New ideas on EFT approach to nuclear system

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Effective field theory for nuclear system

Effective field theory concepts

- Very complicate system, I know no details, but only symmetries of the system.
- Arrange the physics based on separation of scales.
- To get quantum correction (higher order) effect, need renormalization in most cases.
- EFT only make sense only if renormalization group (RG) invariance is satisfied.

Mathematically:

observables

order

Residual, ~O(1) if: 1. EFT works
2. $\Lambda \geq \overline{\Lambda}_{EFT}$

$$\mathcal{O}(k, p_{typ}; \Lambda; \overline{\Lambda}_{EFT}) = \sum_i^n \left(\frac{k, p_{typ}}{\overline{\Lambda}_{EFT}}\right)^i \mathcal{O}_i(k, p_{typ}; \overline{\Lambda}_{EFT}) + \mathscr{C}_n(\Lambda; k, p_{typ}, \overline{\Lambda}_{EFT}) \left(\frac{k, p_{typ}}{\overline{\Lambda}_{EFT}}\right)^{n+1}$$

cutoff

Breakdown scale
(given by 1st meson not included)

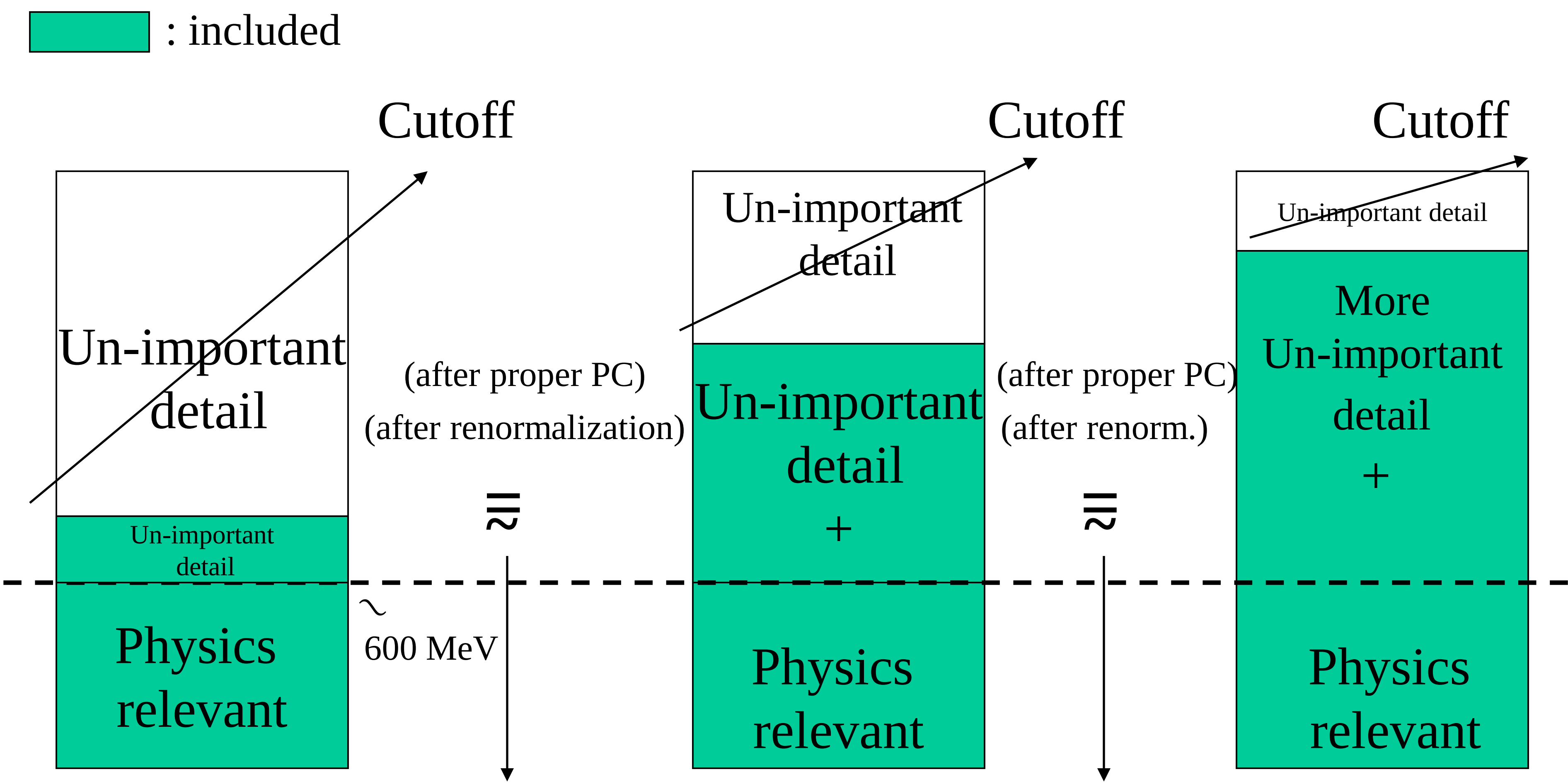
residual cutoff-dep.

No cutoff here! ***=> physics cannot dep. on cutoff !***

H. W. Griesshammer, arXiv:1511.00490v3 [nucl-th].

Lepage plot: subtract at two Λ's to extract "n+1"

Renormalization group (RG)



***Only source of error**: given by the high order terms.

If not so, → **the power counting isn't completely correct!**

(un-important are not really unimportant)

Part I: Nuclear Force

EFT on NN: Weinberg's proposal

- Spontaneous symmetry breaking:

  $SU_L(2) \times SU_R(2) \rightarrow SU_V(2)$ (chiral sym)

- Write down all possible terms in Lagrangian allowed by symmetry.

↓

Chiral perturbation theory: works in ππ, πN, but NN is too strong (infrared enhancement), still have open issues.

Conventional way: Weinberg prescription

Epelbaum, Entem, Machleidt, Kaiser, Meissner, … etc., ~**90% of the people**

- **Arrange diagrams base on Weinberg's prescription (WPP):** each derivative on the Lagrangian terms is always suppressed by the underlying scale of chiral EFT, $M_{hi} \sim m_\sigma$.
- **Iterate potential to all order (in L.S. or Schrodinger eq.), with an ultraviolet Λ.**

Carried out to $N^4LO(Q^5/M^5_{hi})$

D. R. Entem, N. Kaiser, R. Machleidt and Y. Nosyk, PRC 92, 064001.

P. Reinert, H. Krebs and E. Epelbaum, arXiv:1711.08821.

$V(N^{n\geq2}LO)$ performs as good as high accuracy $V_{\text{CDBonn, AV18, etc.,…}}$, if keep **500<Λ<875 MeV** (or, recently, **Λ=350~500** MeV).

Conventional power counting

Epelbaum, Entem, Machleidt, Kaiser, Meissner, … etc., ~**90% of the people**

Hope:

(one to one correspondence)

V up to LO, NLO, NNLO, N3LO, N4LO, :

NN amplitude

LO

NLO

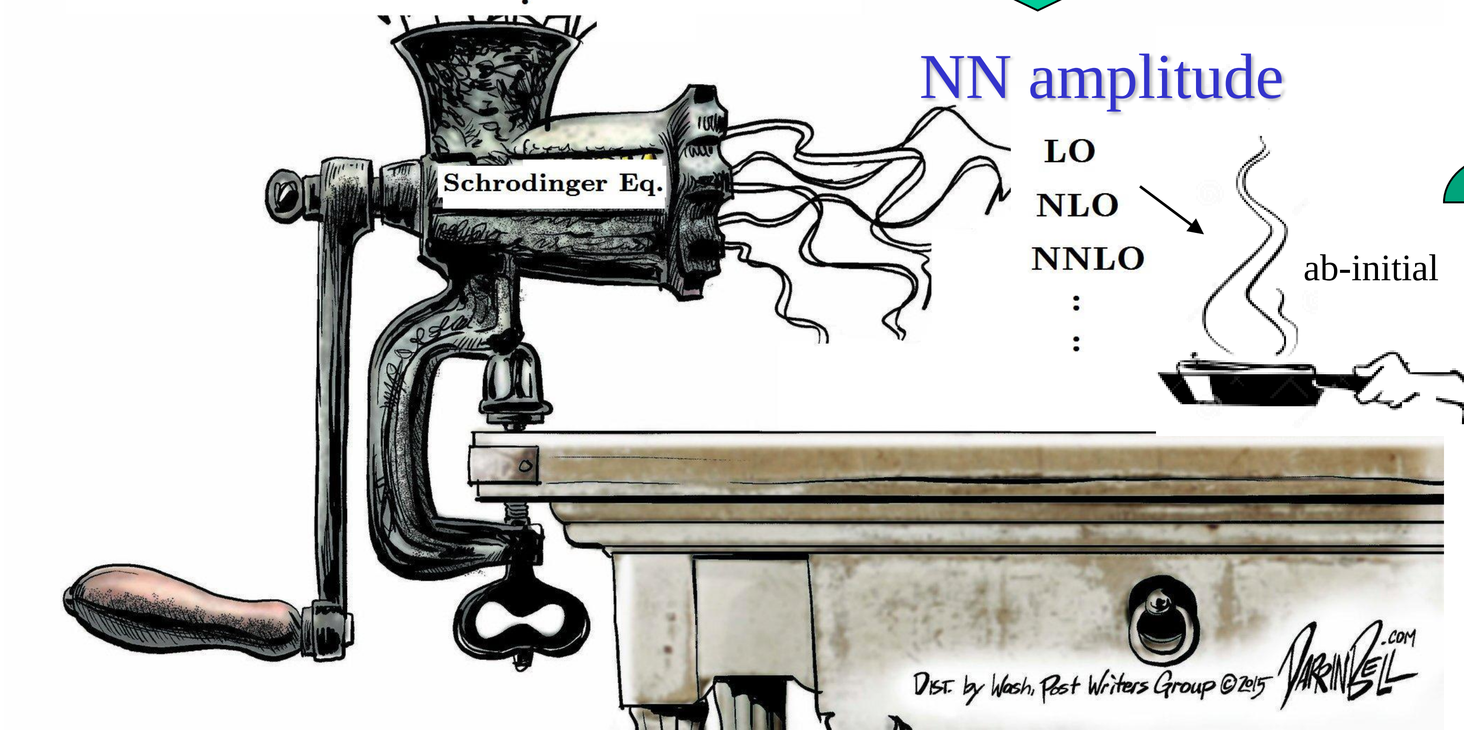
NNLO

:

:

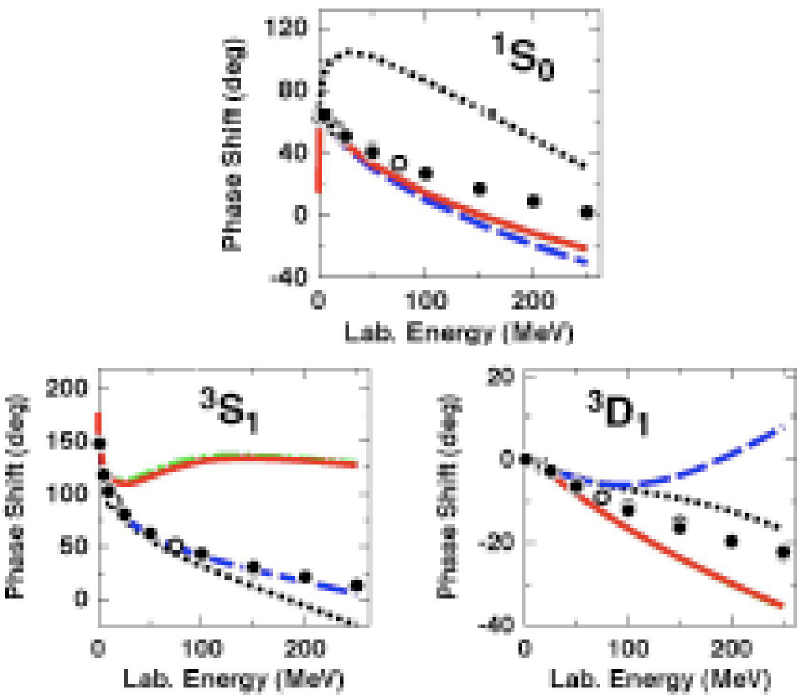
ab-initial

Properties of nuclei



Problems in RG

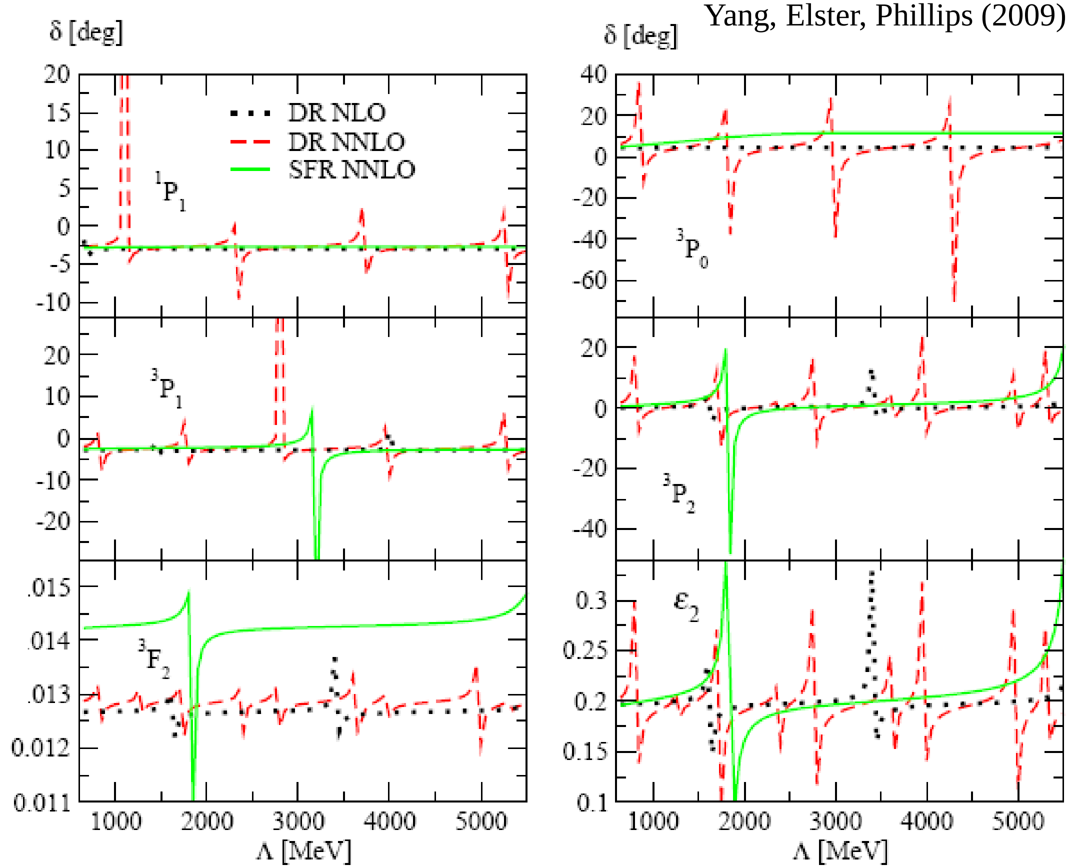
- Singular attractive potentials demand contact terms. (Nogga, Timmermans, van Kolck (2005))
- Beyond LO: Has RG problem at Λ>1 GeV (due to iterate to all order)



N^{3}LO(Q^4)

Ch. Zeoli R. Machleidt D. R. Entem (2012)

Yang, Elster, Phillips (2009)



Why is that a problem?

- Very complicate system, I know no details, but only symmetries of the system.
- Arrange the physics based on separation of scales.
- To get quantum correction (higher order) effect, need renormalization in most cases.
- EFT only make sense only if renormalization group (RG) invariance is satisfied.

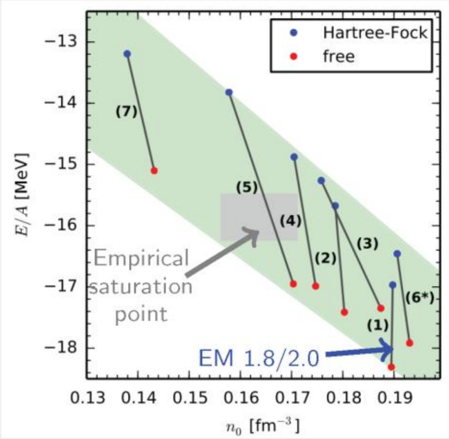
Physics cannot dep. on cutoff

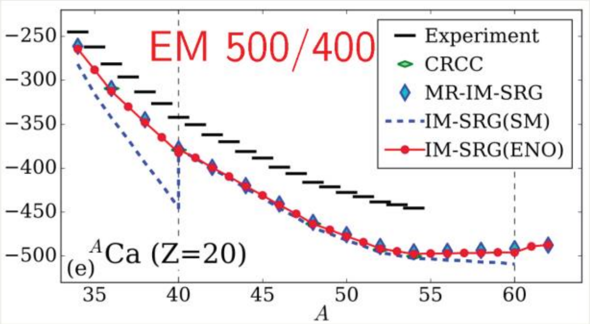
Some indications in nuclear structure

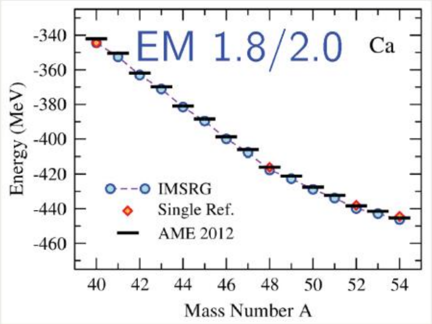
TRIUMF

One "magical" chiral (or chiral-inspired) interaction

	EM 500/400	EM 1.8/2.0
NN	N^3LO	same
	$\Lambda_{2N} = 500$ MeV	same
	non-local regulator	same
	fit to NN scattering, ^2H	same
	$\lambda_{SRG} = 1.88$ fm^{-1}	$\approx$ same
3N	N^2LO	same
	$\Lambda_{3N} = 400$ MeV	$\approx$ same
	local regulator	**non-local regulator**
	fit to ^3H BE, $t_{1/2}$	**fit to ^3H BE, ^4He r_{ch}**
	consistently SRG evolved	**no SRG for 3N**







- Neither interaction is fully consistent

  however. . .

- Saturation properties appear important for finite nuclei

Hebeler et al. PRC(R) (2011), Drischler et al. PRC (2016), Simonis et al. (in prep.)

Talk by R. S. Stroberg, ESNT workshop 2017

New power counting Long & Yang, (2010-2012)

Main idea

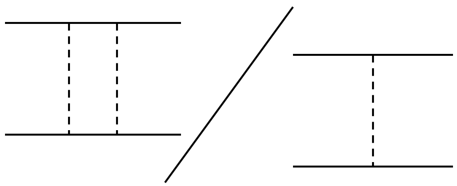
- In EFT, terms in the Lagrangian need *not* all go to the calculations (we have infinity terms, need to cut somewhere), and need *not* be treated non-perturbatively→ Only power counting decides.

New power counting

Long & Yang, (2010-2012)

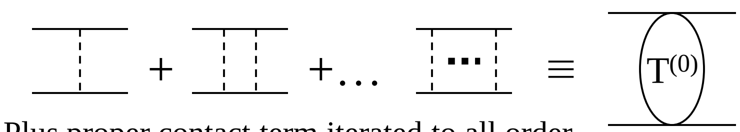
LO: Still iterate to all order (at least for most *l*<2).

Reason: van Kolck, Bedaque,… etc.



$$\sim \frac{g_A^2 M}{8\pi f_\pi^2} Q$$

Thus, at LO:

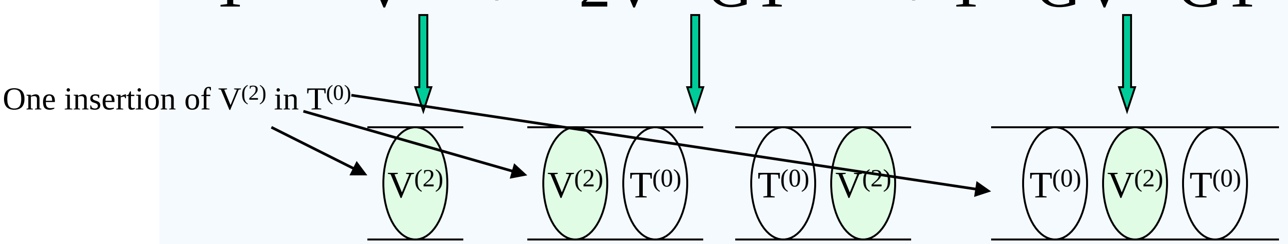


Plus proper contact term iterated to all order

Start at NLO, do perturbation. $(T = T^{(0)} + T^{(1)} + T^{(2)} + T^{(3)} + \ldots)$

If $V^{(1)}$ is absent:

$$T^{(2)} = V^{(2)} + 2V^{(2)}GT^{(0)} + T^{(0)}GV^{(2)}GT^{(0)}.$$



$$G \equiv \frac{2M_N}{\pi} \int_0^\Lambda \frac{p^2 dp}{p_0^2 - p^2 + i\varepsilon}$$

$$T^{(3)} = V^{(3)} + 2V^{(3)}GT^{(0)} + T^{(0)}GV^{(3)}GT^{(0)}.$$

$$V^{(n)} = V^{(n)}_{Long} + V^{(n)}_{Short};$$

$$V^{(n)}_{Long}: \text{pion-exchange at } O\left(\left(\frac{\mathrm{Q}}{\mathrm{M_{hi}}}\right)^{\mathrm{n}} \right)$$

$V^{(n)}_{Short}$: counter terms, $\underbrace{C_0 + C_2 q^2 + C_4 q^4 +}_{}$ ...

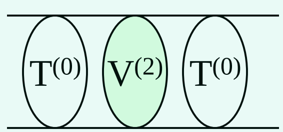
value of C's decided from renormalization

3 types of counter terms (determined by RG)

1. Primordial: Those renormalize the pion-exchange diagrams.

   (always there if survived from partial-wave decomposition)

2. Distorted –wave counter terms

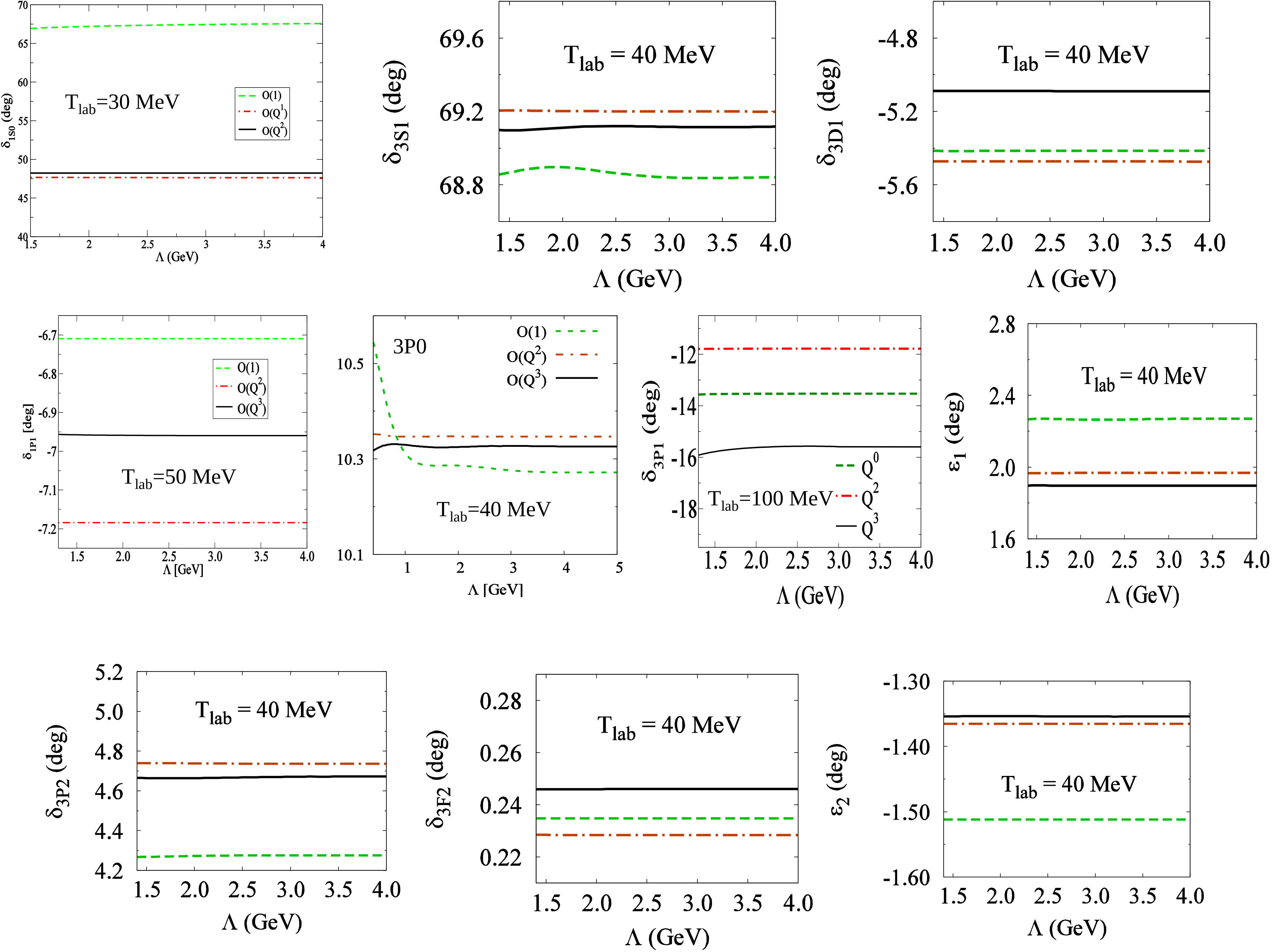


could diverge more than Q^2

3. Residual counter terms: Decided by the requirement from RG.

e.g., if $|T^{(n)}(k;\Lambda) - T^{(n)}(k;\infty)| \geq O(\frac{Q^{n+2}}{M_{hi}^{n+2}})$, then need V^{n+1}_{Short} at order n+1.

Results
(All RG-invariant)



T_lab=30 MeV
δ_1S0 (deg)
Λ (GeV)
O(1)
O(Q^1)
O(Q^2)
T_lab = 40 MeV
δ_3S1 (deg)
Λ (GeV)
T_lab = 40 MeV
δ_3D1 (deg)
Λ (GeV)
T_lab=50 MeV
δ_1P1 [deg]
Λ [GeV]
O(1)
O(Q^2)
O(Q^3)
3P0
O(1)
O(Q^2)
O(Q^3)
T_lab=40 MeV
Λ [GeV]
δ_3P1 (deg)
T_lab=100 MeV
Q^0
Q^2
Q^3
Λ (GeV)
T_lab = 40 MeV
ε_1 (deg)
Λ (GeV)
T_lab = 40 MeV
δ_3P2 (deg)
Λ (GeV)
T_lab = 40 MeV
δ_3F2 (deg)
Λ (GeV)
T_lab = 40 MeV
ε_2 (deg)
Λ (GeV)

To be continue… in nuclear structure calculations

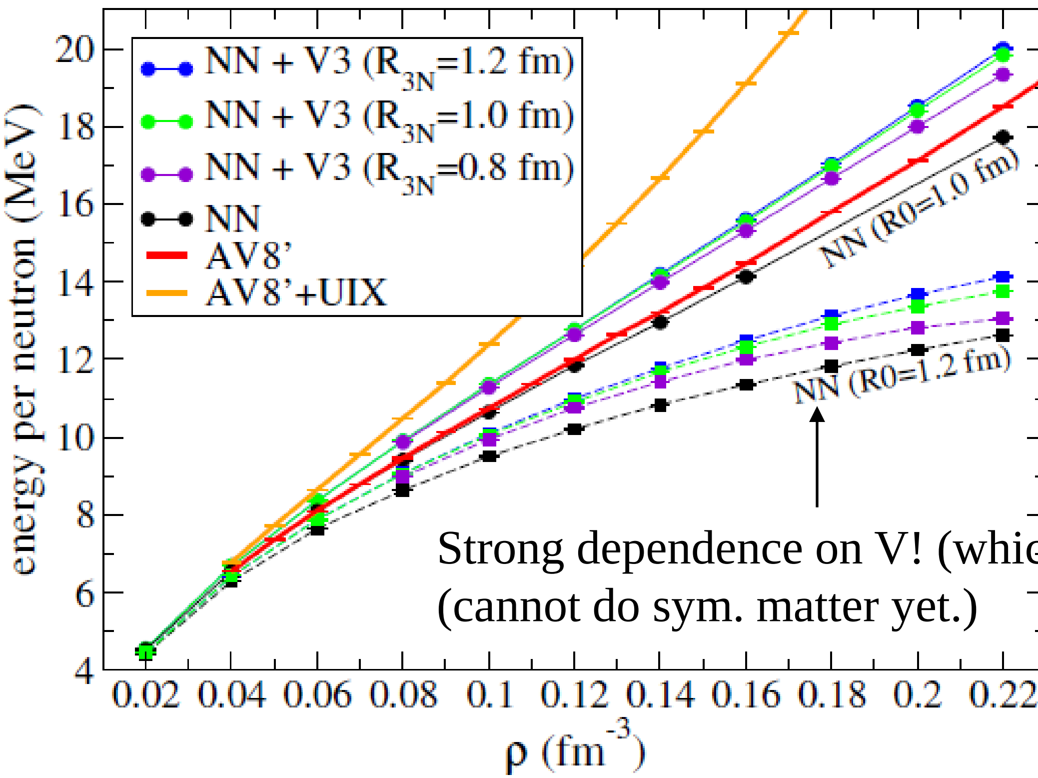
(with A. Ekstrom, C. Fossen, G. Hagen)

Part II: EFT approach to energy density functional (EDF)

Motivation (to do EDF)

Nuclear matter: ab-inito

Equation of state of neutron matter at N^2LO.



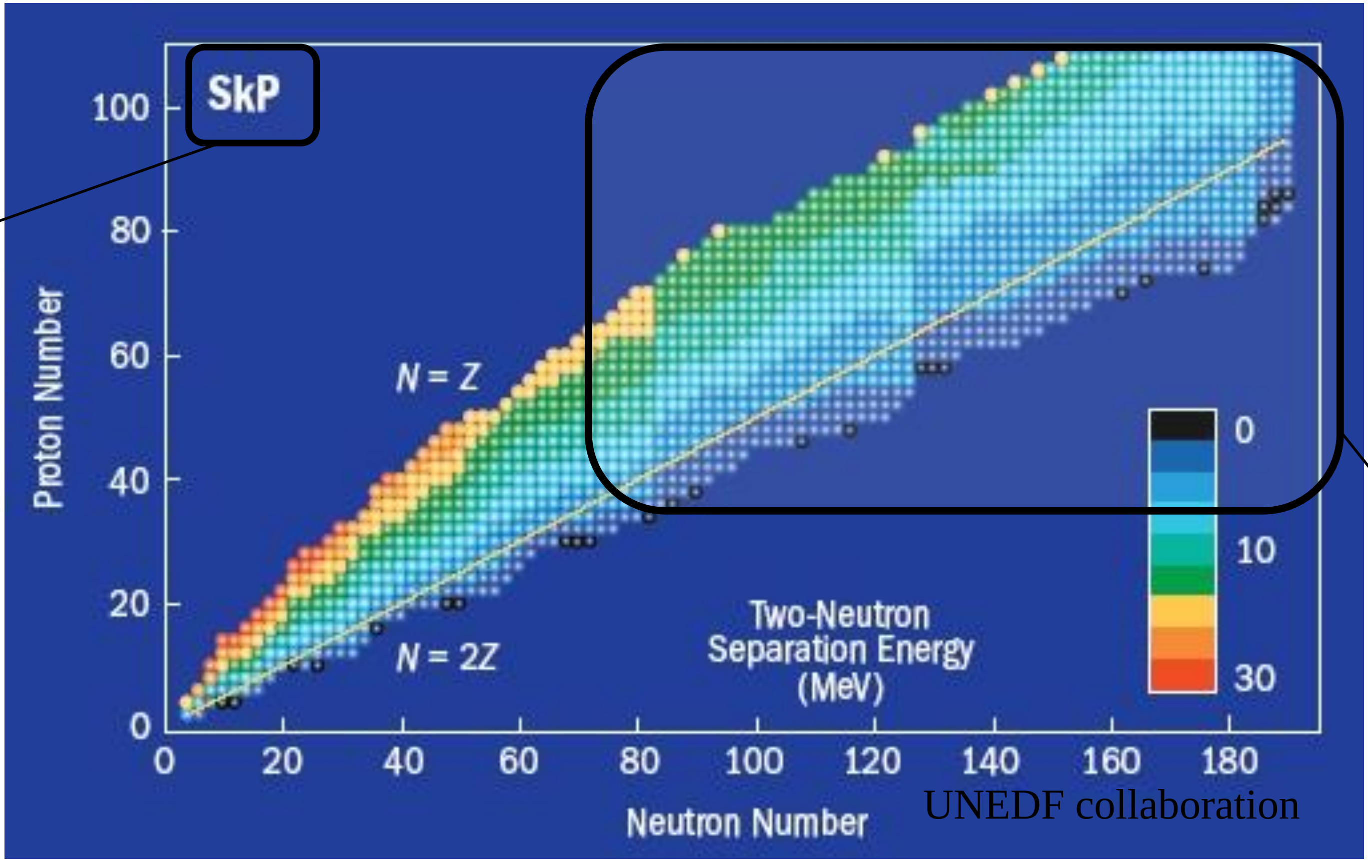
Strong dependence on V! (which does not have RG)
(cannot do sym. matter yet.)

S. Gandolfi, talk in ESNT workshop, 2017

- Even with the correct power counting, it could be that one needs to go to very high order for the N^iLO interaction to have small enough theoretical error for many-body system.

On the other hand…

Mean field with Skyrme-type



Skyrme-type interaction works o.k. (able to do the fitting in EDF framework)

No way to get with ab-initio!

Need to think about other expansion (than on NN d.o.f.).

Interaction & mean field EoS

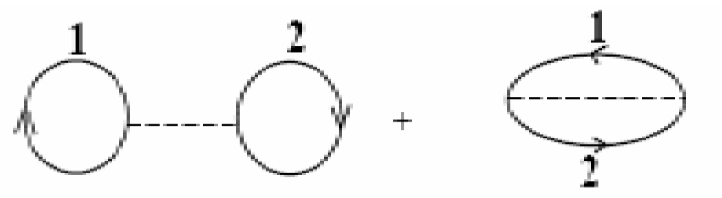
Interaction: Skyrme without spin-orbit

$$v = \underbrace{t_0(1+x_0P_\sigma)}_{S-wave\ \mathrm{O(0)}} + \frac{1}{2}\underbrace{t_1(1+x_1P_\sigma)(k'^2+k^2)}_{S-wave\ \mathrm{O(q^2)}} + \underbrace{t_2(1+x_2P_\sigma)\mathbf{k}'\cdot\mathbf{k}}_{p-wave\ \mathrm{O(q^2)}}$$

$$+\frac{1}{6}\underbrace{t_3(1+x_3P_\sigma)\rho^\alpha}_{s-wave,\ \text{higher body}}. \qquad \mathrm{P}_\sigma = \frac{1}{2}(1+\sigma_1\cdot\sigma_2)$$

No pion! Like pionless EFT, except for the density-dependent term.

$$EoS: \quad \frac{E}{A} \propto \frac{1}{\rho}\int_0^{k_{F1}} d^3\mathbf{k}_1 \int_0^{k_{F2}} d^3\mathbf{k}_2 v$$



$$\left(\mathbf{k} = \frac{\mathbf{k}_1 - \mathbf{k}_2}{2}, \mathbf{k}' = \frac{\mathbf{k}_1 - \mathbf{k}_2}{2} + \mathbf{q}\right)$$

Disadvantages of current EDF approach

- The effective interaction is model-dep. (versions of Skyrme >20) =>lack of predictive power.
- Divergence occurs when goes beyond MF.

It would be good if one can find an EFT for it

Very difficult problem,…, let’s first look at what we already knew.

We already knew: case 1 (expansion on $\mathbf{k_N a}$)

Could do 'strict' EFT:

Pure neutron matter at very low density ($k_N a<1$, $\rho<10^{-6}$ fm^{-3}).

Lee & Yang formula (1957) describes the dilute system.

$$\frac{E_{NM}}{A}=\frac{\hbar^2 k_N^2}{2m}\left[\underbrace{\frac{3}{5}}_{K.E.}+\underbrace{\frac{2}{3\pi}(k_N a)}_{\text{analog to } t_0 \text{ term}}+\underbrace{\frac{4}{35}(11-2\ln 2)(k_N a)^2}_{\textit{automatically} \text{ recover in } 2^{nd} \text{ of } t_0}+\underbrace{O(k_N^3)}_{\textit{higher} \text{ order}}\right]$$

Skyrme completely wrong here!

=> Can be re-derived by EFT with matching to ERE

E.g., L. Platter, H. Hammer, Ulf. Meissner, Nucl.Phys. A714 (2003), 250-264,
H. Hammer and R.J. Furnstahl, Nucl.Phys. A678 (2000) 277-294.

'EFT-inspired'

Tricks to extend to higher **ρ(up to 0.3 fm^{-3})**

Already discussed in Marcella's talk

See also: P.Papakonstantinou et al, arXiv:1606.04219.

What we already knew: case 2 (expansion on $1/(k_N a)$)

Unitarity limit

- For $a \to \infty$, scale invariance gives $$\frac{E}{E_{FG}} = \xi(a_s k_F, r_e k_F)$$
- Nuclear system not far from unitarity.

  $|a_s = -18.9\ fm| >>$ range of interaction

'EFT-inspired' treatment

Neutron matter only

Expansion in $(a_s k_F)^{-1}$ + resum+input from ab-initio (QMC) calculations.

D Lacroix, Phys. Rev. A 94, 043614 (2016).
D Lacroix, A. Boulet, M. Grasso, C. J. Yang, PRC 95, 054306 (2017) .
A. Boulet and D Lacroix, arXiv:1709.05160

Strict EFT maybe possible (within certain range of ρ)

C.J. Yang and U. van Kolck, in preparation.

Unitarity limit: Formula

D Lacroix, A. Boulet, M. Grasso, C. J. Yang, PRC 95, 054306 (2017) .

The proposed functional for Neutron matter

$$\frac{E}{E_{\mathrm{FG}}} = 1 - \frac{U_0}{1-(a_s k_F)^{-1}U_1} + \frac{R_0(r_e k_F)}{[1-R_1(ak_F)^{-1}]\,[1-R_1(a_s k_F)^{-1}+R_2(r_e k_F)]}$$

No free parameters: U_i, R_i from QMC data (with $V_{\mathrm{unitarity}}$)

Validity: $\frac{1}{|a_s|} < k_F < \frac{1}{R} \Rightarrow 4*10^{-6} < \rho < 0.002[\mathrm{fm}^{-3}]$, or higher if there's an extra suppression in the coefficient in front of the range.

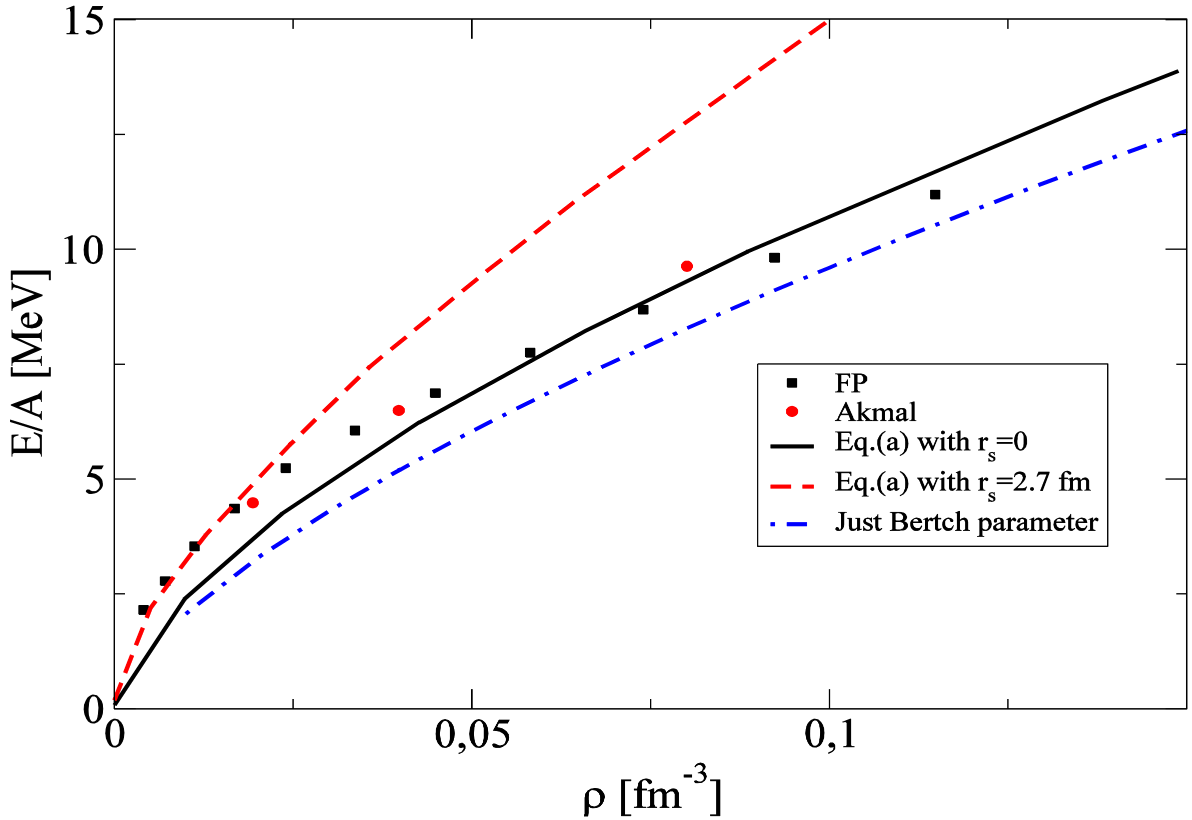
↓

The lower limit ($4*10^{-6}$) is exactly where Skyrme breakdown.

Hint: Skyrme is an UT-like expansion.

Unitarity limit: Results

D Lacroix, A. Boulet, M. Grasso, C. J. Yang, PRC 95, 054306 (2017) .



•Nuclear systems are not too far from the unitarity limit.

•Just a few more parameters might be sufficient to describe data up to ρ=0.3 fm^{-3}, this explains why Skyrme works!

Lessons

In both cases, the interactions are very simple, i.e., Skyrme-like.

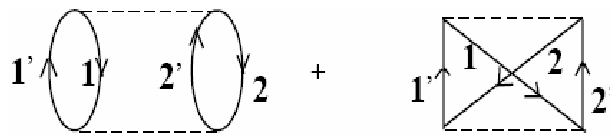
↓

Choose Skyrme-like interaction as the starting point (leading order) for EFT-based approach

↓

Tasks:
1. Need to include higher order corrections
2. Check renormalization
3. Check power counting

2nd order: nuclear matter



← No DR! use cutoff

$$\frac{\Delta E^{(2)}_{sym(l=0)}}{A} = -\frac{mk_F^4}{110880\hbar^2\pi^4}\left\{\begin{array}{c}\left[\begin{array}{c}-6534+1188ln[2]+3564\lambda-19602\lambda^3-5940\lambda^5\\ +(1782-20790\lambda^4)ln[\frac{\lambda-1}{\lambda+1}]\\ +(24948\lambda^5-5940\lambda^7)ln[\frac{\lambda^2-1}{\lambda^2}]\end{array}\right]\widetilde{T}_{03}^2\\ +\left[\begin{array}{c}-\quad 14696+2112ln[2]+5280\lambda-2860\lambda^3\\ -48840\lambda^5-18480\lambda^7+(2640-55440\lambda^6)ln[\frac{\lambda-1}{\lambda+1}]\\ +(71280\lambda^7-18480\lambda^9)ln[\frac{\lambda^2-1}{\lambda^2}]\end{array}\right]k_F^2\widetilde{T}_{03}\widetilde{T}_1\\ +\left[\begin{array}{c}-9886+1128ln[2]+\ 2520\lambda+147\lambda^3-3654\lambda^5\\ -35280\lambda^7-15120\lambda^9+(1260-41580\lambda^8)ln[\frac{\lambda-1}{\lambda+1}]\\ +(55440\lambda^9-15120\lambda^{11})ln[\frac{\lambda^2-1}{\lambda^2}]\end{array}\right]k_F^4\widetilde{T}_1^2\end{array}\right\}$$

Diverge as Λ^5

$$\frac{\Delta E^{(2)}_{sym(l=1)}}{A} = -\frac{mk_F^8}{73920\hbar^2\pi^4}\left\{\left[\begin{array}{c}-1033+156ln[2]+420\lambda+140\lambda^3-840\lambda^5\\ -5880\lambda^7-2520\lambda^9+(-210+6930\lambda^8)ln[\frac{\lambda-1}{\lambda+1}]\\ +(9240\lambda^9-2520\lambda^{11})ln[\frac{\lambda^2-1}{\lambda^2}]\end{array}\right]\widetilde{T}_2^2\right\},$$

$$\frac{\Delta E^{(2)}_{neutr(l=0)}}{A} = -\frac{mk_{F_N}^4}{166320\hbar^2\pi^4}\left\{\begin{array}{c}\left[\begin{array}{c}-6534+1188ln[2]+3564\lambda-19602\lambda^3-5940\lambda^5\\ +(1782-20790\lambda^4)ln[\frac{\lambda-1}{\lambda+1}]\\ +(24948\lambda^5-5940\lambda^7)ln[\frac{\lambda^2-1}{\lambda^2}]\end{array}\right]T_{03}^2\\ +\left[\begin{array}{c}-\quad 14696+2112ln[2]+5280\lambda-2860\lambda^3\\ -48840\lambda^5-18480\lambda^7+(2640-55440\lambda^6)ln[\frac{\lambda-1}{\lambda+1}]\\ +(71280\lambda^7-18480\lambda^9)ln[\frac{\lambda^2-1}{\lambda^2}]\end{array}\right]k_{F_N}^2 T_{03}T_1\\ +\left[\begin{array}{c}-9886+1128ln[2]+\ 2520\lambda+147\lambda^3-3654\lambda^5\\ -35280\lambda^7-15120\lambda^9+(1260-41580\lambda^8)ln[\frac{\lambda-1}{\lambda+1}]\\ +(55440\lambda^9-15120\lambda^{11})ln[\frac{\lambda^2-1}{\lambda^2}]\end{array}\right]k_{F_N}^4 T_1^2\end{array}\right\}$$

Diverge as Λ^5

$$\frac{\Delta E^{(2)}_{neutr(l=1)}}{A} = -\frac{mk_{F_N}^8}{110880\hbar^2\pi^4}\left\{\left[\begin{array}{c}-1033+156ln[2]+420\lambda+140\lambda^3-840\lambda^5\\ -5880\lambda^7-2520\lambda^9+(-210+6930\lambda^8)ln[\frac{\lambda-1}{\lambda+1}]\\ +(9240\lambda^9-2520\lambda^{11})ln[\frac{\lambda^2-1}{\lambda^2}]\end{array}\right]T_2^2\right\},$$

Renormalization:
Check renormalizability

- Treatment I:

Absorb divergence into redefinition of parameters.

- Treatment II:

Add counter terms correspond to the divergences.

Treatment I:

No new term added, use special cases of α and t_i

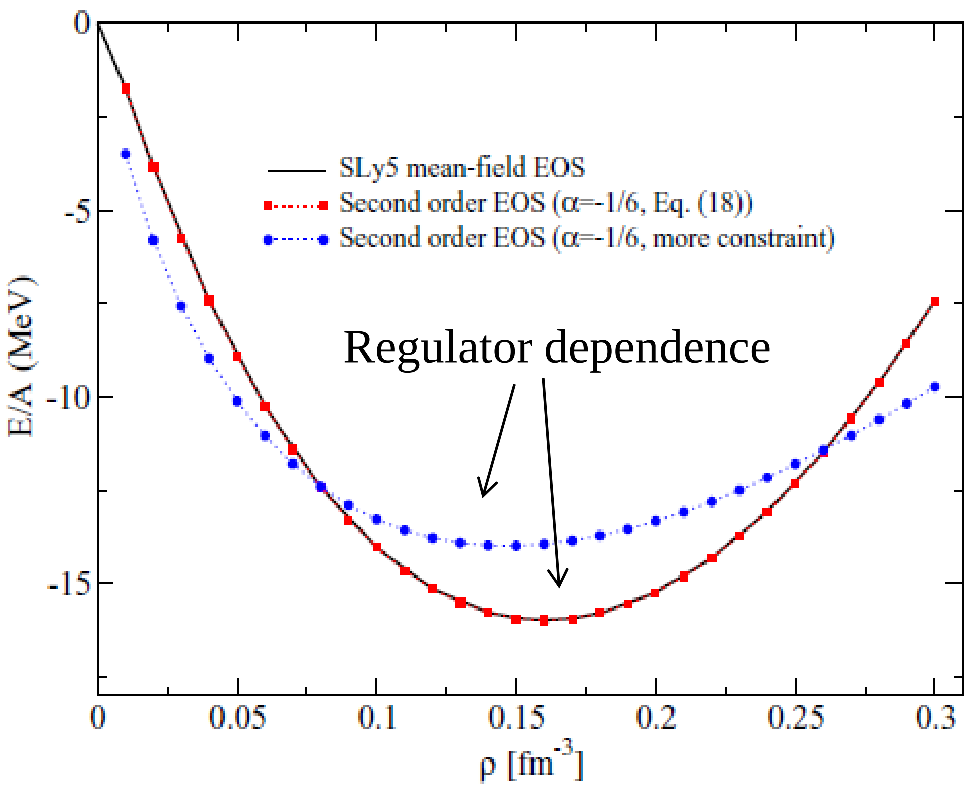
C.J. Yang, M. Grasso, K. Moghrabi, U van Kolck, PRC 95, 054325 (2017)

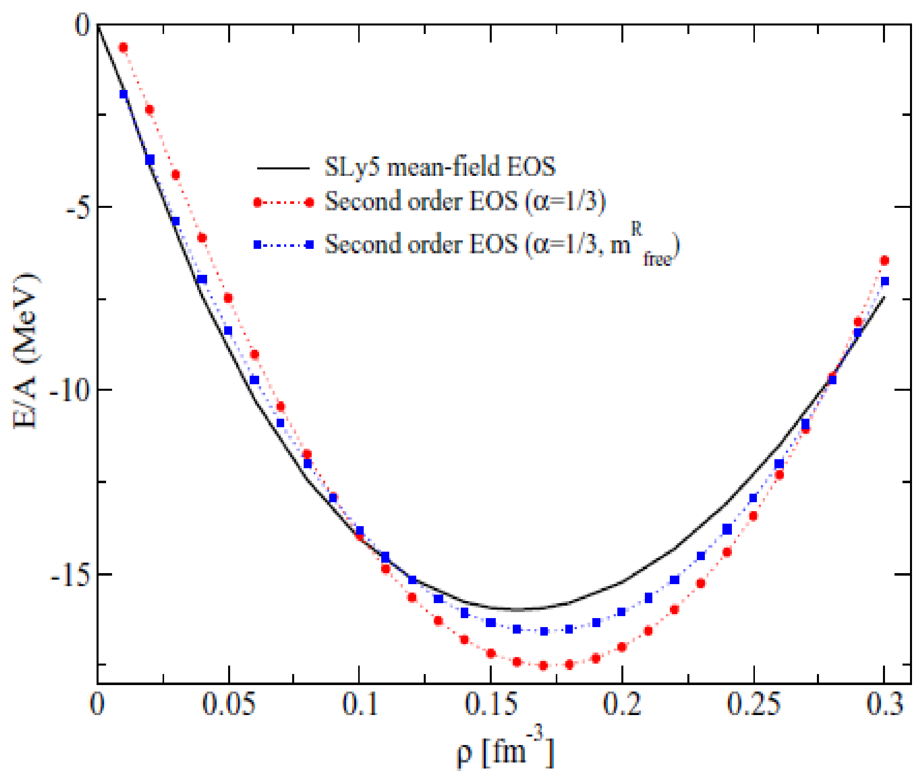
Results: $\alpha=-1/6$

m^R (MeV)	t_0^R (MeV fm^3)	T_3^R (MeV fm$^{5/2}$)	x_0^R	x_3^R	χ^2
591.9	793.15	−1570.8	1.465	−0.1759	<0.1

Results: $\alpha=1/3$

m (MeV)	t_0^R (MeV fm^3)	T_3^R (MeV fm^4)	$\lvert x_3^R \rvert$	χ^2
939	−1244.1	247.11	$<10^{-4}$	1364
23845	−580.16	46.248	$<10^{-2}$	188





Lessons

1. The leading order quite possible just contains only t_0-t_3 terms.

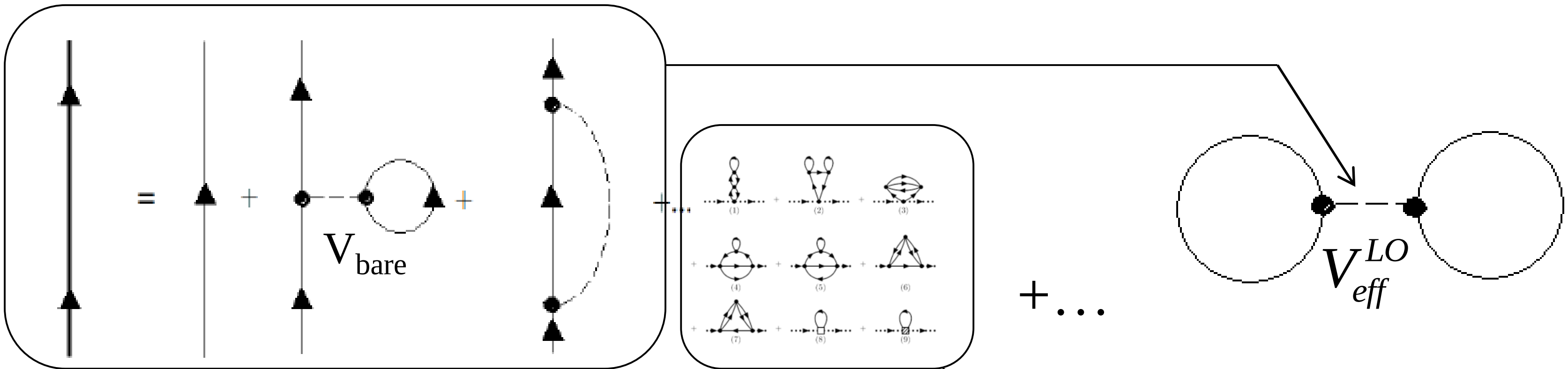
2. However, the regulator dependence tells us the power counting cannot be established in this way.

More general consideration (adding counter terms at NLO):

C.J. Yang, M. Grasso, D. Lacroix, PRC 96, 034318 (2017)

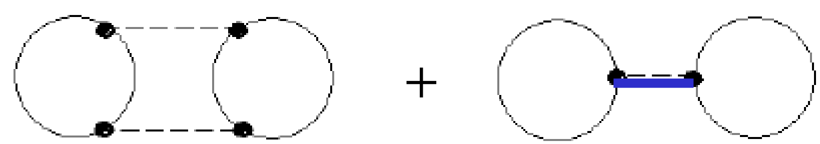
Diagrammatic explanation of the idea

Dressing of propagator→V_{eff}



Leading order (LO)

Then, NLO includes (at least):

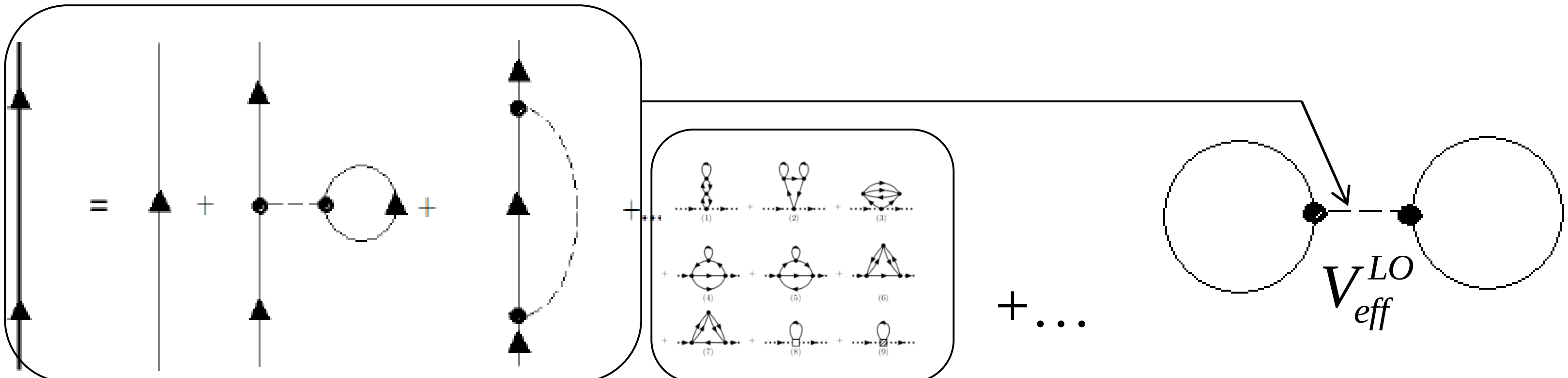


$\underbrace{V_{eff}^{LO} G V_{eff}^{LO}}$ *diverge* at least in Λk_F^3

V_{eff}^{NLO} new counter term(s)

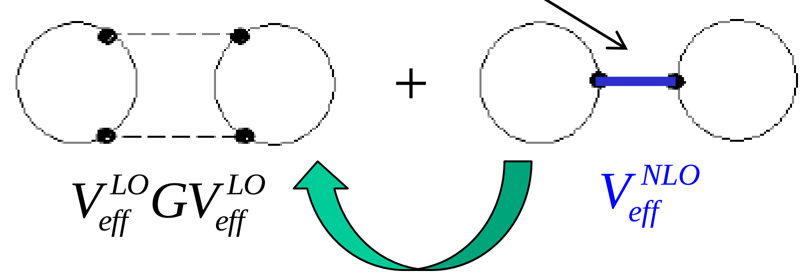
$V_{eff}^{Sly5} G V_{eff}^{Sly5}$ evaluated in: C.J. Yang, M. Grasso, X. Roca-Maza, G. Colo, and K. Moghrabi, PhysRevC.94.034311

Dressing of propagator→V_{eff}



Leading order (LO)

Then, NLO includes:



* V_{eff}^{NLO} contains (at least) contact terms to renormalize $V_{eff}^{LO} G V_{eff}^{LO}$.

Counter term part of the NLO potential

V_{eff}^{NLO} : For t_0-t_3 model, the divergence from $V_{eff}^{LO} G V_{eff}^{LO}$ is:

$\underbrace{O(k_F^3), O(k_F^{3+3\alpha})}_{k_F^n - dep.\text{ appears in } MF}$, $\underbrace{O(k_F^{3+6\alpha})}_{new\ k_F^n - dep.}$. **3 different k_F-dep.**

If want to keep α free, =>Minimun contact term required: $Ck_F^{3+6\alpha}$.

Most general case: $Ak_F^3, Bk_F^{3+3\alpha}, Ck_F^{3+6\alpha}$.

In infinite matter, k_F^{3n} in-distinguishable with $3\pi^2\rho$

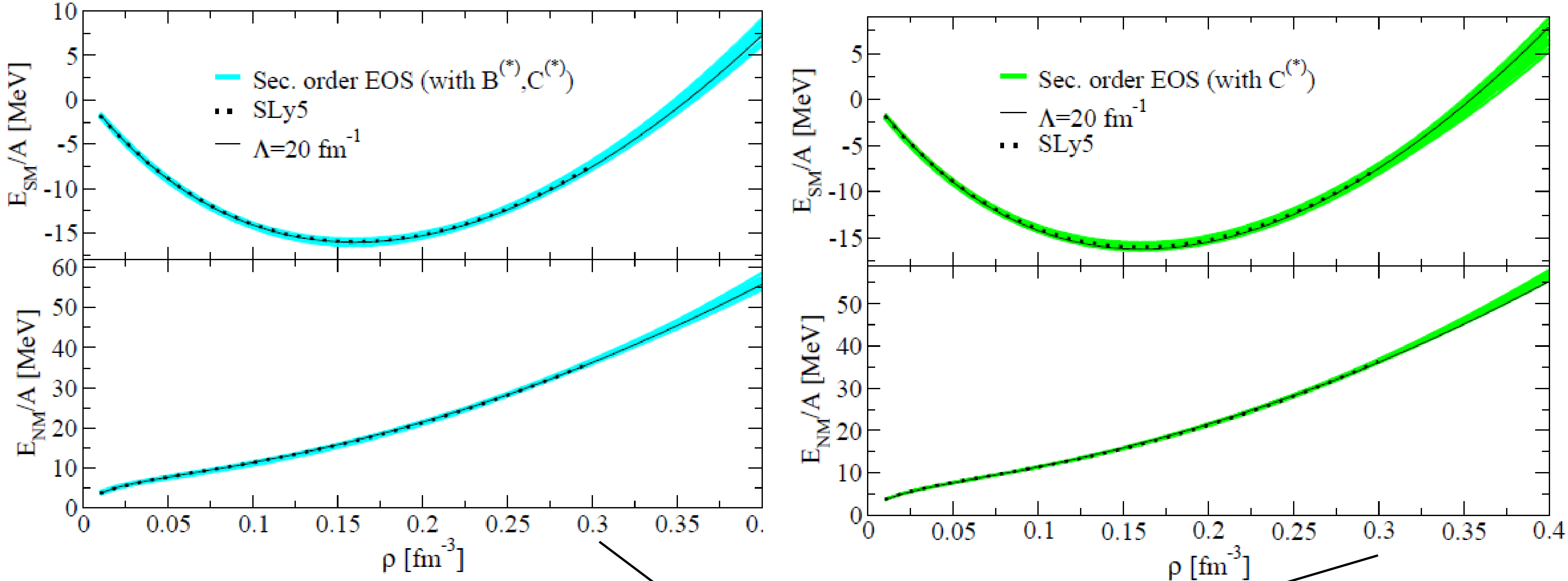
=> k_F^n-term in EOS *could* originated (at interaction level) from:

$(k - k')^{n-3v-3}\rho^v$,

where v is an extra parameter to be decided in the fitting to finite nuclei.

NLO results (based on t_0-t_3 as LO)
$\alpha<1/6$ case

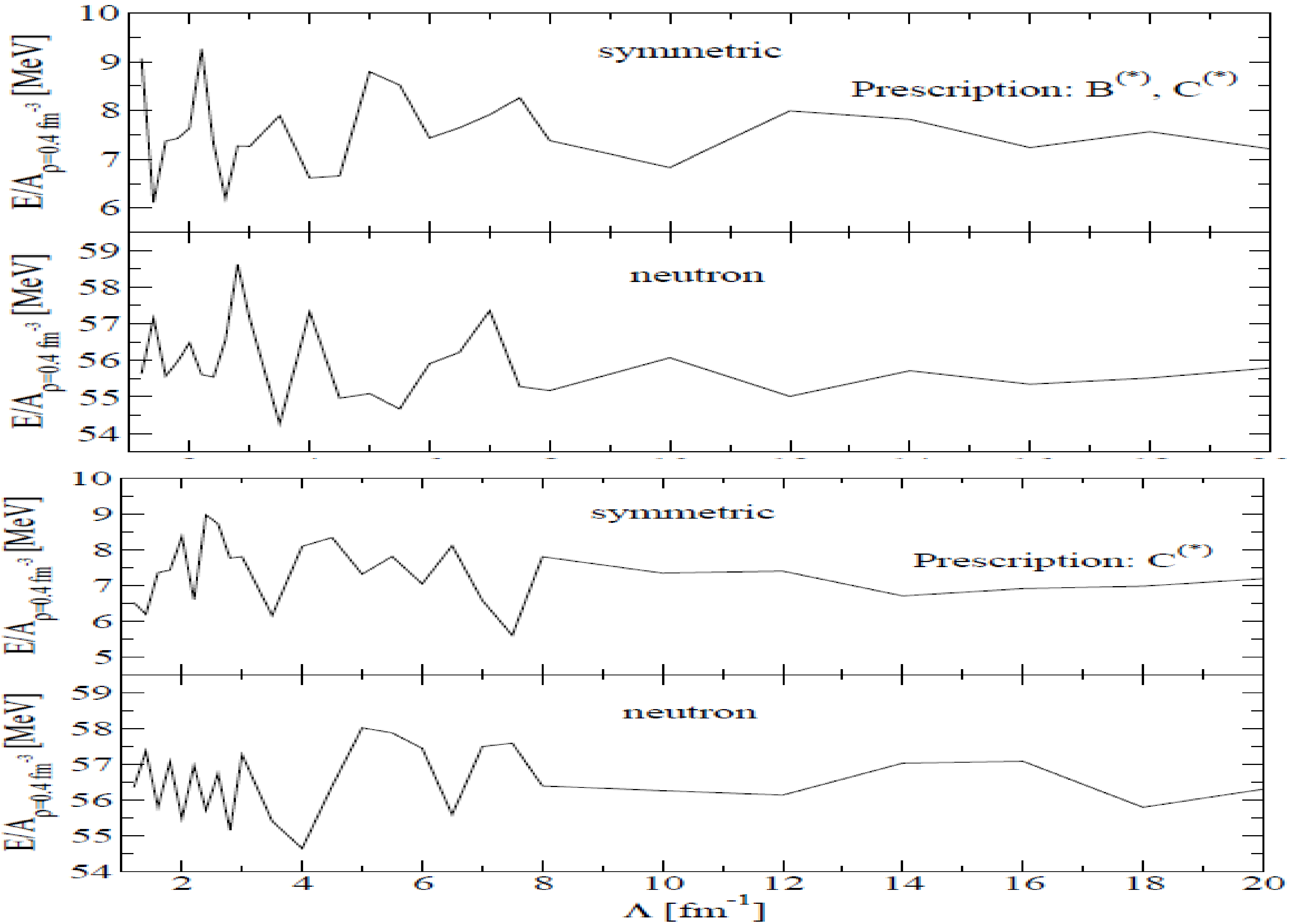
Color band: Λ=1.2~20 fm^{-1}



LECs fitted up to 0.3 fm^{-1}

Similar results (with different counter terms) tell us that the regulator-dependence is eliminated by adding counter terms!

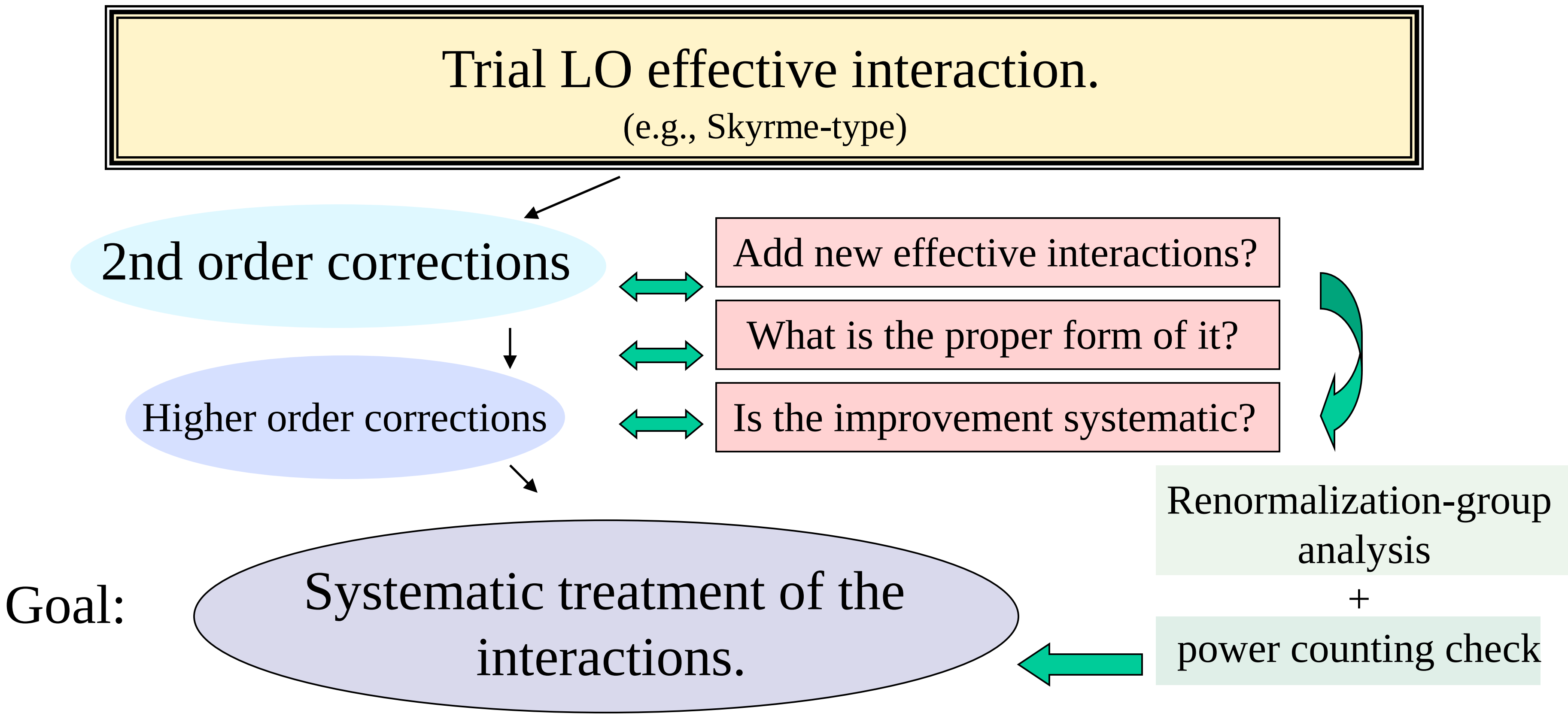
Renormalization group (RG) check at ρ=0.4 fm^{-1}



Scheme for EFT in EDF

or whatever the name it is

Try to bridge EFT ideas/techniques to mean field (and beyond) within EDF framework.



Thank you!

Brainstorming I

- **Any alternative suggestion of LO interaction?**

❖ Could it be derived from more microscope/fundamental theories?

❖ Use multiple density-dep. term at LO?

❖ What's the upper and lower bound value for α (if any)?

❖ Should we keep α independent of cutoff ?

Brainstorming II

- How to do the same (2nd order) for finite nuclei?

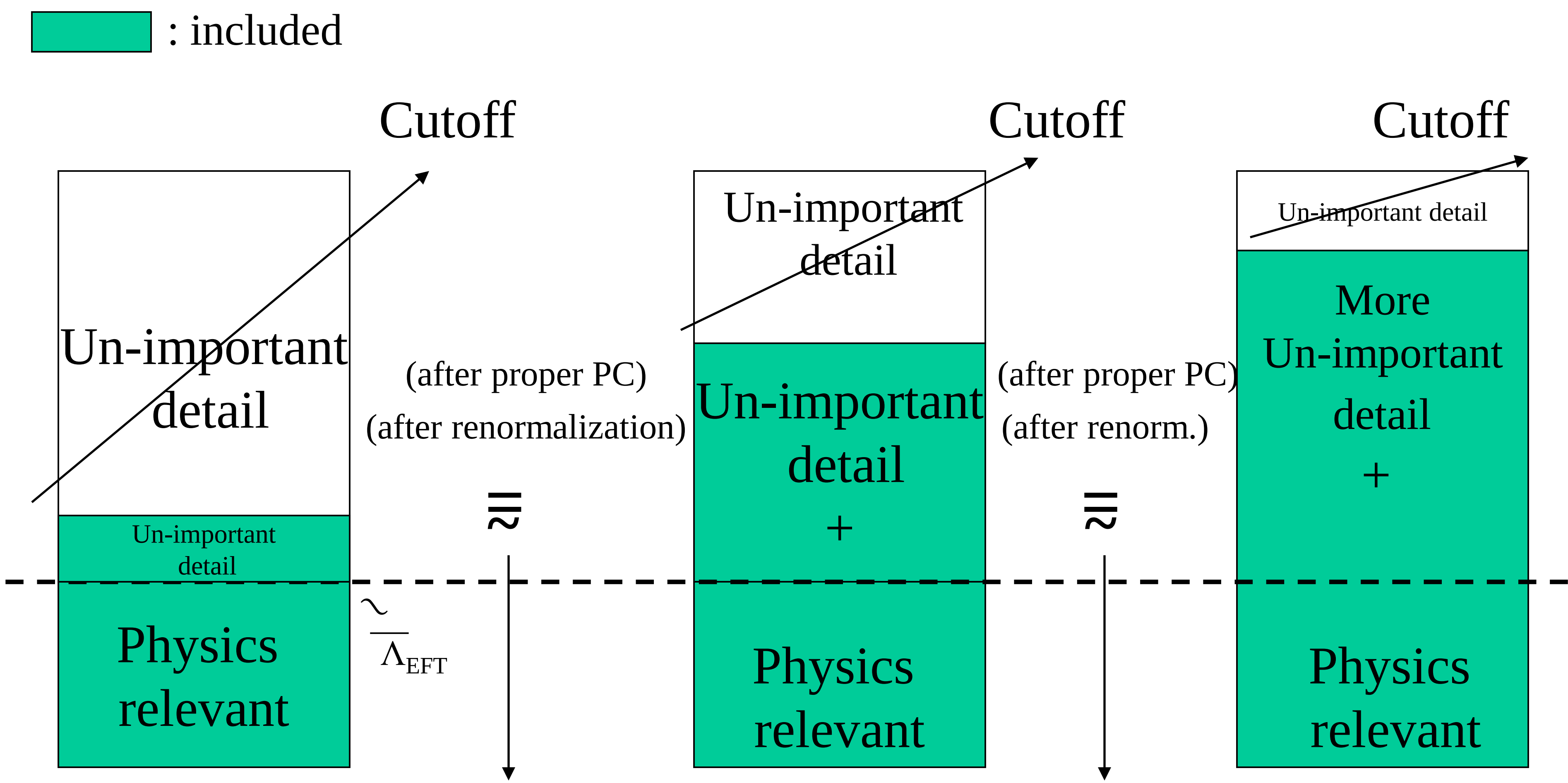
  M. Brenna, G. Colo, X. Roca-Maza PRC 90, 044316 (2014)

- Any idea to extend the EFT built from unitarity limit to symmetric matter.

  Should we have different power counting between pure neutron and symmetric matter?

Back up slides

Renormalization group (RG)



***Only source of error**: given by the high order terms.

If not so, → **the power counting isn't completely correct!**

(unimportant are not really unimportant)

Interaction & mean field EoS

Interaction: Skyrme without spin-orbit

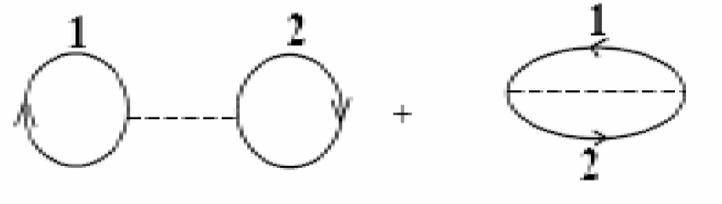
$$v = \underbrace{t_0(1+x_0P_\sigma)}_{S-wave\ \mathrm{O(0)}} + \frac{1}{2}\underbrace{t_1(1+x_1P_\sigma)(k'^2+k^2)}_{S-wave\ \mathrm{O(q^2)}} + \underbrace{t_2(1+x_2P_\sigma)\mathbf{k}'\cdot\mathbf{k}}_{p-wave\ \mathrm{O(q^2)}}$$

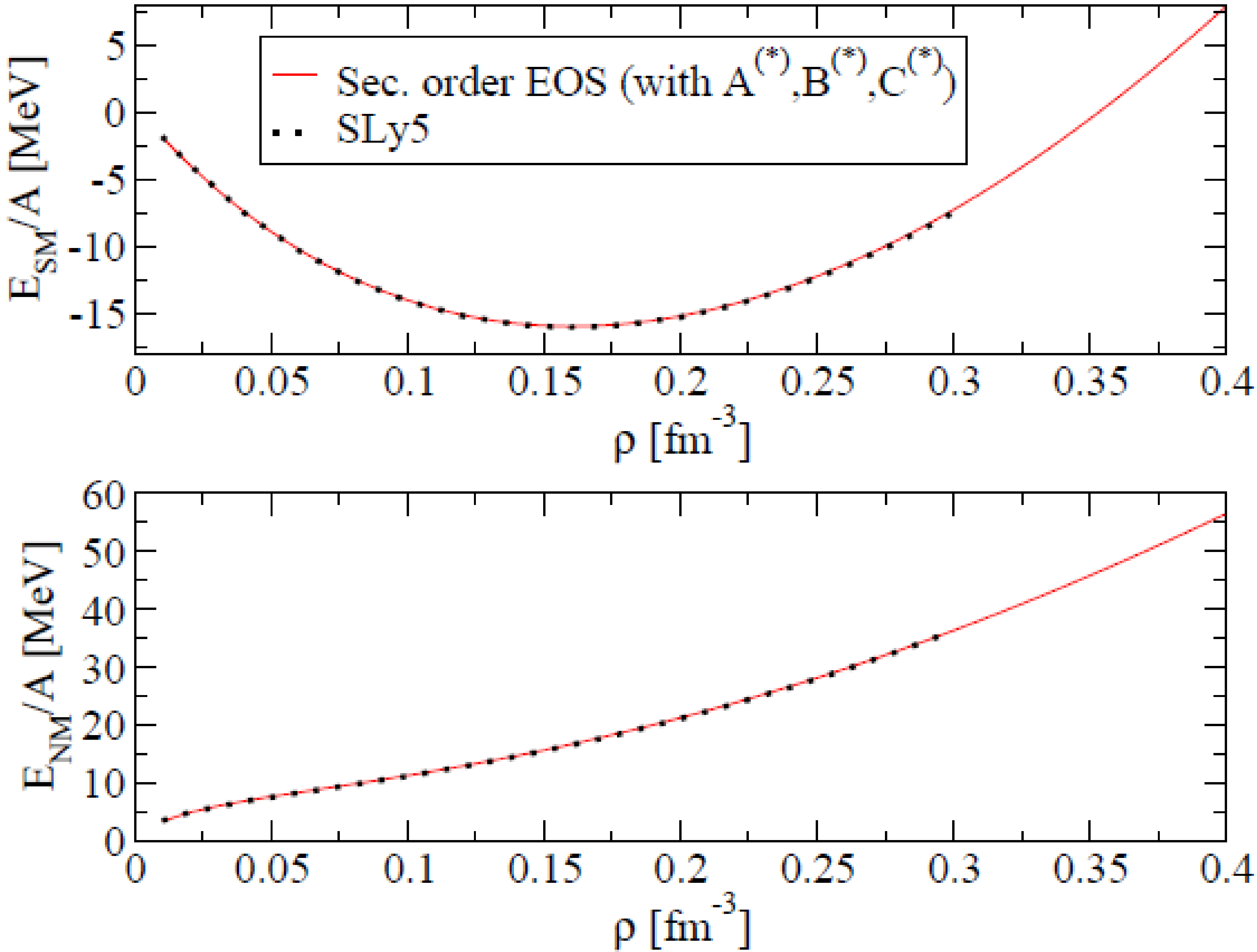
$$+\frac{1}{6}\underbrace{t_3(1+x_3P_\sigma)\rho^\alpha}_{s-wave,\ \text{higher body}}. \qquad \mathrm{P}_\sigma = \frac{1}{2}(1+\sigma_1\cdot\sigma_2)$$

No pion! Like pionless EFT, except for the density-dependent term.

$$EoS: \quad \frac{E}{A} \propto \frac{1}{\rho}\int_0^{k_{F1}} d^3\mathbf{k}_1 \int_0^{k_{F2}} d^3\mathbf{k}_2 v$$

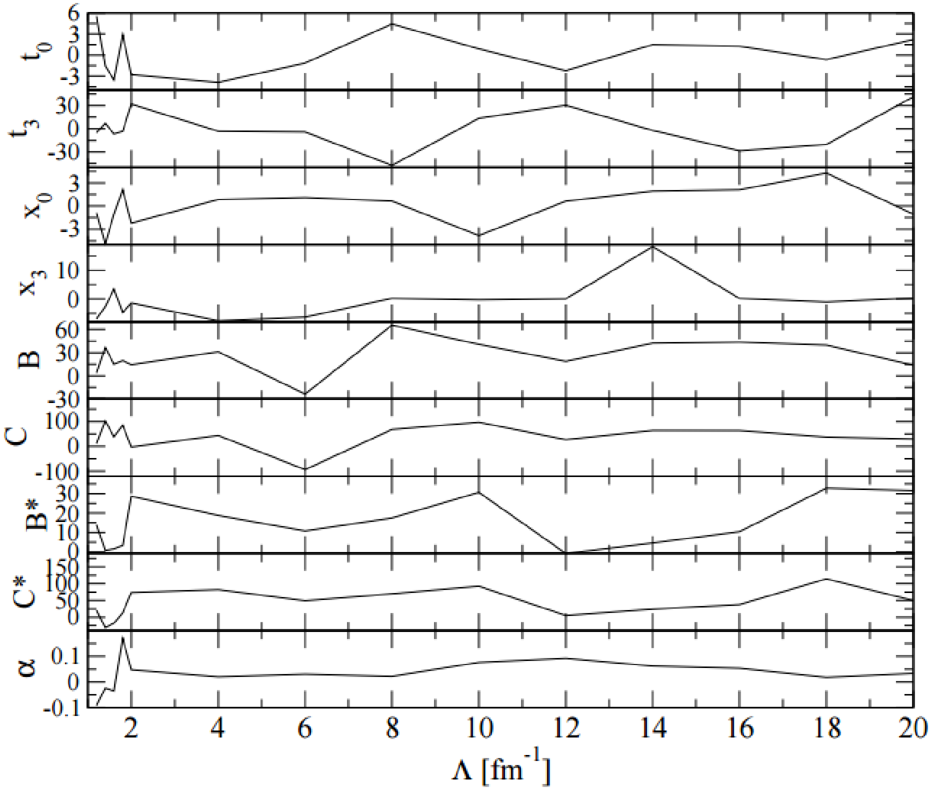
$$\left(\mathbf{k} = \frac{\mathbf{k}_1 - \mathbf{k}_2}{2}, \mathbf{k}' = \frac{\mathbf{k}_1 - \mathbf{k}_2}{2} + \mathbf{q}\right)$$

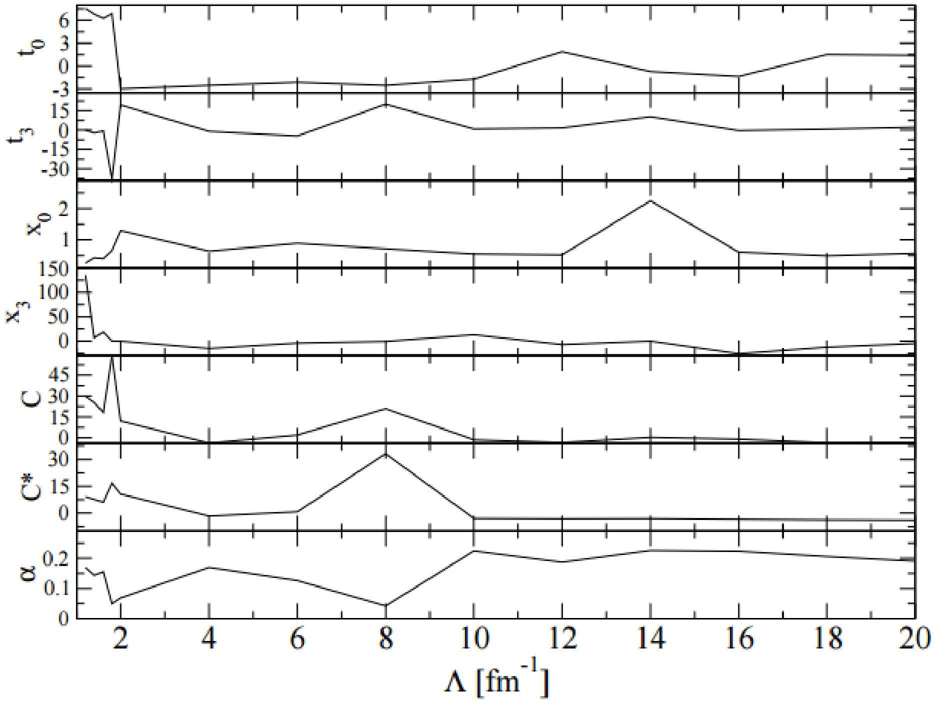




Sec. order EOS (with A(*),B(*),C(*))
SLy5
E_SM/A [MeV]
ρ [fm^-3]
E_NM/A [MeV]
ρ [fm^-3]

Parameters v.s. cutoff





Further link between Skyrme and unitarity limit

Compare unitarity expansion:

$$\frac{E}{E_{\rm FG}} = 1 - \frac{U_0}{1-(a_s k_F)^{-1}U_1} + \frac{R_0(r_e k_F)}{[1-R_1(ak_F)^{-1}]\,[1-R_1(a_s k_F)^{-1}+R_2(r_e k_F)]}$$

to low ρ expansion:

$$\frac{E^{(1)}}{E_{\rm FG}} = \frac{10}{9\pi}(\nu-1)(k_F a_s) + (\nu-1)\frac{1}{6\pi}(k_F r_e)(k_F a_s)^2 \text{ (here } \nu\text{=2)}$$

can be rewritten in terms of t_i and x_i in Skyrme

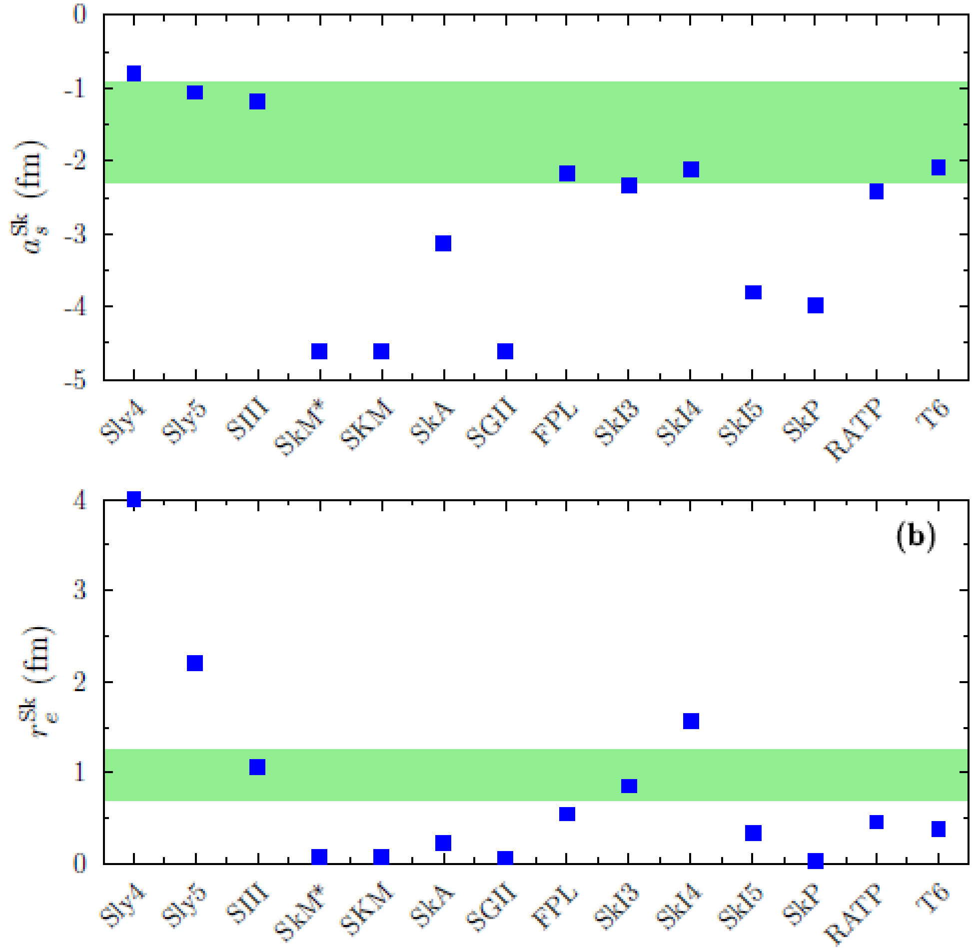
For the first few terms to match each other b/w the above Eqs., then the bare a_s, r_e in the positive power k_F-expansion become ρ-dep.:

$$\widetilde{a}_s(k_F) = -\frac{1}{k_F}\frac{U_1}{[1-(a_s k_F)^{-1}U_1]}, \qquad \widetilde{r}_e(k_F) = \frac{1}{k_F^3 \widetilde{a}_s^2(k_F)}\frac{R_1^2(r_e k_F)}{1-R_1(a_s k_F)^{-1}][1-R_1(a_s k_F)^{-1}+R_2(r_e k_F)]}.$$

Insert values of U_i, R_i from QMC, and vary k_F within typical density relevant to nuclear system ρ=0.01~0.2 [fm^{-3}], one finds:

$$\begin{cases} -2.3\ \text{fm} \leqslant \tilde{a}_s(\rho) \leqslant -0.92\ \text{fm}, \\ +0.69\ \text{fm} \leqslant \tilde{r}_e(\rho) \leqslant +1.26\ \text{fm}. \end{cases}$$

Compare $\widetilde{a}_s(k_F)$, $\widetilde{r}_e(k_F)$ generated by QMC and by Skyrme t_i, x_i:



Skyrme-like approaches are not far from the unitarity expansion!