Hot and dense neutron-rich matter in supernovae and neutron star mergers

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Y. Lim & J. W. Holt, Phys. Rev. Lett. 121 (2018) 062701.

J. W. Holt & Y. Lim, Phys. Lett. B 784 (2018) 77.

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Multi-messenger astronomy



Observational campaigns of neutron stars



Neutron Star Interior Composition Explorer (NICER)

- Neutron star radii: $\pm 5\%$
- Neutron star masses: $\pm 10\%$
- Combined timing and spectral resolution in the soft X-ray band

First dedicated targets: {
PSR_J0437-4715
PSR_J0030+0451
}

LIGO/VIRGO

- Late-inspiral gravitational waveform related to neutron star tidal deformability
- Poster-merger peak frequency sensitive to neutron star radius















Gandolfi, Carlson & Reddy, PRC (2012)

Nuclear forces from chiral effective field theory

NATURAL SEPARATION OF SCALES

CHIRAL EFFECTIVE FIELD THEORY

Low-energy theory of nucleons and pions



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Priors from chiral EFT EOS calculations

 $\rho E^{(1)} = \frac{1}{2} \sum_{12} n_1 n_2 \langle 12 | (\overline{V}_{NN} + \overline{V}_{NN}^{\text{med}}/3) | 12 \rangle,$ $\rho E^{(2)} = -\frac{1}{4} \sum_{\text{reff}} |\langle 12|\overline{V}_{\text{eff}}|34\rangle|^2 \frac{n_1 n_2 \bar{n}_3 \bar{n}_4}{e_3 + e_4 - e_1 - e_2},$ $\rho E_{\rm pp}^{(3)} = \frac{1}{8} \sum \langle 12 | \overline{V}_{\rm eff} | 34 \rangle \langle 34 | \overline{V}_{\rm eff} | 56 \rangle \langle 56 | \overline{V}_{\rm eff} | 12 \rangle$ $\times \frac{n_1 n_2 n_3 n_4 n_5 n_6}{(e_3 + e_4 - e_1 - e_2)(e_5 + e_6 - e_1 - e_2)},$ $\rho E_{\rm hh}^{(3)} = \frac{1}{8} \sum \langle 12 | \overline{V}_{\rm eff} | 34 \rangle \langle 34 | \overline{V}_{\rm eff} | 56 \rangle \langle 56 | \overline{V}_{\rm eff} | 12 \rangle$ $\times \frac{\bar{n}_1 \bar{n}_2 n_3 n_4 n_5 n_6}{(e_1 + e_2 - e_3 - e_4)(e_1 + e_2 - e_5 - e_6)},$ $\rho E_{\rm ph}^{(3)} = -\sum \langle 12|\overline{V}_{\rm eff}|34\rangle\langle 54|\overline{V}_{\rm eff}|16\rangle\langle 36|\overline{V}_{\rm eff}|52\rangle$ 123 456 $\times \frac{n_1 n_2 n_3 n_4 n_5 n_6}{(e_3 + e_4 - e_1 - e_2)(e_3 + e_6 - e_2 - e_5)},$



Symmetric nuclear matter equation of state



Pure neutron matter uncertainty estimates



Sources of uncertainty

- Scale dependence
- Convergence in many-body perturbation theory

Pure neutron matter convergence in the chiral expansion



Pure neutron matter convergence in the chiral expansion



Equations of state from new potentials up to 5th order in the chiral expansion



<u>Question</u>: How to infer properties of the neutron star equation of state from a precise mass vs. radius measurement?

- Construct a model with parameters \vec{a}
- Bayes' Theorem:



- Strategy:
 - Find useful parametrizations for the equation of state
 - Obtain priors from chiral EFT predictions
 - Use laboratory measurements of finite nuclei to obtain likelihood functions and posteriors

Prior distributions from chiral EFT



Prior distributions from chiral EFT



Equations of state from chiral EFT priors



Likelihood functions for symmetric nuclear matter

• Parametrization: $\frac{E}{A}(\rho, \delta = 0) = \frac{3k_F^2}{10m} + \frac{k_F^3}{9\pi^2} \left(a_0 + a_1\beta + \frac{1}{2}a_2\beta^2 + \frac{1}{6}a_3\beta^3\right)$

 $\langle K \rangle = 232.65 \text{ MeV}$

 $\sigma_K = 7.00 \text{ MeV}$

200

220

240

K (MeV)

260

 Average values of a and full covariance matrix from analysis of 200 Skyrme mean field models fitted to nuclear properties

[M. Dutra et al., PRC (2012)]

0.8

Probability Distribution

0.2

0.0 180

17.0

1 (

0.8

Probability Distribution 9.0

0.2

0.0

 $\langle B \rangle = 15.94 \text{ MeV}$

 $\sigma_{n_0} = 0.149 \; {\rm MeV}$

15.5

16.0

B (MeV)

16.5



Q (MeV)

Parametrizing the isospin-asymmetry dependence

$$\frac{E}{A}(\rho,\delta_{np}) = A_0(\rho) + S_2(\rho)\delta_{np}^2 + \underbrace{\sum_{n=2}^{\infty}(S_{2n} + L_{2n}\ln|\delta_{np}|)\delta_{np}^{2n}}_{\text{small}}$$

- Traditionally expand symmetry energy about saturation density: $S_2(\rho) = J + L\left(\frac{\rho - \rho_0}{3\rho_0}\right) + \frac{1}{2}K_{sym}\left(\frac{\rho - \rho_0}{3\rho_0}\right)^2 + \cdots$
- In Fermi liquid theory, symmetry energy related to 2 Landau parameters:

$$S_2(k_F) = \frac{k_F^2}{6m} + \frac{k_F^3}{9\pi^2} \left[3f_0'(k_F) - f_1(k_F)\right]$$

The expression leads to two correlation equations [Holt & Lim, PLB (2018)]:

$$L = 3\rho_0 \left. \frac{dS_2}{d\rho} \right|_{\rho_0} = 3J - S_0 + \frac{\rho_0}{6} \left(3k_F \frac{df'_0}{dk_F} - k_F \frac{df_1}{dk_F} \right) \right|_{k_F^0}$$
$$K_{\text{sym}} = 9\rho_0^2 \left. \frac{d^2S_2}{d\rho^2} \right|_{\rho_0} = 4L - 12J + 2S_0 + \frac{\rho_0}{6} \left(3k_F^2 \frac{d^2f'_0}{dk_F^2} - k_F^2 \frac{d^2f_1}{dk_F^2} \right) \right|_{k_F^0}$$

Parametrizing the zero-temperature equation of state

• Expand about a small reference Fermi momentum k_r :

$$S_{2}(k_{F}) = \frac{k_{F}^{2}}{6m} + \frac{k_{F}^{3}}{9\pi^{2}} \left[3f_{0}'(k_{F}) - f_{1}(k_{F})\right]$$
$$= \frac{k_{F}^{2}}{6m} + \frac{k_{F}^{3}}{9\pi^{2}} \left[c_{0} + c_{1}\beta + \frac{1}{2}c_{2}\beta^{2}\right] \qquad \beta = \frac{k_{F} - k_{r}}{k_{r}}$$

- At low densities, Fermi liquid parameters should be well constrained by chiral effective field theory
- Logarithmic terms $\sim \log(1 + 4k_F^2/m_\pi^2)$ in the symmetry energy require $k_r > 0.9 \,\mathrm{fm}^{-1}$ for the Taylor series to be convergent at saturation density
- Perturbation theory expansion breaks down below similar scale

Therefore choose
$$egin{array}{c} eta_0 = rac{k_F^0 - k_r}{k_r} \simeq 0.5 \end{array}$$

Symmetry energy slope parameter [Holt & Lim, PLB (2018)] :

$$L = 3\rho_0 \left. \frac{dS_2}{d\rho} \right|_{\rho_0} = 3J - S_0 + \frac{\rho_0}{6} \left(3k_F \frac{df_0'}{dk_F} - k_F \frac{df_1}{dk_F} \right) \right|_{k_F^0}$$
$$= (3+\gamma)J - (1+\gamma)S_0 - \gamma \frac{\rho_0}{6} \left(c_0 - \eta_1 c_1 + \eta_1 c_2 \right)$$

Likewise for the symmetry energy incompressibility [Holt & Lim, PLB (2018)]:

$$\begin{split} K_{\text{sym}} &= 9\rho_0^2 \left. \frac{d^2 S_2}{d\rho^2} \right|_{\rho_0} = 4L - 12J + 2S_0 + \frac{\rho_0}{6} \left(3k_F^2 \frac{d^2 f_0'}{dk_F^2} - k_F^2 \frac{d^2 f_1}{dk_F^2} \right) \right|_{k_F^0} \\ &= 5\gamma J - (5\gamma + 2)S_0 - 5\gamma \frac{\rho_0}{6} \left(c_0 - \eta_2 c_1 + \eta_2 c_2 \right) \end{split}$$

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$$\gamma = 3.7 \qquad \eta_1 = -0.08$$

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$$= 5\gamma J - (5\gamma + 2)S_0 - 5\gamma \frac{\rho_0}{6} \left(c_0 - \eta_2 c_1 + \eta_2 c_2 \right)$$
$$\eta_2 = -0.16$$

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$$L = 3\rho_0 \left. \frac{dS_2}{d\rho} \right|_{\rho_0} = 3J - S_0 + \frac{\rho_0}{6} \left(3k_F \frac{df'_0}{dk_F} - k_F \frac{df_1}{dk_F} \right) \right|_{k_F^0}$$

= (3 + \gamma) J - (1 + \gamma) S_0 - \gamma \frac{\rho_0}{6} (c_0 - \eta_1 c_1 + \eta_1 c_2)
Universal slope Model-dependent scale shift

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Comparison to chiral EFT results



- NLO, N2LO, and N3LO potentials (plus N2LO three-body force)
- Predicted correlation slopes agree well with explicit chiral EFT results

Validity of ansatz



Validity of ansatz



 Better theory constraints on low-density Fermi liquid parameters may reduce correlation uncertainties

"Global" parametrization of symmetry energy

$$S_2(\rho) = \sum_{i=0}^{3} b_i \left(\frac{\rho}{\rho_0}\right)^{(i+2)/3}$$

$$b_0 = S_0 \qquad b_2 = -K_{\text{sym}} + 5L - 15J + 3S_0$$

$$b_1 = \frac{1}{2}K_{\text{sym}} - 3L + 10J - 3S_0 \qquad b_3 = \frac{1}{2}K_{\text{sym}} - 2L + 6J - S_0$$



Likelihood functions for pure neutron matter

• Parametrization:
$$\frac{E}{A}(\rho, \delta = 1) = 2^{2/3} \frac{3k_F^2}{10m} + \frac{k_F^3}{9\pi^2} \left(b_0 + b_1\beta + \frac{1}{2}b_2\beta^2 + \frac{1}{6}b_3\beta^3\right)$$

$$S_{2}(\rho) = \frac{k_{F}^{2}}{6m} + \frac{k_{F}^{3}}{9\pi^{2}} \underbrace{\left(c_{0} + c_{1}\beta + \frac{1}{2}c_{2}\beta^{2} + \frac{1}{6}c_{3}\beta^{3}\right)}_{Correlations among J, L, K_{sym}}$$

$$L = (3 + \gamma)J - (1 + \gamma)S_{0} - \gamma \frac{\rho_{0}}{6} (c_{0} - \eta_{1}c_{1} + \eta_{1}c_{2})$$

$$K_{sym} = 5\gamma J - (5\gamma + 2)S_{0} - 5\gamma \frac{\rho_{0}}{6} (c_{0} - \eta_{2}c_{1} + \eta_{2}c_{2})$$

$$\int_{Correlations among J, L, K_{sym}} \int_{Correlations J,$$

 Derive likelihood functions involving {b₀, b₁, b₂, b₃} for subsequent Bayesian posterior probability distribution

Equations of state from posterior probability distributions













SCALES IN CORE-COLLAPSE SUPERNOVAE



FINITE-TEMPERATURE EQUATIONS OF STATE

Lattimer & Swesty, 1991

- Point-like effective interactions + liquid drop model
- Significantly underpredicts neutron matter energy density

Shen et al., 1998

Relativistic mean field theory + Thomas-Fermi approximation



Goal: first microscopic EoS for astrophysical simulations + consistent neutrino cross sections

THERMODYNAMIC EQUATION OF STATE

$$F(\mu_0, T) = F_0(\mu_0, T) + \lambda \Omega_1(\mu_0, T) + \lambda^2 \left(\Omega_2(\mu_0, T) - \frac{1}{2} \frac{(\partial \Omega_1 / \partial \mu_0)^2}{\partial^2 \Omega_0 / \partial \mu_0^2} \right) + \mathcal{O}(\lambda^3)$$



Dense set of mesh points in parameter space needed (~30,000,000 configurations)

High precision for second-order numerical derivatives

BENCHMARK: LIQUID-GAS PHASE TRANSITION



Experiment (compound nucleus & multifragmentation) [J. B. Elliott et al., PRC (2013)

 $T_c = 17.9 \pm 0.4 \,\mathrm{MeV}$ $\rho_c = 0.06 \pm 0.02 \,\mathrm{fm}^{-3}$ $P_c = 0.31 \pm 0.07 \,\mathrm{MeV} \,\mathrm{fm}^{-3}$

BENCHMARK: NEUTRON MATTER VIRIAL EXPANSION



NEW BULK MATTER EQUATIONS OF STATE

Tabulation I	Tabulation II
$0.0002 < ho < 0.32~{ m fm}^{-3}$	$10^{-6} < \rho < 0.18 \ {\rm fm}^{-3}$
$\Delta ho=0.0002~{ m fm}^{-3}$	Variable $\Delta \rho$
$0.2 < T < 50~{\rm MeV}$	$-1.08 < \log_{10} T < 1.52$
$\Delta T=0.2~{ m MeV}$	$\Delta \log_{10} T = 0.04$
$0 \leq Y_p \leq 0.5$	$0 \leq Y_p \leq 0.5$

Du, Steiner, and <u>Holt</u>, arXiv:1802.09710

Medium-dependent bulk energies then included in a modified liquid-drop formalism to construct the EOS for inhomogeneous matter [Furusawa et al., ApJ (2013)]

Summary and outlook

- New era of major observational campaigns to study the properties of neutron stars
- Complementary theoretical models with accurate nuclear physics inputs needed to guide and interpret observations
- Combine properties of finite nuclei with "model independent" predictions from chiral EFT to obtain posterior distribution function for model parameters
- Ultra high-density matter a challenging frontier for *any* theoretical, experimental, or observation investigation