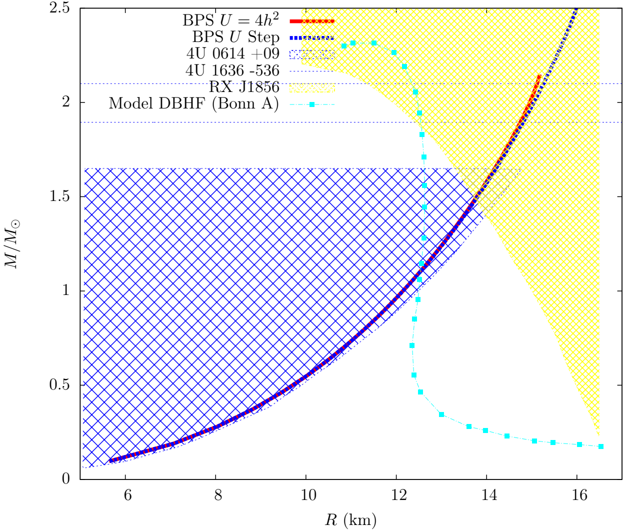
$$\begin{array}{lcl} \mathcal{U} = \Theta & \to & M_{max} = 4.1 \\ \mathcal{U}_{step} & \to & M_{max} = 3.29 \\ \mathcal{U}_{\pi} & \to & M_{max} = 3.34 \\ \mathcal{U}_{\pi}^{2} & \to & M_{max} = 2.15 \end{array}$$

  $\to$ compatible with exp. data
  $\to$ compatible with $M = 2.5$ v.difficult for other EFT

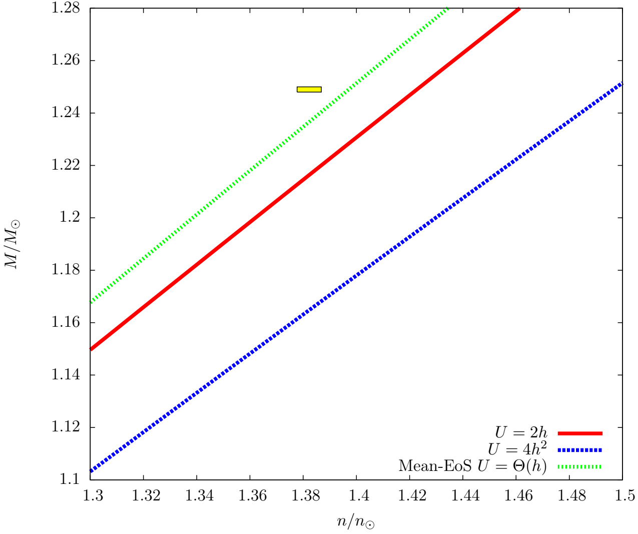
- $M - R$ curve qualitatively different
  $\to$ EoS approaches the max. stiff EoS
  $\to$ light NS: **crust** important

- full FT vs. mean-field
  $\to$ quantitatively differ
  $\to$ true M_{max} is lower than predicted by MF

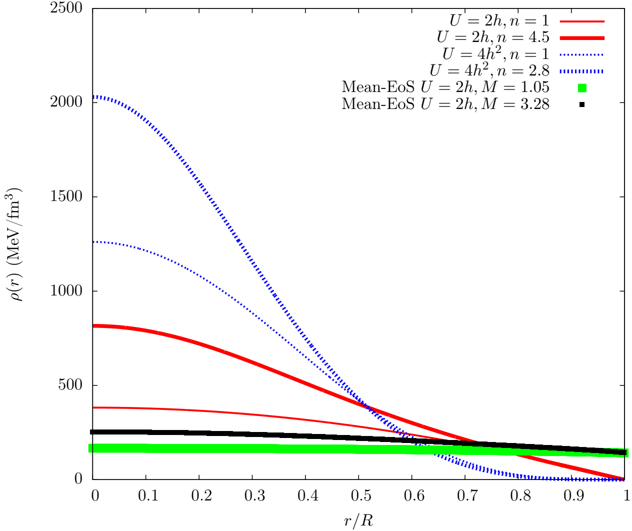
- **results: mass-radius relation**



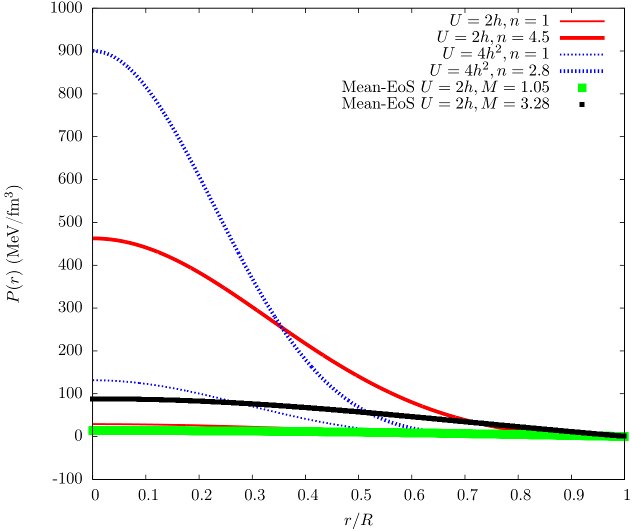
- **results: the double pulsar J0737-3093**



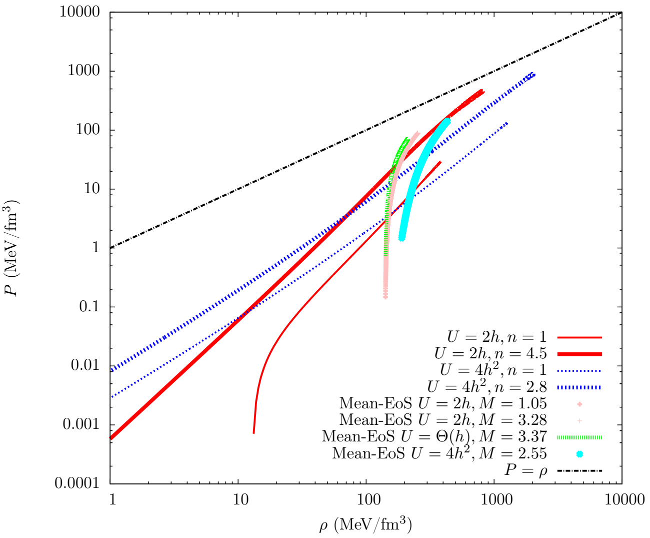
- **results: local energy density**



- **results: local pressure**



- **results: local Equation of State**



- no unique EoS

  $\rightarrow$ on-shell EoS: polytropic

$$p \sim a\,\epsilon^{b}$$

  $\rightarrow$ where $a(B)$, $b(B)$

- **inverse TOV questionable**

  $\rightarrow$ inhomogeneities important

- **role of the potential?**

Beyond the BPS Skyrme limit

crust

model independent crust

- **crust** crystal/solid part

  mass: $M_{crust} = 1\% M$
  radius: light NS - important

  important for many phenomena: pulsar glitches
  X-ray burst
  gamma-ray flares of magnetars
  torsional oscillations
  cooling

- crust EoS $\rightarrow$ complicated, model dependent, small number
  $\downarrow$
  in Skyrme model: $\mathcal{L}_{240}$ + ED

model independent crust

- approx. $M_{crust} \ll M$ Zdunik, Fortin, Haensel (16)

hydrostatic equilibrium TOV in the **crust**: $0 < P < P_{cc}$

$$\frac{dP}{dr} = -\left(\rho + \frac{P}{c^2}\right)\left(1 - \frac{2Gm(r)}{rc^2}\right)^{-1}\left(\frac{Gm(r)}{r^2} + 4\pi G r \frac{P}{c^2}\right)$$

$$\Downarrow \; m(r) = M,\; \frac{4\pi r^2 P}{mc^2} \ll 1$$

$$\frac{dP}{c^2\rho + P} = -\frac{GM}{c^2}\frac{dr}{r^2(1 - 2GM/rc^2)}$$

$\rightarrow \chi(P) = \int_0^P \frac{dP'}{c^2\rho + P'}$ solely by the EoS of the crust

but

$$\chi[P(r)] = \frac{1}{2}\ln\frac{1 - r_g/R}{1 - r_g/r}, \quad r_g = 2GM/c^2$$

$\rightarrow$ baryon chemical potential $\mu = \frac{d\rho}{dn}$ or $\mu = \frac{P + \rho c^2}{n}$

$$\frac{dP}{c^2\rho + P} = \frac{d\mu}{\mu} \quad \Rightarrow \quad \chi(P) = \ln\frac{\mu(P)}{\mu_0}$$

$\mu_0 = \mu(P = 0) = 930.4\text{MeV}$

model independent crust

- only knowledge of $\mu_{cc} = \mu(P_{cc})$ required
- rest follow from the core properties

- mass-radius

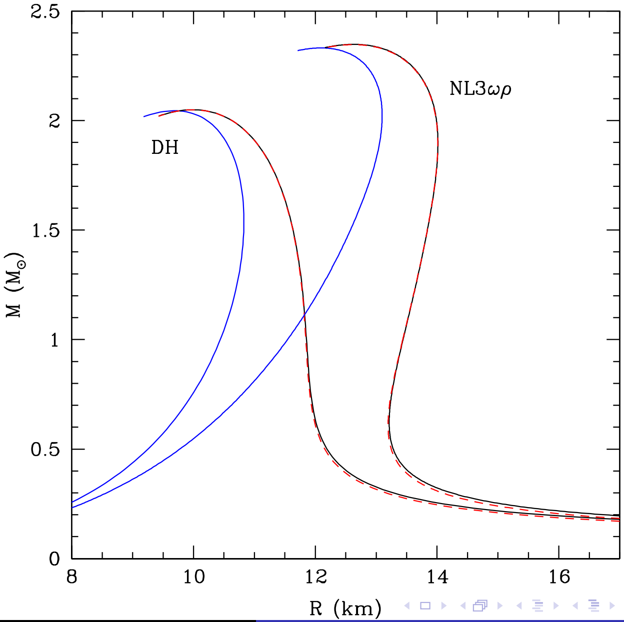
$$R = \frac{R_{core}}{1 - \left(\frac{\mu_{cc}^2}{\mu_0^2} - 1\right)\left(\frac{R_{core}c^2}{2GM} - 1\right)}$$

- the crust thickness

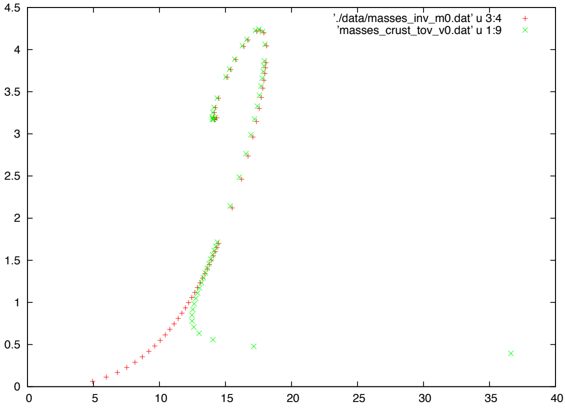
$$l = \phi R_{core} \frac{1 - 2GM/R_{core}c^2}{1 - \phi(1 - 2GM/R_{core}c^2)}$$

$$\phi = \left(\frac{\mu_{cc}^2}{\mu_0^2} - 1\right) \frac{R_{core}c^2}{2GM}$$

model independent crust



BPS Skyrme with the crust Θ-step-function potential



summary

- neutron stars in the Skyrme Model
  - the BPS Skyrme model limit = liquid core

    $\rightarrow$ very promising M_{max}, R_{max}, $M(R)$

    $\rightarrow$ non-barotropic nuclear fluid
    $\rightarrow$ the inverse TOV?

  - **beyond** the BPS limit

    $\rightarrow$ crust (model independent results)
    $\rightarrow$ core/crust: phases $\rightarrow$ pasta, lasagne, meat ball

the Skyrme model - unified EFT for nuclear matter at all scales from $B = 1$ to $B \sim 10^{58}$

perfect fluid - exact thermodynamics

- **particle (baryon) density EoS**

$$\rho_B = \mathcal{B}_0$$

  - generically non-constant (beyond MF)

$$\rho_B = \frac{\nu}{\lambda\pi^2}\sqrt{\mathcal{U} + \frac{P}{\nu^2}}$$

  - *on-shell*

$$\rho_B = \rho_B(P, \vec{x})$$

no universal $\varepsilon = \varepsilon(P)$, $\rho_B = \rho_B(P)$

universal relation - *off-shell* and *non-MF*

$$\varepsilon + P = 2\lambda^2\pi^4\rho_B^2$$

- **baryon chemical potential**

definition: $\varepsilon + P = \rho\mu \qquad \Rightarrow \qquad \mu_B = 2\lambda^2\pi^4\rho_B$

  - off-shell
  - universal, potential independent
  - non-MF (local)

perfect fluid - exact thermodynamics

- generically **exact (non-mean field) thermodynamics**
  - ϵ, ρ_B non-constant generically non-constant
  - non-barotropic fluid
  - no universal EoS

- **mean-field limit**
  - MF averages $\bar{\epsilon}$, $\bar{\rho}_B$

$$\bar{\epsilon} = \frac{E_{06}}{V}, \qquad \bar{\rho} = \frac{B}{V}$$

  - *universal* (geometrical) EoS
    - E_{06}, V, $\bar{\epsilon}$, $\bar{\rho}_B$
      - known as functions of P FT pressure
      - no need for solutions!
      - only $\mathcal{U}$ matters

$$E_{06}(P) = B\pi^2\lambda\mu \left\langle \frac{2\mathcal{U} + P/\mu}{\sqrt{\mathcal{U} + P/\mu}} \right\rangle, \qquad V(P) = B\pi^2 \frac{\lambda}{\mu} \left\langle \frac{1}{\sqrt{\mathcal{U} + P/\mu}} \right\rangle$$

- FT pressure is the pressure

$$P = -\frac{dE_{06}}{dV}$$

micro (FT) thermodynamics = macro thermodynamics

comparison **full** vs. **mean-field**