Nuclear matter from skyrmion crystal approach in magnetic field

Mamiya Kawaguchi (Nagoya University)

collaborator :
Yong-Liang Ma (Jilin University)
Shinya Matsuzaki (Jilin University, Nagoya University)
arXiv:1804.09015 [nucl-th].

Outline

- Introduction
- Our work

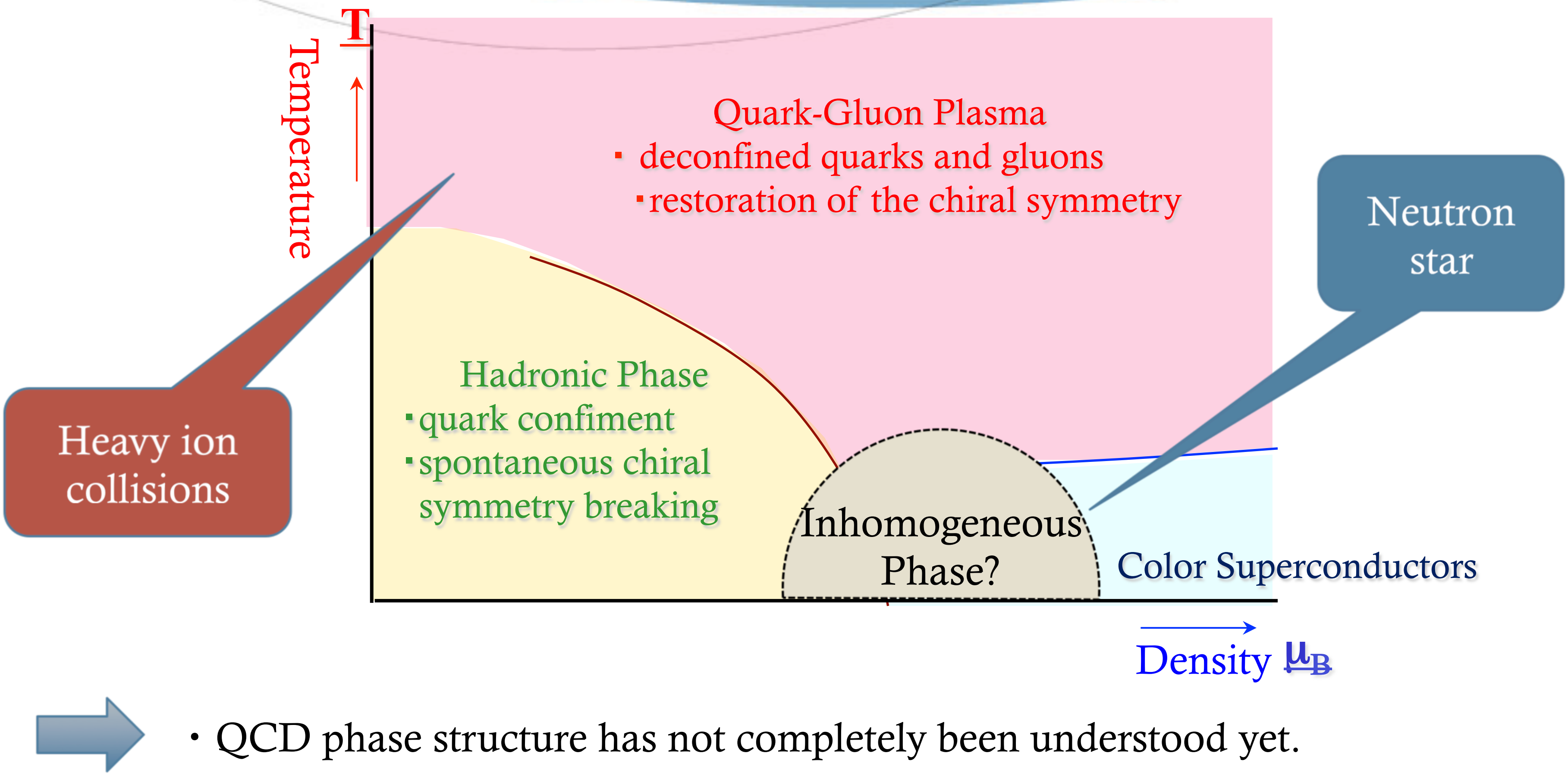
  - Short review of skyrmoin and skyrmion crystal

  - Skyrmion crystal in a magnetic field

  -(Chiral soliton lattice effect on Skyrmion crystal)
- Summary

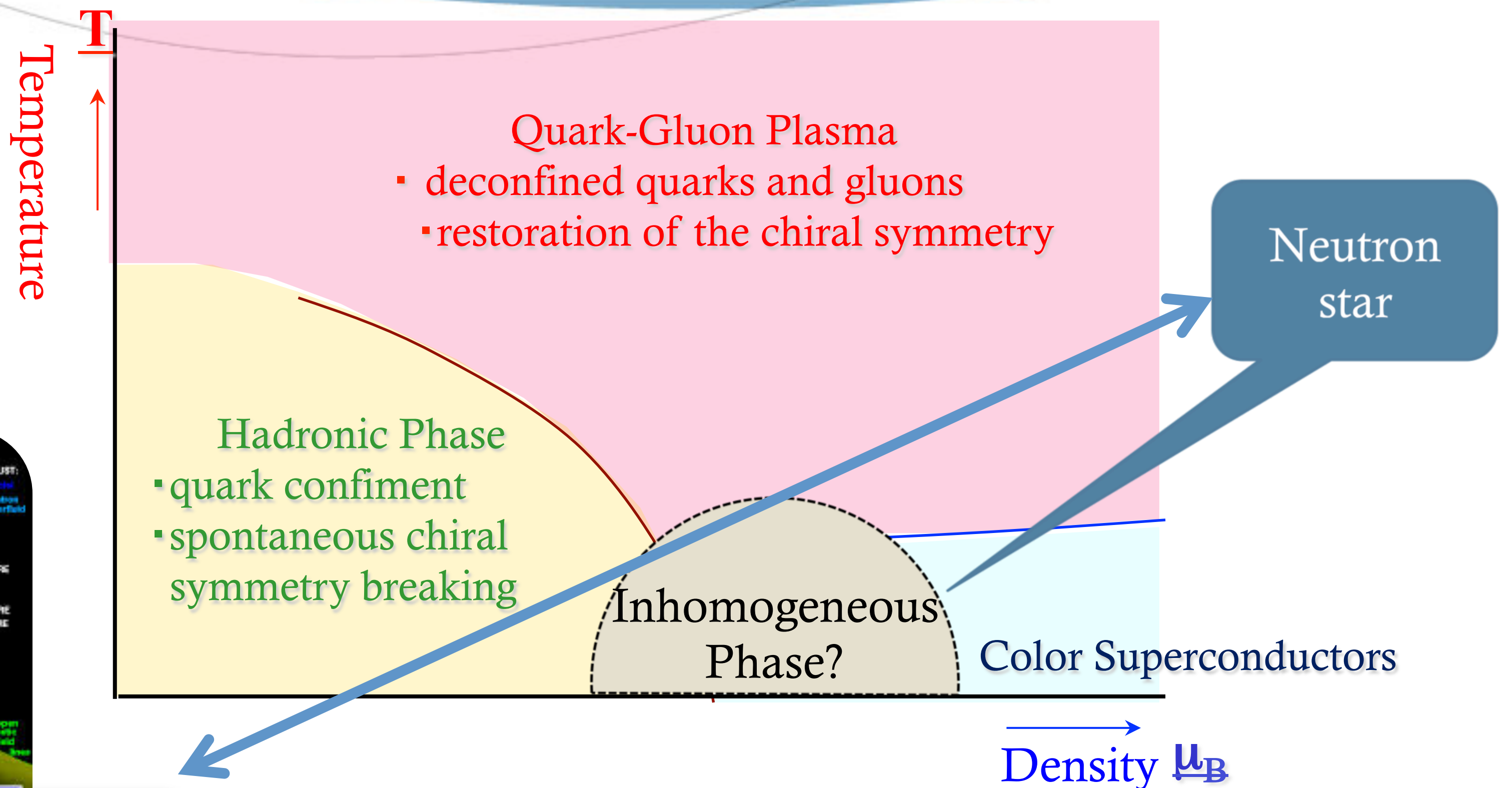
1. Introduction

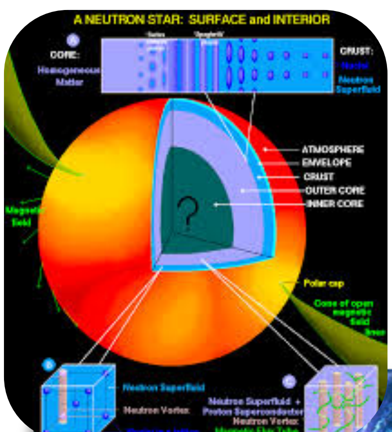
QCD phase structure

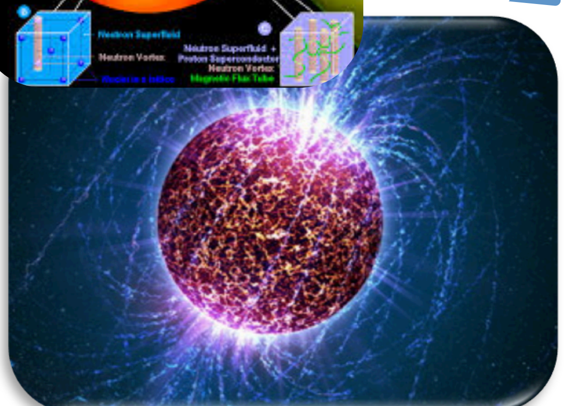


- QCD phase structure has not completely been understood yet.
- Does phase diagram have any other axis ?

QCD phase structure







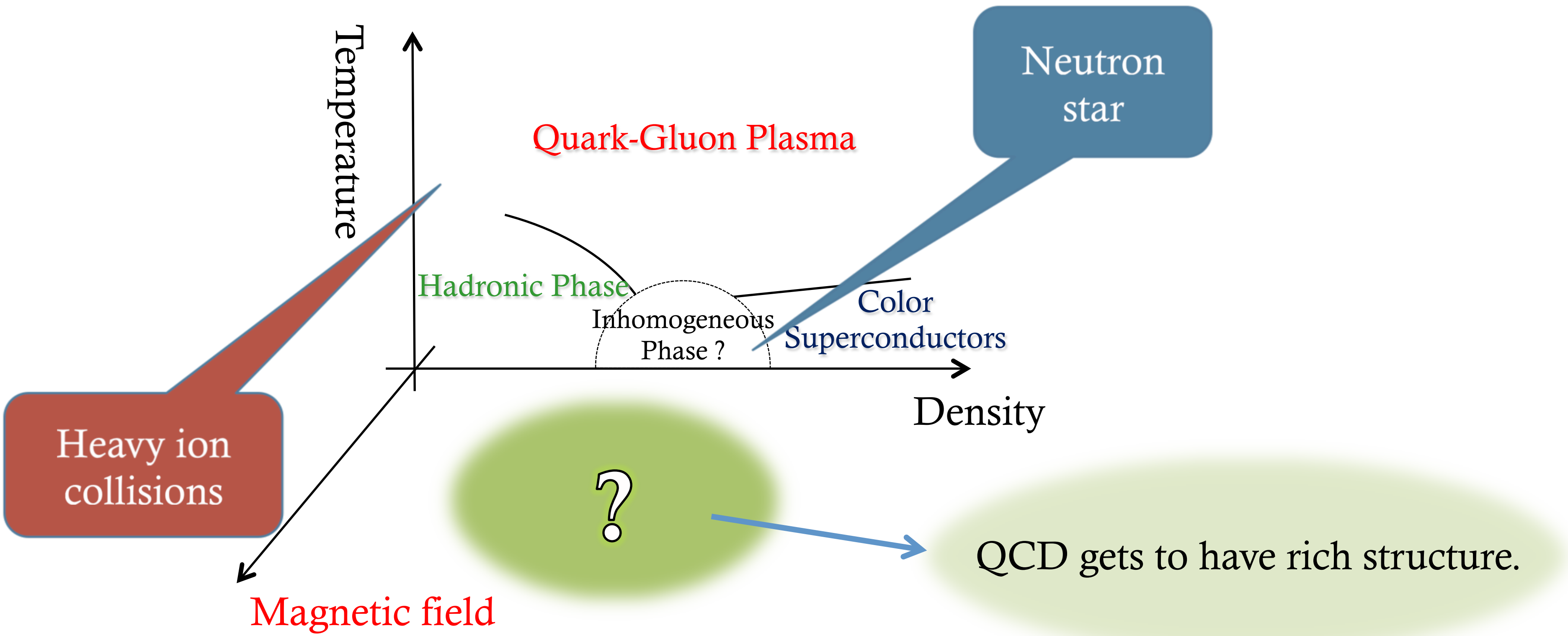
Magnetar has an extremely strong magnetic field.

Magnetar (surface)	10^{15}Gauss
Magnetar (inner core)	10^{18}Gauss

It seems that the hadron properties are changed dramatically in a magnetic field.

Need eB-axis for QCD phase structure

QCD phase diagram includes only temperature and density. ⇒ It would be important to add an axis along the magnetic field to QCD phase diagram.



Purpose of my study is to get the new insight for understanding the phase structure of QCD through such an extreme condition.

High density region and Strong magnetic field

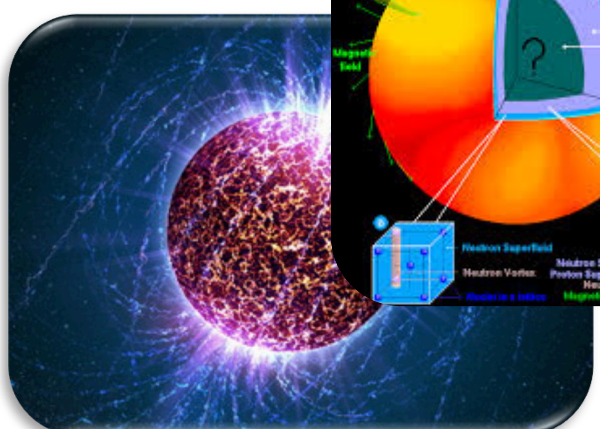
How to tackle to QCD phase structure

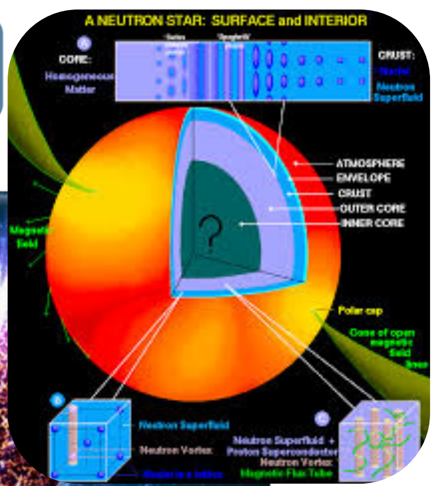
Summarize the above…

The purpose of my research is to extract the new aspect of QCD phase structure through extreme conditions, i.e. high density region with a magnetic field.

To accomplish my purpose, I focus on a baryonic matter with a strong magnetic field.

Neutron Star





How to tackle to QCD phase structure

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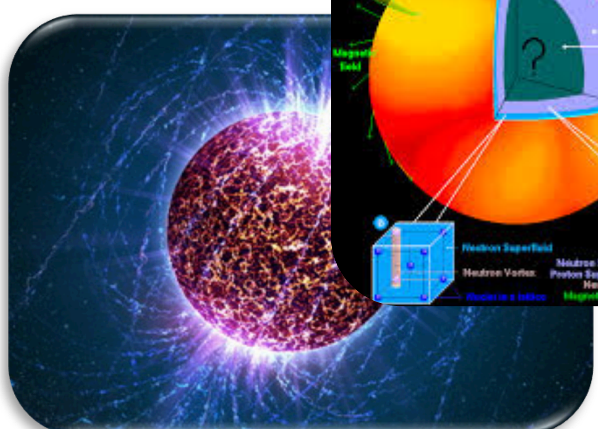
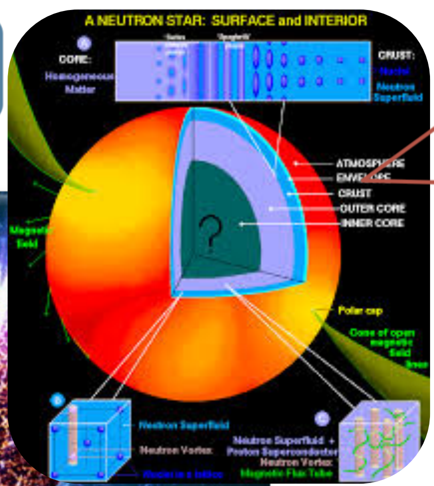
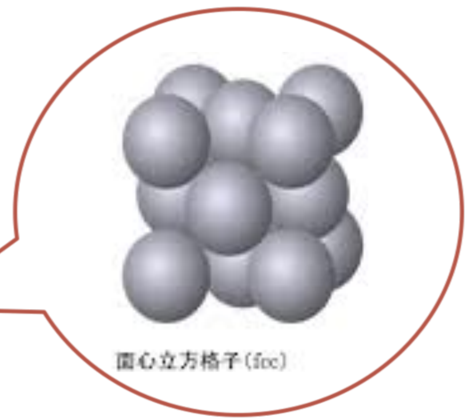
The purpose of my research is to extract the new aspect of QCD phase structure through extreme conditions, i.e. high density region with a magnetic field.

To accomplish my purpose, I focus on a baryonic matter with a strong magnetic field.
Assume that the nuclear matter consists of crystals of baryon.

Neutron Star







Face centered cubic

How to tackle to QCD phase structure

Summarize the above…

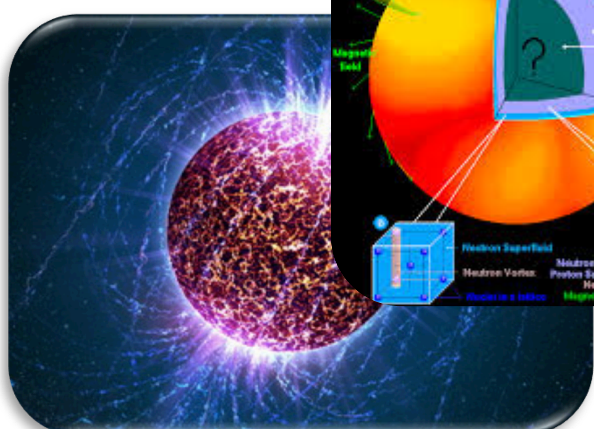
The purpose of my research is to extract the new aspect of QCD phase structure through extreme conditions, i.e. high density region with a magnetic field.

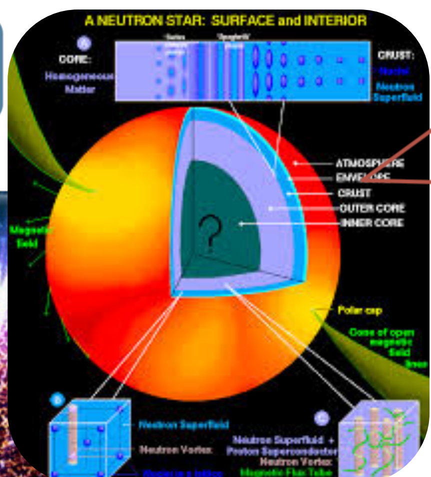
↓

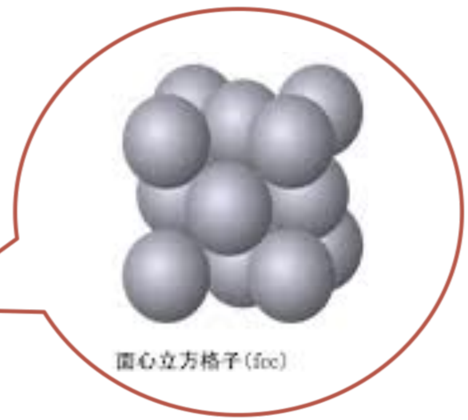
In this study, we employ the skyrmion crystal model.

Skyrmion is identified as baryon while respecting the chiral symmetry.

Neutron Star







= Skyrmions

Face centered cubic

*In high density region crystalline description for nuclear matter could be valid, because in such a region nucleons are nearly compressed at certain fixed position, being treated as static objects (solitons).

How to tackle to QCD phase structure

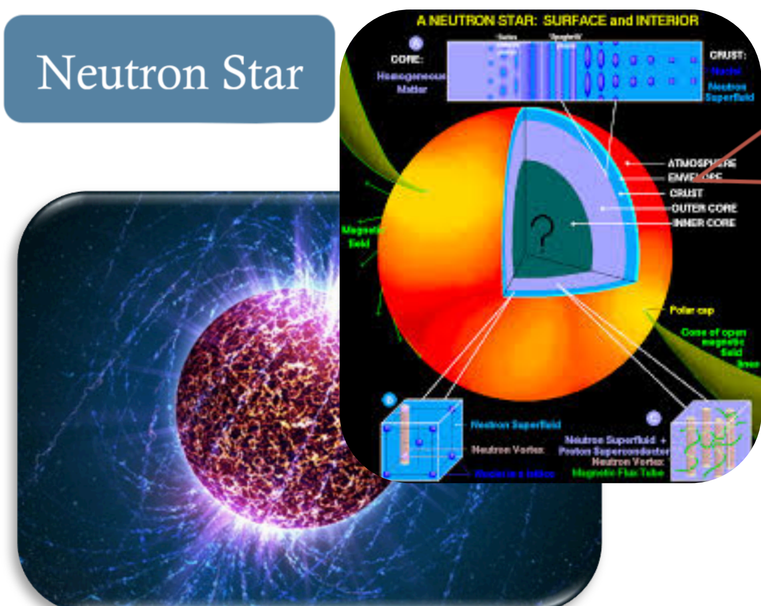
Summarize the above…

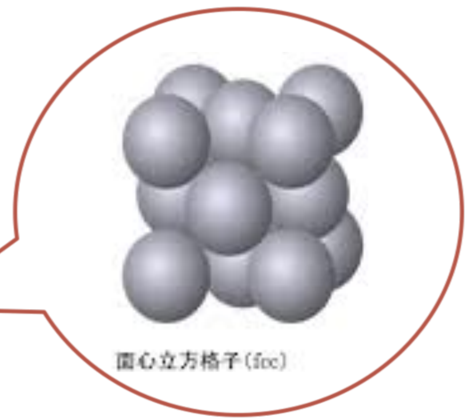
The purpose of my research is to extract the new aspect of QCD phase structure through extreme conditions, i.e. high density region with a magnetic field.

By applying a magnetic field, we study the nuclear matter properties to get the new insight for understanding QCD .

Magnetic field

Neutron Star





Face centered cubic

=

Skyrmions

To get new insight for understanding QCD

Our work

(Short review of skyrmion)

Short review of skyrmion

T. H. R. Skyrme, Proc. Roy. Soc. Lond. A260 (1961) 127;
Nucl. Phys. 31 (1962) 556;
I. Zahed and G. E. Brown, Phys. Rept., 142 (1986) 1.

Skyrme model Lagrangian based on the chiral symmetry $U = \exp[i\pi^a\tau^a/F_\pi]$

$$\mathcal{L}_{\rm Skyr} = \frac{f_\pi^2}{4}{\rm tr}[\partial_\mu U \partial^\mu U^\dagger] + \frac{1}{32e^2}{\rm tr}\left\{[U^\dagger\partial_\mu U, U^\dagger\partial_\nu U][U^\dagger\partial^\mu U, U^\dagger\partial^\nu U]\right\}$$

- Invariant under chiral transformation $U \to g_L U g_R^\dagger$

To describe baryon-physics, we give the hedgehog ansatz, $U = \exp[i\hat{x}^i\tau^i F(r)]$.

In topology

The ansatz is denoted as the nontrivial map $U(x)\ :\ R^3 \to S^3$

This maps constitute the third homotopy group $\pi_3(S^3) = Z$.

Winding number (baryon number) : $B = \int d^3x j_B^0 = 1$

boundary condition

$F(0) = \pi,\ F(\infty) = 0$

Baryon current : $j_B^\mu = \frac{1}{24\pi^2}\epsilon^{\mu\nu\rho\sigma}{\rm tr}\left[(\partial_\nu U\cdot U^\dagger)(\partial_\rho U\cdot U^\dagger)(\partial_\sigma U\cdot U^\dagger)\right]$

The hedgehog ansats is characterized by the winding number (baryon number).
→Skyrme model describes “baryon”

Short review of skyrmion

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- Invariant under chiral transformation $U \to g_L U g_R^\dagger$

To describe baryon-physics, we give the hedgehog ansatz, $U = \exp[i\hat{x}^i\tau^i F(r)]$.

through the numerical calculation….

Baryon properties

- Energy of skyrmion (baryon)

$$M_{\rm Skyrm} = -\int d^3x \mathcal{L}_{\rm Skyrm}$$

$$M_{\rm Skyr} \sim 1150[{\rm MeV}]$$

- Isoscalar charge radius of a nucleon

$$r_0 = 0.66\,{\rm fm}$$

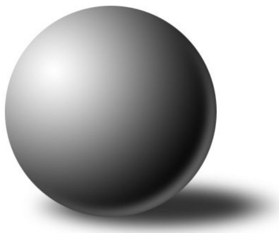
$$(r_0^{\rm (exp)} = 0.877 \pm 0.005\,{\rm fm})$$

Input parameter

$f_\pi = 93{\rm MeV}$ (experimenta value)

$e \sim 6$ (determined from $\rho \to \pi\pi$)

Skyrmion (nucleon) is the finite size particle.



*Observables are acceptable at the leading $\mathcal{O}(N_c)$.

Our work

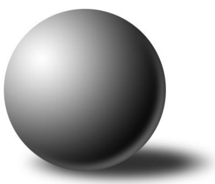
So far, I just showed the "isolated skyrmoin (= baryon)".
Let's move on "skyrmoins (=baryonic matter)"

(Short review of skyrmion crystal)

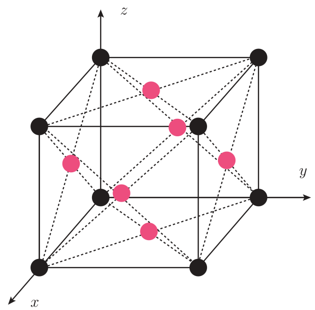
Short review of skyrmion crystal

To investigate the baryonic matter properties, we put skyrmions onto crystal lattice

Skyrmion



Put skyrmions onto crystal lattice



Skyrmion crystal

面心立方格子(fcc)

I. Klebanov, Nucl. Phys. B262(1985) 133-143

H. J. Lee, B. Y. Park, D. P. Min, M. Rho and V. Vento, Nucl. Phys. A bf 723, 427 (2003)

Short review of skyrmion crystal

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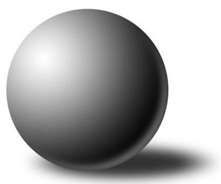
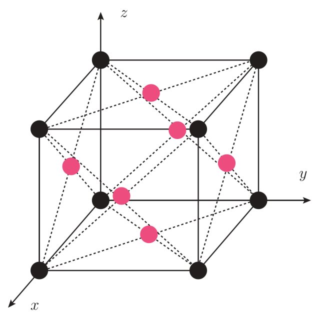
Skyrmion



Put skyrmions onto crystal lattice



Skyrmion crystal

面心立方格子(fcc)

Identify skyrmion crystal as baryonic matter.

I. Klebanov, Nucl. Phys. B262(1985) 133-143

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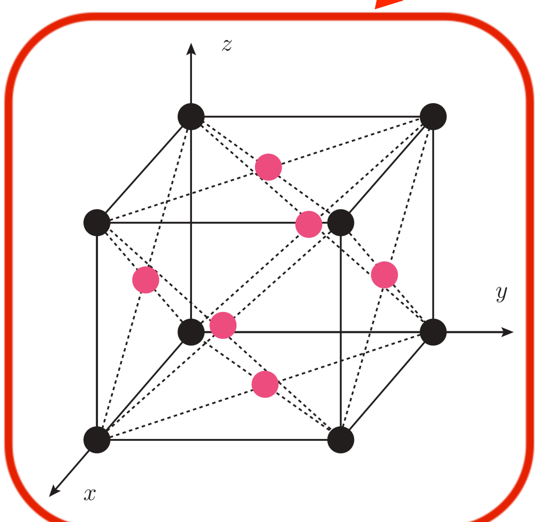
Specifically choose the face centered cubic in our work.

- Put skyrmions onto the face centered cubic(FCC) crystal
- A single FCC crystal has the volume size $(2L)^3$ and contains 4 skyrmions.



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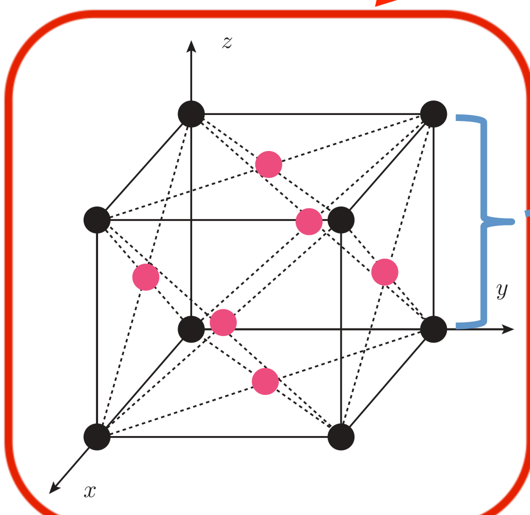

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Lattice size

- Baryonic matter density: $\rho = 4/(2L)^3$

As the lattice size is changed,
the baryonic matter density is also changed.

Short review of skyrmion crystal

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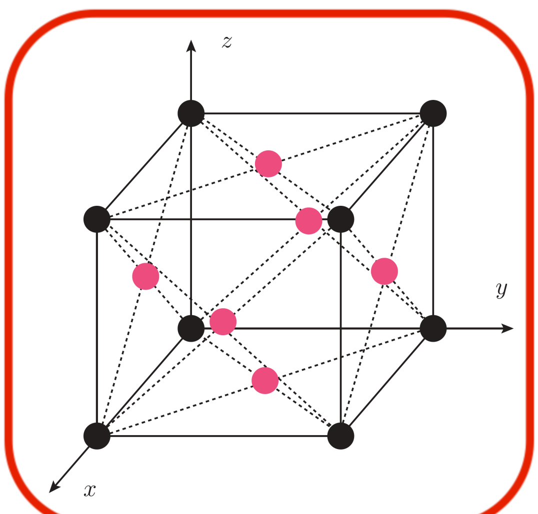
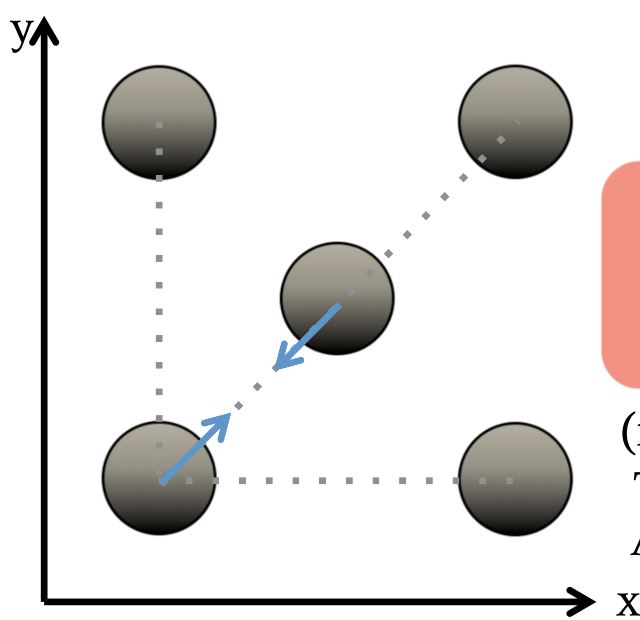
How is the skyrmion-skyrmion interaction going?





Focus on the x-y plane

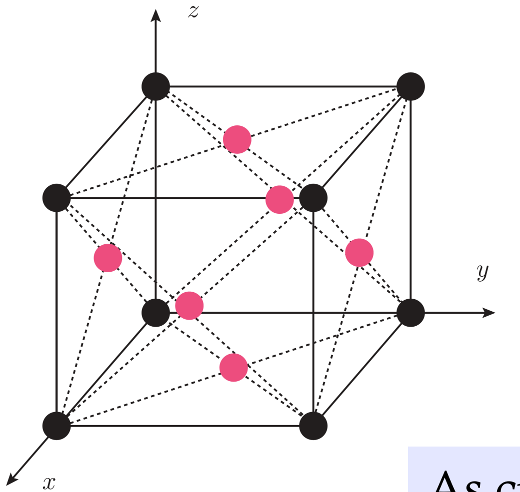
In skyrmion crystal approach, nearest skyrmions get the strongest attractive interaction

(for more on this, please see T. H. R. Skyrme, Proc. Roy. Soc. Lond. A 260 (1961) 127.)

Short review of skyrmion crystal

The skyrmion approach has a characteristic phenomena.

On the premise of this work
skyrmions are put onto a FCC crystal.



As crystal size is changed to be small,
interesting phenomena happens.
*Baryonic matter density: $\rho = 4/(2L)^3$

Short review of skyrmion crystal

The skyrmion approach has a characteristic phenomena which is the topological phase transition between the skyrmion and the half-skyrmion phase.

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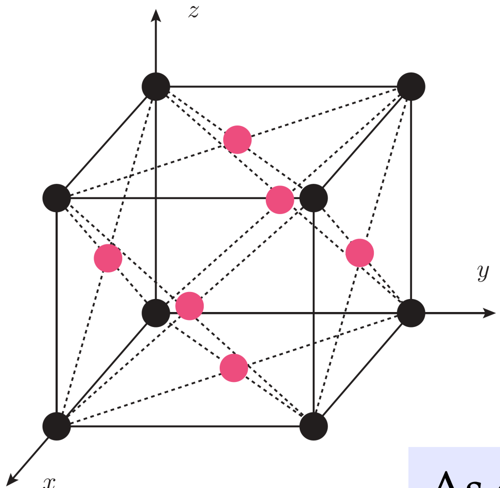

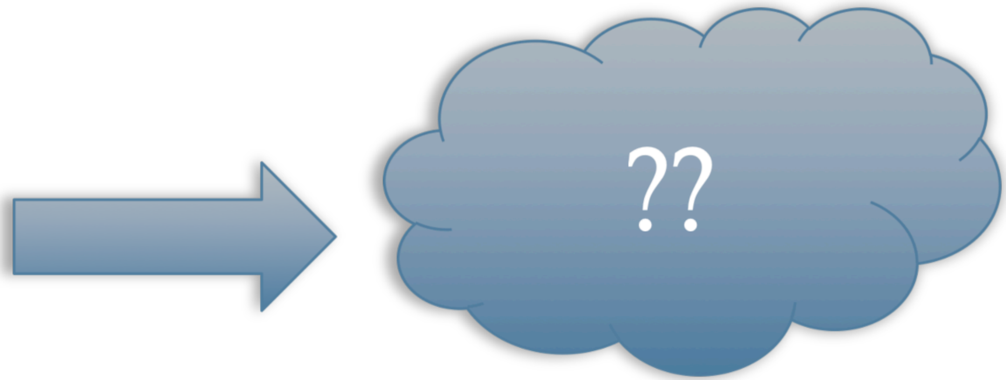

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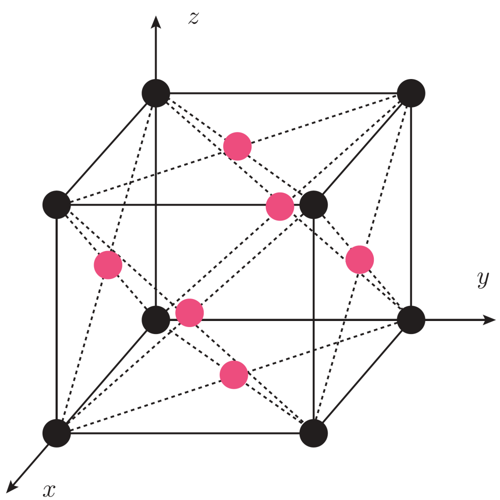
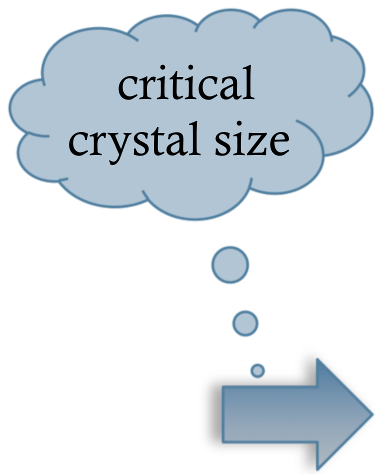
Short review of skyrmion crystal

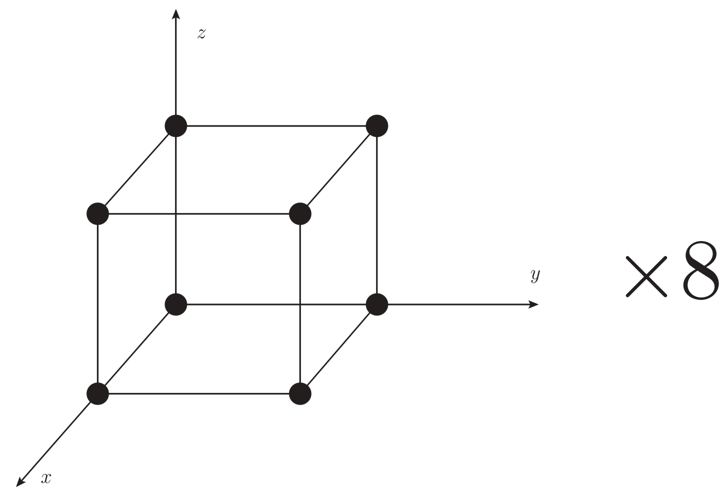
The skyrmion approach has a characteristic phenomena which is the topological phase transition between the skyrmion and the half-skyrmion phase.

- A FCC crystal with volume size $(2L)^3$ contains 4 skyrmions





- A crystal lattice with volume size $(2L)^3$ has 8 cubic-centered (CC) crystals.
- A single CC contains 1 skyrmion.



* Baryon number is conserved even if this system undergoes the topological phase transition.

Short review of skyrmion crystal

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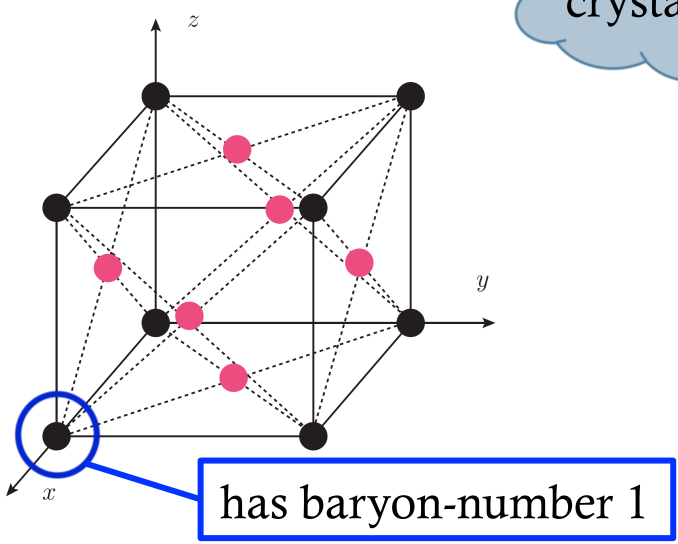

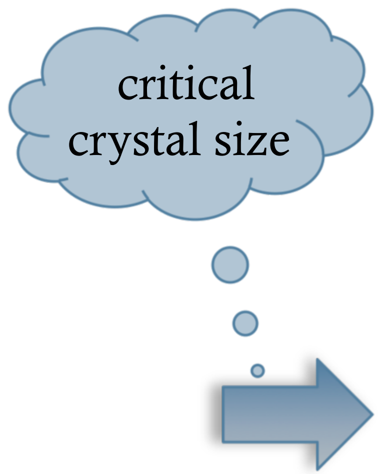

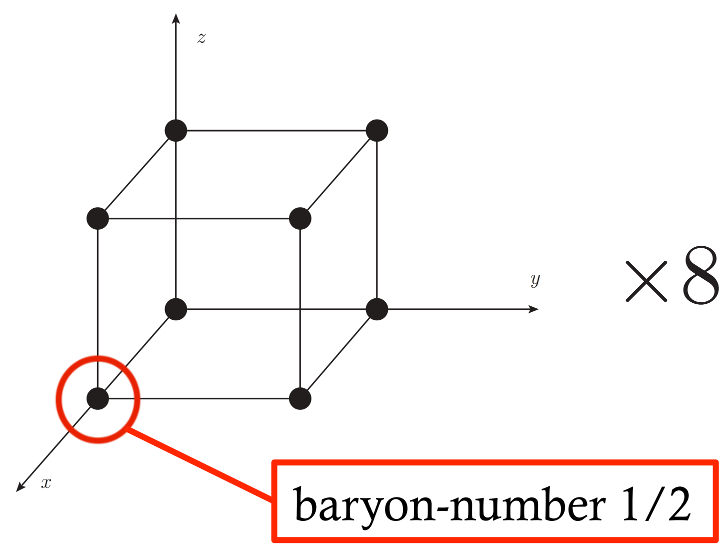

Skyrmion phase

Half-skyrmion phase

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Short review of skyrmion crystal

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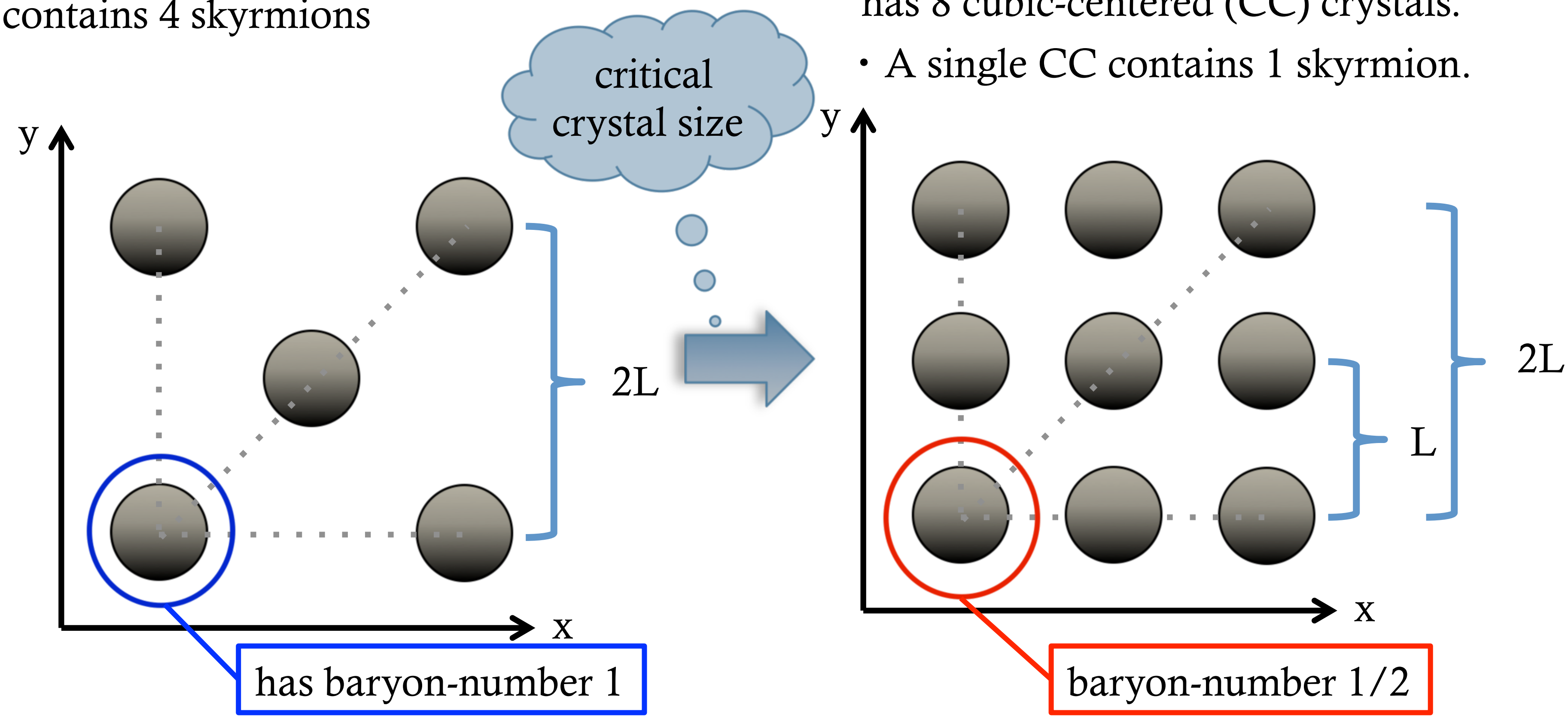
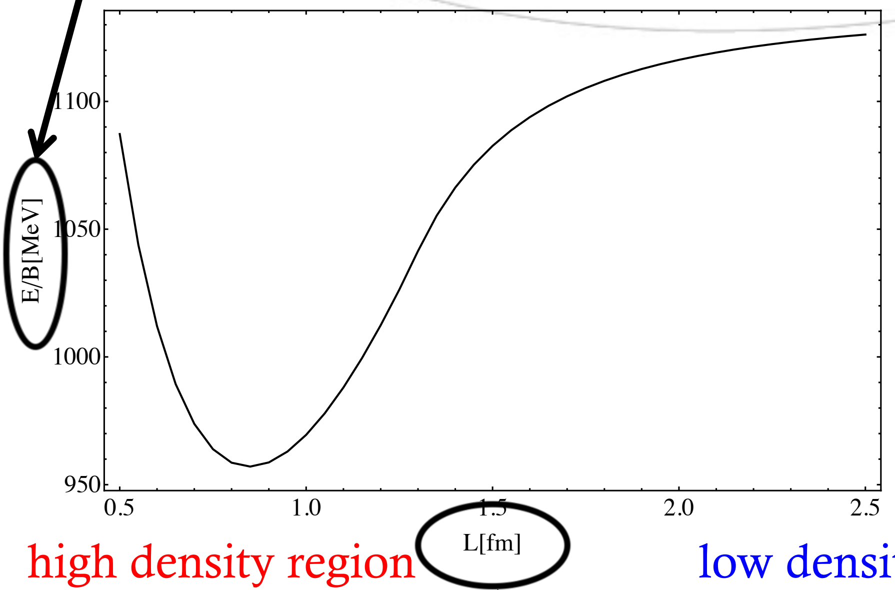


Let's check skyrmion crystal properties through the numerical calculation

Short review of skyrmion crystal

Baryon energy per skyrmion



high density region

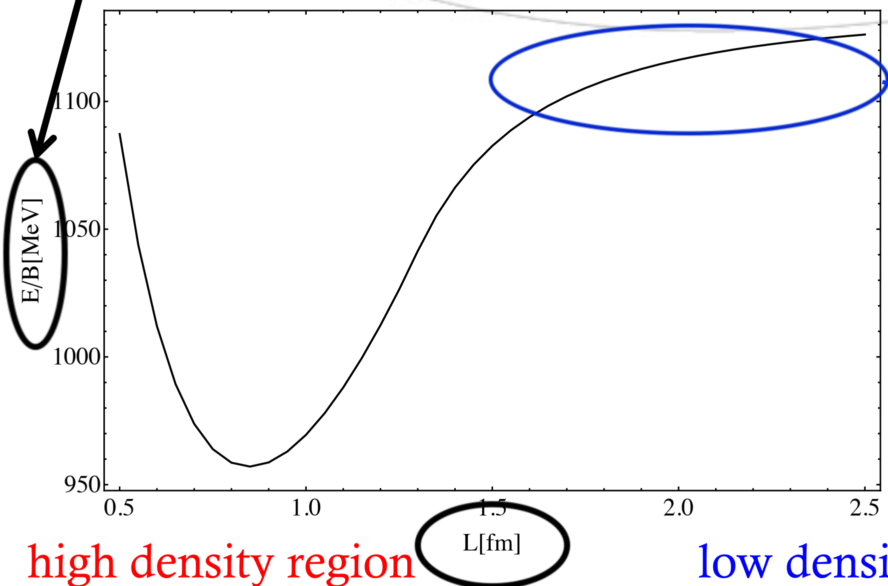
low density region

Lattice size, L[fm]

Baryonic matter density: $\rho = 4/(2L)^3$

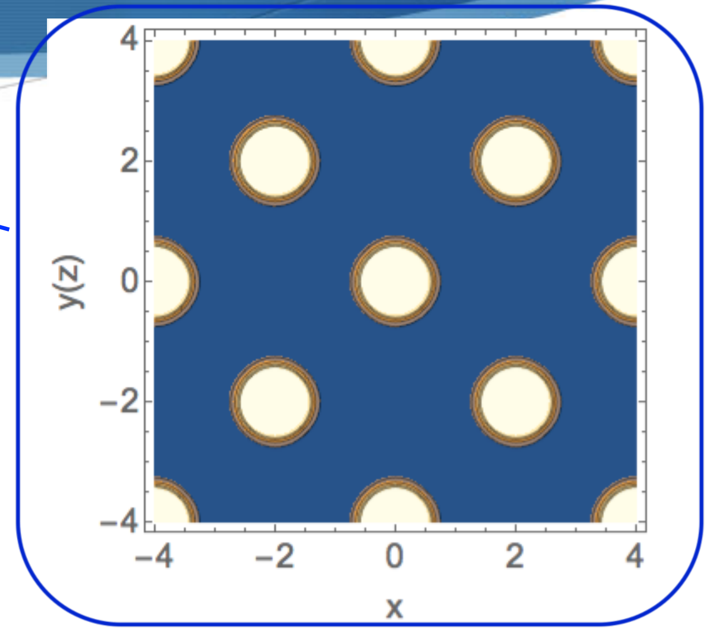
Short review of skyrmion crystal

Baryon energy per skyrmion



high density region

low density region



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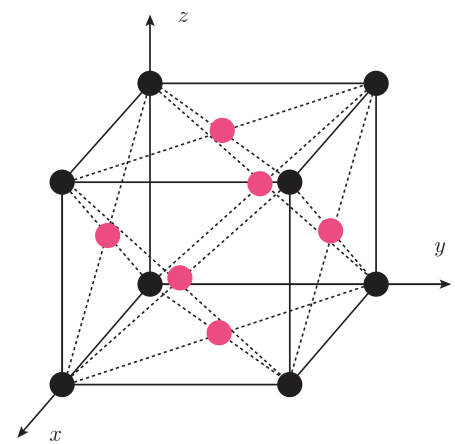


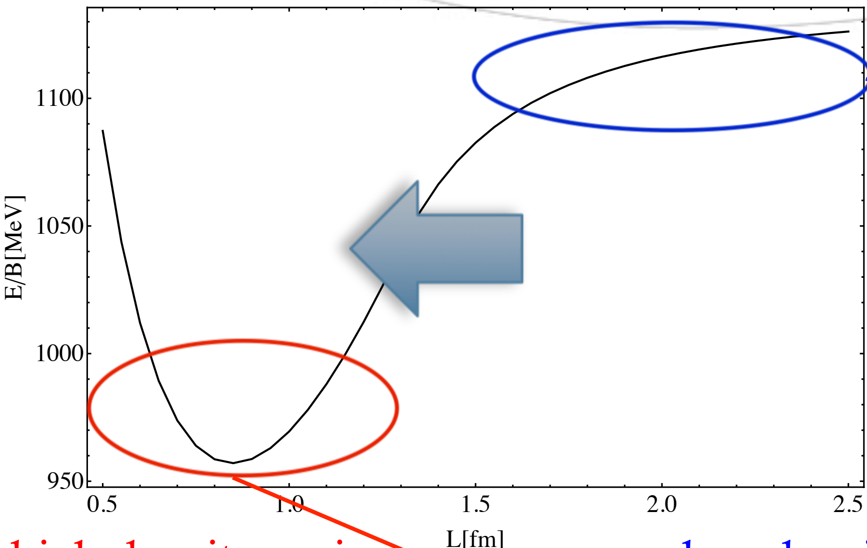
Winding number(Baryon number density)

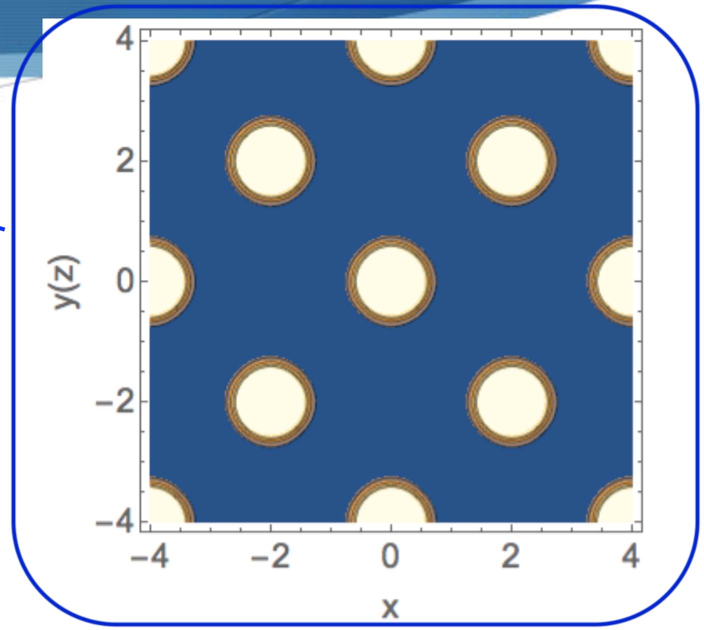
$$\rho_B = \frac{1}{24\pi^2}\epsilon^{0\nu\rho\sigma}\mathrm{tr}\left[(\partial_\nu U\cdot U^\dagger)(\partial_\rho U\cdot U^\dagger)(\partial_\sigma U\cdot U^\dagger)\right]$$

It is able to reproduce FCC crystal numerically.

Short review of skyrmion crystal

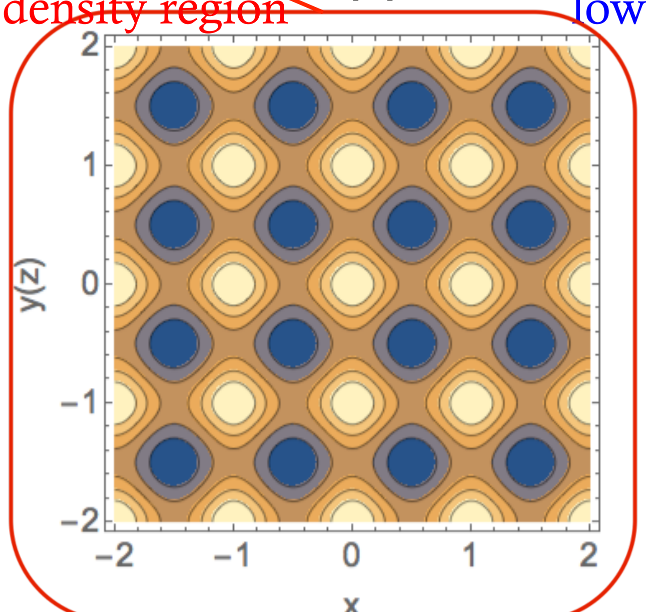
Baryon energy per skyrmion

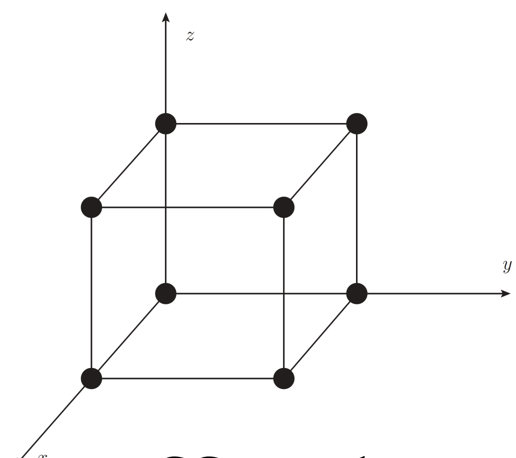




high density region

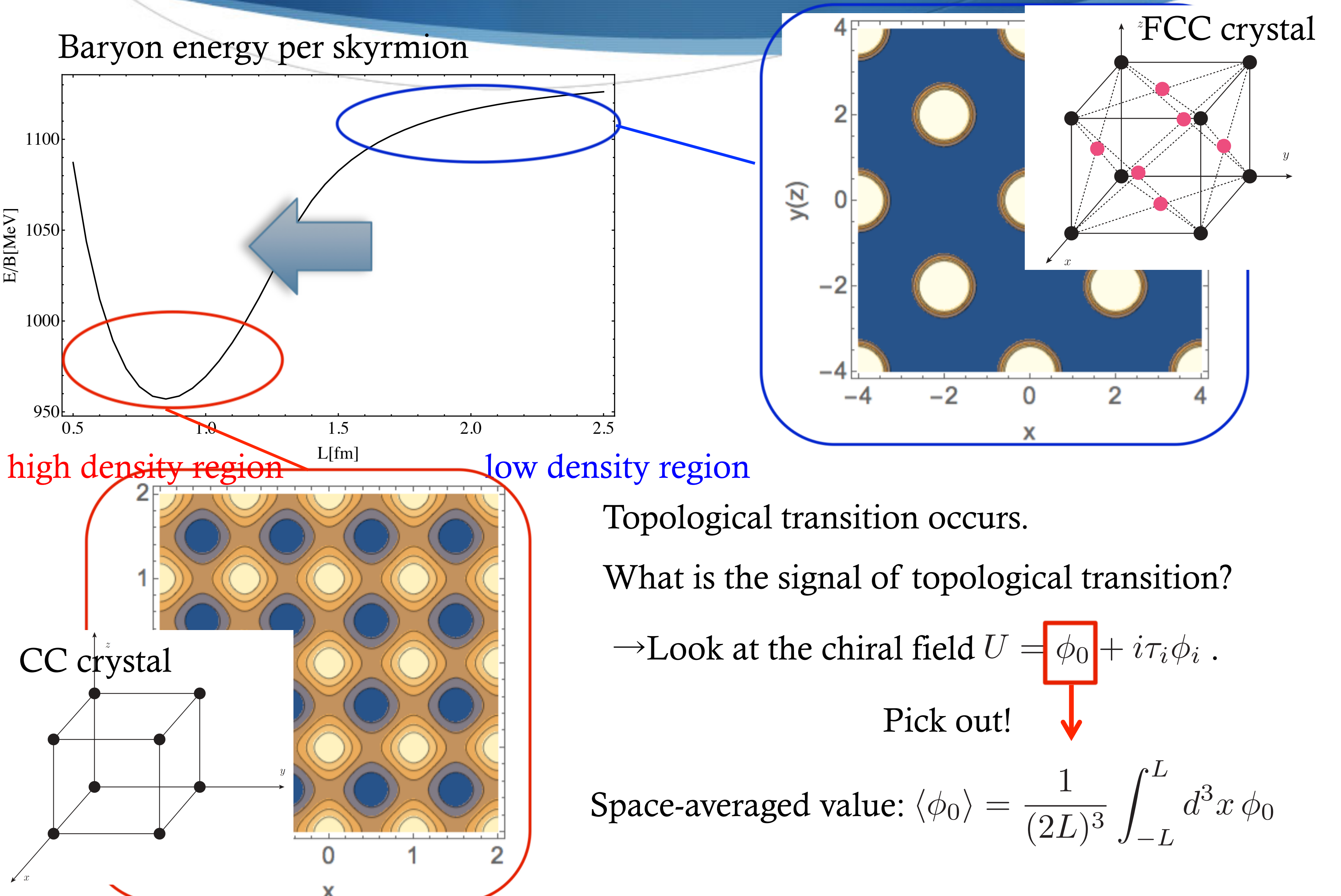
low density region





CC crystal

Short review of skyrmion crystal



Topological transition occurs.

What is the signal of topological transition?

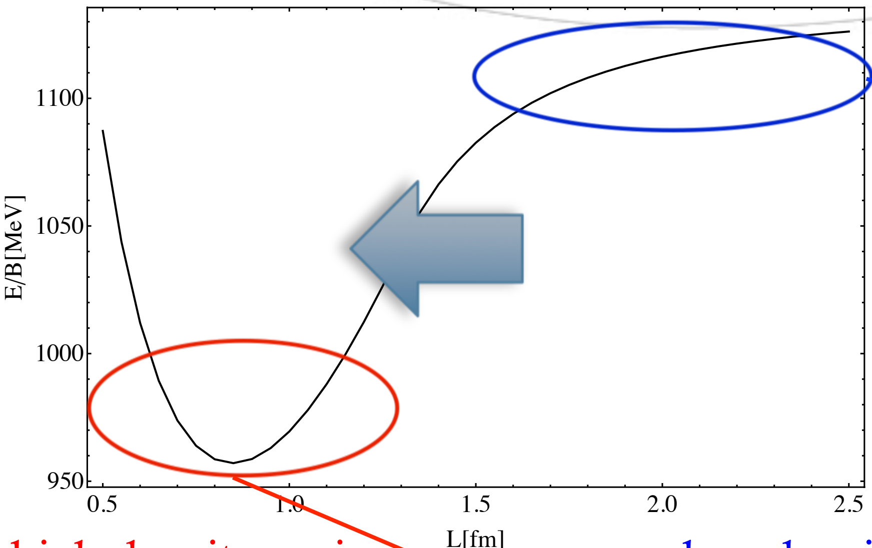
→Look at the chiral field $U = \phi_0 + i\tau_i\phi_i$.

Pick out!

Space-averaged value: $\langle\phi_0\rangle = \frac{1}{(2L)^3}\int_{-L}^{L} d^3x\,\phi_0$

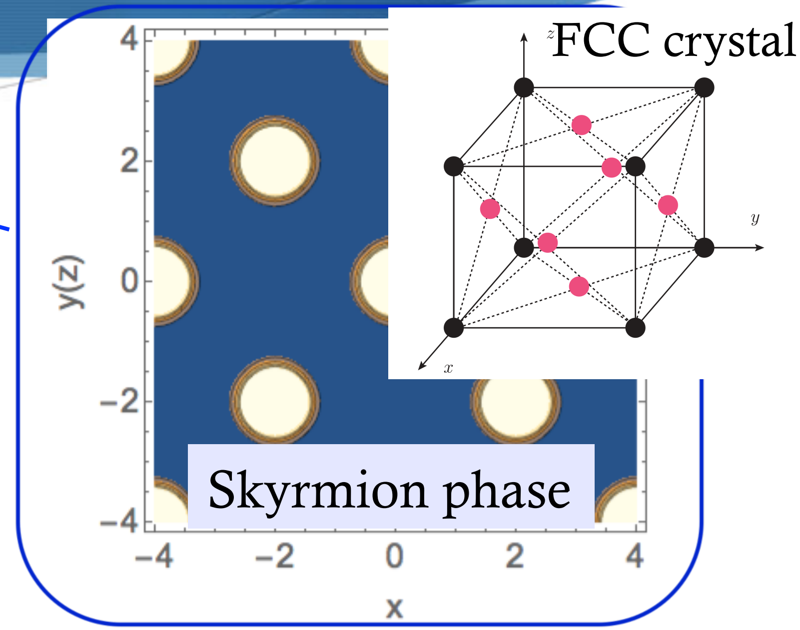
Short review of skyrmion crystal

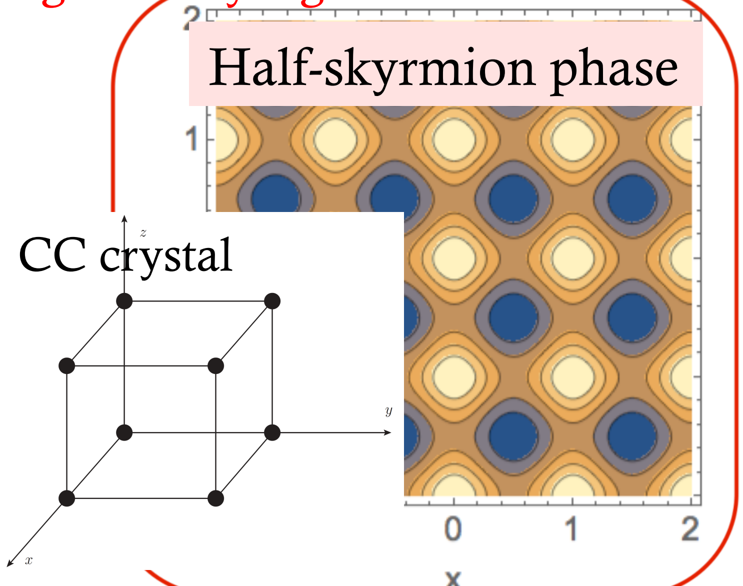
Baryon energy per skyrmion

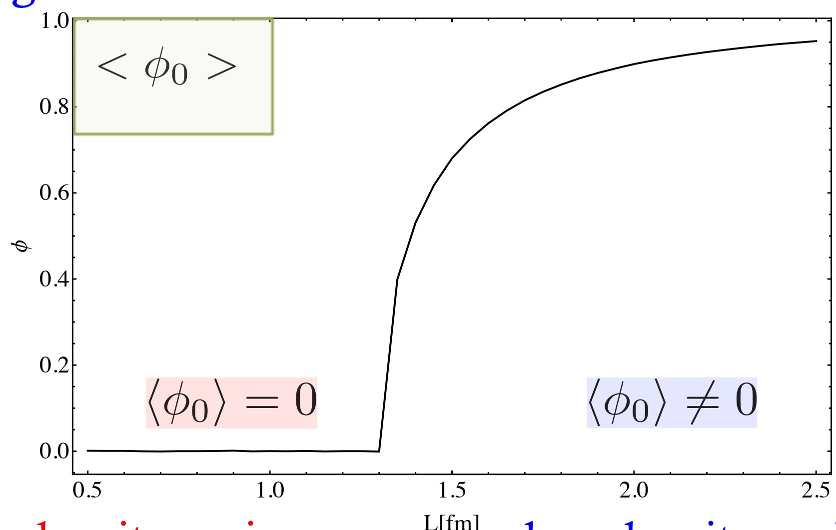


high density region

low density region



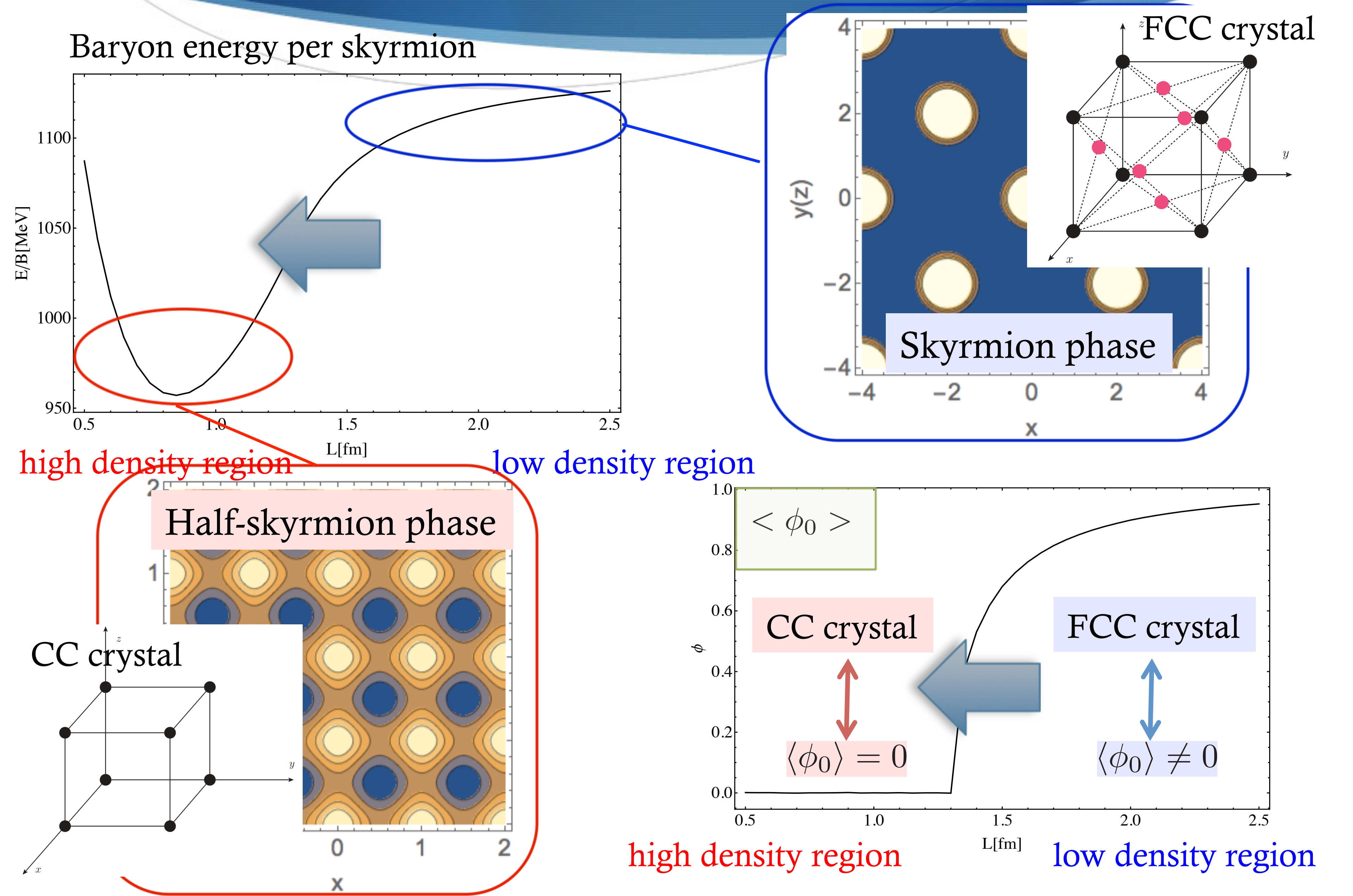




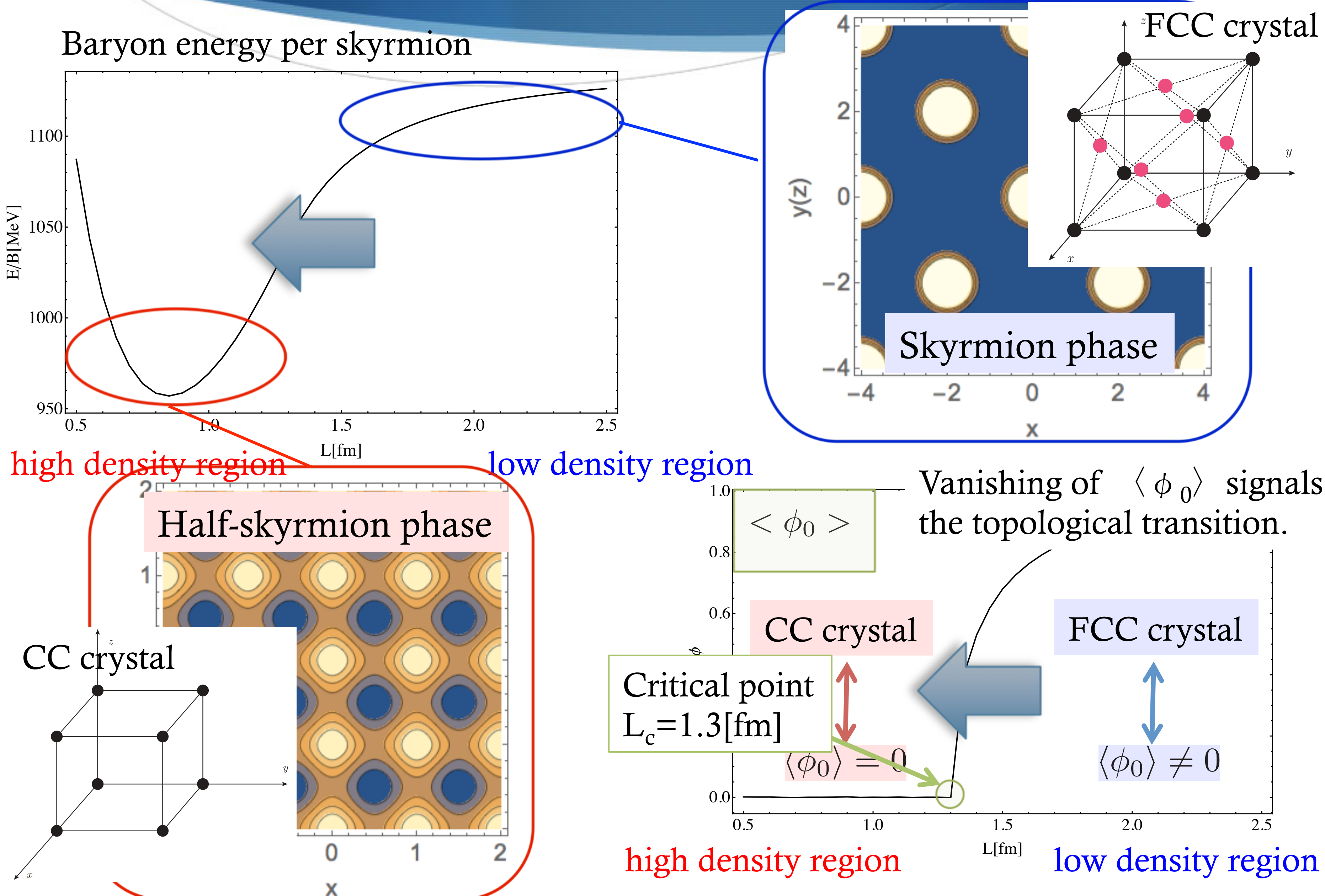
high density region

low density region

Short review of skyrmion crystal



Short review of skyrmion crystal

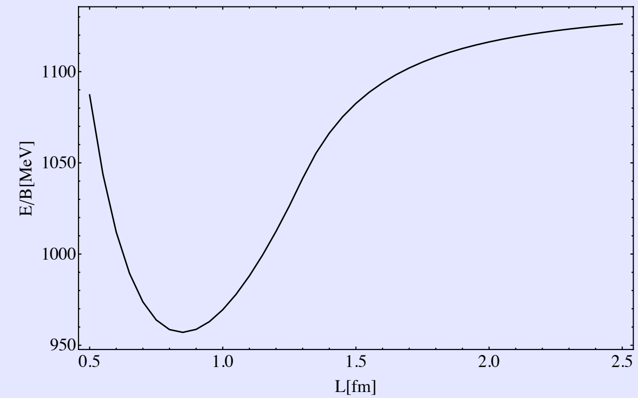


What happens in magnetic field?

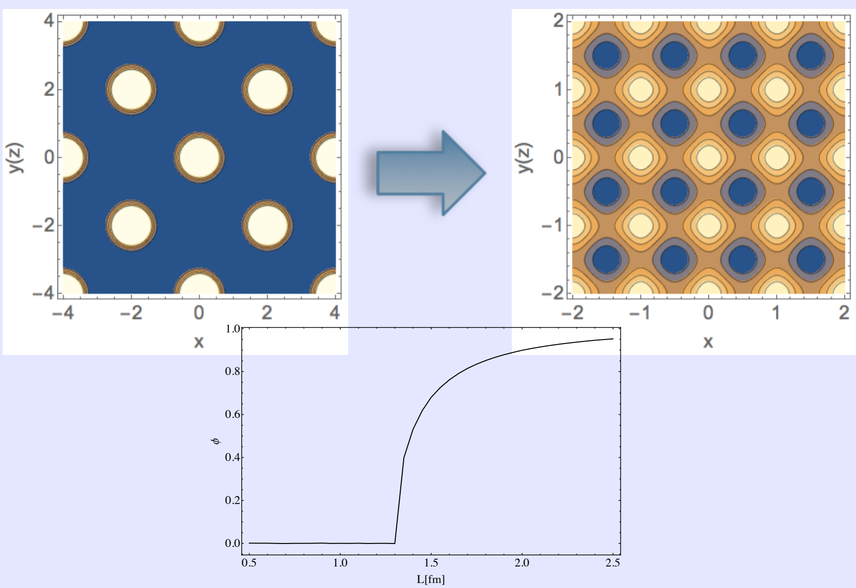
Magnetic field

Deformation of skyrmion configuration

Baryon energy per skyrmion



Topological transition



By applying magnetic field,
what changes in crystal properties?

Skyrmion crystal in a magnetic field

M. K., Y. L. Ma and S. Matsuzaki,
``Magnetic field effect on nuclear matter from skyrmion crystal model,"
arXiv:1804.09015 [nucl-th].

Skyrmion crystal in a magnetic field

Replace the derivative operator with the gauge covariant one

*Constant magnetic field along z-axis

$$\partial_\mu U \to D_\mu U = \partial_\mu U - i\mathcal{L}_\mu U + iU\mathcal{R}_\mu \qquad \mathcal{L}_\mu = \mathcal{R}_\mu = eQ_{\mathrm{em}}A_\mu$$

$$\mathcal{L}_{\mathrm{Skyr}} = \frac{f_\pi^2}{4}\mathrm{tr}[\partial_\mu U \partial^\mu U^\dagger] + \frac{1}{32e^2}\mathrm{tr}\left\{[U^\dagger\partial_\mu U, U^\dagger\partial_\nu U][U^\dagger\partial^\mu U, U^\dagger\partial^\nu U]\right\}$$

Skyrmion crystal in a magnetic field

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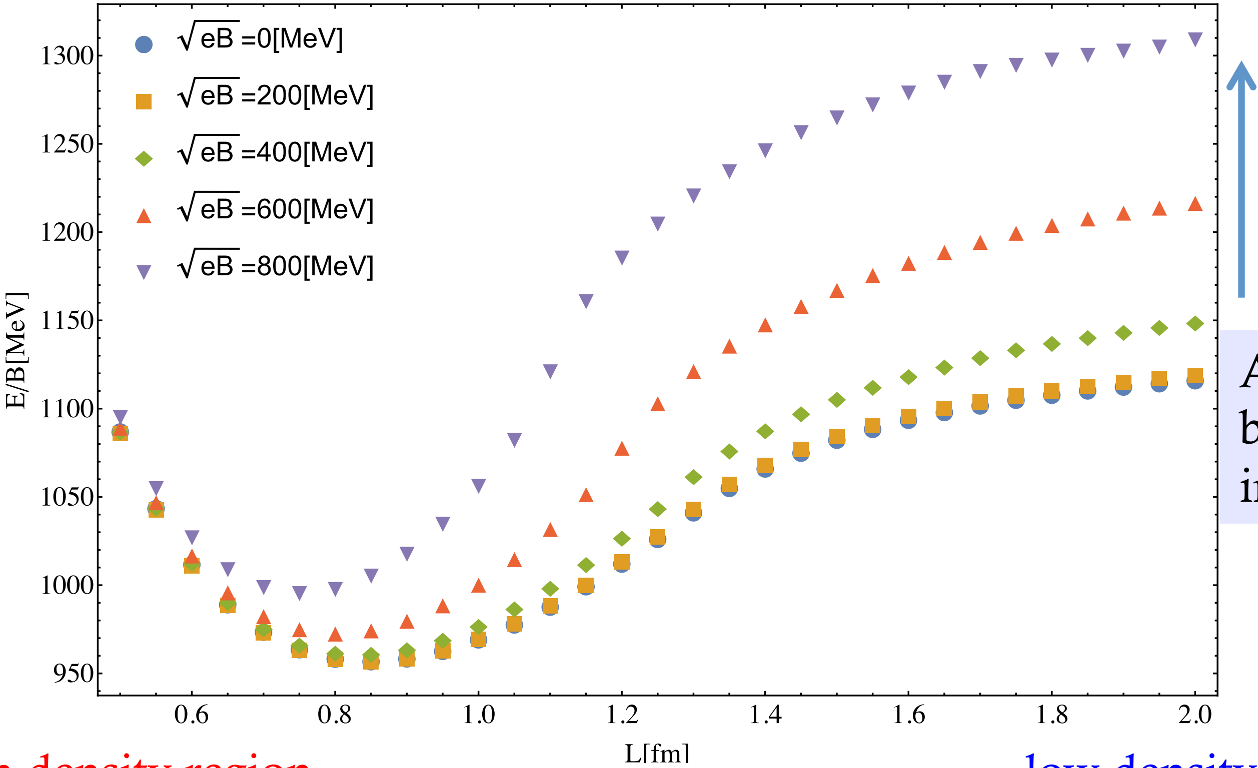
Baryon energy per skyrmion



Magnetic field eB

As magnetic field increases, baryon (skyrmion) energy increases.

high density region

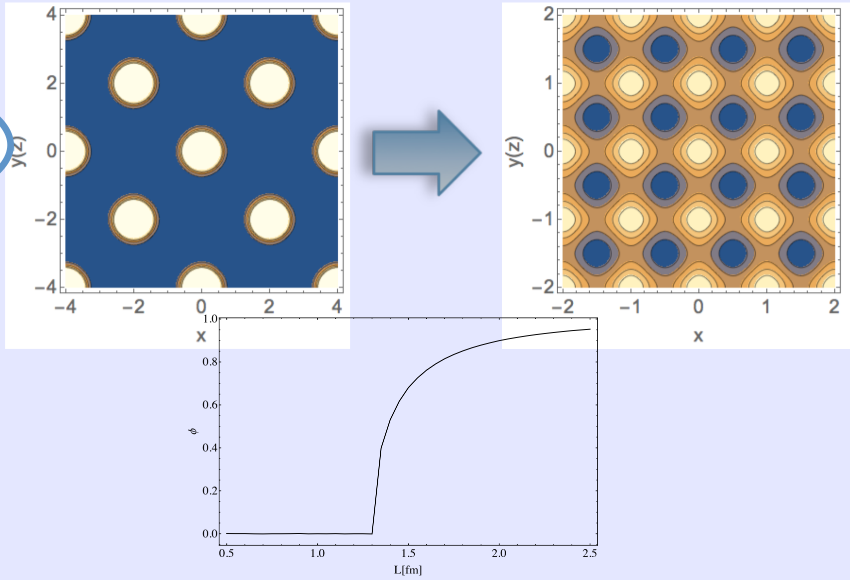
low density region

$\langle \phi_0 \rangle$ in a magnetic field

Magnetic effect on $< \phi_0 >$

Vanishing of $\langle \phi_0 \rangle$ signals the topological transition.

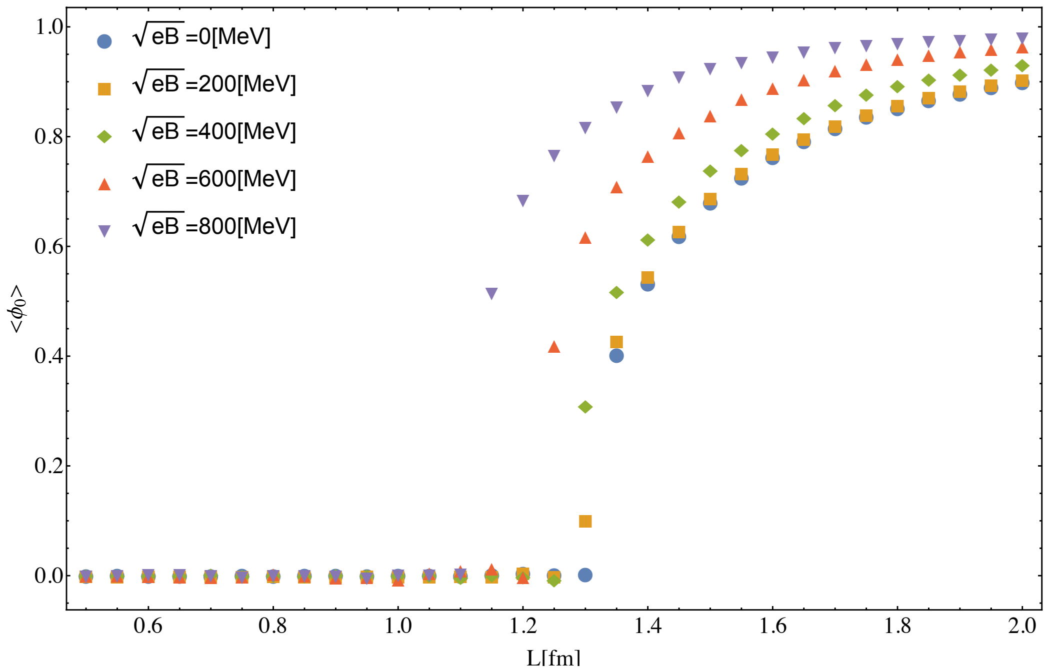
Topological transition



$\langle \phi_0 \rangle$ in a magnetic field

Magnetic effect on $< \phi_0 >$

Magnetic field eB



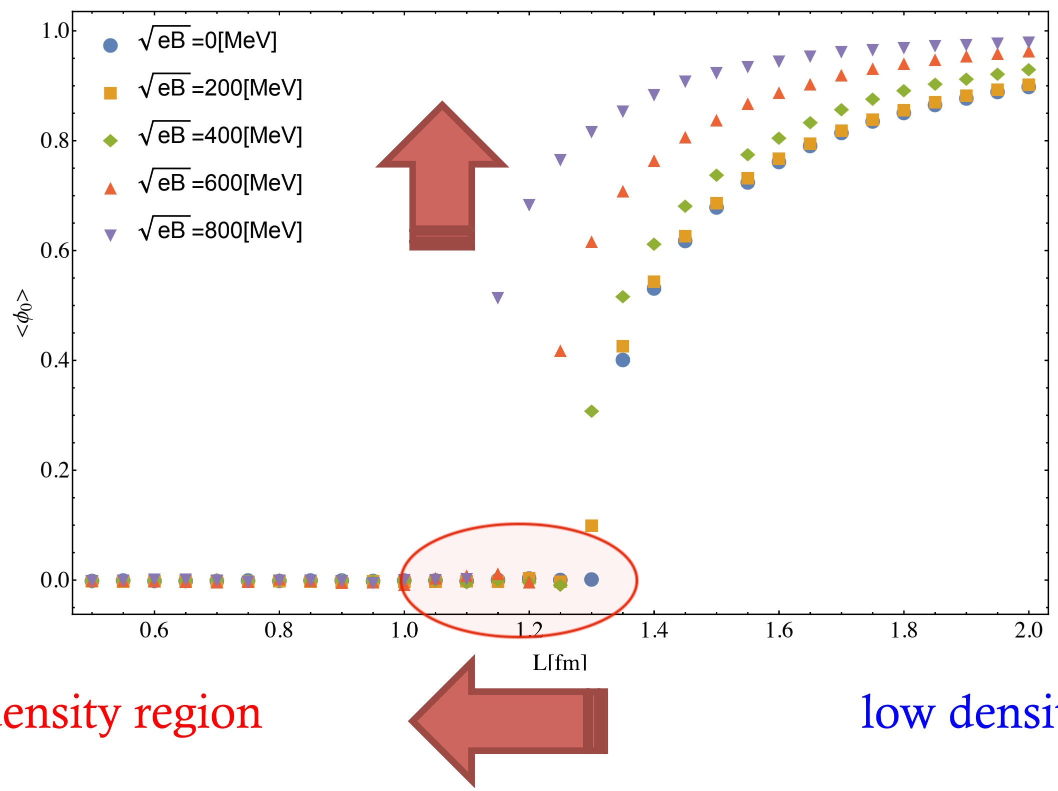
high density region

low density region

⟨ ϕ_0 ⟩ in a magnetic field

Magnetic effect on $< \phi_0 >$

Magnetic field eB



high density region

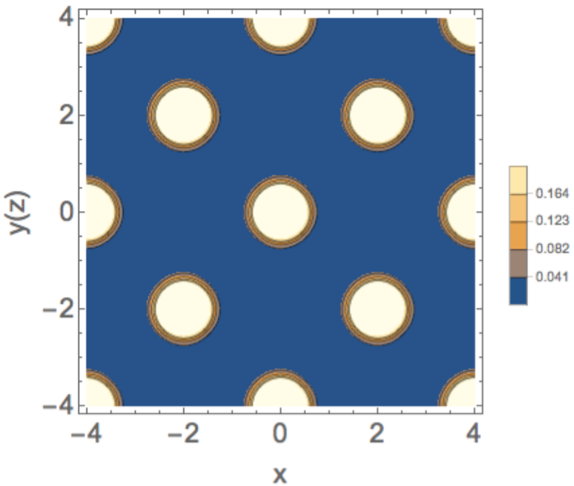
low density region

As the magnetic field increases, the topological transition point is shifted to a high density region and the value of ⟨ ϕ_0 ⟩ gets larger.

Deformation of the skyrmion configuration

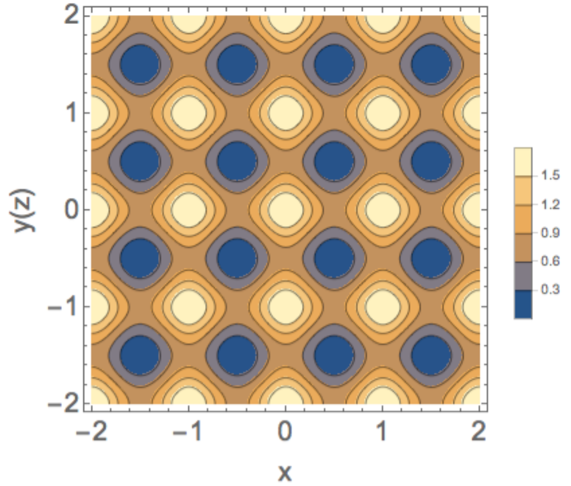
Skyrmion phase

$L = 2.0[\text{fm}]$



Half-skyrmion phase

$L = 1.0[\text{fm}]$



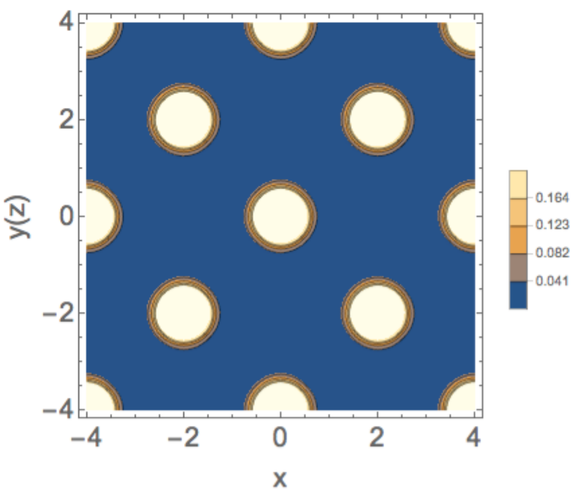
Skyrmion configuration is described by the baryon number density-distribution.

$$\rho_B = \frac{1}{24\pi^2}\epsilon^{0\nu\rho\sigma}\text{tr}\left[(\partial_\nu U\cdot U^\dagger)(\partial_\rho U\cdot U^\dagger)(\partial_\sigma U\cdot U^\dagger)\right]$$

Deformation of the skyrmion configuration

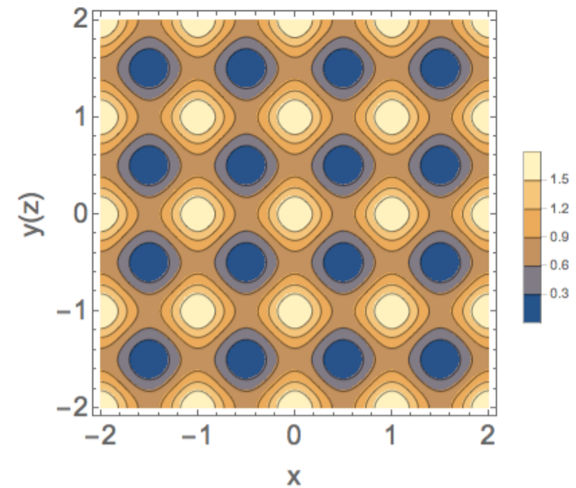
Skyrmion phase

$L = 2.0[\mathrm{fm}]$



Half-skyrmion phase

$L = 1.0[\mathrm{fm}]$



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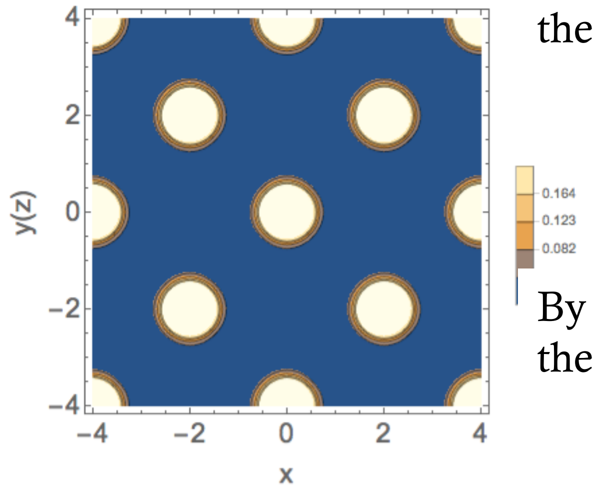
Induced charge by magnetic field

$$+\frac{1}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\left[ie(\partial_\nu A_\rho)Q_E(\partial_\sigma U\cdot U^\dagger + U^\dagger\partial_\sigma U) + ieA_\nu Q_E(\partial_\rho U\partial_\sigma U^\dagger - \partial_\rho U^\dagger\partial_\sigma U)\right]$$

Deformation of the skyrmion configuration

Skyrmion phase

$L = 2.0[\mathrm{fm}]$



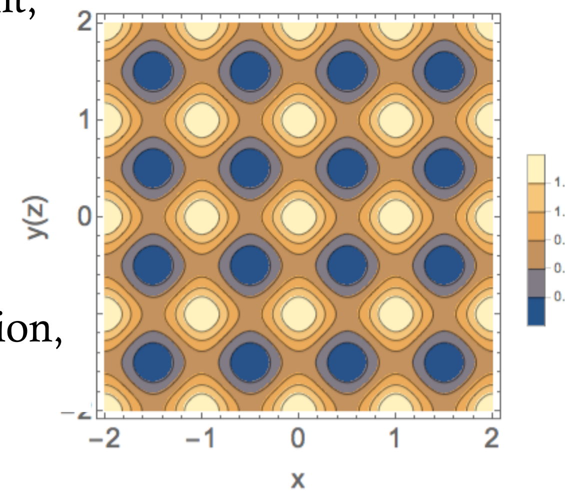
Even if the magnetic field is present, the baryon number is conserved.

$$N_B = \int_{\mathrm{cube}} d^3x \rho_B$$
$$= 4$$

By performing the spacial integration, the induced charge goes to zero.

Half-skyrmion phase

$L = 1.0[\mathrm{fm}]$



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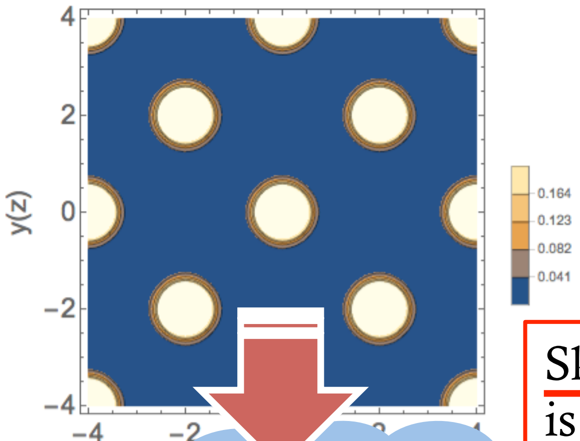
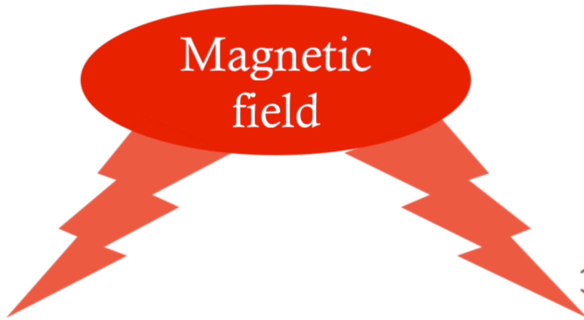
Deformation of the skyrmion configuration

Skyrmion phase

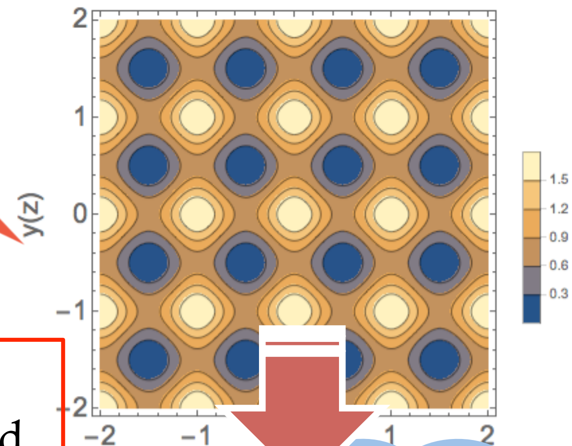
$L = 2.0[\mathrm{fm}]$

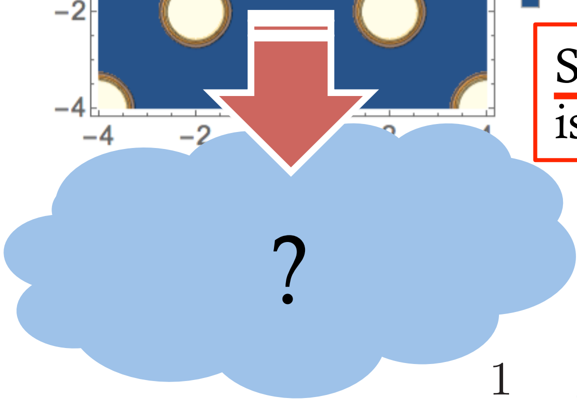




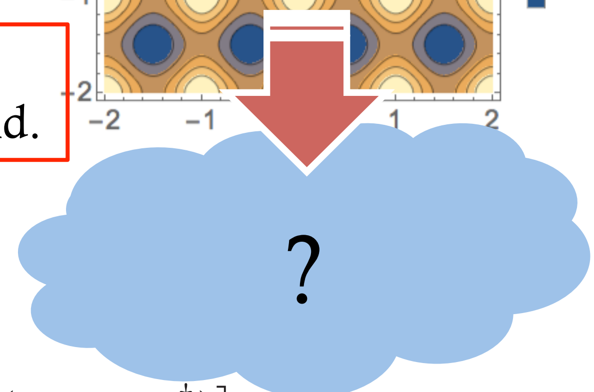
Half-skyrmion phase

$L = 1.0[\mathrm{fm}]$





Skyrmion configuration
is deformed by a magnetic field.



$$\rho_B = \frac{1}{24\pi^2}\epsilon^{0\nu\rho\sigma}\mathrm{tr}\left[(\partial_\nu U\cdot U^\dagger)(\partial_\rho U\cdot U^\dagger)(\partial_\sigma U\cdot U^\dagger)\right]$$

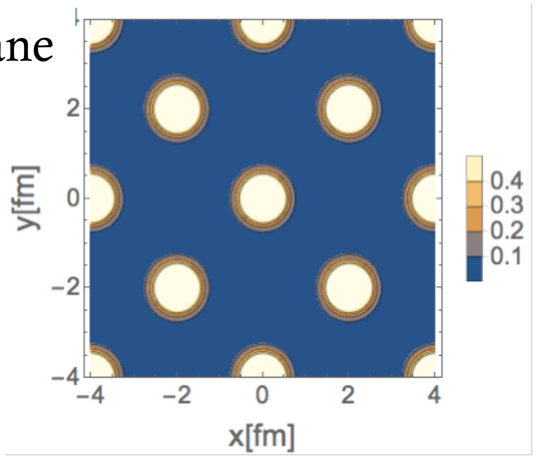
$$+\frac{1}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\left[ie(\partial_\nu A_\rho)Q_E(\partial_\sigma U\cdot U^\dagger + U^\dagger\partial_\sigma U)\right.$$
$$\left. + ieA_\nu Q_E(\partial_\rho U\partial_\sigma U^\dagger - \partial_\rho U^\dagger\partial_\sigma U)\right]$$

Deformation of the skyrmion configuration

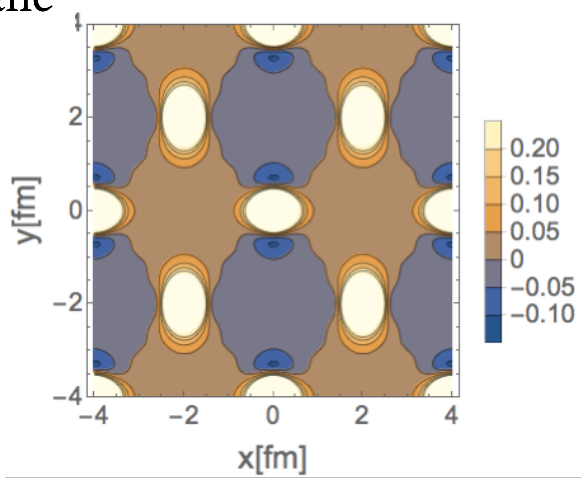
$\sqrt{eB} = 400[\mathrm{MeV}]$

$L = 2.0[\mathrm{fm}]$ Skyrmion phase

x-y plane



x-z plane



Deformation of the skyrmion configuration

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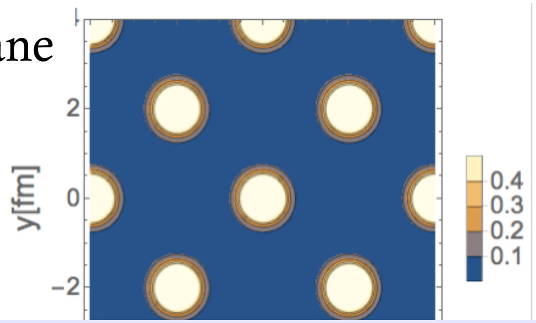
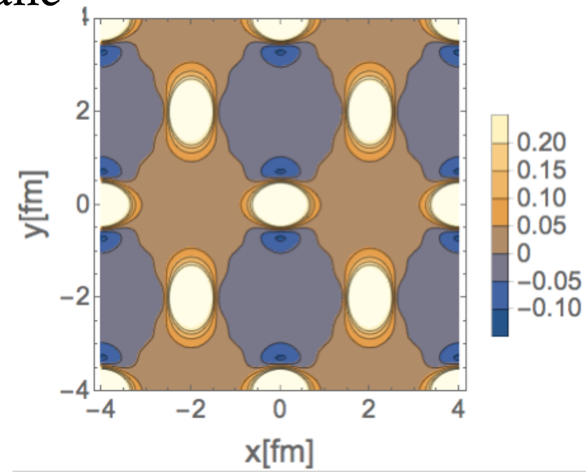
x-y plane



Single baryon shape is deformed to be an elliptic form.

x-z plane



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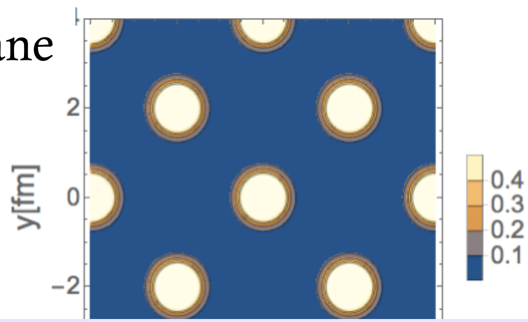
$L = 2.0[\mathrm{fm}]$ Skyrmion phase

x-y plane



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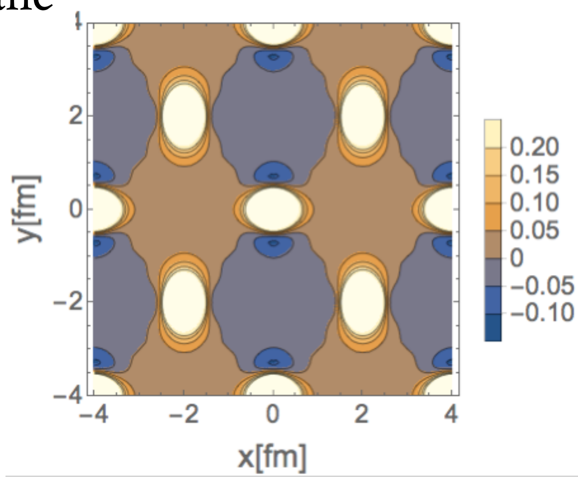
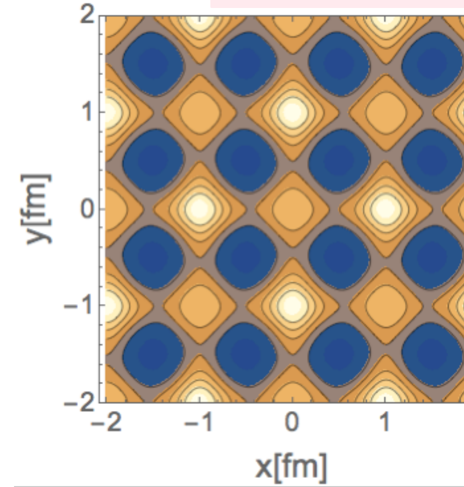


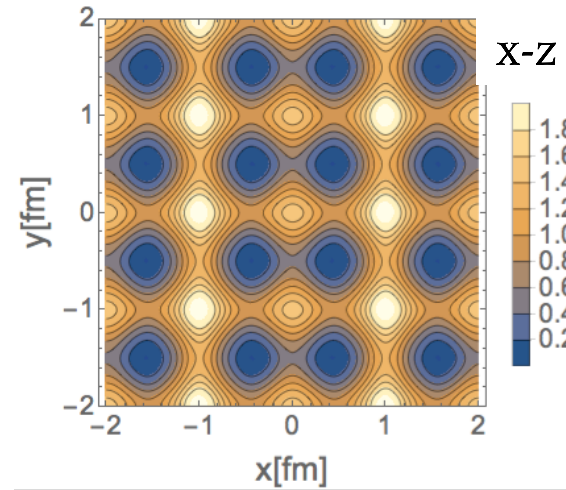
L_c=1.25[fm]

$L = 1.0[\mathrm{fm}]$ Half-skyrmion phase

x-y plane



x-z plane

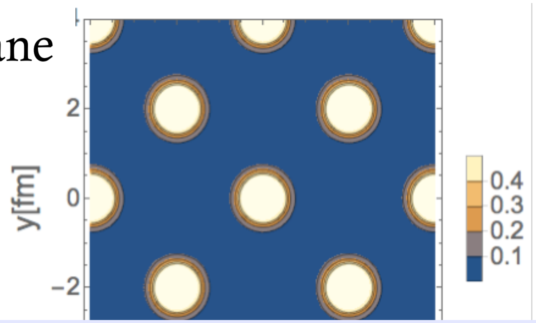


Deformation of the skyrmion configuration

$\sqrt{eB} = 400[\mathrm{MeV}]$

$L = 2.0[\mathrm{fm}]$ Skyrmion phase

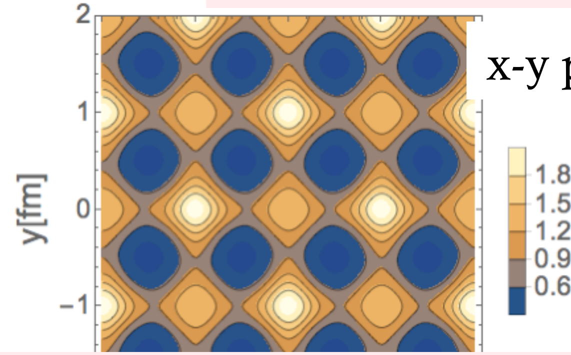
x-y plane



L_c=1.25[fm]

$L = 1.0[\mathrm{fm}]$ Half-skyrmion phase

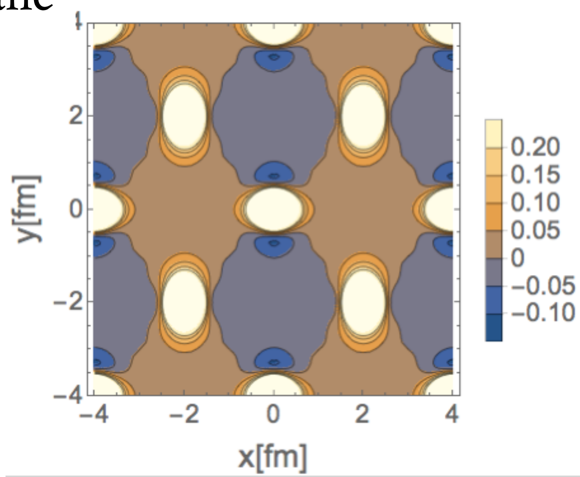
x-y plane



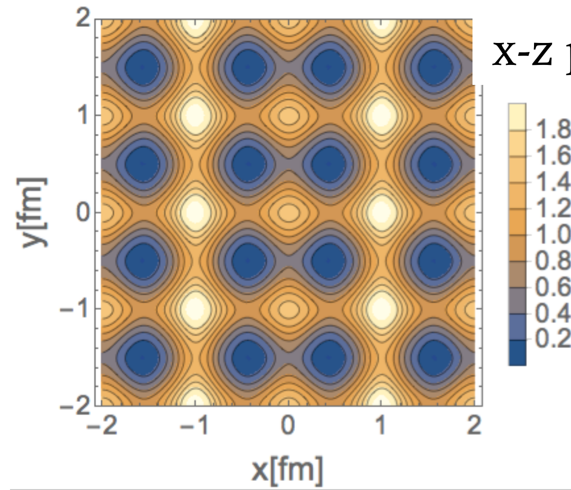
Single baryon shape is deformed to be an elliptic form.

CC structure is strongly effected by a magnetic field.

x-z plane



x-z plane

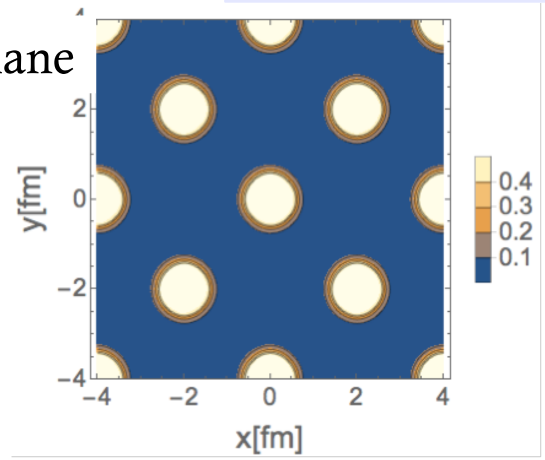


Deformation of the skyrmion configuration

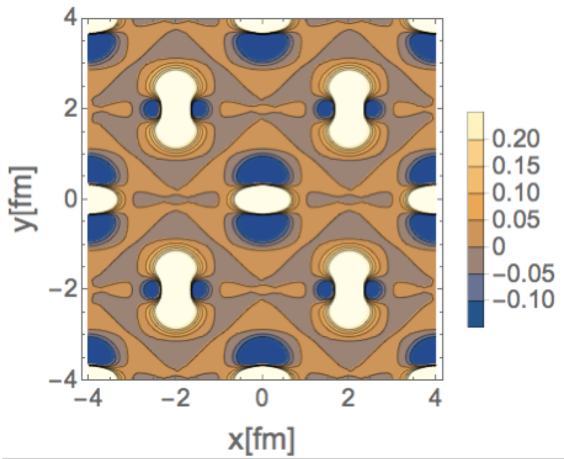
$\sqrt{eB} = 800[\mathrm{MeV}]$

$L = 2.0[\mathrm{fm}]$ Skyrmion phase

x-y plane



x-z plane

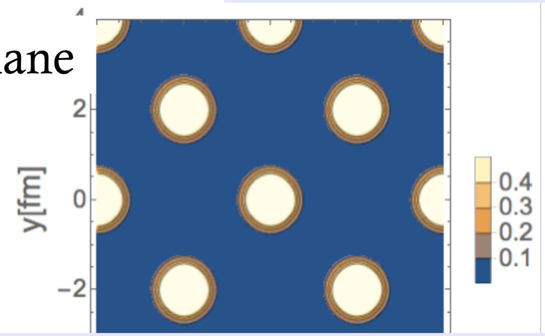


Deformation of the skyrmion configuration

$\sqrt{eB} = 800[\mathrm{MeV}]$

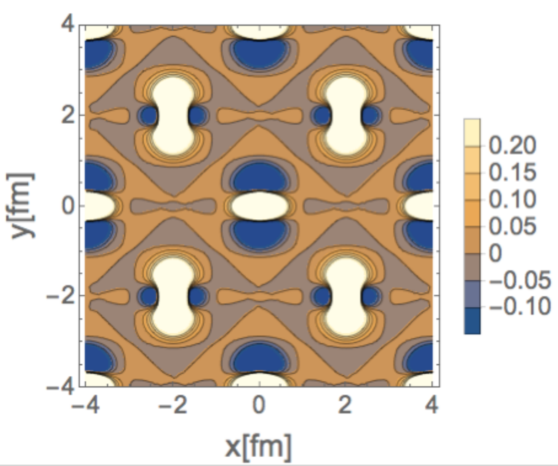
$L = 2.0[\mathrm{fm}]$ Skyrmion phase

x-y plane



Single baryon shape is deformed to be an elliptic form.

x-z plane



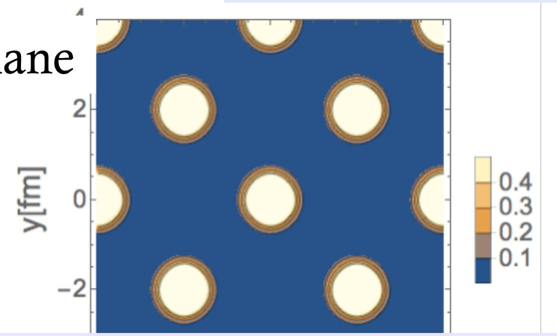
Deformation of the skyrmion configuration

$\sqrt{eB} = 800[\text{MeV}]$

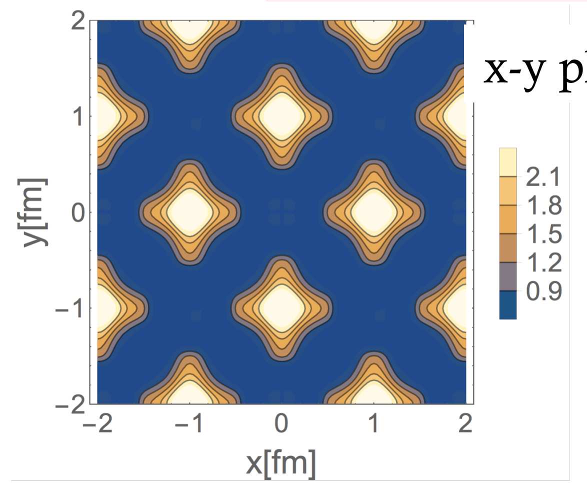
$L = 2.0[\text{fm}]$ Skyrmion phase

$L = 1.0[\text{fm}]$ Half-skyrmion phase

x-y plane



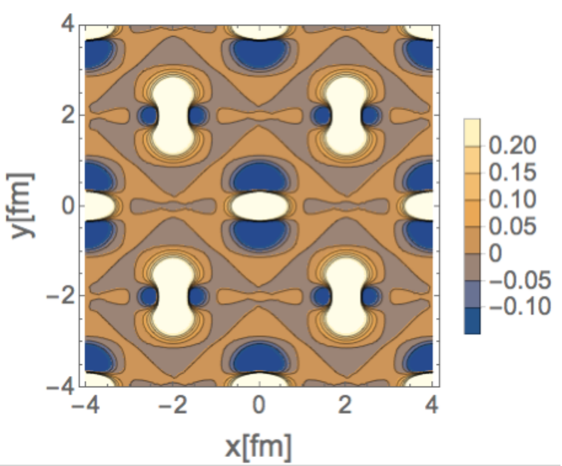
L_c=1.1[fm]

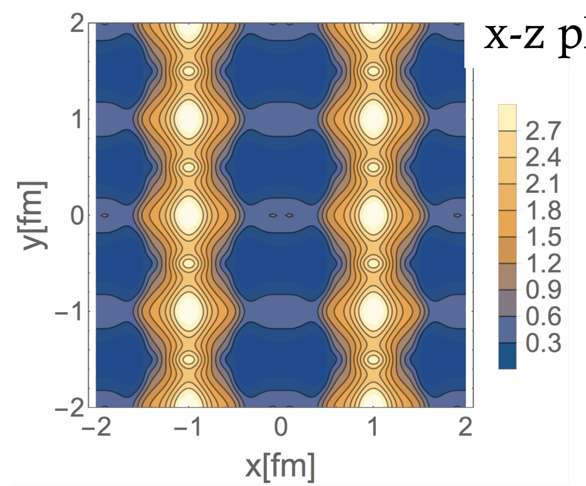


x-y plane

Single baryon shape is deformed to be an elliptic form.

x-z plane





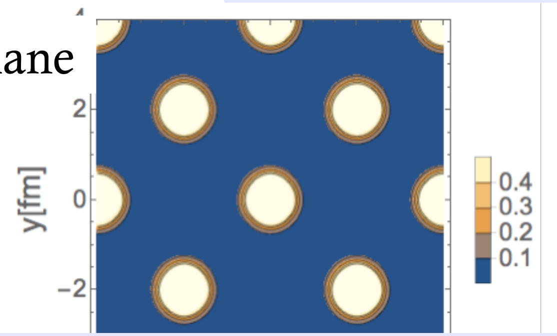
x-z plane

Deformation of the skyrmion configuration

$\sqrt{eB} = 800[\mathrm{MeV}]$

$L = 2.0[\mathrm{fm}]$ Skyrmion phase

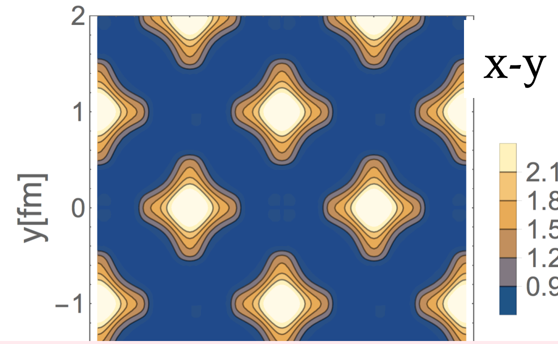
x-y plane



L_c=1.1[fm]

$L = 1.0[\mathrm{fm}]$ Half-skyrmion phase

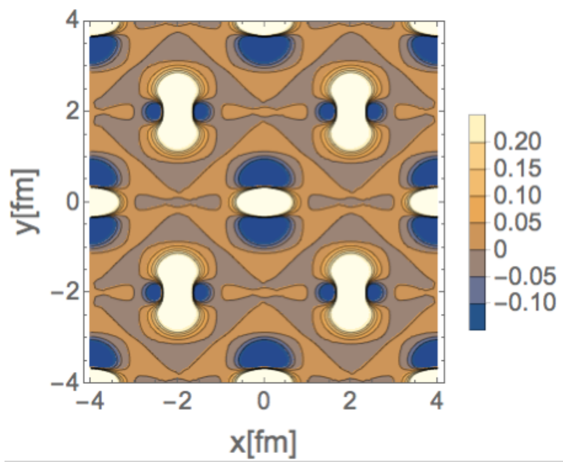
x-y plane



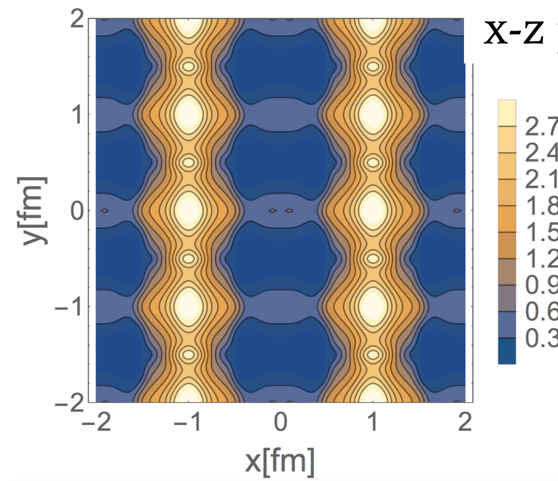
Single baryon shape is deformed to be an elliptic form.

CC structure gets completely lost for a large magnetic field.

x-z plane



x-z plane



Summary 1

Discussed the magnetic effect on the baryonic matter based on the skyrmion crystal approach.

- As magnetic field increases, baryon (skyrmion) energy increases for any crystal size.
- As the magnetic field increases, the topological transition point is shifted to a high density region and the value of $\langle \phi_0 \rangle$ gets larger.
  → Magnetic effect plays the role of a catalyzer for the topological transition.
- Magnetic field distorts the skyrmion crystal structure.
  - Low density region : Single baryon shape is deformed to be an elliptic form.
  - Highr density region : CC structure is strongly effected by a magnetic field. In particularly, CC structure gets completely lost for a large magnetic field.

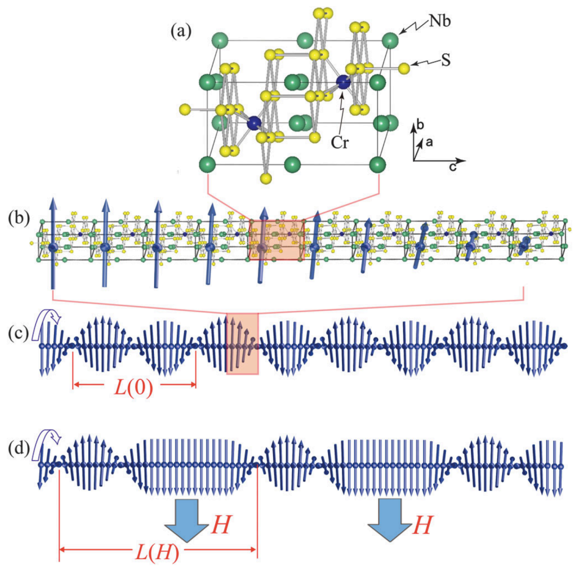
The results obtained in this study might be realized in the deep interior of compact stars.

Chiral soliton lattice in Skyrmion crystal

M. K., Y. L. Ma and S. Matsuzaki,
in preparation.

Chiral soliton lattice

The Chiral Soliton Lattice (CSL) is a periodic and parity-violating topological soliton, which have been studied in condensed-matter systems such as chiral magnets.



Y. Togawa, *et al.*, Phys. Rev. Lett. **108**, 107202 (2012).

- Chiral magnets has helical spin structure.
- The structure is changed by a magnetic field.

$$\mathcal{H} = -J\sum_{i,j} S_i \cdot S_j + D \cdot \sum_{i,j} S_i \times S_j - \tilde{H} \cdot \sum_i S_i$$

$$\vec{S}(z) = \sum_i \vec{S}_i \delta(z - z_i) = S\vec{n}(z)$$

$$\vec{n}(z) = \left(\cos\theta(z), \sin\theta(z)\cos\phi(z), \sin\theta(z)\cos\phi(z)\right)$$

Goes like Sine-Gordon equation, which has topological solution.

$$\mathcal{H} = \frac{JS^2}{a_o}\int_0^L dz\left[\frac{1}{2}(\partial_z\theta)^2 + \frac{1}{2}\sin^2\theta(\partial_z\phi)^2 - Q_0\sin^2\theta(\partial_z\phi) - m^2\sin\theta\cos\phi\right]$$

Spin structure is expressed as the topological solution.

Chiral soliton lattice

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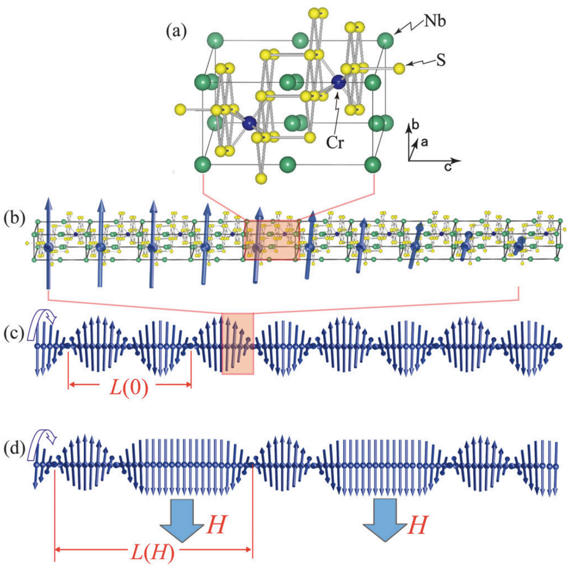


Y. Togawa, *et al.*,
Phys. Rev. Lett. **108**, 107202 (2012).

- Chiral magnets has helical spin structure.
- The structure is changed by a magnetic field.

Chiral soliton Lattice

$$\pi = 0$$
$$\phi = 2\arccos\left[-\mathrm{sn}\left(\frac{m_\pi z}{k}, k\right)\right]$$

Goes like Sine-Gordon equation, which has topological solution.

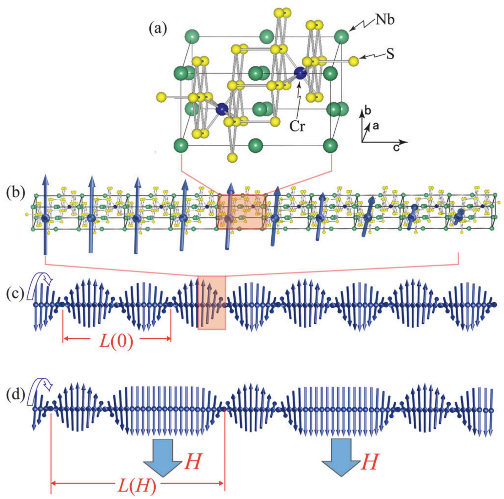
$$\mathcal{H} = \frac{JS^2}{a_o}\int_0^L dz\left[\frac{1}{2}(\partial_z\theta)^2 + \frac{1}{2}\sin^2\theta(\partial_z\phi)^2 - Q_0\sin^2\theta(\partial_z\phi) - m^2\sin\theta\cos\phi\right]$$

Spin structure is expressed as the topological solution.

Chiral soliton lattice

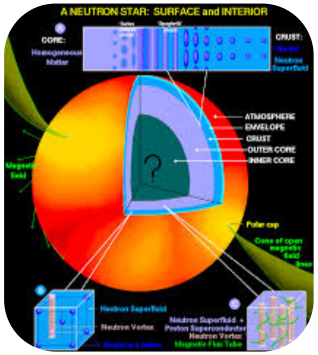
Given the fact that the CSL structure was discovered in condense matter physics, some attempts have been made to adapt the idea of CSL to QCD in the high energy physics.

Condense matter



- chiral magnet
- spin(electron)

QCD



- nuclear matter
- classical pion solution

The CSL effect on properties of baryonic matter based on the skyrmion crystal.
(the deformation of the baryonic matter structure has not fully been examined in the hadron physics.)

CSL in skyrmion crystal

Introduce the CSL into the chiral field U as the fluctuating of the neutral pion field.

Skyrmion crystal

Chiral soliton lattice

$$
\begin{aligned}
U &= \breve{u}\bar{U}\breve{u} \\
\breve{u} &= \exp\left[i\frac{\pi_3(z)\tau_3}{2}\right]
\end{aligned}
$$

*To match the conventional chiral-soliton lattice picture, we take the pion field as a one-dimensional configuration.

By substituting the chiral field with the CSL into the skyrme Lagrangian, the Lagrangian goes like

$$
\mathcal{L} = \mathcal{L}_{\rm mat} + \mathcal{L}_{\rm pion}
$$

$$
\mathcal{L}_{\rm mat} = \frac{f_\pi^2}{4}\mathrm{tr}[\partial_\mu \bar{U}\partial^\mu \bar{U}^\dagger] + \frac{1}{32g^2}\mathrm{tr}\left\{[\bar{U}^\dagger\partial_\mu\bar{U},\bar{U}^\dagger\partial_\nu\bar{U}][\bar{U}^\dagger\partial^\mu\bar{U},\bar{U}^\dagger\partial^\nu\bar{U}]\right\}
$$

$$
\mathcal{L}_{\rm pion} = -\frac{1}{2}A(\partial_z\pi_3)^2 + \left(f_\pi^2 m_\pi^2 B\right)\cos\left(\frac{\pi_3}{f_\pi}\right)
$$

(Sine-Gordon equation)

Chiral soliton lattice

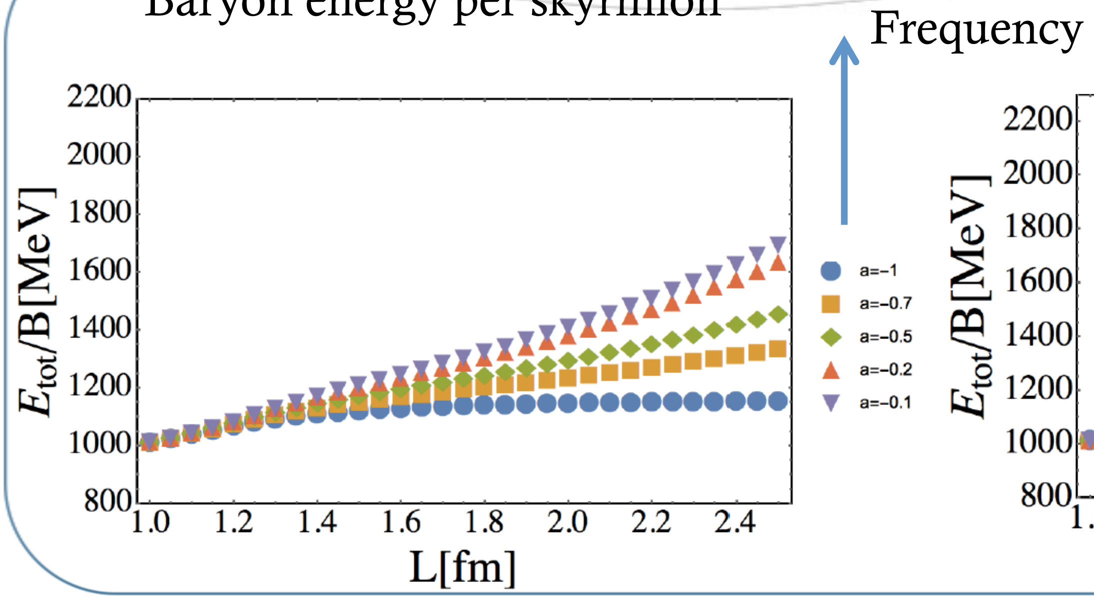
$$
\tilde{\pi}_3 = 2\tilde{f}_\pi^*\arccos\left[-\mathrm{sn}\left(\frac{m_\pi^* z}{k}, k\right)\right]
$$

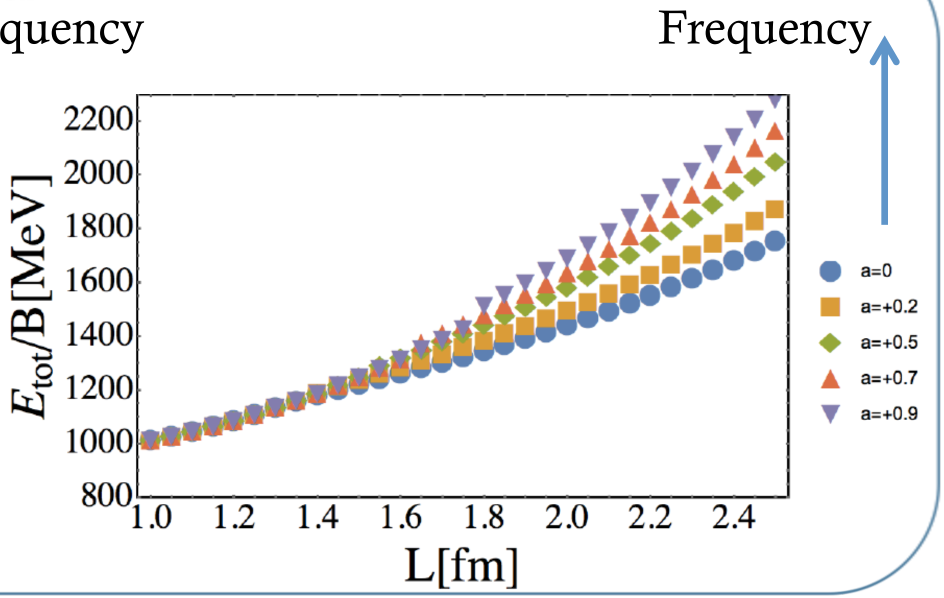
$$
\pi_1(S_1) = Z
$$

*k (0<k<1) is elliptic modulus

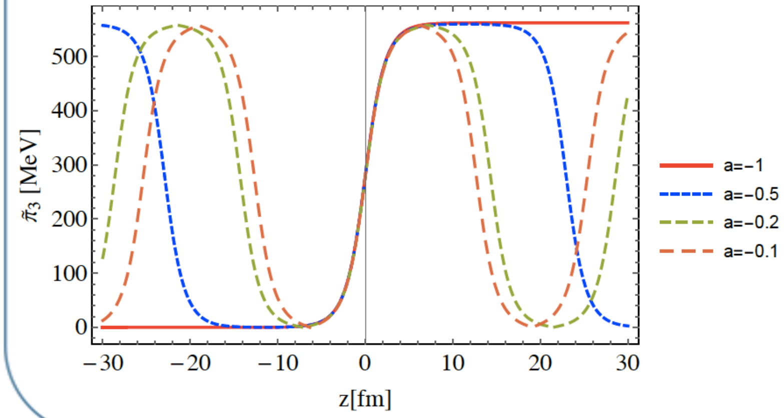
CSL in skyrmion crystal

Baryon energy per skyrmion





Profile of chiral soliton lattice (L=2.5[fm])



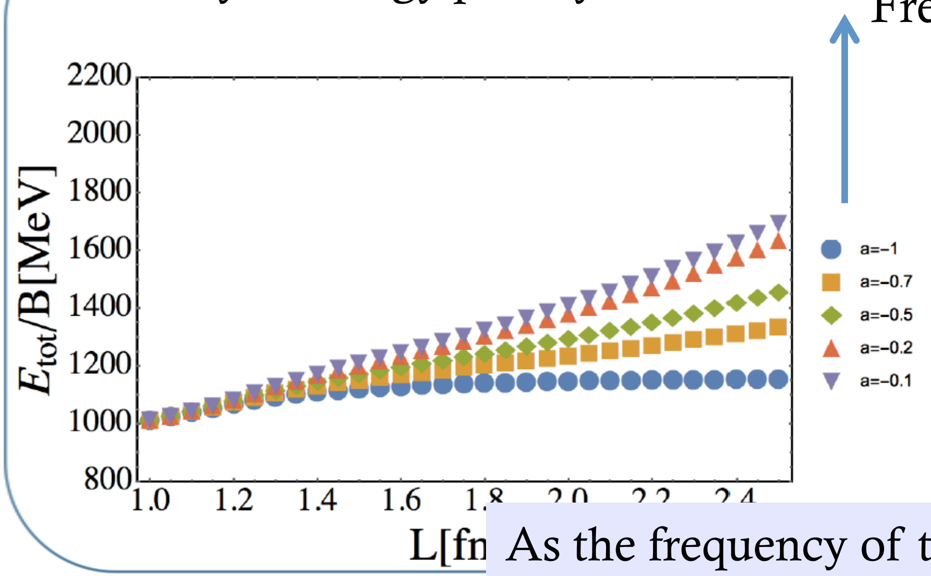
$$\tilde{\pi}_3 = 2\tilde{f}_\pi^* \arccos\left[-\mathrm{sn}\left(\frac{m_\pi^* z}{k}, k\right)\right]$$

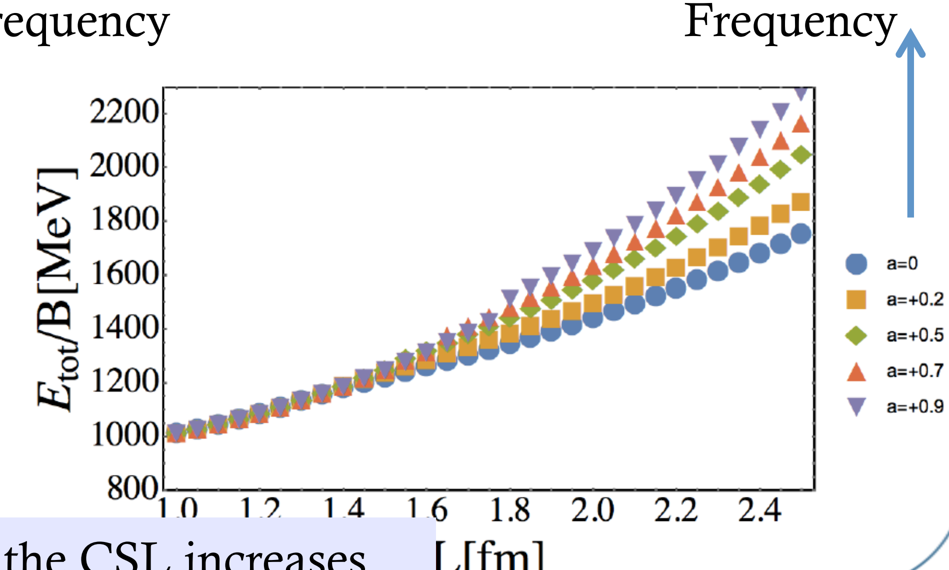
$$a = \left[4\frac{E(k)}{k^2 K(k)} + \left(1 - \frac{2}{k^2}\right)\right]$$

*As the parameter a increases,
the period of the CSL becomes short.

CSL in skyrmion crystal

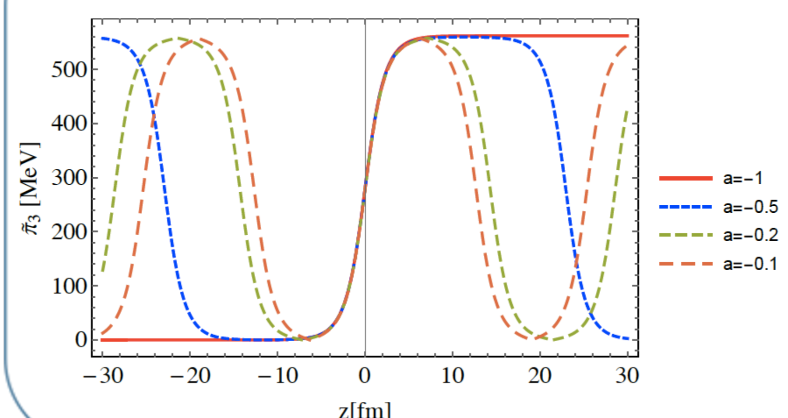
Baryon energy per skyrmion





As the frequency of the CSL increases,
the baryon (skyrmion) energy increases.

Profile of chiral soliton lattice (L=2.5[fm])

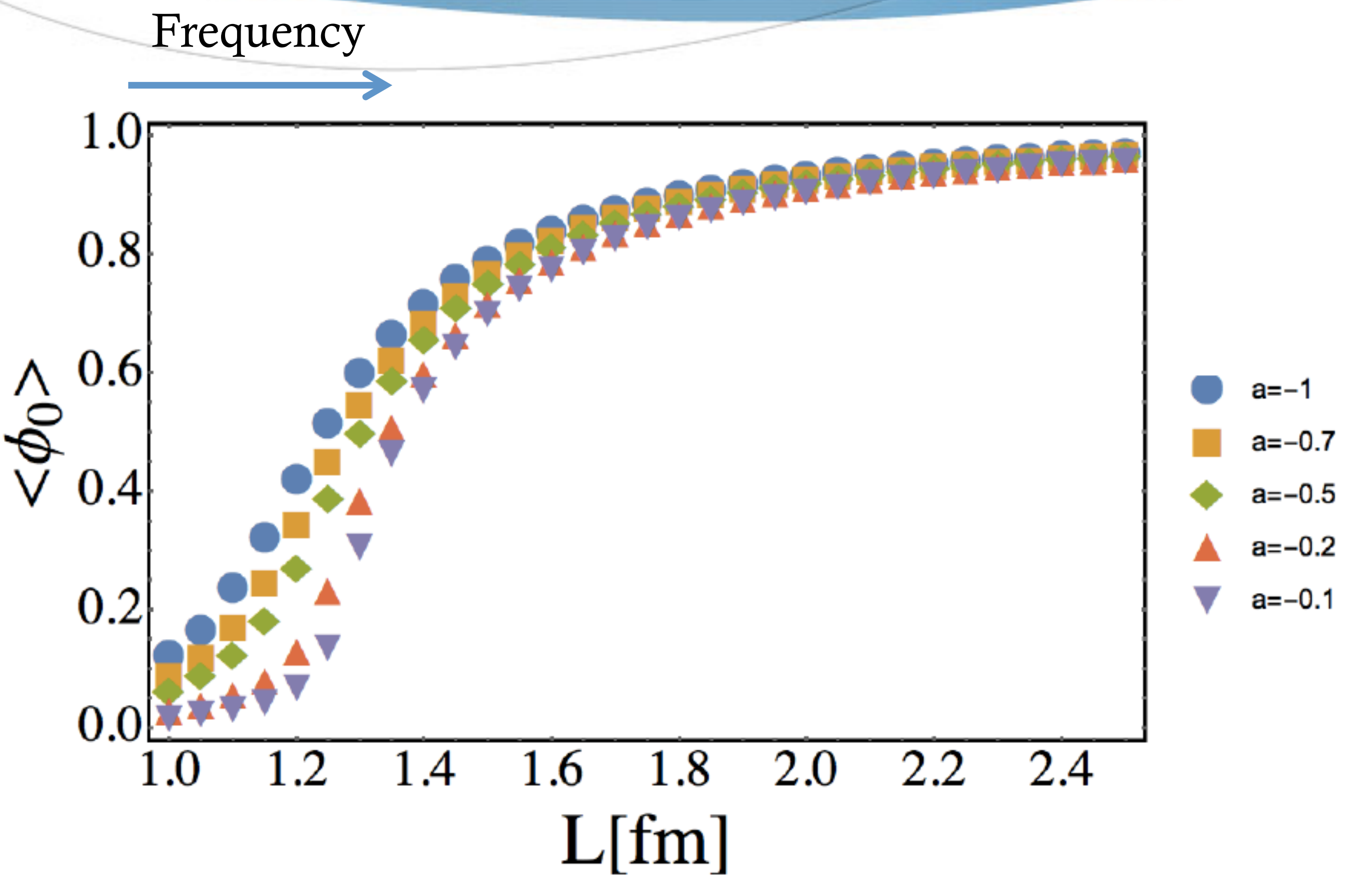


$$\tilde{\pi}_3 = 2 f_\pi^* \arccos\left[-\mathrm{sn}\left(\frac{m_\pi^* z}{k}, k\right)\right]$$

$$a = \left[4\frac{E(k)}{k^2 K(k)} + \left(1 - \frac{2}{k^2}\right)\right]$$

*As the parameter a increases,
the period of the CSL becomes short.

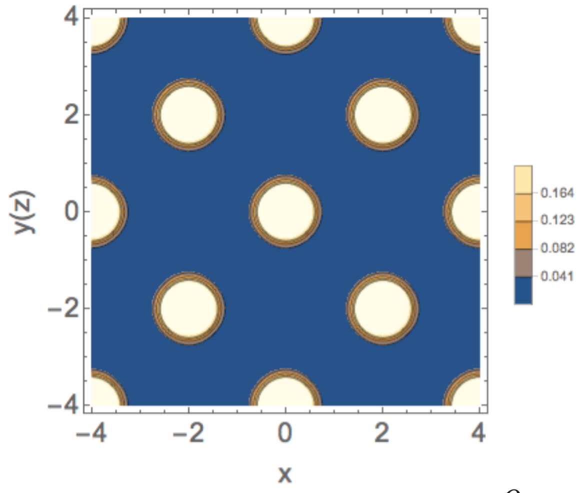
CSL effect on $\langle \phi_0 \rangle$



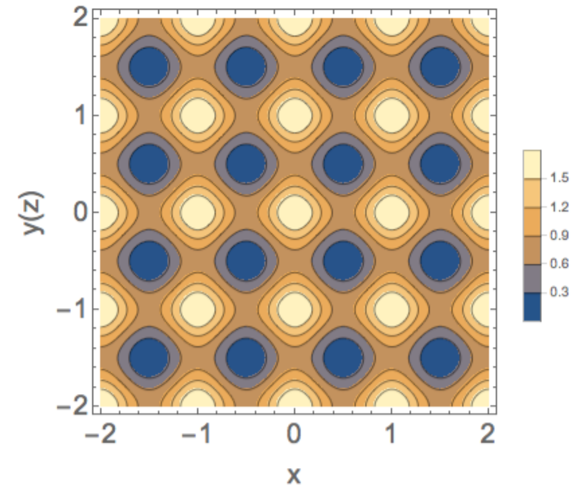
As the frequency of the CSL increases, the value of $\langle \phi_0 \rangle$ gets smaller for any crystal size.
→ the CSL causes an inverse catalysis for the topological phase transition.

Deformation of the skyrmion configuration

Low density region



High density region



Skyrmion configuration is deformed by the CSL.

$$\rho_B = \rho_W + \rho_{\text{ind}}$$

$$\rho_W = \frac{1}{24\pi^2}\epsilon^{0\nu\rho\sigma}\text{tr}\left[(\partial_\nu\bar{U}\cdot\bar{U}^\dagger)(\partial_\rho\bar{U}\cdot\bar{U}^\dagger)(\partial_\sigma\bar{U}\cdot\bar{U}^\dagger)\right]$$

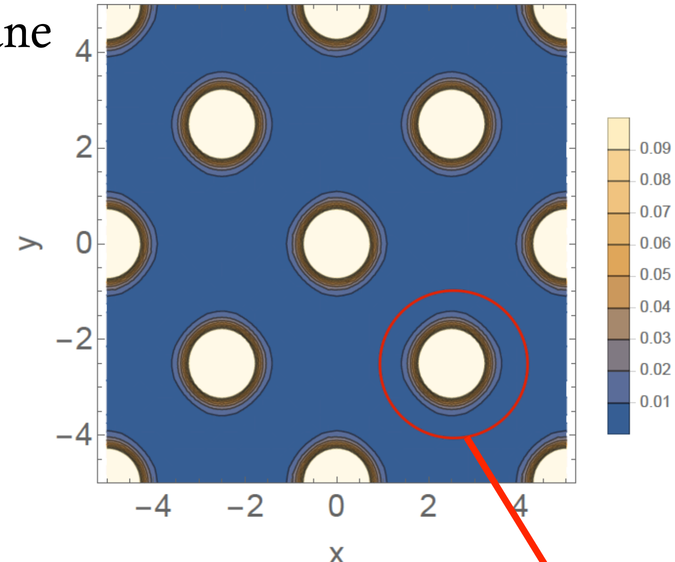
$$\rho_{\text{ind}} = \frac{1}{2\pi^2}\frac{\partial_z\tilde{\pi}(z)}{f_\pi^*}\Big(\partial_x\phi_1\partial_y\phi_2 - \partial_y\phi_1\partial_x\phi_2\Big),$$

Induced charge by the CSL

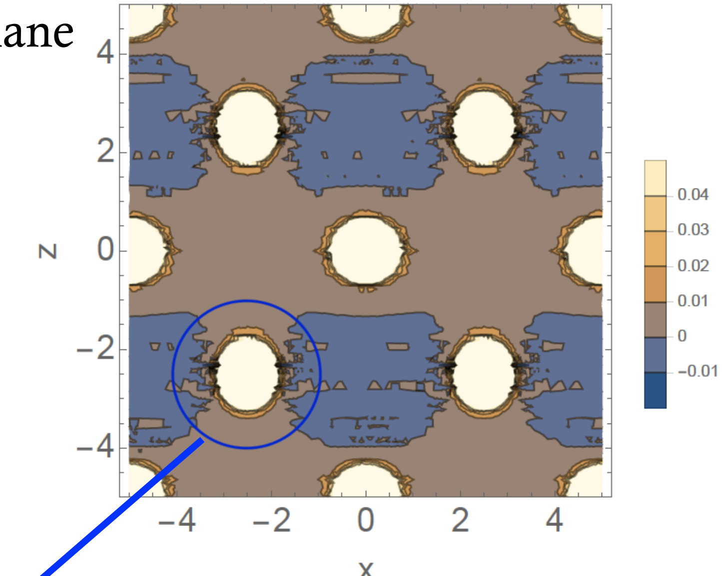
Deformation of the skyrmion configuration

Low density region (a= -0.1)

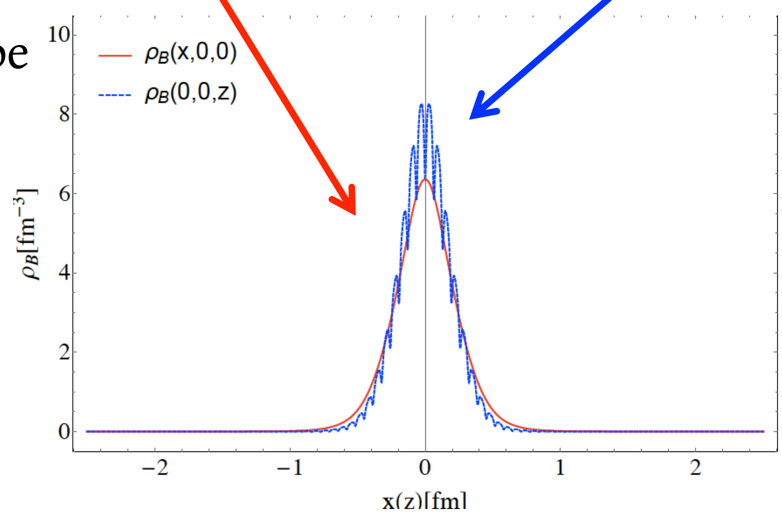
x-y plane



x-z plane



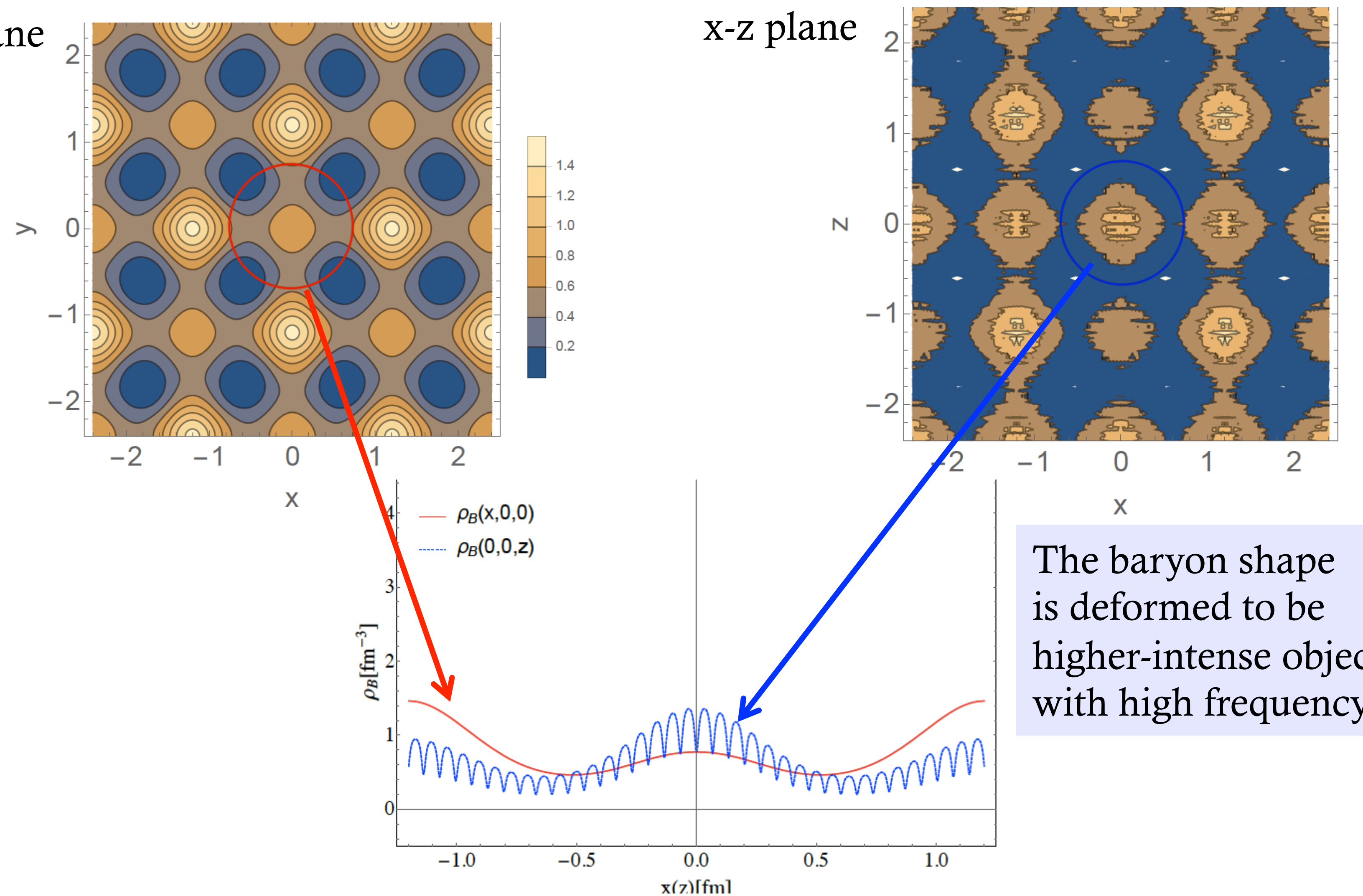
On single baryon shape



Single baryon shape is deformed to be higher-intense objects with high frequency.

Deformation of the skyrmion configuration

High density region



The baryon shape is deformed to be higher-intense objects with high frequency.

Summary 2

Discussed the chiral soliton effect on the skyrmion crystal approach.

・As the frequency of the CSL increases, the skyrmion energy increases for any crystal size.

・As the frequency of the CSL increases, the value of $\langle \phi_0 \rangle$ gets smaller for any crystal size.
 → the CSL causes an inverse catalysis for the topological phase transition.

・CSL makes the single-baryon shape deformed to be highly oscillating.

Those findings might be relevant to deeper understanding in condensed-matter systems as well as in compact stars.

Thank you very much!

back up

normal nuclear density $n_0 = 0.17/[\mathrm{fm}^3]$ corresponding to $L \sim 1.43[\mathrm{fm}]$

$L = 1.3\mathrm{fm}$ corresponds to $1.3n_0$

$L = 1.135\mathrm{fm}$ corresponds to $2n_0$

Skyrmion crystal

On the construction of the fcc crystal

Expand the chiral field by Fourier series.

$$U = \phi_0 + i\tau_i\phi_i$$

$$\phi_a = \frac{\bar{\phi}_a}{\sqrt{\bar{\phi}_b\bar{\phi}_b}}$$

$$\begin{aligned}
\bar{\phi}_0 &= \sum_{a,b,c} \bar{\beta}_{abc} \cos(a\pi x/L)\cos(b\pi y/L)\cos(c\pi z/L) \\
\bar{\phi}_1 &= \sum_{h,k,l} \bar{\alpha}^{(1)}_{hkl} \sin(h\pi x/L)\cos(k\pi y/L)\cos(l\pi z/L) \\
\bar{\phi}_2 &= \sum_{h,k,l} \bar{\alpha}^{(2)}_{hkl} \cos(l\pi x/L)\sin(h\pi y/L)\cos(k\pi z/L) \\
\bar{\phi}_3 &= \sum_{h,k,l} \bar{\alpha}^{(3)}_{hkl} \cos(k\pi x/L)\cos(l\pi y/L)\sin(h\pi z/L)
\end{aligned}$$

面心立方格子(fcc)

A fcc crystal has symmetries for crystal structure.

reflection symmetry

$$(x,y,z) \to (-x,y,z): \quad (\sigma,\pi_1,\pi_2,\pi_3) \to (\sigma,-\pi_1,\pi_2,\pi_3)$$

three fold symmetry

$$(x,y,z) \to (z,x,y); \quad (\sigma,\pi_1,\pi_2,\pi_3) \to (\sigma,\pi_3,\pi_1,\pi_2)$$

four fold symmetry

$$(x,y,z) \to (x,z,-y); \quad (\sigma,\pi_1,\pi_2,\pi_3) \to (\sigma,\pi_1,\pi_3,-\pi_2)$$

translation symmetry

$$(x,y,z) \to (x+L,y+L,z): \quad (\sigma,\pi_1,\pi_2,\pi_3) \to (\sigma,-\pi_1,-\pi_2,\pi_3)$$

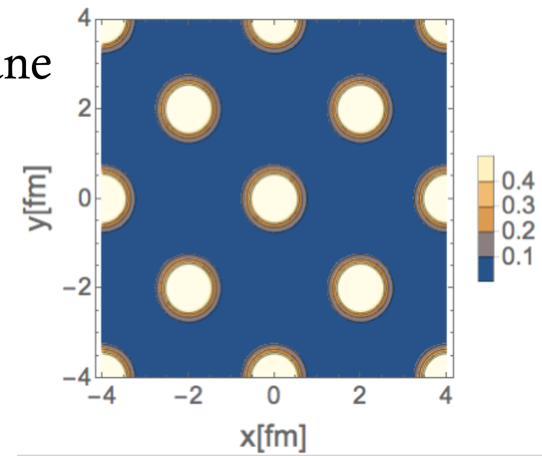
Baryon charge in magnetic field

Baryon number density: $\rho_B(x,y,z) = \rho_W(x,y,z) + \tilde{\rho}_{eB}(x,y,z),$

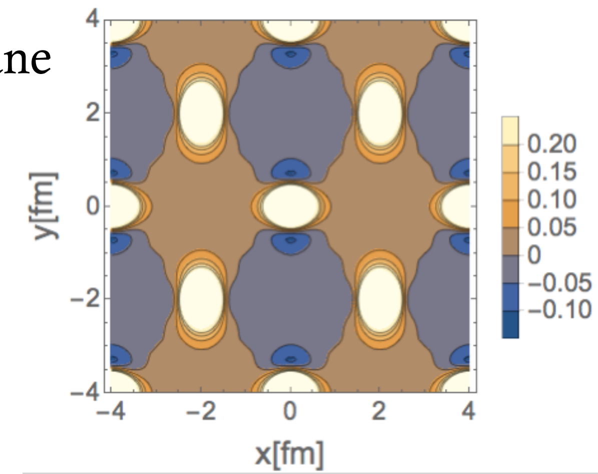
$$\begin{aligned} \text{skyrmion phase(FCC): } \rho_W(x,y,z) &= \rho_W(x+L,y+L,z) = \rho_W(x+L,y,z+L) = \rho_W(x,y+L,z+L), \\ \text{half-skyrmion phase(CC): } \rho_W(x,y,z) &= \rho_W(x+L,y,z) = \rho_W(x,y+L,z) = \rho_W(x,y,z+L). \end{aligned}$$

$$\begin{aligned} \text{skyrmion phase(FCC): } \tilde{\rho}_{eB}(x,y,z) &= \tilde{\rho}_{eB}(x+L,y+L,z) = -\tilde{\rho}_{eB}(x+L,y,z+L) \\ &= -\tilde{\rho}_{eB}(x,y+L,z+L), \\ \text{half-skyrmion phase(CC): } \tilde{\rho}_{eB}(x,y,z) &= -\tilde{\rho}_{eB}(x+L,y,z) = -\tilde{\rho}_{eB}(x,y+L,z) = \tilde{\rho}_{eB}(x,y,z+L). \end{aligned}$$

x-y plane



x-z plane

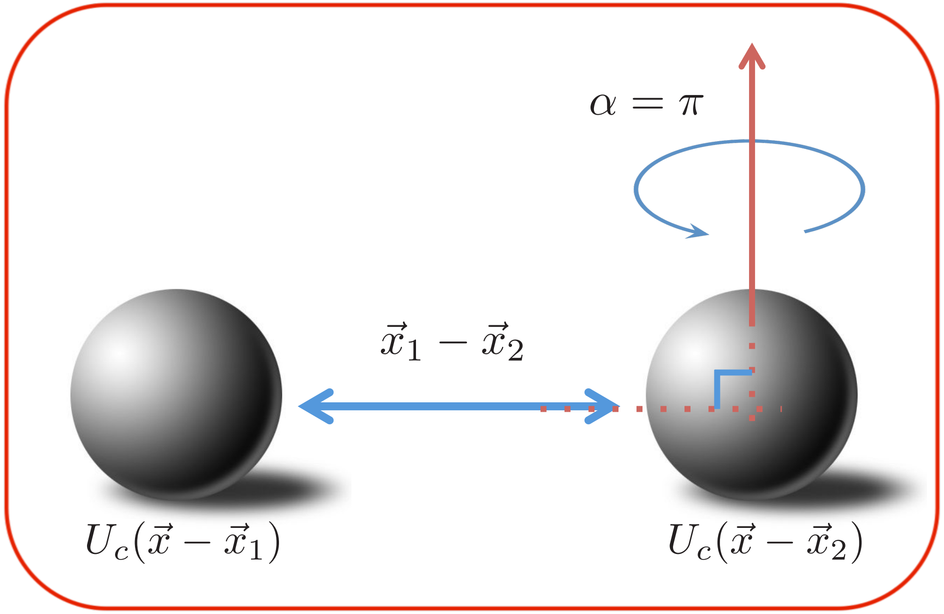


Skyrmion-Skyrmion interaction

Product two hedgehog skyrmions with a relative rotation in spin-isospin space.

$$U_{cc}(\vec{x}, \vec{x}_1, \vec{x}_2) = U_c(\vec{x} + \vec{x}_1) C(\vec{\alpha}) U_c(\vec{x} + \vec{x}_2) C^\dagger(\vec{\alpha}) \qquad C(\alpha) = \exp(i\vec{\alpha} \cdot \vec{\tau}/2)$$

To get the most attractive potential, the pair skyrmions should be arranged in such a way that they should mutually rotate in the isospin space by angle about the axis perpendicular to the line joining them.



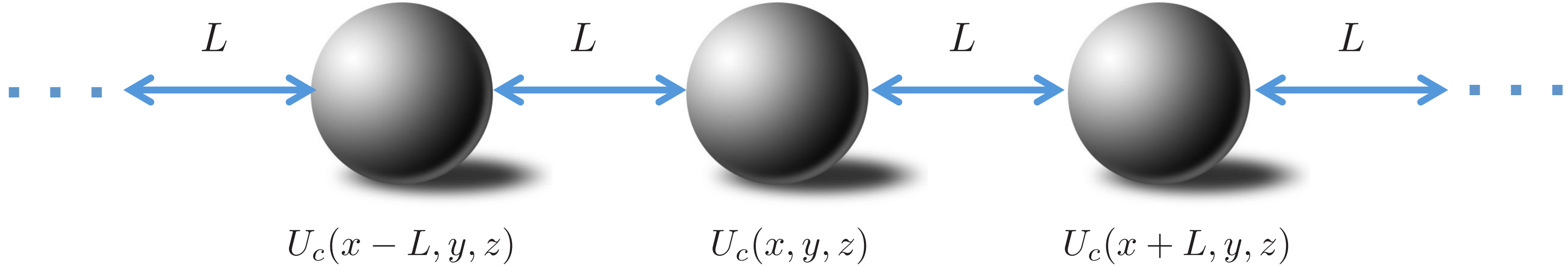
For instance

$$\vec{x}_1 = (0,0,0) \quad \vec{x}_2 = (L,0,0)$$

$$U_{cc}(x,y,z) = U_c(x,y,z) e^{i\pi\tau_y/2} U_c(x+L,y,z) e^{-i\pi\tau_y/2}$$

$$U_c(x,y,z) = e^{i\pi\tau_y/2} U_c(x+L,y,z) e^{-i\pi\tau_y/2}$$

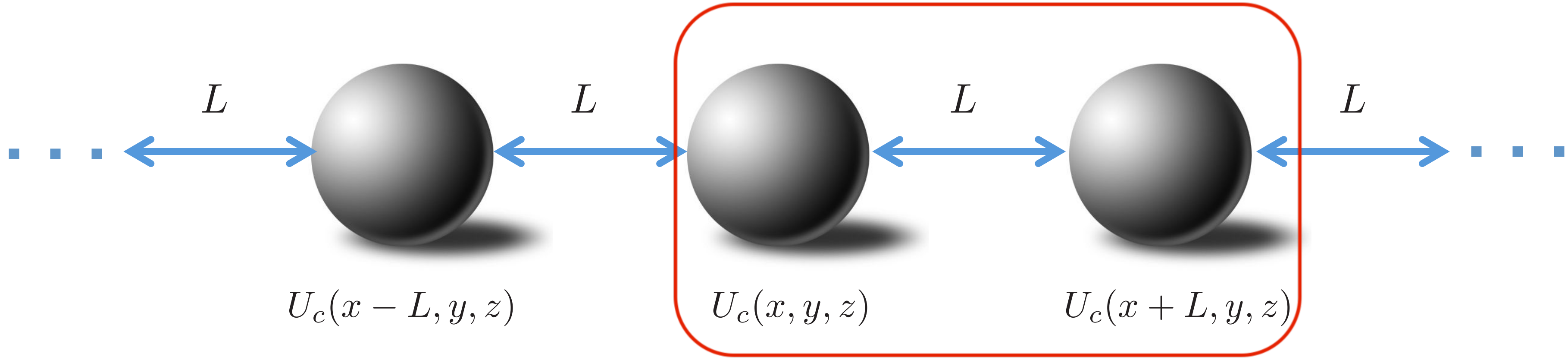
Skyrmion-Skyrmion interaction



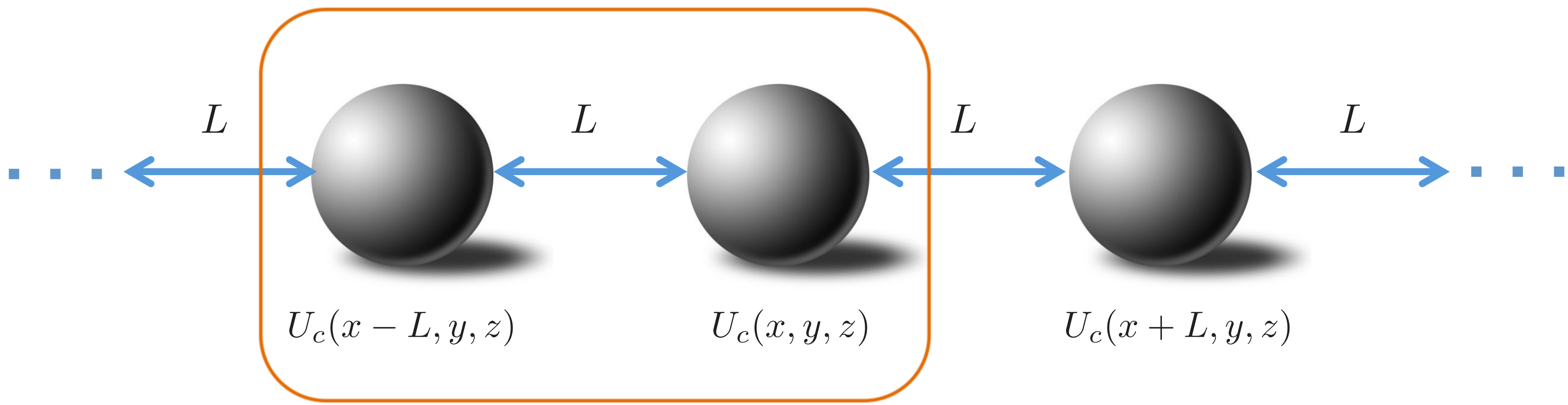
Skyrmion-Skyrmion interaction

$$\vec{x}_1 = (0,0,0) \quad \vec{x}_2 = (L,0,0)$$

$$U_{cc}(x,y,z) = U_c(x,y,z)e^{i\pi\tau_y/2}U_c(x+L,y,z)e^{-i\pi\tau_y/2}$$



Skyrmion-Skyrmion interaction



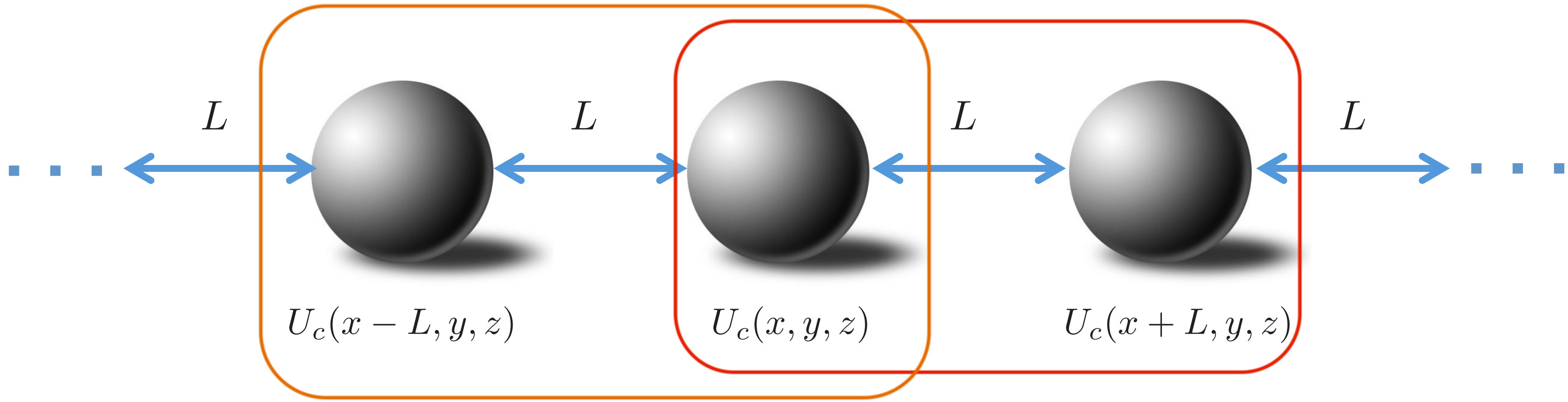
$$\vec{x}_1 = (0,0,0) \quad \vec{x}_2 = (L,0,0)$$

$$U_{cc}(x,y,z) = U_c(x,y,z)e^{i\pi\tau_y/2}U_c(x+L,y,z)e^{-i\pi\tau_y/2}$$

Skyrmion-Skyrmion interaction

$$\vec{x}_1 = (0,0,0) \quad \vec{x}_2 = (L,0,0)$$

$$U_{cc}(x,y,z) = U_c(x,y,z)e^{i\pi\tau_y/2}U_c(x+L,y,z)e^{-i\pi\tau_y/2}$$



$$\vec{x}_1 = (0,0,0) \quad \vec{x}_2 = (L,0,0)$$

$$U_{cc}(x,y,z) = U_c(x,y,z)e^{i\pi\tau_y/2}U_c(x+L,y,z)e^{-i\pi\tau_y/2}$$

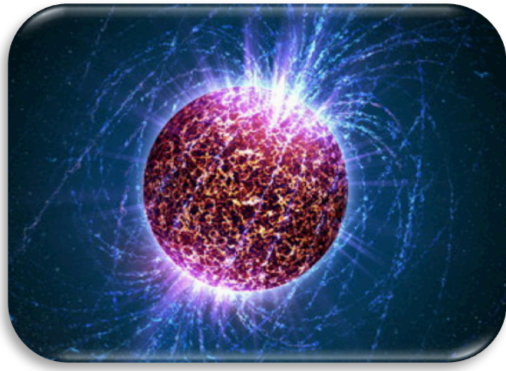
$$\Rightarrow \quad U_c(x,y,z) = e^{i\pi\tau_y/2}U_c(x+L,y,z)e^{-i\pi\tau_y/2}$$

Pion domain wall

Pion domain wall is the classical solution of pion field.

$$\mathcal{L} = \frac{1}{2} f_\pi^2 (\partial_\mu \pi)^2 - m_\pi^2 f_\pi^2 \cos \pi \qquad \pi = 4 f_\pi \arctan[\exp(m_\pi z)] \qquad \pi_1(S_1) = Z$$

$\rightarrow$ The magnetization is induced by the neutral pion domain wall.



The emergent magnetic field would reach QCD scale,
which suggests that the quantum anomaly can be a microscopic origin of the magnetars (highly magnetized neutron stars).

M. Eto, K. Hashimoto and T. Hatsuda, Phys. Rev. D 88, 081701 (2013)

Little is known about the pion domain wall effects on the nuclear matter in terms of crystal structure.

Few studies have focused on (this point).

By applying pion domain wall to skyrmion crystal approach,
what changes occur in crystal properties?