Abelian Decomposition and Monopole Condensation in QCD

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September 15, 2018

Mass Generation in QCD

1. What makes QCD so different from QED?
2. How can we simplify QCD?
3. How can we generate the confinement of color?
4. What are the observable consequences?

History

- In 1974 Nambu and Mandelstam conjectured the monopole condensation as the confinement mechanism.
- In 1977 Savvidy calculated the SU(2) QCD effective action and obtained the Savvidy vacuum.
- In 1980 the Abelian decomposition of QCD was proposed. Independently, in 1981 't Hooft conjectured the Abelian dominance.
- The lattice QCD was able to obtain the linear confining potential numerically, but unable to tell what is the confinement mechanism.

Recent Progress

1. For the first time the lattice QCD calculation could pinpoint the monopole is responsible for the confinement gauge independently.

2. A new calculation of the QCD effective action which generates the stable monopole condensation and the dimensional transmutation was made.

3. The quark and chromon model has been proposed which generalizes the quark model to provide a new hadron spectroscopy.

Contents

A. Motivation

- To prove the Abelian dominance we have to know what is the Abelian part. How do we identify the Abelian part?
- Proton is made of three quarks, but obviously it contains gluons to bind them. But the quark model tells that it has no valence gluon. If so, what is the binding gluon and how can we separate it?

B. Abelian decomposition of SU(2) QCD

- Let $(\hat{n}_1, \hat{n}_2, \hat{n}_3 = \hat{n})$ be an orthonormal basis and $\hat{n}$ be the Abelian direction. Impose the isometry to obtain the restricted potential $\hat{A}_\mu$,

$$D_\mu \hat{n} = \partial_\mu \hat{n} + g\vec{A}_\mu \times \hat{n} = 0,$$

$$\vec{A}_\mu \to \hat{A}_\mu = A_\mu \hat{n} - \frac{1}{g}\hat{n} \times \partial_\mu \hat{n} = \mathcal{A}_\mu + \mathcal{C}_\mu,$$

$$\mathcal{A}_\mu = A_\mu \hat{n}, \quad \mathcal{C}_\mu = -\frac{1}{g}\hat{n} \times \partial_\mu \hat{n}, \quad A_\mu = \hat{n} \cdot \vec{A}_\mu.$$

$\hat{A}_\mu$ is Abelian but has a dual structure, made of the non-topological (Maxwellian) $\mathcal{A}_\mu$ which describes the neutral binding gluon and the topological (Diracian) $\mathcal{C}_\mu$ which describes the non-Abelian monopole.

- Obtain the gauge independent Abelian decomposition

$$\vec{A}_\mu = \hat{A}_\mu + \vec{X}_\mu, \quad (\hat{n} \cdot \vec{X}_\mu = 0).$$
$$\vec{F}_{\mu\nu} = \hat{F}_{\mu\nu} + \hat{D}_\mu \vec{X}_\nu - \hat{D}_\nu \vec{X}_\mu + g \vec{X}_\mu \times \vec{X}_\nu,$$
$$\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu + g \hat{A}_\mu \times \hat{A}_\nu = (F_{\mu\nu} + H_{\mu\nu})\hat{n},$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,\ H_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu,\ C_\mu = -\frac{1}{g}\hat{n}_1 \cdot \partial_\mu \hat{n}_2.$$

1. $\hat{A}_\mu$ has the full SU(2) gauge degrees of freedom.
2. $\vec{X}_\mu$ transforms covariantly and describes the colored valence gluon.

Two Types of Gluons!

Restricted QCD (RCD): Abelian QCD

- Define RCD which describes the Abelian sub-dynamics with $\hat{A}_\mu$,

$$\mathcal{L}_{RCD} = -\frac{1}{4}\hat{F}_{\mu\nu}{}^2 = -\frac{1}{4}(F_{\mu\nu} + H_{\mu\nu})^2$$
$$= -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2g}F_{\mu\nu}\hat{n}\cdot(\partial_\mu\hat{n}\times\partial_\nu\hat{n}) - \frac{1}{4g^2}(\partial_\mu\hat{n}\times\partial_\nu\hat{n})^2.$$

  It has the full SU(2) gauge symmetry yet is simpler than QCD, and has a dual structure with two potentials $\mathcal{A}_\mu$ and $\mathcal{C}_\mu$.

- $\hat{n}$ describes the monopole topology $\pi_2(S^2)$ and the vacuum topology $\pi_3(S^2)$.

"Non-Abelian" Dirac theory

Extended QCD (ECD)

- Adding the valence gluon we have ECD

$$\mathcal{L}_{ECD} = -\frac{1}{4}\vec{F}_{\mu\nu}^2 = -\frac{1}{4}\hat{F}_{\mu\nu}^2$$

$$-\frac{1}{4}(\hat{D}_\mu \vec{X}_\nu - \hat{D}_\nu \vec{X}_\mu)^2 - \frac{g}{2}\hat{F}_{\mu\nu} \cdot (\vec{X}_\mu \times \vec{X}_\nu) - \frac{g^2}{4}(\vec{X}_\mu \times \vec{X}_\nu)^2.$$

1. QCD can be interpreted as RCD made of $\hat{A}_\mu$ which has $\vec{X}_\mu$ as colored source.
2. This puts QCD to the background field formalism, with $\hat{A}_\mu$ and $\vec{X}_\mu$ as classical background and quantum fluctuation.
3. $\hat{n}$ describes the topological, not dynamical, degree.

- ECD has two gauge symmetries, the classical (background) gauge symmetry

$$\delta \hat{A}_\mu = \frac{1}{g}\hat{D}_\mu \vec{\alpha}, \quad \delta \vec{X}_\mu = -\vec{\alpha} \times \vec{X}_\mu,$$

as well as the quantum (fast) gauge symmetry

$$\delta \hat{A}_\mu = \frac{1}{g}(\hat{n} \cdot D_\mu \vec{\alpha})\hat{n}, \quad \delta \vec{X}_\mu = \frac{1}{g}\hat{n} \times (D_\mu \vec{\alpha} \times \hat{n}).$$

- This justifies us to call the colorless binding gluon the neuron and the colored valence gluon the chromon, and generaliizes the quark model to the quark and chromon model.

C. Skyrme Theory from QCD

- The Skyrme theory is described by

$$\mathcal{L}_{\mathcal{S}} = -\frac{\mu^2}{4}\Big[\frac{1}{2}(\partial_\mu\omega)^2 + 2\sin^2\frac{\omega}{2}(\partial_\mu\hat{n})^2\Big]$$
$$-\frac{\alpha}{16}\Big[\sin^2\frac{\omega}{2}(\partial_\mu\omega\partial_\nu\hat{n} - \partial_\nu\omega\partial_\mu\hat{n})^2 + 4\sin^4\frac{\omega}{2}(\partial_\mu\hat{n}\times\partial_\nu\hat{n})^2\Big]$$
$$= -\frac{\mu^2}{2}\big((\partial_\mu\sigma)^2 + (\partial_\mu\vec{\pi})^2\big)$$
$$-\frac{\alpha}{4}\big((\partial_\mu\sigma\partial_\nu\vec{\pi} - \partial_\nu\sigma\partial_\mu\vec{\pi})^2 + (\partial_\mu\vec{\pi}\times\partial_\nu\vec{\pi})^2\big),$$

where $\sigma = \cos\dfrac{\omega}{2}$ and $\vec{\pi} = \hat{n}\sin\dfrac{\omega}{2}$ are the sigma and pion fields.

- With $\omega = \pi$, it reduces to Skyrme-Faddeev theory which describes the core dynamics of Skyrme theory and contains Faddeev-Niemi knot (twisted magnetic vortex ring) given by the $\pi_3(S^2)$ topology

$$\mathcal{L}_{SF} = -\frac{\mu^2}{2}(\partial_\mu \hat{n} \times \partial_\nu \hat{n})^2 - \frac{\alpha}{4}(\partial_\mu \hat{n})^2.$$

- The Skyrme theory has a deep connection to QCD. With $\hat{A}_\mu = -\frac{1}{g}\hat{n} \times \partial_\mu \hat{n} = \mathcal{C}_\mu$ the massive SU(2) QCD reduces to Skyrme-Faddeev theory (with $\mu = mg$ and $\alpha = 1/g^2$),

$$\mathcal{L}_{QCD} = -\frac{1}{4}\hat{F}_{\mu\nu}^2 - \frac{m^2}{2}\hat{A}_\mu^2 \Rightarrow -\frac{\mu^2}{2}(\partial_\mu \hat{n} \times \partial_\nu \hat{n})^2 - \frac{\alpha}{4}(\partial_\mu \hat{n})^2.$$

Abelian decomposition!

- Moreover, activating the sigma field we recover the Skyrme theory as a massive QCD interacting with the sigma field

$$\mathcal{L}_{\mathcal{S}} = -\frac{1}{4}\Big[(1-\sigma^2)^2 \hat{F}_{\mu\nu}^2 + (\partial_\mu \sigma \hat{A}_\nu - \partial_\nu \sigma \hat{A}_\mu)^2\Big]$$
$$-\frac{m^2}{2}\Big[\frac{(\partial_\mu \sigma)^2}{1-\sigma^2} + (1-\sigma^2)\hat{A}_\mu^2\Big].$$

- Since $\hat{A}_\mu$ describes the Wu-Yang monopole, the Skyrme theory can be interpreted as a theory of monopole in which the magnetic flux is confined by the built-in Meissner effect, where $\vec{\pi} = \hat{n}\sin\dfrac{\omega}{2}$ describes the dressed monopole, not pion.

- In fact, the non-spherically symmetric skyrmions should be viewed as monopoles, because they have the $\pi_2(S^2)$ topology given by the rational map of $\hat{n}$.

- So the skyrmion carries two topological numbers (b, m), the baryon number b given by $\pi_3(S^3)$ and the monopole number m given by $\pi_2(S^2)$.

- Moreover, the Skyrme theory has the knots as well as the skyrmions. This necessitates a totally new interpretation of Skyrme theory.

For Skyrme theory see PRL 87, 252001 (2001), EPJC 77, 88 (2017), and IJMPA 33, 1830006 (2018)

D. SU(3) QCD

- Since the SU(3) QCD has two Abelian directions, the Abelian projection is given by two magnetic symmetries,

$$D_\mu \hat{n} = 0, \qquad D_\mu \hat{n}' = 0, \qquad (\hat{n}^2 = \hat{n}'^2 = 1)$$

where $\hat{n}$ and $\hat{n}' = \hat{n} * \hat{n}$ are λ_3-like and λ_8-like octet unit vectors.

- With this we have the following Abelian decomposition,

$$\vec{A}_\mu = \hat{A}_\mu + \vec{X}_\mu, \quad \hat{A}_\mu = A_\mu \hat{n} + A'_\mu \hat{n}' - \frac{1}{g}(\hat{n} \times \partial_\mu \hat{n} + \hat{n}' \times \partial_\mu \hat{n}')$$
$$A_\mu = \hat{n} \cdot \vec{A}_\mu, \quad A'_\mu = \hat{n}' \cdot \vec{A}_\mu, \quad \hat{n} \cdot \vec{X}_\mu = \hat{n}' \cdot \vec{X}_\mu = 0.$$

- $\hat{A}_\mu$ can be expressed by the three neurons of SU(2) subgroups in Weyl symmetric form

$$\hat{A}_\mu = \sum_p \frac{2}{3}\hat{A}^p_\mu, \quad (p = 1, 2, 3),$$

$$\hat{A}^p_\mu = A^p_\mu \hat{n}^p - \frac{1}{g}\hat{n}^p \times \partial_\mu \hat{n}^p = \mathcal{A}^p_\mu + \mathcal{C}^p_\mu,$$

$$A^1_\mu = A_\mu, \quad A^2_\mu = -\frac{1}{2}A_\mu + \frac{\sqrt{3}}{2}A'_\mu, \quad A^3_\mu = -\frac{1}{2}A_\mu - \frac{\sqrt{3}}{2}A'_\mu,$$

$$\hat{n}^1 = \hat{n}, \quad \hat{n}^2 = -\frac{1}{2}\hat{n} + \frac{\sqrt{3}}{2}\hat{n}', \quad \hat{n}^3 = -\frac{1}{2}\hat{n} - \frac{\sqrt{3}}{2}\hat{n}'.$$

- With this we have the Abelian decomposition of SU(3) QCD,

$$\vec{A}_\mu = \hat{A}_\mu + \vec{X}_\mu = \sum_p (\frac{2}{3}\hat{A}^p_\mu + \vec{W}^p_\mu), \quad \vec{X}_\mu = \sum_p \vec{W}^p_\mu,$$

$$\vec{W}^1_\mu = X^1_\mu \hat{n}_1 + X^2_\mu \hat{n}_2, \ \vec{W}^2_\mu = X^6_\mu \hat{n}_6 + X^7_\mu \hat{n}_7, \ \vec{W}^3_\mu = X^4_\mu \hat{n}_4 + X^5_\mu \hat{n}_5.$$

- $\vec{W}^p_\mu$ can be expressed by red, blue, and green chromons of SU(2) subgroups (R_μ, B_μ, G_μ),

$$R_\mu = \frac{X^1_\mu + iX^2_\mu}{\sqrt{2}}, \ B_\mu = \frac{X^6_\mu + iX^7_\mu}{\sqrt{2}}, \ G_\mu = \frac{X^4_\mu + iX^5_\mu}{\sqrt{2}}.$$

But unlike $\hat{A}^p_\mu$, they are mutually independent.

- From this we have the Weyl symmetric SU(3) RCD and ECD

$$\mathcal{L}_{RCD} = -\sum_p \frac{1}{6}(\hat{F}^p_{\mu\nu})^2,$$

$$\mathcal{L}_{ECD} = -\sum_p \Big\{\frac{1}{6}(\hat{F}^p_{\mu\nu})^2 + \frac{1}{4}(\hat{D}^p_\mu \vec{W}^p_\nu - \hat{D}^p_\nu \vec{W}^p_\mu)^2$$

$$+\frac{g}{2}\hat{F}^p_{\mu\nu} \cdot (\vec{W}^p_\mu \times \vec{W}^p_\nu)\Big\} - \sum_{p,q} \frac{g^2}{4}(\vec{W}^p_\mu \times \vec{W}^q_\mu)^2$$

$$-\sum_{p,q,r} \frac{g}{2}(\hat{D}^p_\mu \vec{W}^p_\nu - \hat{D}^p_\nu \vec{W}^p_\mu) \cdot (\vec{W}^q_\mu \times \vec{W}^r_\mu)$$

$$-\sum_{p\neq q} \frac{g^2}{4}\big[(\vec{W}^p_\mu \times \vec{W}^q_\nu) \cdot (\vec{W}^q_\mu \times \vec{W}^p_\nu) + (\vec{W}^p_\mu \times \vec{W}^p_\nu) \cdot (\vec{W}^q_\mu \times \vec{W}^q_\nu)\big].$$

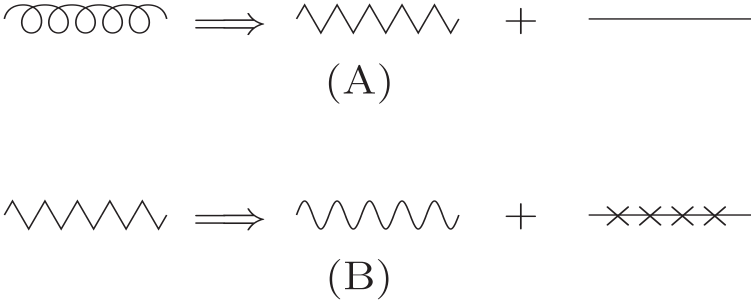

Figure: The gauge independent Abelian decomposition of QCD potential. (A) decomposes it to the restricted part and the chromon, and (B) decomposes the restricted part to the neuron and monopole.

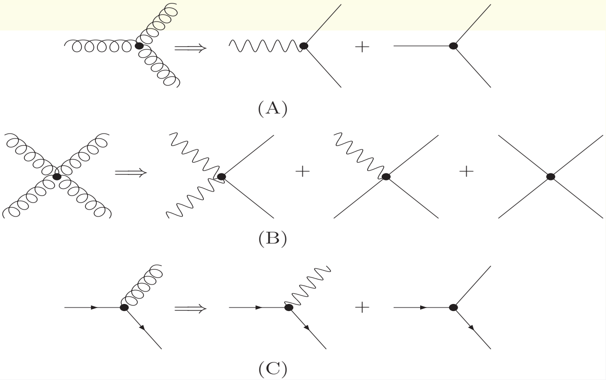

Figure: The Abelian decomposition of Feynman diagrams in SU(3) QCD. Notice that the monopole does not appear in the diagram because it describes a topological degree.

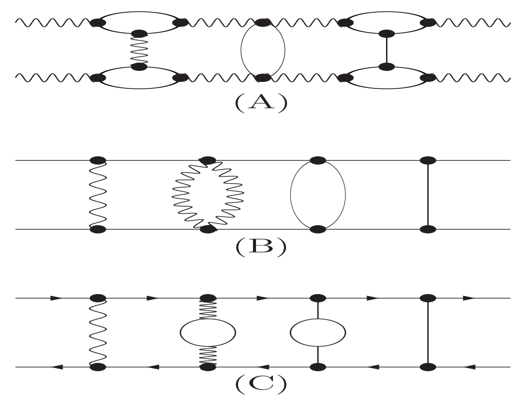

Figure: The possible Feynman diagrams of the neuron and chromon bindings. Two neuron binding is shown in (A), two chromon binding is shown in (B). In comparison the quark-antiquark binding is shown in (C).

E. Abelian Dominance versus Monopole Dominance

- With the Abelian decomposition we can prove the Abelian dominance, that the restricted potential generates the confining force in the Wilson loop integral.
- Moreover, implementing the Abelian decomposition on lattice, we can demonstrate the monopole dominance, that the monopole is responsible for the confinement.
- But this does not tell how the monopole confines the color.

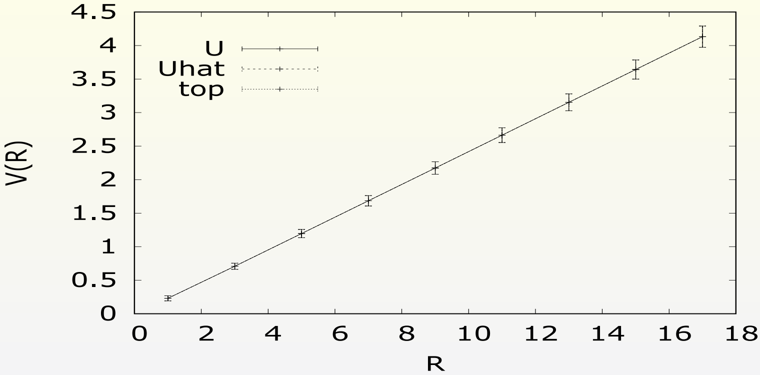

Figure: The Abelian dominance versus the monopole dominance in the lattice calculation. Here (U, Uhat, top) represent the full, Abelian, and monopole potentials.

A. Color Reflction Symmetry (CRS)

- Fixing $\hat{n}$ breaks the gauge symmetry. But there exists the residual symmetry, the color reflection symmetry, which plays the role of the non-Abelian gauge symmetry.

- Under the color reflection $(\hat{n}_1, \hat{n}_2, \hat{n}) \rightarrow (-\hat{n}_1, \hat{n}_2, -\hat{n})$ the isometry condition $D_\mu \hat{n} = 0$ does not change, but $\hat{A}_\mu$ and $\vec{X}_\mu$ change,

$$\hat{A}_\mu \rightarrow \hat{A}_\mu^{(c)} = -A_\mu \hat{n} - \frac{1}{g} \hat{n} \times \partial_\mu \hat{n} = -\mathcal{A}_\mu + \mathcal{C}_\mu,$$

$$\vec{X}_\mu \rightarrow \vec{X}_\mu^{(c)} = -X_\mu^1 \, \hat{n}_1 + X_\mu^2 \, \hat{n}_2.$$

- So we have two different Abelian decompositions with the same isometry

$$\vec{A}_\mu = \hat{A}_\mu + \vec{X}_\mu, \qquad \vec{A}_\mu = \hat{A}^{(c)}_\mu + \vec{X}^{(c)}_\mu.$$

- This has deep implication which makes QCD fundamentally different from QED:

1. The monopole is color reflection invariant, and becomes an ideal candidate of QCD vacuum.
2. The chromon and anti-chromon always come in pair and play exactly the same amount of role in QCD.

- For SU(3) QCD the color reflection group is made of 24 elements given by

$$C_{ab} = D_a R_b, \quad (a = 1, 2, 3, 4; b = 1, 2, ..., 6)$$

$$D_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad D_2 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$D_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, D_4 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

$$R_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad R_2 = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$R_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, \quad R_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$R_5 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad R_6 = \begin{pmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix},$$

where the four D-matrices form the diagonal subgroup.

B. Decomposition of Gluon Octet

- The neuron and chromon transform separately under CRS. The neuron triplet $(A_\mu^1, A_\mu^1, A_\mu^3)$ has no anti-neuron partner, and transforms as follows

$$
\begin{aligned}
R_2: &\quad (A_\mu^1, A_\mu^2, A_\mu^3) \rightarrow -(A_\mu^1, A_\mu^3, A_\mu^2), \\
R_3: &\quad (A_\mu^1, A_\mu^2, A_\mu^3) \rightarrow -(A_\mu^3, A_\mu^2, A_\mu^1), \\
R_4: &\quad (A_\mu^1, A_\mu^2, A_\mu^3) \rightarrow -(A_\mu^2, A_\mu^1, A_\mu^3), \\
R_5: &\quad (A_\mu^1, A_\mu^2, A_\mu^3) \rightarrow \ \ (A_\mu^3, A_\mu^1, A_\mu^2), \\
R_6: &\quad (A_\mu^1, A_\mu^2, A_\mu^3) \rightarrow \ \ (A_\mu^2, A_\mu^3, A_\mu^1).
\end{aligned}
$$

- The chromon sextet $(R_\mu, B_\mu, G_\mu, \bar{R}_\mu, \bar{B}_\mu, \bar{G}_\mu)$ has the anti-chromon partner, and transforms as follows

$$
\begin{aligned}
R_2 : (R_\mu, B_\mu, G_\mu, \bar{R}_\mu, \bar{B}_\mu, \bar{G}_\mu) &\to \ \ (\bar{R}_\mu, \bar{G}_\mu, \bar{B}_\mu, R_\mu, G_\mu, B_\mu), \\
R_3 : (R_\mu, B_\mu, G_\mu, \bar{R}_\mu, \bar{B}_\mu, \bar{G}_\mu) &\to -(\bar{G}_\mu, \bar{B}_\mu, \bar{R}_\mu, G_\mu, B_\mu, R_\mu), \\
R_4 : (R_\mu, B_\mu, G_\mu, \bar{R}_\mu, \bar{B}_\mu, \bar{G}_\mu) &\to -(\bar{B}_\mu, \bar{R}_\mu, \bar{G}_\mu, B_\mu, R_\mu, G_\mu), \\
R_5 : (R_\mu, B_\mu, G_\mu, \bar{R}_\mu, \bar{B}_\mu, \bar{G}_\mu) &\to -(G_\mu, R_\mu, B_\mu, \bar{G}_\mu, \bar{R}_\mu, \bar{B}_\mu), \\
R_6 : (R_\mu, B_\mu, G_\mu, \bar{R}_\mu, \bar{B}_\mu, \bar{G}_\mu) &\to -(B_\mu, G_\mu, R_\mu, \bar{B}_\mu, \bar{G}_\mu, \bar{R}_\mu).
\end{aligned}
$$

- So the color reflection invariant combination of neuron triplet and chromon sextet of CRS, not the gluon octet, become the physical states.

Quark and Chromon Model

A. Savvidy Action of SU(2) QCD: A Review

- Savvidy calculated the one-loop effective action of SU(2) QCD and obtained the Savvidy vacuum, integrating out gluons in a constant magnetic background.

- But the separation of the classical and quantum parts was ad hoc. More seriously, it had two critical defects:

  1. The Savvidy vacuum was unstable.
  2. It was not gauge invariant nor parity conserving.

B. Gauge Invariant Effective Action

- To remove these defects, we need the followings.

1. Start from ECD and and treat RCD as the classical part. Choose the color reflection invariant and parity conserving monopole background

$$\hat{F}^{(b)}_{\mu\nu} = \bar{H}_{\mu\nu}\hat{n}, \quad \bar{H}_{\mu\nu} = H\delta^1_{[\mu}\delta^2_{\nu]}.$$

2. Integrate out the chromon pair $\vec{X}_\mu$ and $\vec{X}^{(c)}_\mu$ simultaneously, imposing the color reflection invariance.

- Adopting the quantum gauge condition $\bar{D}_\mu \vec{X}_\mu = 0$ we have

$$\exp\left[iS_{eff}(\hat{A}_\mu)\right] \simeq \int \mathcal{D}\vec{X}_\mu \mathcal{D}\vec{X}_\mu^{(c)} \mathcal{D}\vec{c}\mathcal{D}\vec{c}^*$$

$$\exp\Big\{ -i\int \big[\frac{1}{4}\hat{F}_{\mu\nu}^2 + \frac{1}{4}(\hat{D}_\mu \vec{X}_\nu - \hat{D}_\nu \vec{X}_\mu)^2 + \frac{g}{2}\hat{F}_{\mu\nu} \cdot (\vec{X}_\mu \times \vec{X}_\nu)$$

$$+\vec{c}^* \bar{D}_\mu D_\mu \vec{c} + \frac{1}{2\xi}(\bar{D}_\mu \vec{X}_\mu)^2\big] d^4x \Big\},$$

where $\vec{c}$ and $\vec{c}^*$ are the ghost fields.

- Under the color reflection the eigenvalues of the chromon functional determinant change to

$$2gH(n + \frac{1}{2} \mp S_3) + k^2 \rightarrow 2gH(n + \frac{1}{2} \pm S_3) + k^2.$$

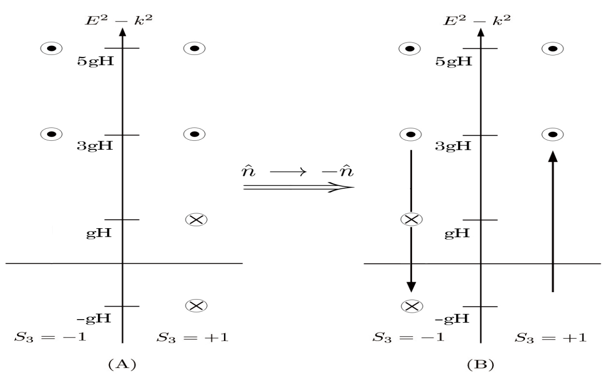

Figure: The gauge invariant eigenvalues of the chromon functional determinant. Notice that the C-projection excludes the lowest two eigenmodes, in particular the tachyonic modes.

- So we must make the C-projection to exclude the lowest two eigenmodes, in particular the tachyonic mode. With this we have

$$\ln \text{Det}^{1/2}\ K = \ln \text{Det}\ [(-\hat{D}^2 + 2gH)(-\hat{D}^2 + 2gH)],$$
$$\Delta\mathcal{L} = \frac{1}{16\pi^2}\int_0^\infty \frac{dt}{t^{3-\epsilon}} \frac{gHt/\mu^2}{\sinh(gHt/\mu^2)}$$
$$\times\Big[\exp(-2gHt/\mu^2)+\exp(-2gHt/\mu^2)-1\Big].$$

- Just like the GSO-projection which removes tachyons and assures supersymmetry and modular invariance in string theory, the C-projection removes the tachyonic modes and restores the stability of the monopole condensation in QCD.

No Infra-red Divergence!

C. Monopole Condensation and Asymptotic Freedom

- With the C-projection the effective potential becomes real

$$V = \frac{H^2}{2}\Big[1 + \frac{11g^2}{24\pi^2}\big(\ln\frac{gH}{\mu^2} - c\big)\Big].$$

- Define the running coupling $\bar{g}$ by $\displaystyle \frac{\partial^2 V}{\partial H^2}\Big|_{H=\bar{\mu}^2} = \frac{g^2}{\bar{g}^2}$ and find

$$\frac{1}{\bar{g}^2} = \frac{1}{g^2} + \frac{11}{24\pi^2}\big(\ln\frac{\bar{\mu}^2}{\mu^2} - c + \frac{3}{2}\big), \quad \beta(\bar{\mu}) = \bar{\mu}\frac{\partial \bar{g}}{\partial \bar{\mu}} = -\frac{11\bar{g}^3}{24\pi^2}.$$

Asymptotic Freedom

- The renormalized potential has the non-trivial minimum

$$V_{ren} = \frac{H^2}{2}\Big[1 + \frac{11\bar{g}^2}{24\pi^2}\big(\ln\frac{H}{\bar{\mu}^2} - \frac{3}{2}\big)\Big],$$

$$\langle H \rangle = \bar{\mu}^2 \exp\Big(-\frac{24\pi^2}{11\bar{g}^2} + 1\Big).$$

This is the Savvidy potential without the imaginary part.

Dynamical Symmetry Breaking!

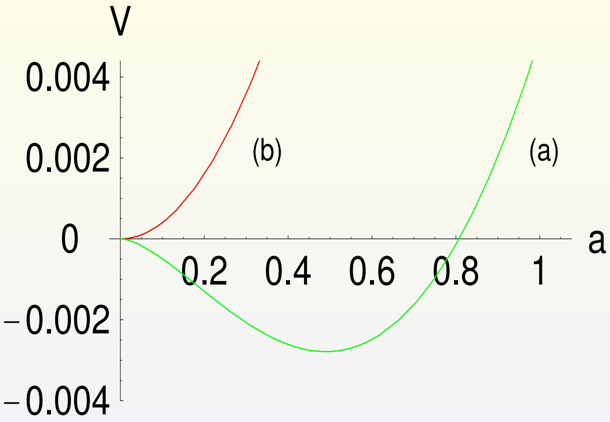

Figure: The one-loop effective potential of SU(2) QCD. Here (a) and (b) represent the effective potential and the classical potential.

- In general for arbitray constant monopole background $\bar{H}_{\mu\nu}$ we find

$$\mathcal{L}_{eff} = \begin{cases} -\dfrac{H^2}{2} - \dfrac{11g^2H^2}{48\pi^2}\Big(\ln\dfrac{gH}{\mu^2} - c\Big), & E = 0 \\ -\dfrac{E^2}{2} + \dfrac{11g^2E^2}{48\pi^2}\Big(\ln\dfrac{gE}{\mu^2} - c\Big) - i\dfrac{11g^2E^2}{96\pi}, & H = 0 \end{cases}$$

$$c = 1 - \ln 2 - \frac{24}{11}\zeta'(-1, \frac{3}{2}) = 0.94556...$$

- The negative imaginary part in the chromo-electric background tells that the chromo-electric field annihilates the chromon pairs. This is the origin of the asymptotic freedom.

- The old calculations calculated the effective action of Maxwell's theory coupled to massless charged gluon. This is a sick theory, not QCD.

- In physics something is wrong when we encounter tachyons.

  1. In Higgs mechanism we have tachyon when we choose the false vacuum.
  2. In NSR string we have tachyonic vacuum when we do not make the GSO-projection.

- With the Weyl symmetry of SU(3) ECD we have

$$\exp\Big[iS_{eff}(\hat{A}_\mu)\Big] \simeq \sum_p \int \mathcal{D}\vec{W}^p_\mu \mathcal{D}\vec{W}^{(c)p}_\mu \mathcal{D}\vec{c}^p \mathcal{D}\vec{c}^{*p}$$

$$\exp\Big\{-i\int\Big[\frac{1}{6}(\hat{F}^p_{\mu\nu})^2 + \frac{1}{4}(\hat{D}^p_\mu \vec{W}^p_\nu - \hat{D}^p_\nu \vec{W}^p_\mu)^2 + \frac{g}{2}\hat{F}^p_{\mu\nu}\cdot(\vec{W}^p_\mu \times \vec{W}^p_\nu)$$

$$-\vec{c}^{*p}\bar{D}^p_\mu D^p_\mu \vec{c}^p - \frac{1}{2\xi}(\bar{D}^p_\mu \vec{W}^p_\mu)^2\Big]d^4x\Big\},$$

at one-loop level. This allows us to calculate the effective action of SU(3) QCD from that of SU(2) QCD.

- With the C-projection we have

$$\Delta S = i\sum_p \ln \mathrm{Det}(-D_p^2 + 2gH_p)(-D_p^2 + 2gH_p)$$
$$+i\sum_p \ln \mathrm{Det}(-D_p^2 - 2igE_p)(-D_p^2 - 2igE_p)$$
$$-2i\sum_p \ln \mathrm{Det}(-D_p^2),$$

and

$$\Delta\mathcal{L} = \lim_{\epsilon\to 0}\frac{g^2}{8\pi^2}\sum_p \int_0^\infty \frac{dt}{t^{3-\epsilon}}\frac{H_p E_p t^2/\mu^4}{\sinh(gH_p t/\mu^2)\sin(gE_p t/\mu^2)}$$
$$\Big[\exp(-2gH_p t/\mu^2) + \exp(+2igE_p t/\mu^2) - 1\Big].$$

- So we have the Weyl symmetric SU(3) QCD effective Lagrangian

$$\mathcal{L}_{eff} = \begin{cases} -\sum_p \Big(\frac{H_p^2}{3} + \frac{11 g^2 H_p^2}{48\pi^2} (\ln \frac{g H_p}{\mu^2} - c) \Big), & (E_p = 0) \\ \sum_p \Big(\frac{E_p^2}{3} + \frac{11 g^2 E_p^2}{48\pi^2} (\ln \frac{g E_p}{\mu^2} - c) \\ \qquad - i \frac{11 g^2}{96\pi} E_p^2 \Big). & (H_p = 0) \end{cases}$$

- This assures that the essential features of SU(2) QCD remains the same. In particular, this tells that the chromo-electric field makes the pair annihilation of chromon.

- The effective potential for the monopole background is given by

$$V = \frac{3}{4}\sum_p H_p^2 + \frac{11g^2}{48\pi^2}\sum_p H_p^2 \ln\big(\frac{gH_p}{\mu^2} - c\big).$$

- Although the classical potential depends on one Casimir invariant $\vec{H}_3^2 + \vec{H}_8^2$, the effective potential depends on three Casimir invariants which can be chosen as $H_1,\ H_2,\ H_3$ (or equivalently $|\vec{H}_3|$, $|\vec{H}_8|$, and the angle θ between $\vec{H}_3$ and $\vec{H}_8$).

- Define the renormalized coupling $\bar{g}$ by

$$\forall_p \quad \frac{\partial^2 V}{\partial H_p^2}\Big|_{H_1=H_2=H_3=\bar{\mu}^2} = \frac{g^2}{\bar{g}^2},$$

and find

$$\frac{1}{\bar{g}^2} = \frac{1}{g^2} + \frac{11}{16\pi^2}\big(\ln\frac{\bar{\mu}^2}{\mu^2} - c + \frac{5}{4}\big), \quad \beta(\bar{\mu}) = \bar{\mu}\frac{\partial \bar{g}}{\partial \bar{\mu}} = -\frac{11\bar{g}^3}{16\pi^2}.$$

Asymptotic Freedom!

- The renormalized potential

$$V_{ren} = \sum_p \Big(\frac{3}{4} H_p^2 + \frac{11\bar{g}^2}{48\pi^2} H_p^2 \ln\big(\frac{\bar{g} H_p}{\bar{\mu}^2} - \frac{5}{4}\big)\Big).$$

has the unique minimum

$$V_{min} = -\frac{11\bar{\mu}^4}{32\pi^2} \exp\big(-\frac{32\pi^2}{11\bar{g}^2} + \frac{3}{2}\big)$$

at the Weyl symmetric vacuum

$$\langle H_1 \rangle = \langle H_2 \rangle = \langle H_3 \rangle = \frac{\bar{\mu}^2}{\bar{g}} \exp\big(-\frac{16\pi^2}{11\bar{g}^2} + \frac{3}{4}\big).$$

Mass Gap!

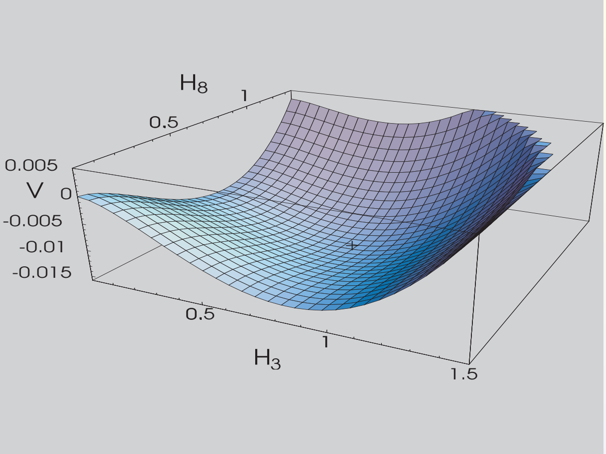

Figure: The effective potential with $\cos\theta = 0$, which has a unique minimum at $H = H' = H_0$ (or $H_1 = H_2 = H_3 = H_0$).

A. Two Types of Gluon Jets

- The Abelian decomposition tells that there are two types of gluons, the neurons and chromons, which behave differently. This predicts two types of gluon jets, the neuron jet and chromon jet.
- Experimental varification of two different gluon jets becomes an urgent issue which is as important as the confirmation of the gluon jet.

B. Quark and Chromon Model: Chromoballs and Mixed States

- The Abelian decomposition generalizes the quark model to the quark and chromon model which provides a clear picture of glueballs and their mixing with quarkoniums.
- The model predicts the chromoballs made of chromons. But experimentally, there are not so many candidates of chromoballs.
- There are two reasons for this. Unlike the quarks the chromoballs have intrinsic instability, and often mix with quarkoniums. This makes the identification complicated.

- Nevertheless we can make a systematic mixing analysis of chromoball-quarkonium in 0^{++}, 2^{++}, and 0^{-+} sectors below 2 GeV.

- The result shows that $f_0(1500)$ in the 0^{++} sector, $f_2(1950)$ in the 2^{++} sector, and $\eta(1405)$ and $\eta(1475)$ in the 0^{-+} sector could be identified as predominantly the glueball states.

- The quark and chromon model also predicts the hybrid hadrons made of chromons and quarks, which could be verified experimentally.

C. Monoball: Vacuum fluctuation of Monopole Condensation

- The monopole condensation could generate the quantum fluctuation. This suggests the existence of at least one monoball, the O^{++} vacuum fluctuation mode. $f_0(500)$ could be a possible candidate.
- Unlike all other hadrons, it originates from the QCD vacuum. This makes the experimental verification of the monoball an important issue in QCD.

Summary

- The Abelian decomposition is not just a mathematical proposition. It reveals the important hidden structures which simplifies the QCD dynamics greatly.

- It predicts two types of gluons, decomposes the Feynman diagram, simplifies the gauge symmetry, generalizes the quark model, and allows us to prove the monopole condensation.

- Moreover, it has other applications. It allows us to define gauge invariant canonical quark momentum, and resolve the proton spin crisis problem.

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