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Properties of Nuclei in Chiral Soliton Model (from nucleons to nuclei and neutron stars)

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Motivation (in wider sense)

What is important in the understanding process of surrounding physical reality,

- **a general global understanding or**
- **a deep and directed analysis (an exact and precise description of experimental data)?**

What are the fundamental degrees of freedom (according to reductionism principle),

- **bosons or**
- **fermions?**

Beyond the standard model (e.g. Unification problem) …

- **It seems, it is not possible to put "an unified picture of all fundamental interactions" into the 4-dimensional space-time.**
- **It seems, a gravity is spreading to extra dimensions - e.g. String theory in many dimensional space.**

Basic ingredients and important concepts are

- **strings, brains (fundamental degrees of freedom) have topological nature;**
- **duality (e.g. AdS-CFT correspondence);**
- **compactification of an extra dimension (infinite tower of Kaluza-Klein modes).**

Motivation (in narrower sense)

Strong interactions at low energies are mainly governed by two phenomena:

- **confinement;**
- **spontaneous chiral symmetry breaking.**

Vacuum structure of nonabelian-gauge theory is nontrivial… It allows a variety of topological structures: instantons, monopoles… They can explain

- **spontaneous chiral symmetry breaking (e.g. instantons) and**
- **may be, confinement also (stability of topological structures).**

Direct application of QCD is not yet possible (except lattice QCD). Possible ways to proceed:

- **large Nc approach (theories with a variety of mesonic ingredients) <=> similarity with KK theory;**
- **the topological solitons (fundamental ingredients are bosons) <=> automatically takes into account confinement of quarks;**
- **fermions having a composite structure appear after the quantization (spin has an essentially quantum nature).**

In other words our aim:

- **To construct a framework — "The modelling of nuclear and hadron physics".**

Strategy

How to construct a theoretical framework (model)?

Our guiding principles are

- **simplicity (easy to analyse, transparent, etc…) <=> e.g. small number of terms in the Lagrangian;**
- **relation to phenomenology in an attractive way — as much as possible peculiarities of strong interactions should be taken into account using as less as possible number of parameters;**
- **universality <=> applicability to**
  - **hadron structure and spectrum studies (from light to heavy sector);**
  - **analysis of NN interactions;**
  - **nuclear many body problems <=> nucleonic systems (finite nuclei) and infinite nuclear matter properties (EOS);**
  - **relation to mesonic atoms;**
  - **hadron structure changes in nuclear environment;**
  - **extreme density phenomena (e.g. neutron stars);**
  - **etc.**

Two possible ways (in the sense of choice):

- **to construct a completely new approach;**
- **a bit fresh look to old ideas (e.g. putting a bit more phenomenological information).**

Content

- Topological models (describe structure-full hadrons)
- Medium modifications (interactions with surrounding environment)
- Nucleon in nuclear matter (structure changes due to surrounding environment)
- Nuclear matter (takes into account structure changes of the constituents)
- Neutron stars (extrapolations to high density regions)
- Nucleon in finite nuclei (non-spherical deformations)
- Properties of finite nuclei (example: mirror nuclei)
- Consistency (difference) with (from) other approaches
- Summary and Outlook

Topological Models

Topological models

Why topological models?

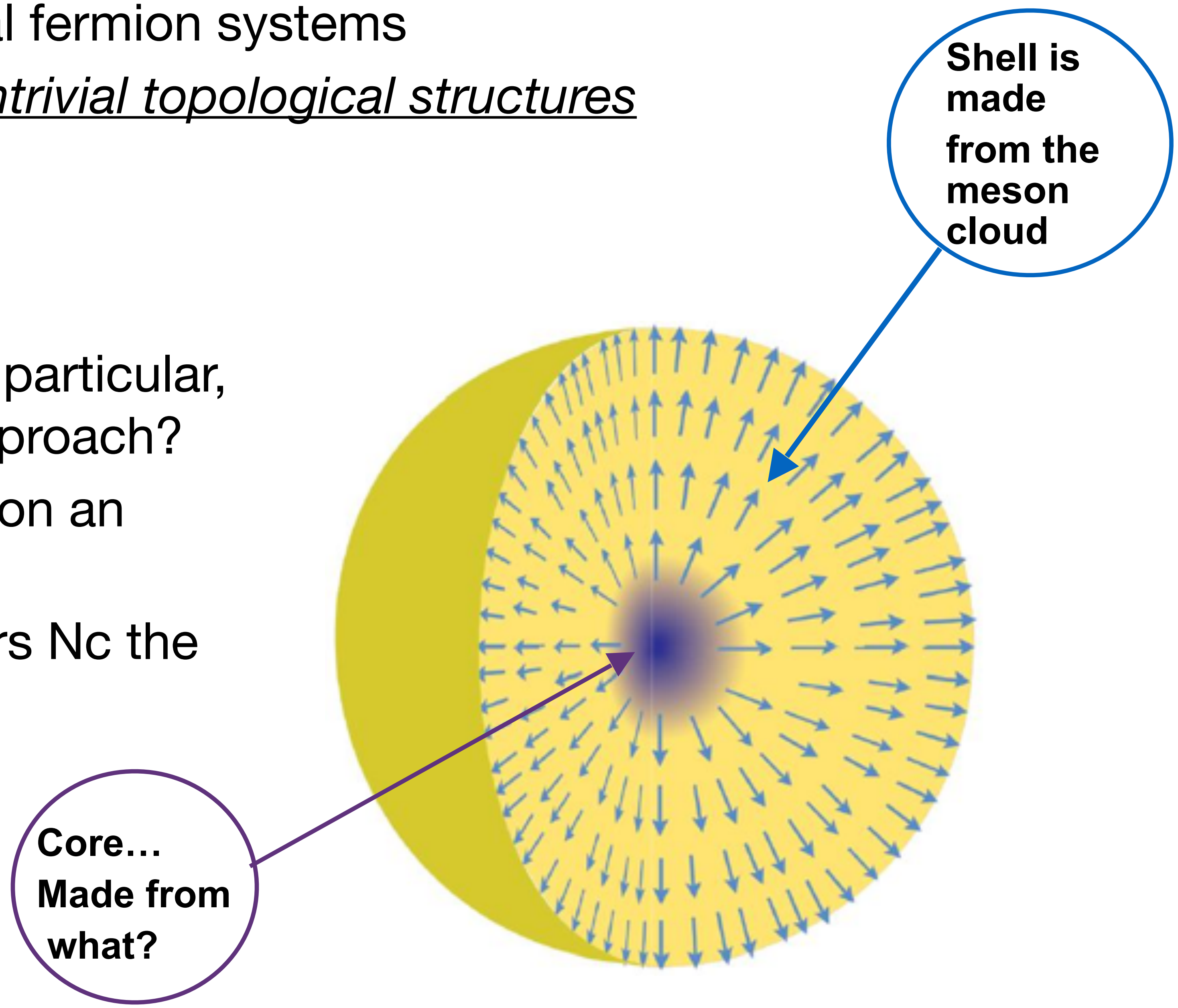
At fundamental level we may have

- fermions -> then bosons are trivial fermion systems
- bosons -> then fermions are *nontrivial topological structures*

Structure

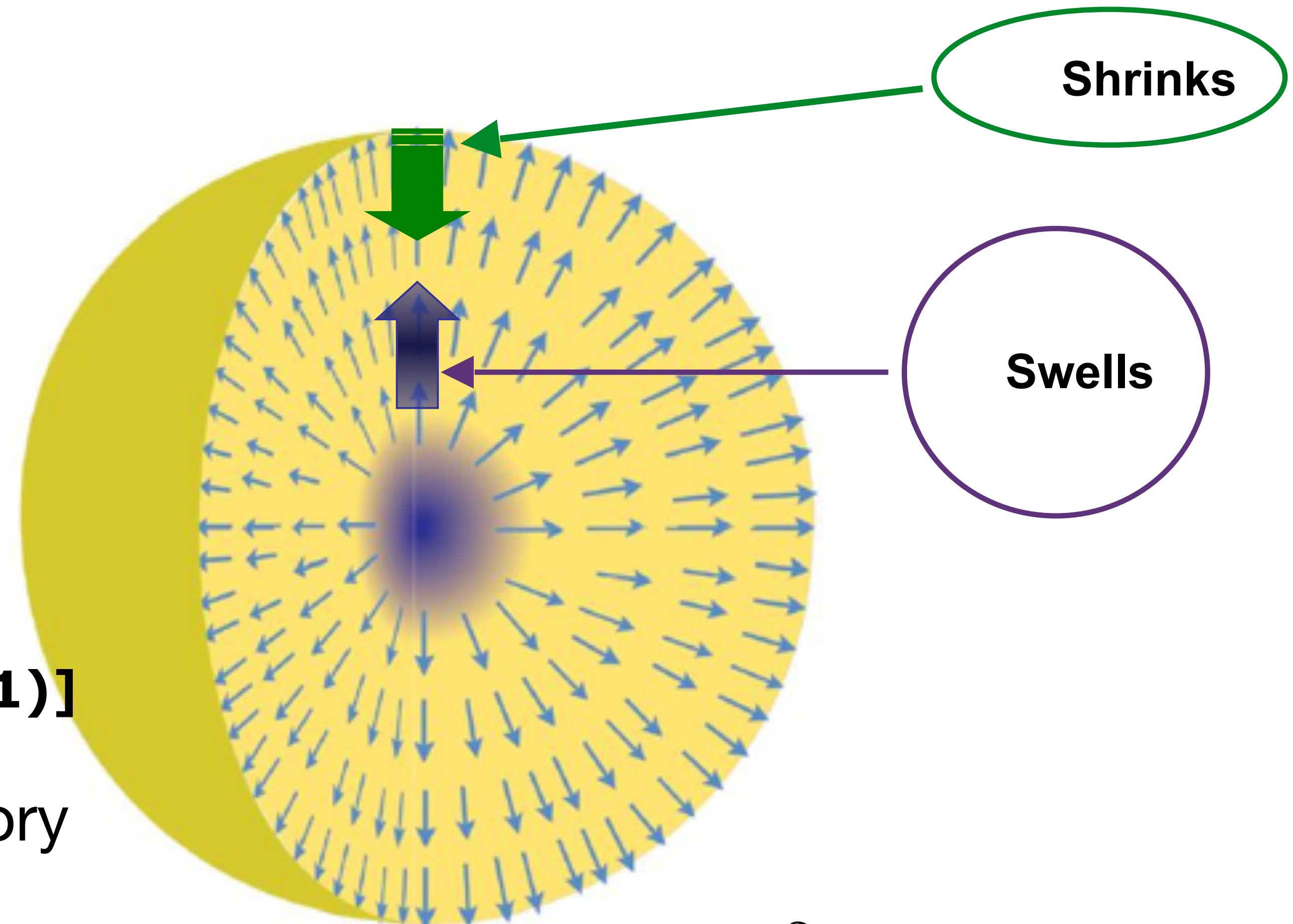
From what is made a nucleon and, in particular, its core in a starting boson picture approach?

- The structure treatment depends on an energy scale
- At the limit of large number colours Nc the core still has the mesonic content



Topological models

Stabilization mechanism



- Soliton has the finite size and the finite energy
- One needs at least two counter terms in the effective (mesonic) Lagrangian

Prototype: Skyrme model

[T.H.R. Skyrme, Pros.Roy.Soc.Lond. A260 (1961)]

- Nonlinear chiral effective meson (pionic) theory

$$\mathcal{L} = \frac{F_\pi^2}{16}\,\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) - \frac{1}{16e^2}\,\mathrm{Tr}\,\left[U^\dagger\partial_\mu U, U^\dagger\partial_\nu U\right]^2$$

Shrinking term — Swelling term

- *Hedgehog* solution (nontrivial mapping)

$$U = \exp\left\{\frac{i\bar{\tau}\,\bar{\pi}}{2F_\pi}\right\} = \exp\{i\bar{\tau}\,\bar{n}F(r)\}$$

Topological models

The free space Lagrangian (which was widely in use)
[G.S.Adkins et al. Nucl.Phys. B228 (1983)]

$$\mathcal{L} = \frac{F_\pi^2}{16}\,\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) - \frac{1}{16e^2}\,\mathrm{Tr}\,\left[U^\dagger\partial_\mu U, U^\dagger\partial_\nu U\right]^2 + \frac{F_\pi^2 m_\pi^2}{16}\,\mathrm{Tr}\left(U + U^\dagger - 2\right)$$

- Nontrivial structure: topologically stable solitons with the corresponding conserved topological number (baryon number) ***A***
- Nucleon is quantized state of the classical soliton-skyrmion which rotates in the ordinary and internal spaces

$$U = \exp\{i\bar{\tau}\,\bar{\pi}/2F_\pi\} = \exp\{i\bar{\tau}\,\bar{n}F(r)\}$$

$$B^\mu = \frac{1}{24\pi^2}\varepsilon^{\mu\nu\alpha\beta}Tr(L_\nu L_\alpha L_\beta) \qquad L_\alpha = U^+\partial_\alpha U$$

$$A = \int d^3 r B^0$$

$$H = M_{cl} + \frac{\bar{S}^2}{2I} = M_{cl} + \frac{\bar{T}^2}{2I},$$

$$|S = T, s, t >= (-1)^{t+T}\sqrt{2T+1}D^{S=T}_{-t,s}(A)$$

Topological models

Topological charge density

$$B^{\mu} = \frac{1}{24\pi^2}\varepsilon^{\mu\nu\alpha\beta} Tr(L_{\nu} L_{\alpha} L_{\beta}) \qquad L_{\alpha} = U^{+}\partial_{\alpha} U \qquad A = \int d^3 r B^0$$

- Symmetric hedgehog (hedgehog ansatz)

$$U = \exp\{i\vec{\tau}\vec{\pi}/2F_{\pi}\} = \exp\{i\vec{\tau}\vec{n}F(r)\}$$

- One of possible deformed hedgehogs (rational map ansatz)

$$U = \exp\{i\vec{\tau}\vec{\pi}/2F_{\pi}\} = \exp\{i\vec{\tau}\vec{N}(\theta,\varphi)F(r)\}$$

- Generalized hedgehog

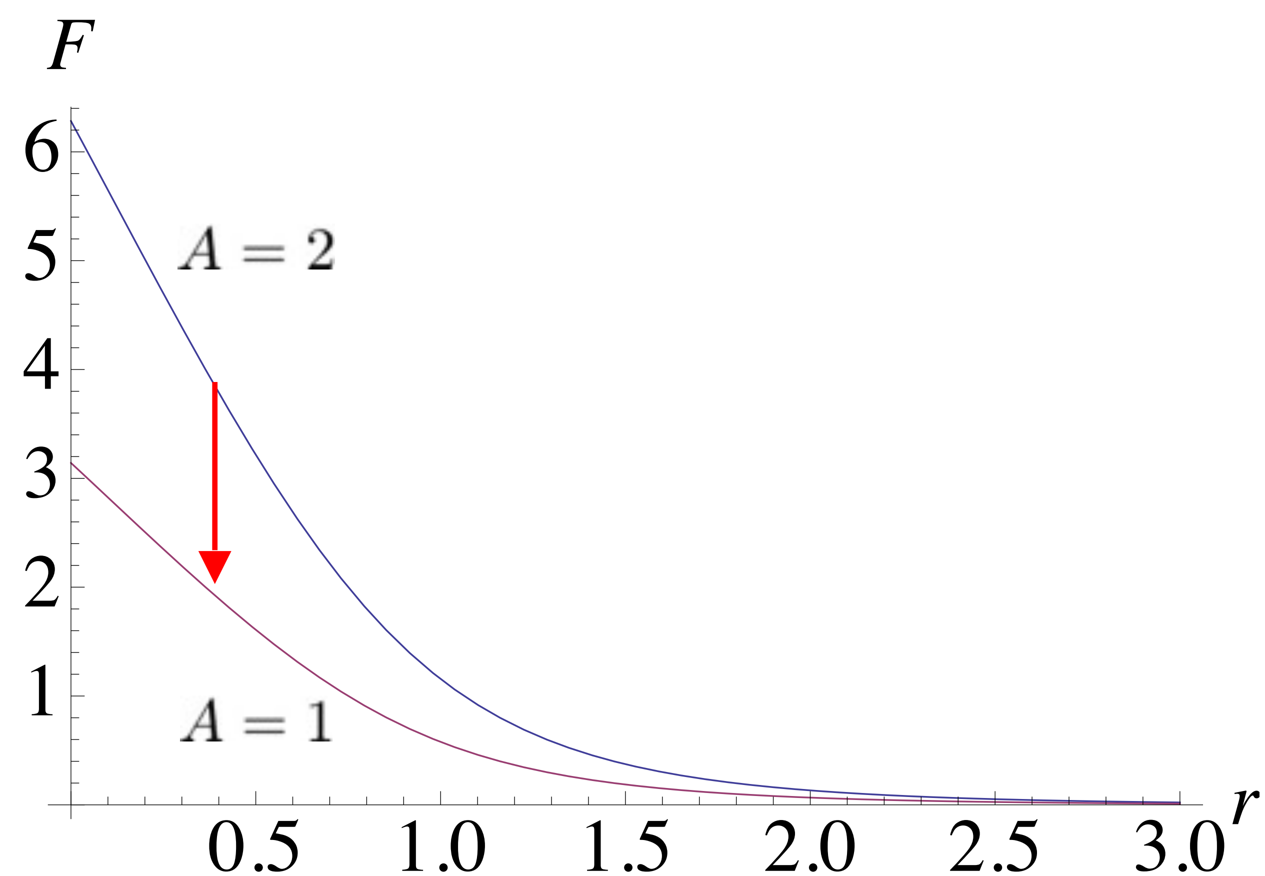
$$U = \exp\{i\vec{\tau}\vec{\pi}/2F_{\pi}\} = \exp\{i\vec{\tau}\vec{N}(r,\theta,\varphi)F(r,\theta,\varphi)\}$$

Many skyrmionic sector (symmetric skyrmion)

$$A = \int d^3 r B_0 = -\frac{1}{\pi}[F(\infty) - F(0)], \qquad F(0) = A\pi, \quad F(\infty) = 0$$

- Classically "unstable"
- Mathematically, there is no smooth transition between topologically distinct sectors
- Physically, it will be an infinite potential barrier between the corresponding solutions, e.g. energy density

$$E_4(r) \sim \frac{\sin^2 F(r)}{r^4}$$



Medium Modifications

Medium modifications

What happens in the nuclear medium?

The possible medium effects

- Deformations (swelling or shrinking, multipole deformations) of nucleons
- Characteristic changes in: effective mass, charge distributions, all possible form factors
- NN interactions may change
- etc.

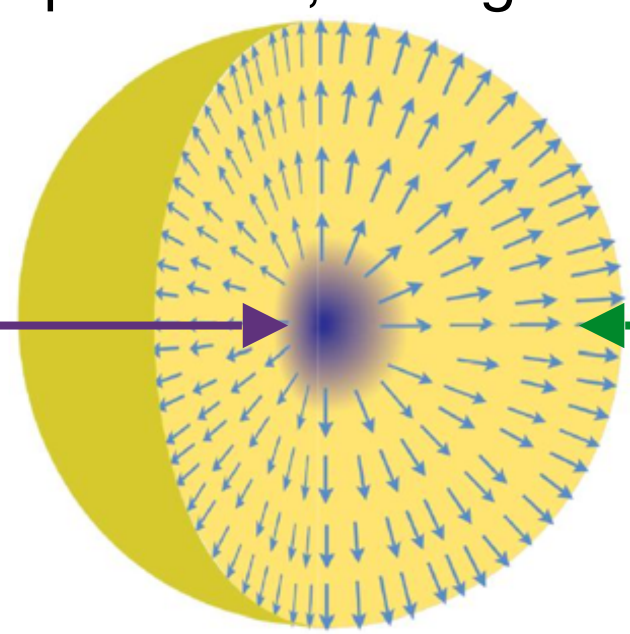
One should be able to describe all those phenomena

Soliton in the nuclear medium (phenomenological way)

- Outer shell modifications (informations from pionic atoms)
- Inner core modifications, in particular, at large densities (nuclear matter properties)

Inner core modifications in the nuclear medium may be related to:

- **vector meson properties in the nuclear medium**
- **nuclear matter properties at saturation density**

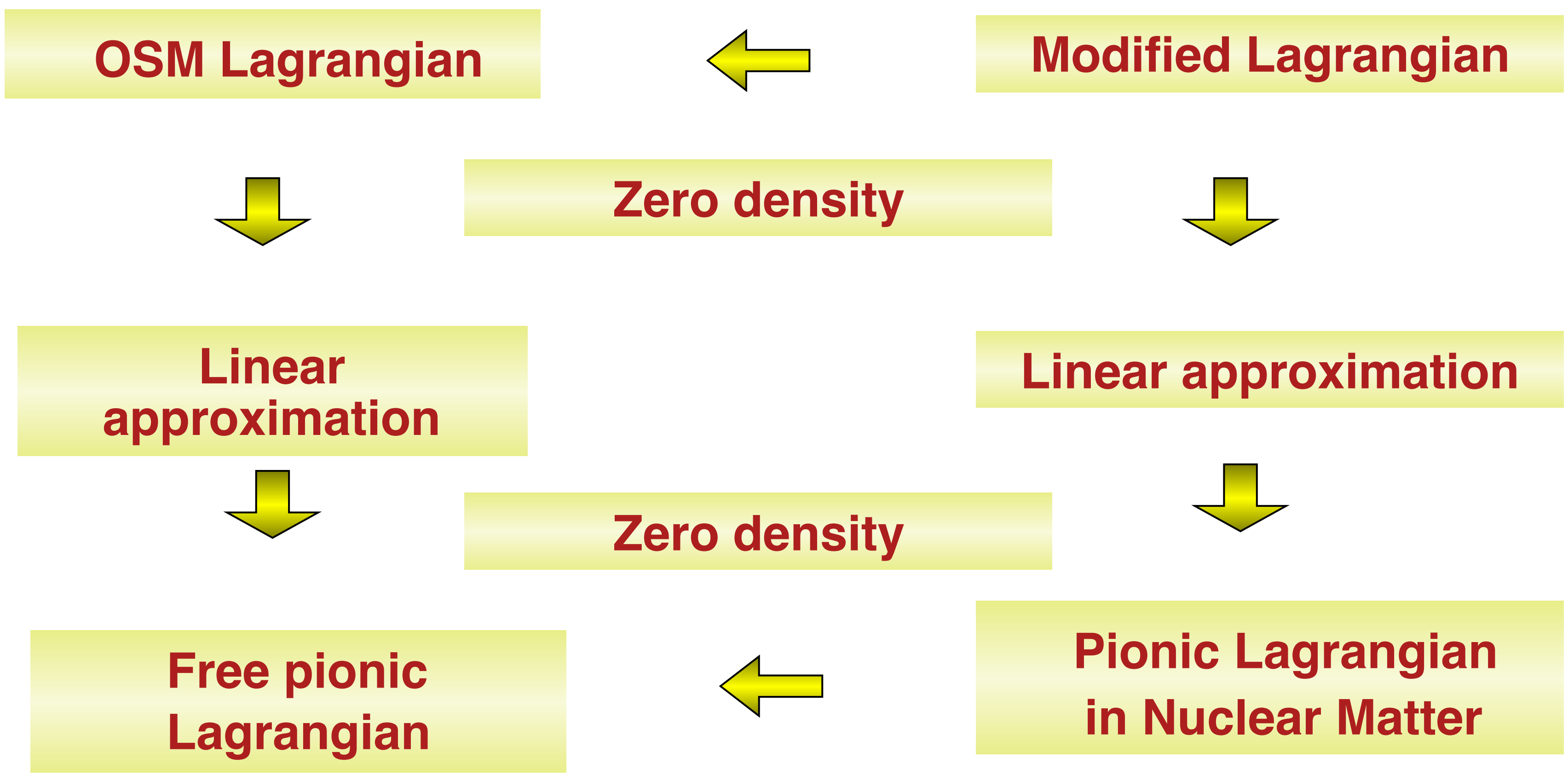


Meson cloud modifications in the nuclear medium:

Pion physics in the nuclear medium

Medium modifications

- Modifications of the mesonic sector modifies the baryonic sector
- Lagrangian must satisfy some limiting conditions



Medium modifications

“Outer shell” modifications

- In free space three types of pions can be treated separately: isospin breaking

$$(\partial^\mu \partial_\mu + m_\pi^2)\vec{\pi}^{(\pm,0)} = 0$$

- In nuclear matter: there are three types of polarization operators

$$(\partial^\mu \partial_\mu + m_\pi^2 + \hat{\Pi}^{(\pm,0)})\vec{\pi}^{(\pm,0)} = 0$$

$$\hat{\Pi}^0 = 2\omega U_{\rm opt} = \chi_s(\rho, b_0, c_0) + \vec{\nabla} \cdot \chi_p(\rho, b_0, c_0)\vec{\nabla}$$

- Optic potential approach: parameters from the pion-nucleon scattering (including the isospin dependents)

	π-atom	$T_\pi = 50$ MeV
$b_0\ [m_\pi^{-1}]$	- 0.03	- 0.04
$b_1\ [m_\pi^{-1}]$	- 0.09	- 0.09
$c_0\ [m_\pi^{-3}]$	0.23	0.25
$c_1\ [m_\pi^{-3}]$	0.15	0.16
g'	0.47	0.47

Medium modifications

“Outer shell” modifications in the Lagrangian
[U.Meissner *et al.*, EPJ A36 (2008)]

$$\mathcal{L}_2^* = \frac{F_\pi^2}{16}\alpha_\tau \mathrm{Tr}\left(\partial_0 U \partial_0 U^\dagger\right) - \frac{F_\pi^2}{16}\alpha_s \mathrm{Tr}\left(\partial_i U \partial_i U^\dagger\right)$$

$$\mathcal{L}_m^* = -\frac{F_\pi^2 m_\pi^2}{16}\alpha_m \mathrm{Tr}\left(2 - U - U^\dagger\right)$$

$$\hat{\Pi}^0 = 2\omega U_{\mathrm{opt}} = \chi_s(\rho, b_0, c_0) + \vec{\nabla}\cdot\chi_p(\rho, b_0, c_0)\vec{\nabla}$$

- Due to the non-locality of optic potential the kinetic term is also modified
- Due to energy and momentum dependence of the optic potential parameters, the following parts of the kinetic term are modified in different forms:
  - Temporal part
  - Space part

	π-atom	$T_\pi = 50$ MeV
b_0 $[m_\pi^{-1}]$	- 0.03	- 0.04
b_1 $[m_\pi^{-1}]$	- 0.09	- 0.09
c_0 $[m_\pi^{-3}]$	0.23	0.25
c_1 $[m_\pi^{-3}]$	0.15	0.16
g'	0.47	0.47

Medium modifications

“Inner core” modifications

[UY & H.Ch. Kim, PRC83 (2011); UY, JKPS62 (2013); UY, PRC88 (2013)]

$$\mathcal{L}_4^* = -\frac{1}{16e^2\zeta_T}\mathrm{Tr}\,\left[U^\dagger\partial_0 U, U^\dagger\partial_i U\right]^2 + \frac{1}{32e^2\zeta_s}\mathrm{Tr}\,\left[U^\dagger\partial_i U, U^\dagger\partial_j U\right]^2$$

may be related to

- Vector meson properties in nuclear matter;
- Nuclear matter properties.

$$\zeta_{T,s} = \zeta_{T,s}(\rho, \delta\rho, \mathrm{parameters})$$

Medium modifications

Final Lagrangian
[UY, JKPS62 (2013); UY, PRC88 (2013)]

Separated into two parts

$$\mathcal{L}^* = \mathcal{L}^*_{\rm sym} + \mathcal{L}^*_{\rm asym}$$

- Isoscalar part

$$\mathcal{L}^*_{\rm sym} = \mathcal{L}^*_2 + \mathcal{L}^*_4 + \mathcal{L}^*_m$$

- Isovector part

$$\mathcal{L}^*_{\rm asym} = \mathcal{L}^*_{\delta m} + \mathcal{L}^*_{\delta \rho}$$

- **Nuclear matter stabilization**
- **Asymmetric matter properties**

$$\mathcal{L}^*_2 = \frac{F_\pi^2}{16}\alpha_\tau {\rm Tr}\left(\partial_0 U \partial_0 U^\dagger\right) - \frac{F_\pi^2}{16}\alpha_s {\rm Tr}\left(\partial_i U \partial_i U^\dagger\right)$$

$$\mathcal{L}^*_4 = -\frac{1}{16e^2\zeta_\tau}{\rm Tr}\left[U^\dagger\partial_0 U, U^\dagger \partial_i U\right]^2 + \frac{1}{32e^2\zeta_s}{\rm Tr}\left[U^\dagger\partial_i U, U^\dagger \partial_j U\right]^2$$

$$\mathcal{L}^*_m = -\frac{F_\pi^2 m_\pi^2}{16}\alpha_m {\rm Tr}\left(2 - U - U^+\right)$$

$$\mathcal{L}^*_{\delta m} = -\frac{F_\pi^2}{32}\sum_{a=1}^{2}(m^2_{\pi^\pm} - m^2_{\pi^0}){\rm Tr}\left(\tau_a U\right){\rm Tr}\left(\tau_a U^\dagger\right)$$

$$\mathcal{L}^*_{\delta\rho} = -\frac{F_\pi^2}{16} m_\pi \alpha_e\, \varepsilon_{ab3}{\rm Tr}\left(\tau_a U\right){\rm Tr}\left(\tau_b \partial_0 U^\dagger\right)$$

Medium modifications

Reparametrization
[UY, PRC88 (2013)]

- Five density dependent parameters

- Rearrangment (technical simplification)

Outer shell modifications

$$F_{\pi,\tau} \to F^*_{\pi,\tau}, \quad F_{\pi,s} \to F^*_{\pi,s},$$

Core modifications

$$e_\tau \to e^*_\tau, \quad e_s \to e^*_s$$

$$m_\pi \to m^*_\pi,$$

$$1 + C_1\lambda = f_1(\lambda) \equiv \sqrt{\frac{\alpha_p^0}{\zeta_s}},$$

$$1 + C_2\lambda = f_2(\lambda) \equiv \frac{\alpha_s^{00}}{\left(\alpha_p^0\right)^2 \zeta_s},$$

$$1 + C_3\lambda = f_3(\lambda) \equiv \frac{\left(\alpha_p^0 \zeta_s\right)^{3/2}}{\alpha_s^{02}},$$

$$\frac{\alpha_e}{\gamma_s} = f_4\left(\frac{\rho}{\rho_0}\right)\frac{\rho_n - \rho_p}{\rho_0} = \frac{C_4 \dfrac{\rho}{\rho_0}}{1 + C_5 \dfrac{\rho}{\rho_0}} \frac{\rho_n - \rho_p}{\rho_0}$$

Nucleon in Nuclear Matter

Nucleon in nuclear matter

Structure studies 1: Energy momentum tensor

- It allows to address the questions like:
  - How are the total angular momentum and angular momentum of the nucleon shared among its constituents?
  - How are the strong forces experienced by its constituents distributed inside the nucleon?
- EMT form factors in free space studied in lattice QCD, ChPT and in different models (chiral quark soliton model, Skyrme model, etc.)
- We made a further step studying EMT form factors in nuclear matter

Nucleon in nuclear matter

Structure studies 1: Energy momentum tensor

- Definition

$$\langle p'|\hat{T}_{\mu\nu}(0)|p\rangle = \bar{u}(p',\, s')\left[M_2(t)\,\frac{P_\mu P_\nu}{M_N} + J(t)\,\frac{i(P_\mu\sigma_{\nu\rho}+P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M_N} + d_1(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{5M_N}\right]u(p,\, s)\,,$$

- Three form factors give an information about energy distribution, angular momentum distribution and about the stabilization of strong forces inside the nucleon

$$T^*_{00}(r) = \frac{F^{*2}_{\pi,s}}{8}\left(\frac{2\sin^2 F}{r^2} + F'^2\right) + \frac{\sin^2 F}{2\,e^{*2}\,r^2}\left(\frac{\sin^2 F}{r^2} + 2F'^2\right) + \frac{m^{*2}_\pi F^{*2}_{\pi,s}}{4}\,(1-\cos F)\,,$$

$$T^*_{0k}(r,s) = \frac{\epsilon^{klm} r^l s^m}{(\mathrm{s}\times\mathrm{r})^2}\,\rho^*_J(r)\,,$$

$$T^*_{ij}(r) = s^*(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\delta_{ij}\right) + p^*(r)\,\delta_{ij}$$

$$M^*_2(t) - \frac{t}{5M^{*2}_N}\,d^*_1(t) = \frac{1}{M^*_N}\int \mathrm{d}^3r\, T^*_{00}(r)\, j_0(r\sqrt{-t})\,,$$

$$d^*_1(t) = \frac{15M^*_N}{2}\int \mathrm{d}^3r\, p^*(r)\,\frac{j_0(r\sqrt{-t})}{t}\,,$$

$$J^*(t) = 3\int \mathrm{d}^3r\, \rho^*_J(r)\,\frac{j_1(r\sqrt{-t})}{r\sqrt{-t}}\,,$$

$$M^*_2(0) = \frac{1}{M^*_N}\int \mathrm{d}^3r\, T^*_{00}(r) = 1\,, \quad J^*(0) = \int \mathrm{d}^3r\, \rho^*_J(r) = \frac{1}{2}\,.$$

Nucleon in nuclear matter

Structure studies1: Energy momentum tensor related quantities
[H.C.Kim, P. Schweitzer, UY, Phys.Lett. B718 (2012)]

Different quantities related to the nucleon EMT densities and their form factors: $T_{00}^*(0)$ denotes the energy in the center of the nucleon; $\langle r_{00}^2\rangle^*$ and $\langle r_J^2\rangle^*$ depict the mean square radii for the energy and angular momentum densities, respectively; $p^*(0)$ represents the pressure in the center of the nucleon, whereas r_0^* designates the position where the pressure changes its sign; d_1^* is the value of the $d_1^*(t)$ form factor at the zero momentum transfer.

ρ/ρ_0	$T_{00}^*(0)$ [$\mathrm{GeV\,fm^{-3}}$]	$\langle r_{00}^2\rangle^*$ [$\mathrm{fm^2}$]	$\langle r_J^2\rangle^*$ [$\mathrm{fm^2}$]	$p^*(0)$ [$\mathrm{GeV\,fm^{-3}}$]	r_0^* [fm]	d_1^*
0	1.45	0.68	1.09	0.26	0.71	−3.54
0.5	0.96	0.83	1.23	0.18	0.82	−4.30
1.0	0.71	0.95	1.35	0.13	0.90	−4.85

Nucleon in nuclear matter

Structure studies 1: Pressure distribution inside the nucleon in free space and in symmetric matter [H.C.Kim, P. Schweitzer, UY, Phys.Lett. B718 (2012)]

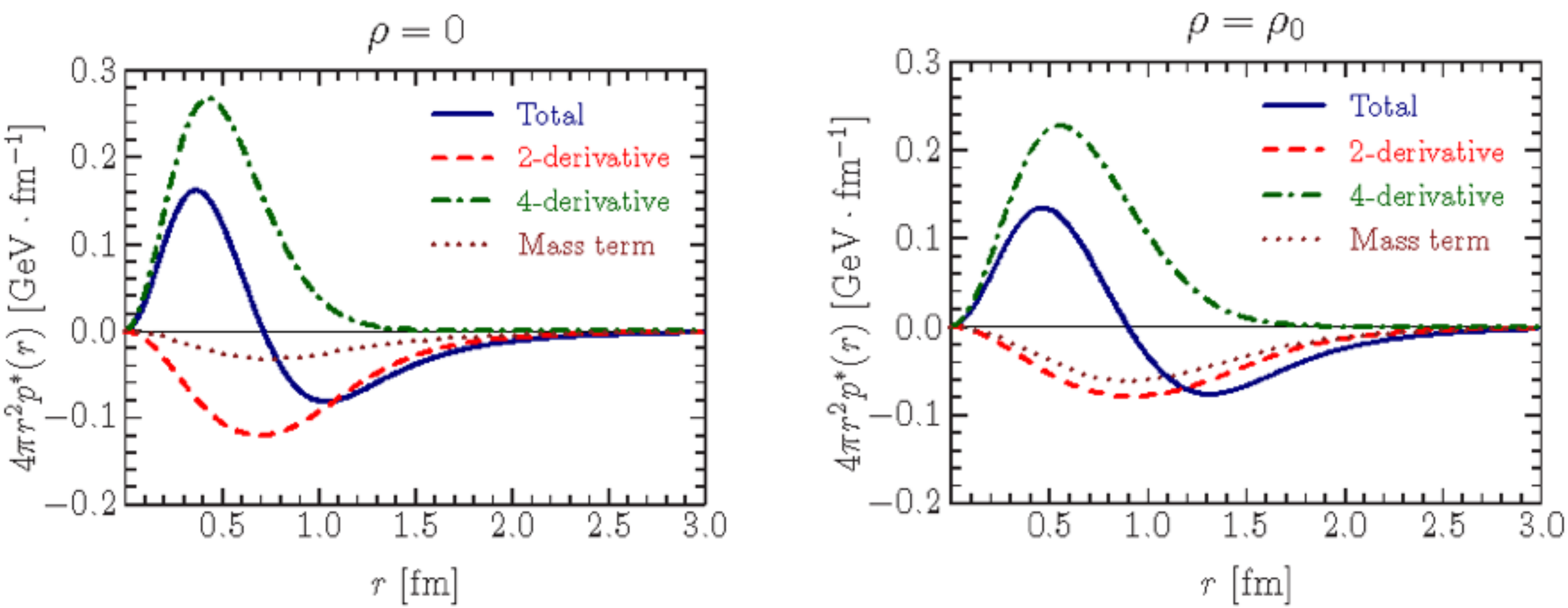


FIG. 3: (Color online) The decomposition of the pressure densities $4\pi r^2 p^*(r)$ as functions of r, in free space ($\rho = 0$) and at $\rho = \rho_0$, in the left and right panels, respectively. The solid curves denote the total pressure densities, the dashed ones represent the contributions of the 2-derivative (kinetic) term, the long-dashed ones are those of the 4-derivative (stabilizing) term, and the dotted ones stand for those of the pion mass term.

Stability and applicability [H.C.Kim, P. Schweitzer, UY, Phys.Lett. B718 (2012)]

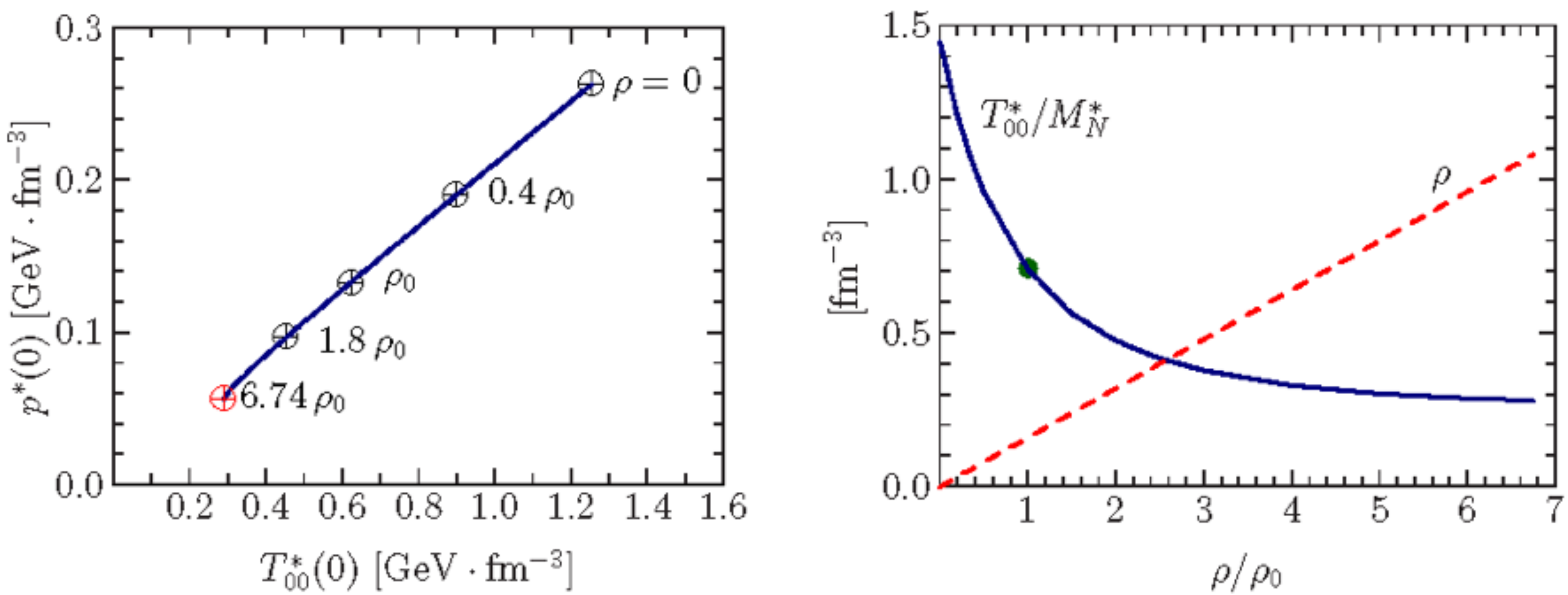


FIG. 5: (Color online) In the left panel, the correlated change of $p^*(0)$ and $T^*_{00}(0)$ drawn with ρ varied. In the right panel, the T^*_{00}/M^*_N and ρ depicted as a function of ρ/ρ_0. The maximal density is given as about $6.74\,\rho_0$, above which the Skyrmion does not exist anymore. The filled circle on the solid curve represents the value of T^*_{00}/M^*_N at normal nuclear matter density.

Nucleon in nuclear matter

Structure studies 2: Transverse EM charge densities

- Definition of EM ff's $\langle N(p',S')|J_\mu^{EM}(0)|N(p,S)\rangle$

$$= \bar{u}_N(p',S')\left[\gamma_\mu F_1^*(q^2) + i\frac{\sigma_{\mu\nu}q^\nu}{2m_N}F_2^*(q^2)\right]u_N(p,S).$$

- These Pauli and Dirac ff's can be expressed by Sachs ff's

$$\begin{aligned} G_E\left(Q^2\right) &= F_1\left(Q^2\right) + \frac{Q^2}{4M_N^2}F_2\left(Q^2\right) \\ G_M\left(Q^2\right) &= F_1\left(Q^2\right) + F_2\left(Q^2\right), \end{aligned}$$

- They give an information about transverse charge distributions inside the nucleon

$$\rho_0^*(b) = \int_0^\infty \frac{Q\,dQ}{2\pi} J_0(bQ) \frac{G_E^*(Q^2) + \tau G_M^*(Q^2)}{1+\tau}$$

$$\rho_T^*(\boldsymbol{b}) = \rho_0^*(b) - \sin(\phi_b - \phi_S) \times \int_0^\infty \frac{Q^2\,dQ}{4\pi m_N} J_1(bQ) \frac{-G_E^*(Q^2) + G_M^*(Q^2)}{1+\tau},$$

$$\boldsymbol{b} = b(\cos\phi_b \hat{\boldsymbol{e}}_x + \sin\phi_b \hat{\boldsymbol{e}}_y)$$

Nucleon in nuclear matter

Structure studies 2: Transverse EM charge densities inside an unpolarized nucleon [UY, H.C.Kim, Phys.Lett. B726 (2013)]

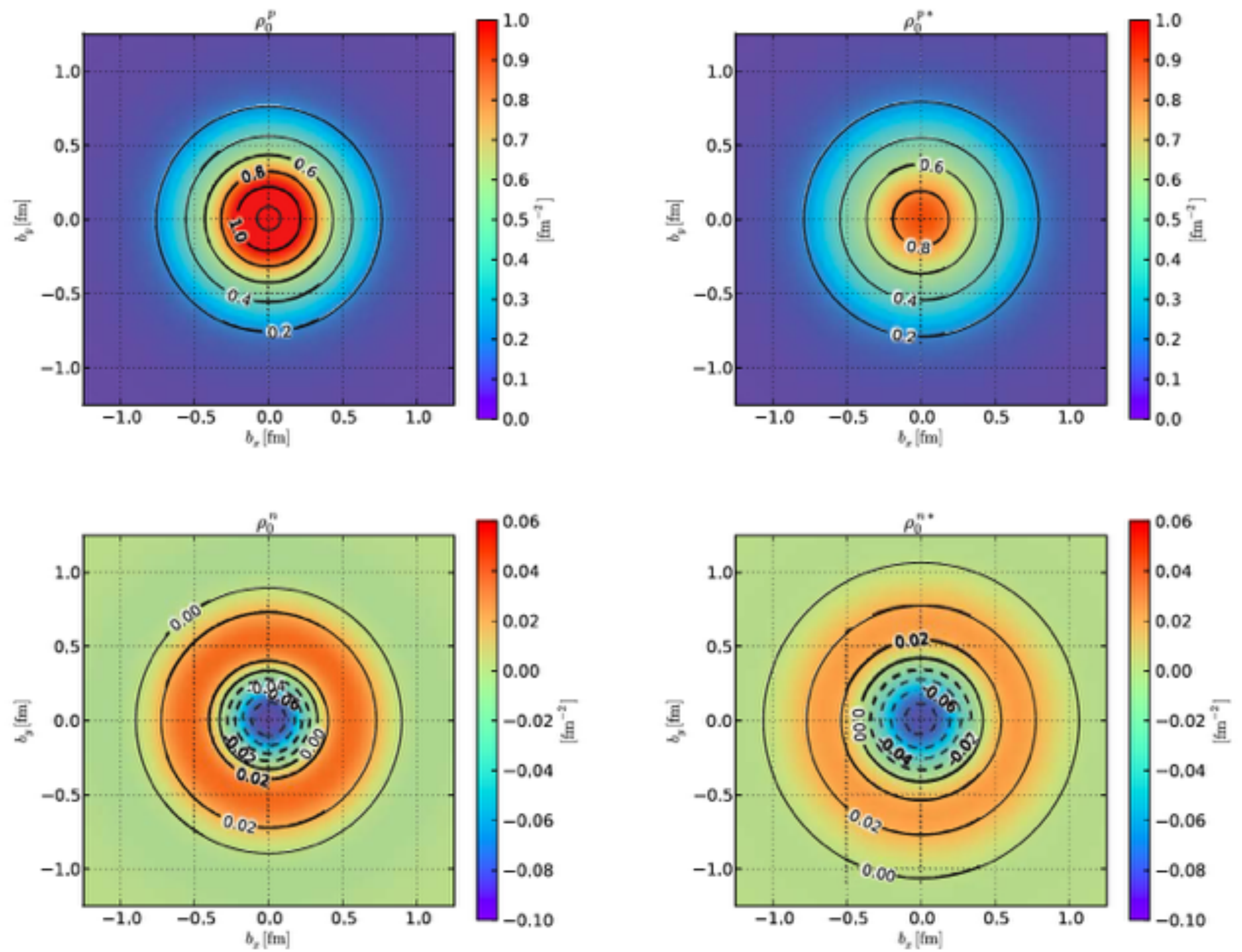


Fig. 3. Quark transverse charge densities inside an unpolarized proton (upper panels) and a neutron (lower panels) in free space (left panels) and at nuclear matter density $\rho_0 = 0.5m_\pi^3$ (right panels).

Structure studies 2: Transverse EM charge densities inside the polarized nucleon [UY, H.C.Kim, Phys.Lett. B726 (2013)]

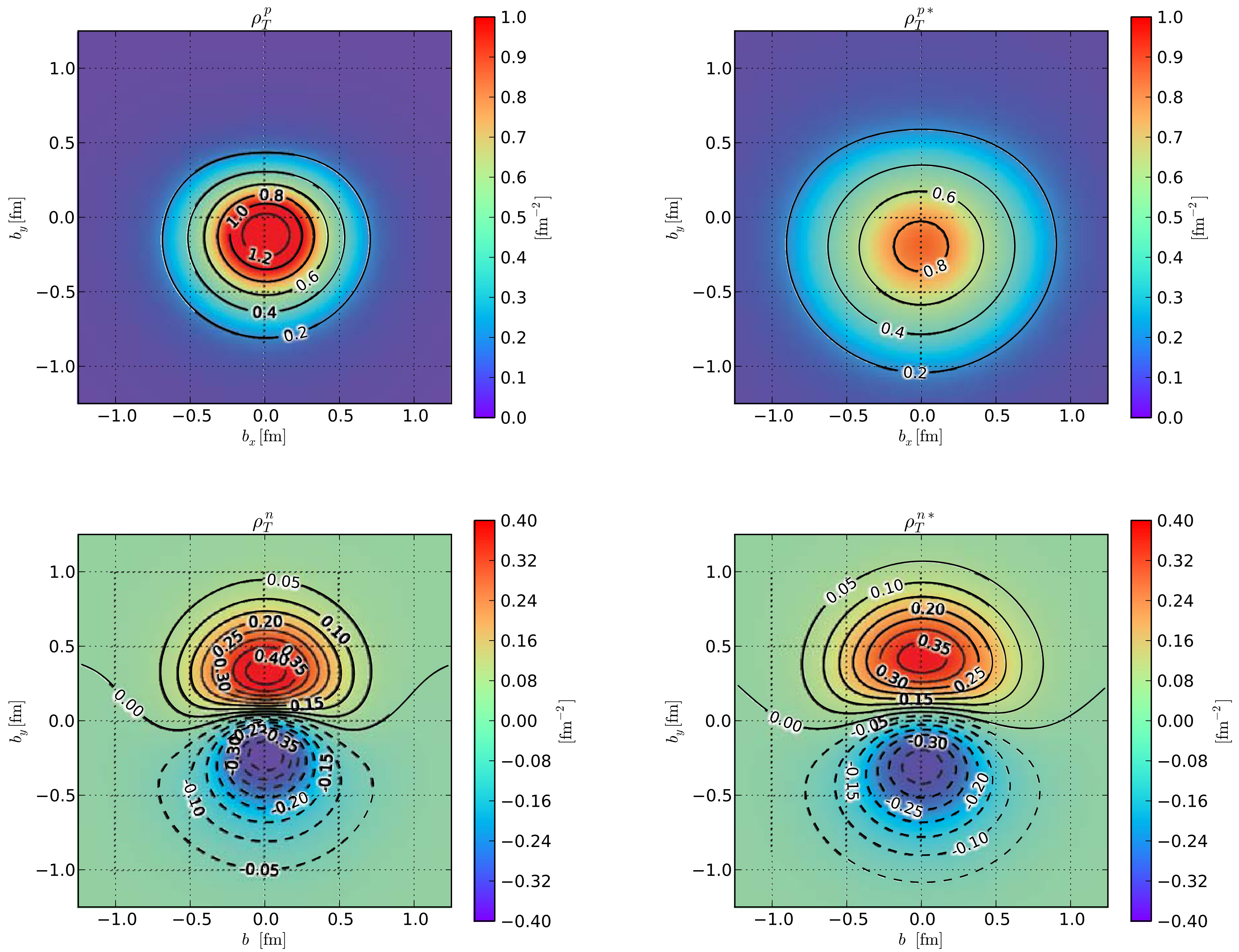


Fig. 4. Transverse charge densities of the proton (upper panels) and neutron (lower panels) in free space (left panels) and in nuclear matter with the density $\rho_0 = 0.5 m_\pi^3$ (right panels).

Nucleon in nuclear matter

Static properties (e.g. mass) [UY, PRC88 (2013)]

- Isoscalar effective mass

$$m^*_{N,s} = M^*_S + \frac{3}{8\Lambda^*} + \frac{\Lambda^*}{2}\left(a^{*2} + \frac{\Lambda^{*2}_{\rm env}}{\Lambda^{*2}}\right)$$

- Isovector effective mass (relevant to: universe evolution in Early stage; stability of drip line nuclei; mirror nuclei; transport in heavy-ion collisions; asymmetric nuclear matter)

$$\Delta m^*_{np} = a^* + \frac{\Lambda^*_{\rm env}}{\Lambda^*}$$

- Effective masses of the nucleons

$$m^*_{n,p} = m^*_{N,s} - \Delta m^*_{np} T_3$$

Nuclear Matter

Nuclear matter

Bethe-Weizsacker formula for the binding energy per nucleon

$$\varepsilon(A,Z) = -a_V + a_S \frac{(N-Z)^2}{A^2} + \square$$

Its terms can be obtained in the framework of present model

We are ready to reproduce

- Volume term
  - considering symmetric infinite nuclear matter
- Asymmetry term
  - considering isospin asymmetric environment
- Surface and Coulomb terms
  - considering the nucleons in a finite volume
- Finite nuclei properties
  - using the local density approximation

The volume term and Symmetry energy

- At infinite nuclear matter approximation the binding energy per nucleon takes the form

$$\varepsilon(\lambda,\delta) = \varepsilon_V(\lambda) + \varepsilon_S\delta^2 + O(\delta^4) \equiv \varepsilon_V(\lambda) + \varepsilon_A(\lambda,\delta)$$

  - λ is normalised nuclear matter density
  - δ is asymmetry parameter
  - ε_S is symmetry energy

- In our model
  - Symmetric matter
  - Asymmetric matter

$$\varepsilon_V(\lambda) = m^*_{N,s}(\lambda,0) - m_N^{\text{free}}$$

$$\varepsilon_A(\lambda,\delta) = \varepsilon(\lambda,\delta) - \varepsilon_V(\lambda)$$
$$= m^*_{N,s}(\lambda,\delta) - m^*_{N,s}(\lambda,0) + m^*_{N,V}(\lambda,\delta)\delta$$

Nuclear matter

Nuclear matter properties

- Symmetric matter properties (pressure, compressibility and third derivative)

$$p = \rho_0 \lambda^2 \left.\frac{\partial \varepsilon_V(\lambda)}{\partial \lambda}\right|_{\lambda=1}, \quad K_0 = 9\rho^2 \left.\frac{\partial^2 \varepsilon_V(\lambda)}{\partial \rho^2}\right|_{\rho=\rho_0} \quad Q = 27\lambda^3 \left.\frac{\partial^3 \varepsilon_V(\lambda)}{\partial \lambda^3}\right|_{\lambda=1}$$

- Symmetry energy properties (coefficient, slop and curvature)

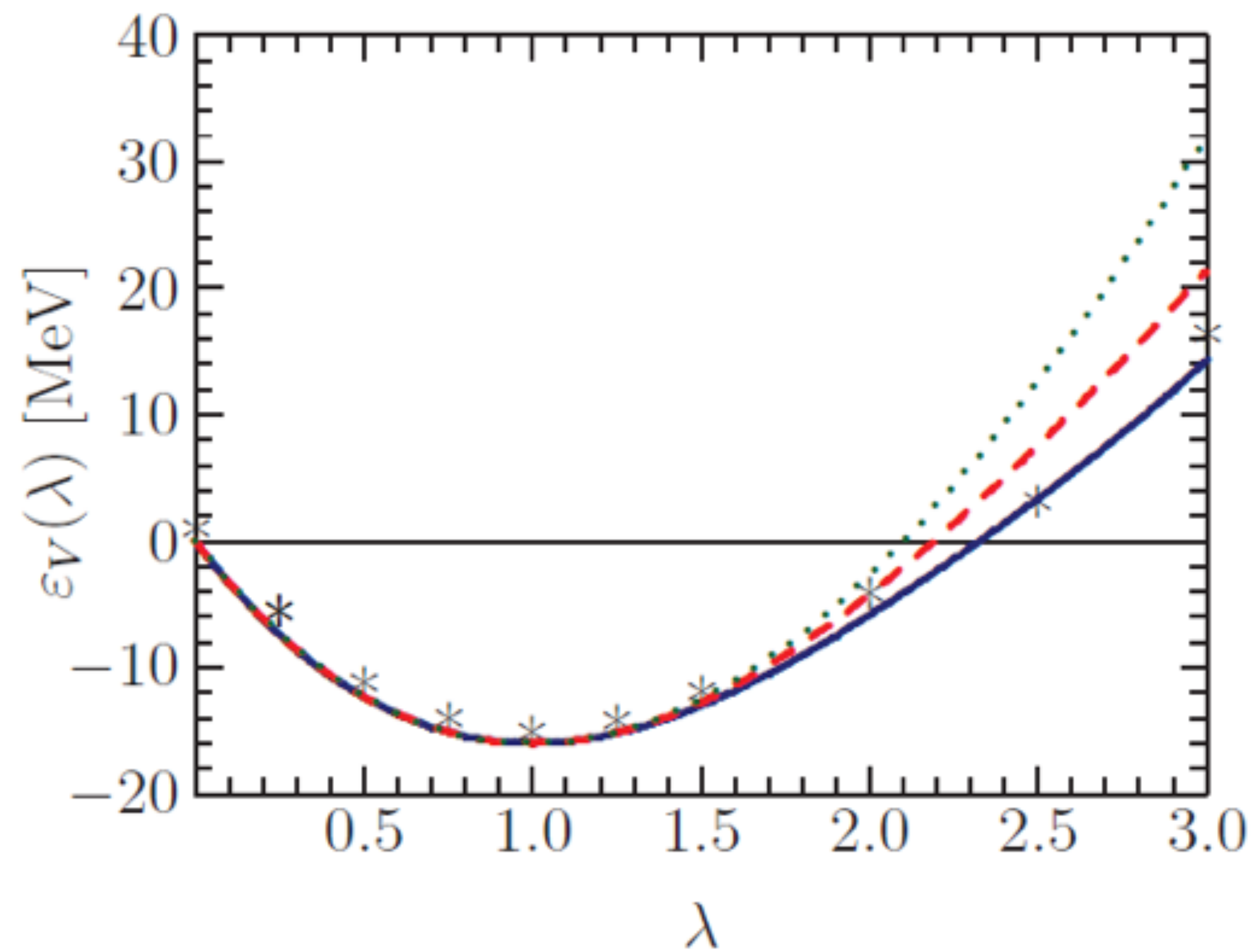
$$\varepsilon_s(\lambda) = \varepsilon_s(1) + \frac{L_s}{3}(\lambda - 1) + \frac{K_S}{18}(\lambda - 1)^2 + \cdots$$

Symmetric matter

Volume energy
[UY, PRC88 (2013)]

- Set I – solid
- Set II – dashed
- Set III – dotted

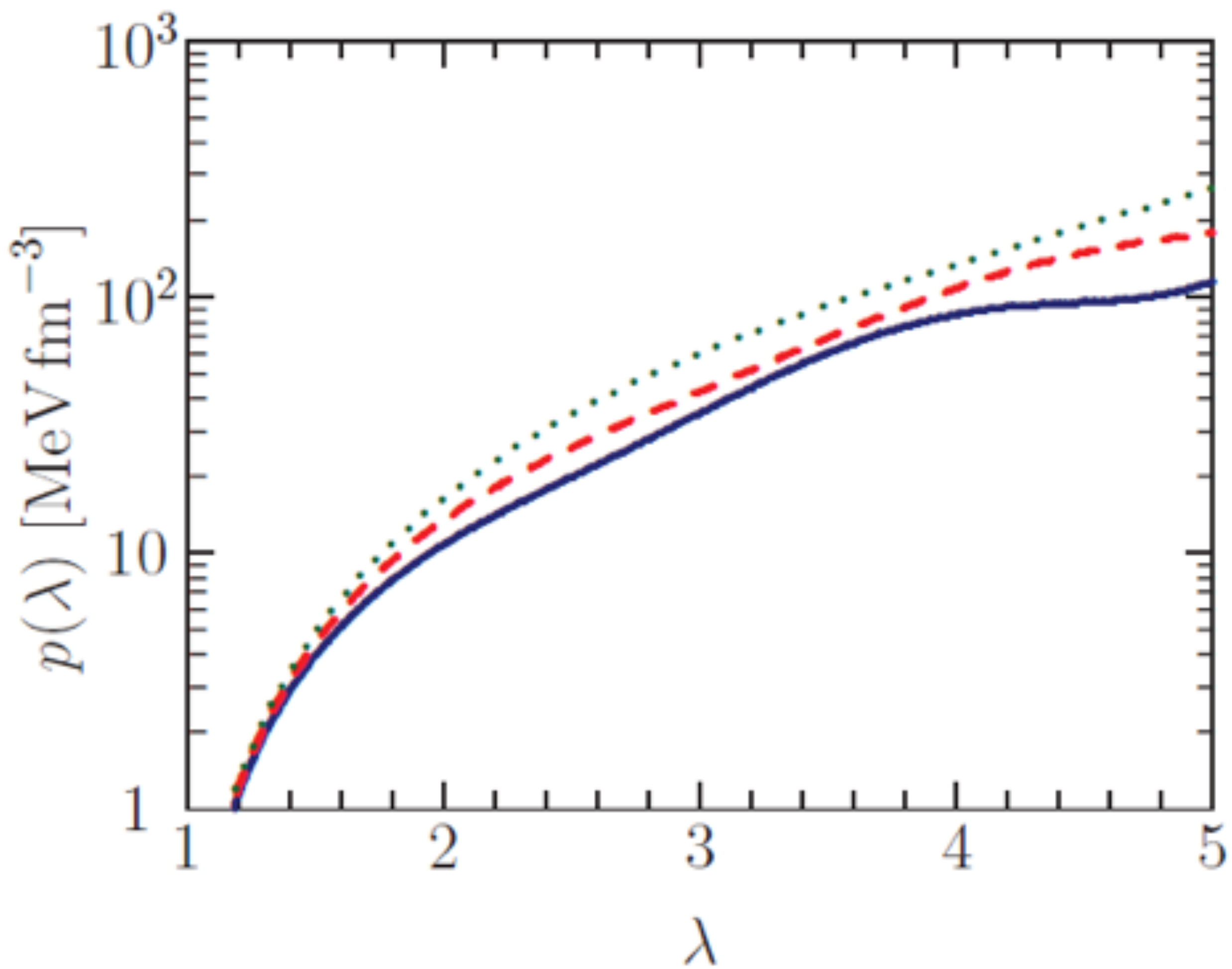
For comparison: Akmal-Pandharipande-Ravenhall (APR) predictions
[PRC 58, 1804 (1998)]
are given by stars.
(From Arigonna 2 body interactions + 3 body interactions)

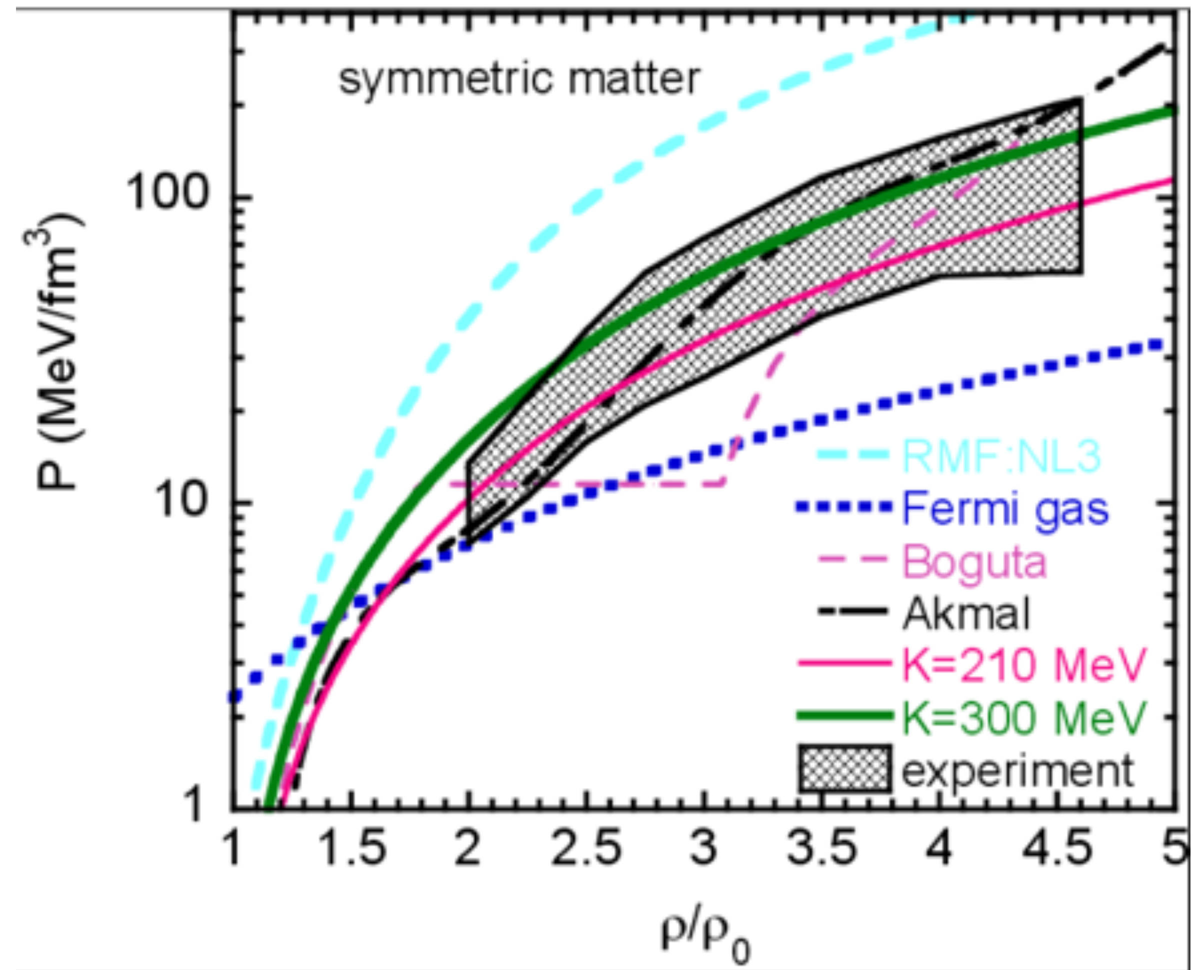


Set	C_1	C_2	C_3	$\varepsilon_V(\rho_0)$ (MeV)	K_0 (MeV)	Q (MeV)
I	−0.279	0.737	1.782	−16	240	−410
II	−0.273	0.643	1.858	−16	250	−279
III	−0.277	0.486	2.124	−16	260	−178

Symmetric matter

Pressure
[UY, PRC88 (2013)]





For comparison: Right figure from
Danielewicz- Lacey-Lynch, Science 298, 1592 (2002).
(Deduced from experimental flow data and simulations studies)

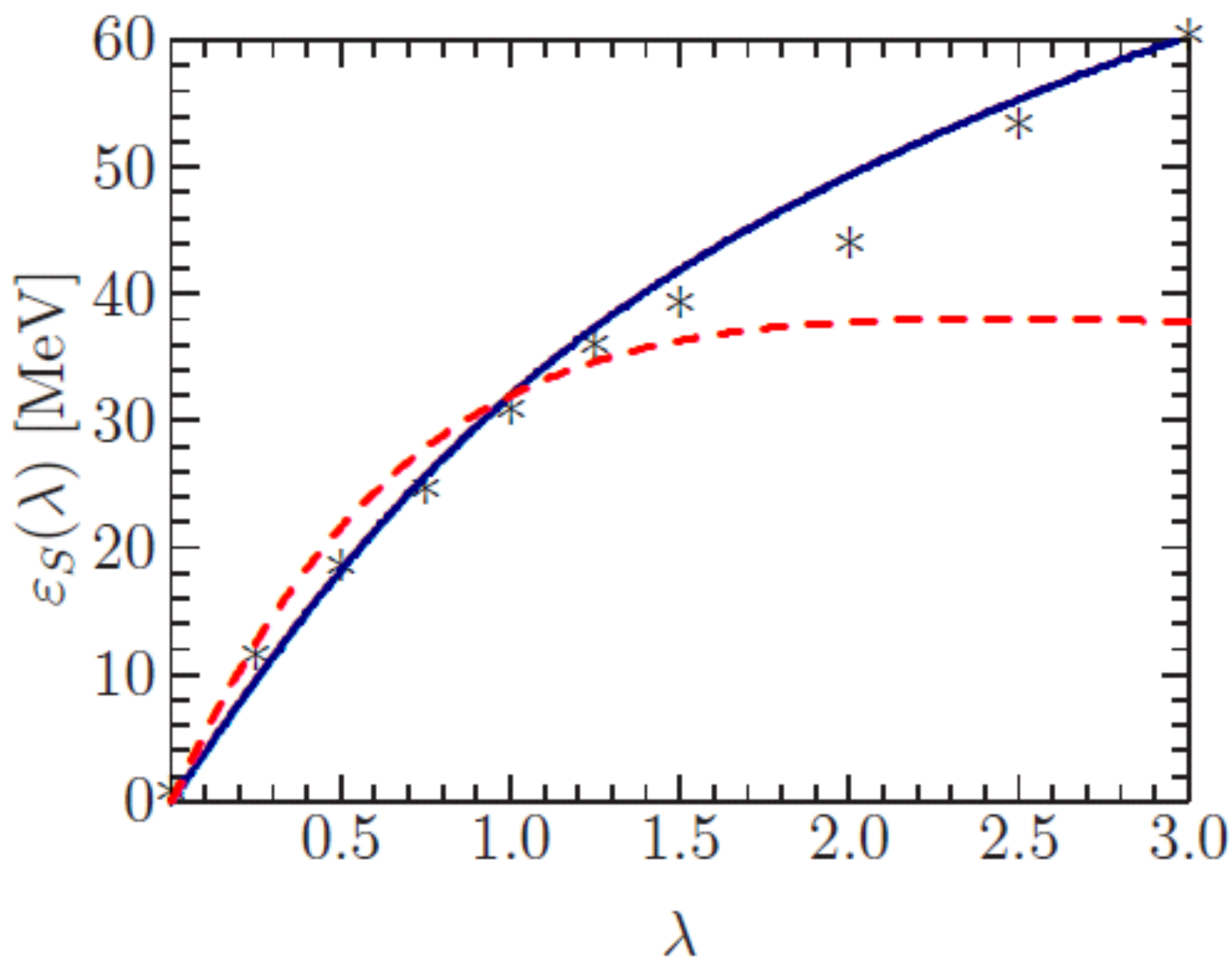
Asymmetric matter

Symmetry energy

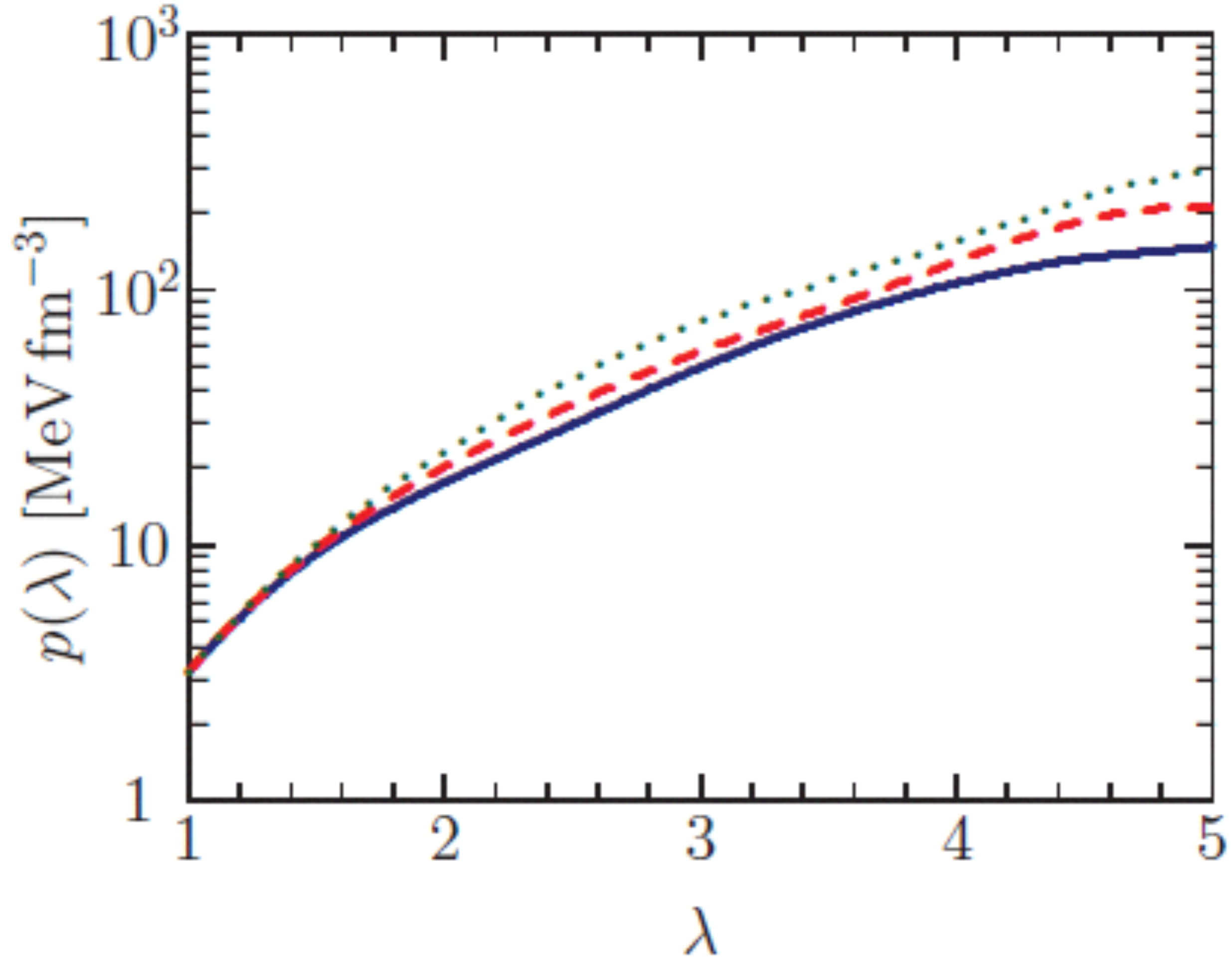
- Solid $L_s = 70\,\mathrm{MeV}$
- Dashed $L_s = 40\,\mathrm{MeV}$

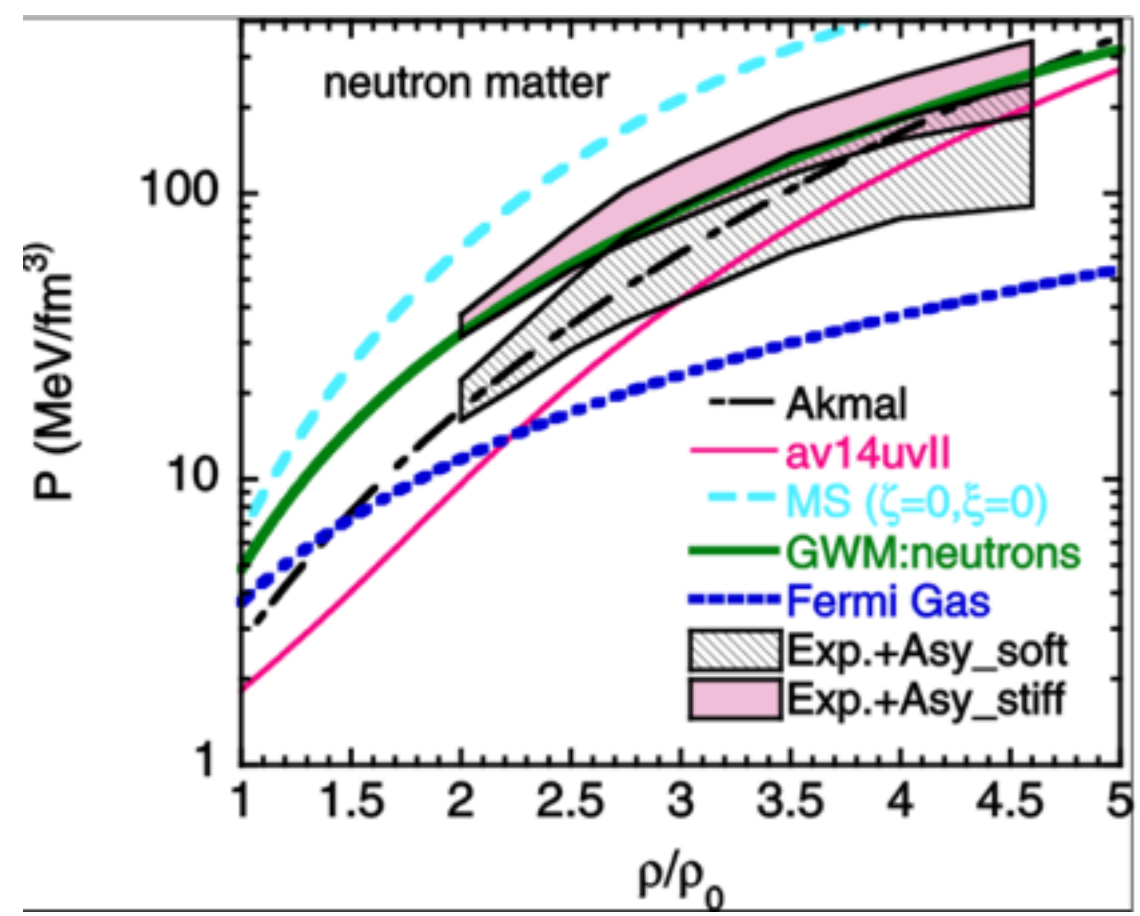
For comparison: Akmal-Pandharipande-Ravenhall (APR) predictions [PRC 58, 1804 (1998)] are given by stars.

(From arigonna 2 body interactions + 3 body interactions)



Pressure in neutron matter
[UY, PRC88 (2013)]





For comparison: Right figure from
Danielewicz- Lacey-Lynch, Science 298, 1592 (2002).
(Deduced from experimental flow data and simulations studies)

Asymmetric matter

Low density behaviour of symmetry energy

For comparison:
Trippa-Colo-Vigezzi
[PRC 77, 061304 (2008)];
From analysis of GDR
(208Pb).

$$23.3 < \varepsilon_s(\rho = 0.1\text{fm}^{-3}) < 24.9\text{ MeV}$$

Consequently one can predict in this model:

$$K_\tau = K_s - 6L_s$$

$$K_{0,2} = K_\tau - \frac{Q}{K_0} L_S$$

$\varepsilon_S(\rho_0)$ [MeV]	L_S [MeV]	K_S [MeV]	K_τ [MeV]	$K_{0,2}$ [MeV]	$\varepsilon_S(0.1\text{fm}^{-3})$ [MeV]
32	40	−181	−301	−257	25.15
32	50	−160	−310	−254	24.15
32	60	−126	−306	−239	23.22
32	70	−80	−290	−211	22.37
32	80	−21	−261	−172	21.57
32	90	50	−220	−119	20.82
32	100	134	−166	−55	20.13

Neutron Stars

Neutron star properties

- **TOV equations**

$$-\frac{dP(r)}{dr} = \frac{G\mathcal{E}(r)\mathcal{M}(r)}{r^2}\left(1-\frac{2G\mathcal{M}(r)}{r}\right)^{-1}\left(1+\frac{P(r)}{\mathcal{E}(r)}\right)\left(1+\frac{4\pi r^3 P(r)}{\mathcal{M}(r)}\right)$$

- **Energy-pressure relation**

$$P = P(\mathcal{E})$$

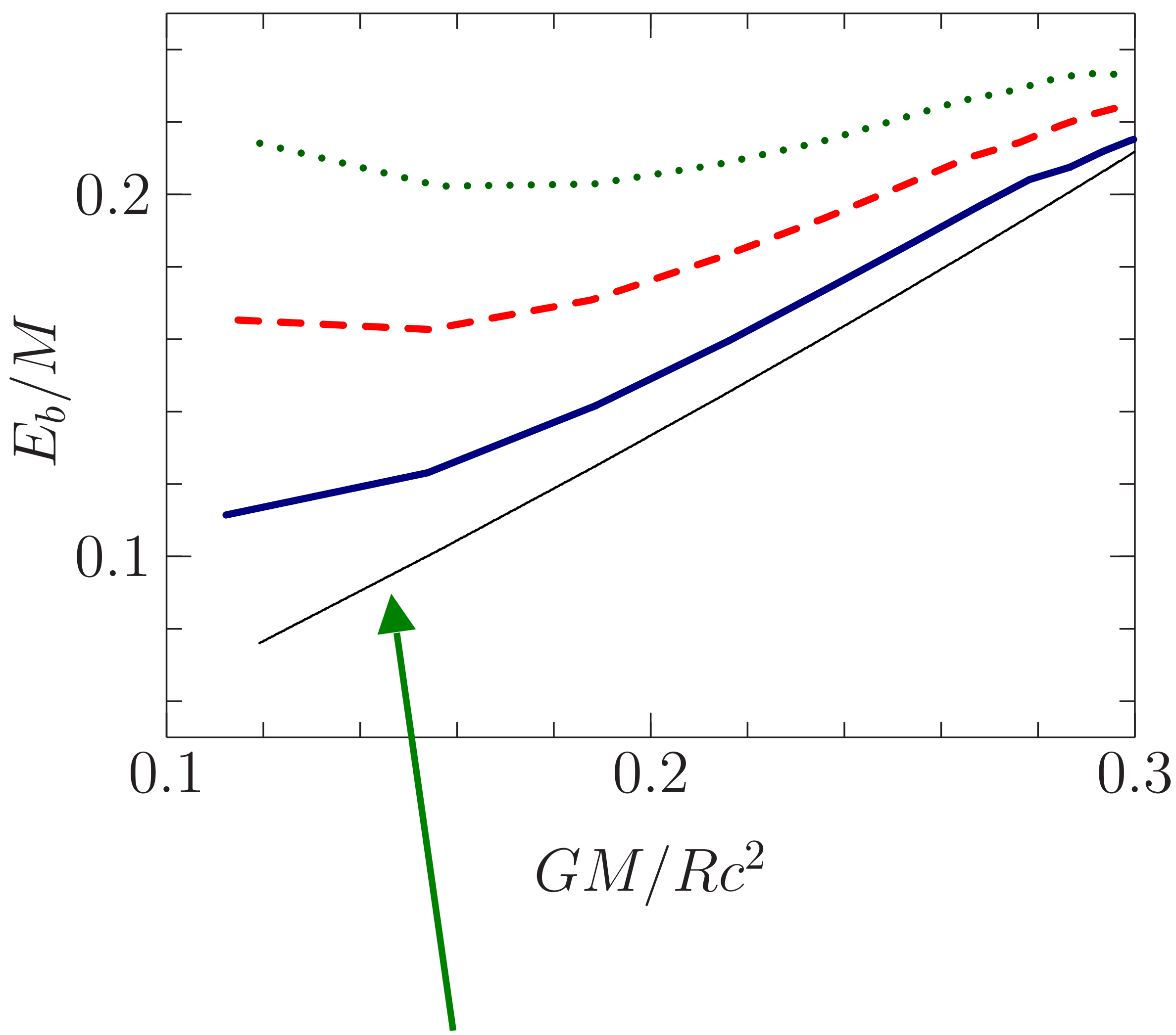
$$P(\lambda) = \rho_0\lambda^2\frac{\partial\varepsilon(\lambda,1)}{\partial\lambda}\,,$$
$$\mathcal{E}(\lambda) = [\varepsilon(\lambda,1)+m_N]\lambda\rho_0.$$

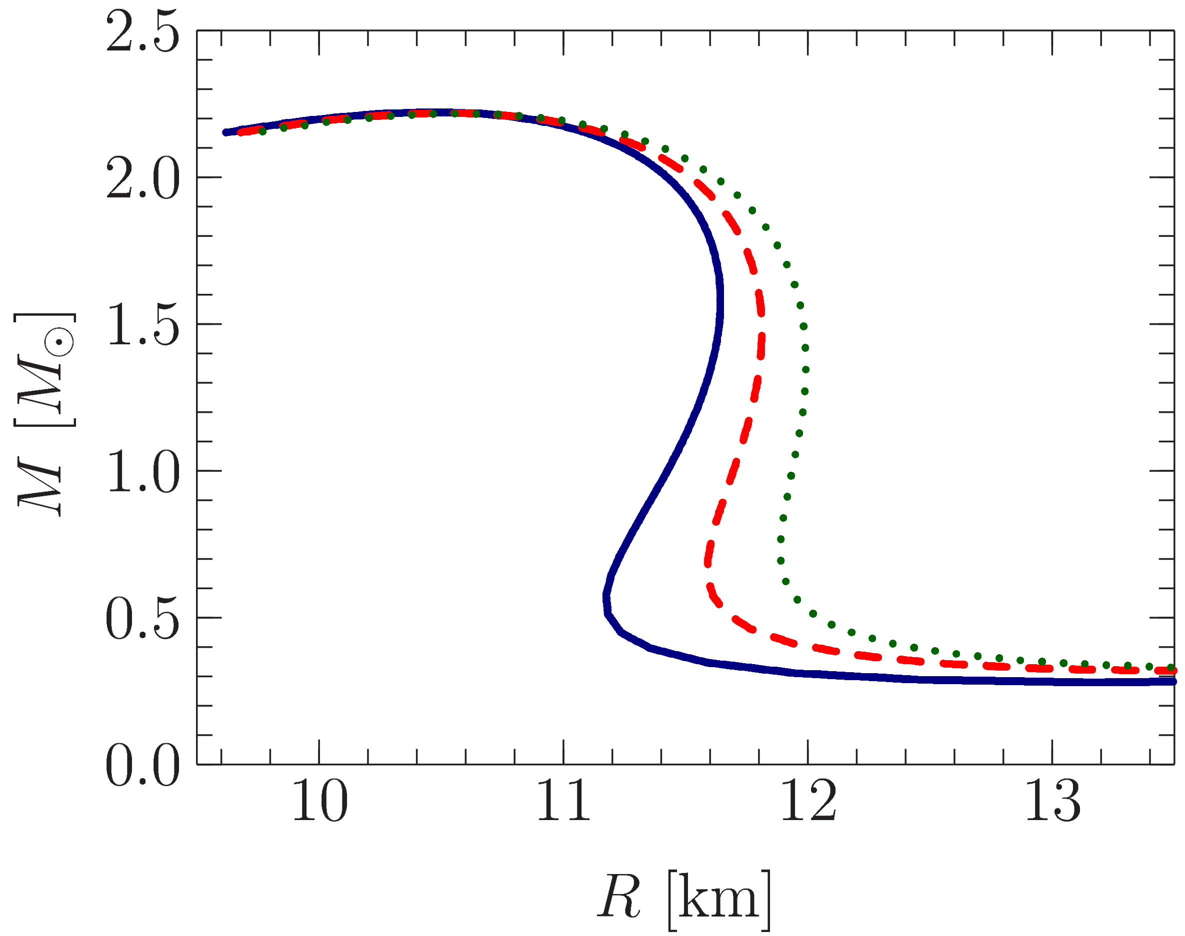
- **Neutron star's mass**

$$\mathcal{M}(r) = 4\pi\int_0^r \mathrm{d}r\, r^2\mathcal{E}(r)\,.$$

Neutron stars

Neutron star properties [UY, PLB749 (2015)]





From Ref. [J.M. Lattimer & M. Prakash, Astrophys. J. 550 (2001)].

Neutron star properties

[UY, PLB749 (2015)]

TABLE III: Properties of the neutron stars from the different sets of parameters (see Tables I and II for the values of parameters): n_c is central number density, ρ_c is central energy-mass density, R is radius of the neutron star, $M_{\max}$ is possible maximal mass, A is number of baryons in the star, E_b is binding energy of the star. In the left panel we represent the neutron star properties corresponding to the maximal mass $M_{\max}$ and in right panel approximately 1.4 solar mass neutron star properties. The last two lines are results from the Ref. [21].

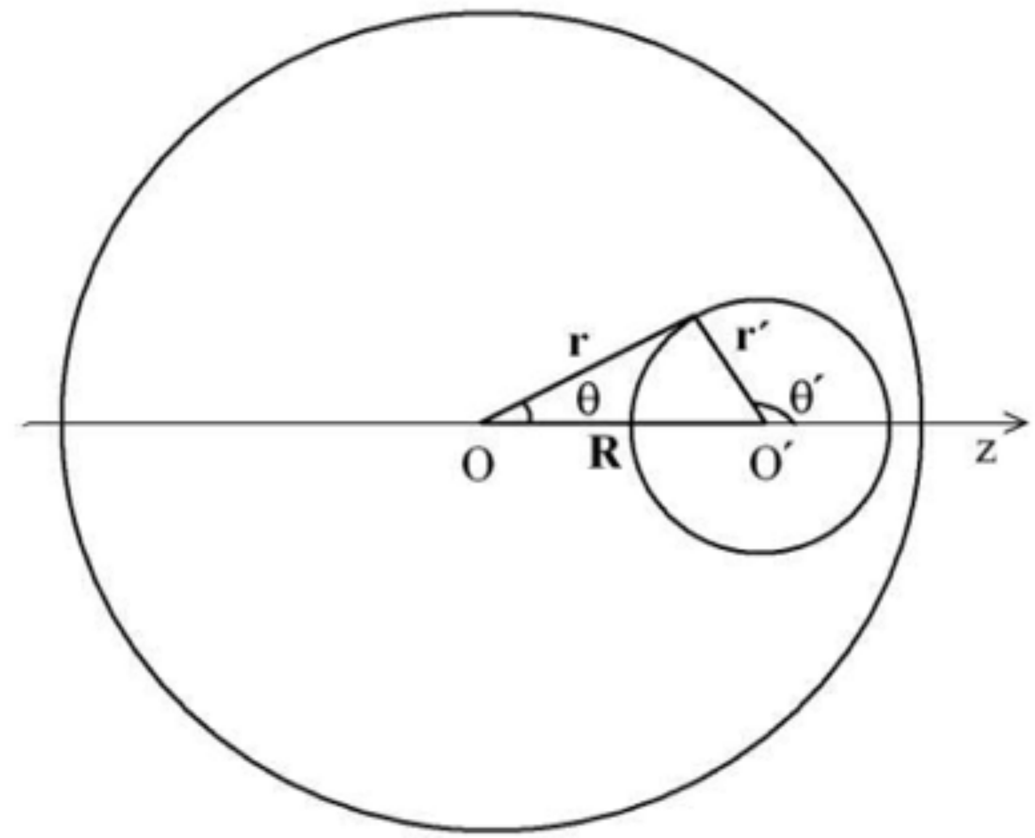
Set	n_c [fm^{-3}]	ρ_c [10^{15}gr/cm^3]	R [km]	$M_{\max}$ [$M_\odot$]	A [10^{57}]	E_b [10^{53}erg]	n_c [fm^{-3}]	ρ_c [10^{15}gr/cm^3]	R [km]	M [$M_\odot$]	A [10^{57}]	E_b [10^{53}erg]
III-a	1.046	2.445	10.498	2.226	3.227	8.721	0.479	0.861	11.587	1.402	1.898	3.503
III-b	1.045	2.444	10.547	2.223	3.216	8.557	0.471	0.861	11.772	1.402	1.895	3.453
III-c	1.037	2.424	10.616	2.221	3.200	8.397	0.460	0.832	11.953	1.402	1.887	3.339
III-d	1.047	2.452	10.494	2.221	3.213	8.598	0.481	0.867	11.619	1.402	1.893	3.422
III-e	1.044	2.440	10.554	2.218	3.203	8.495	0.473	0.858	11.809	1.403	1.890	3.384
III-f	1.040	2.433	10.609	2.216	3.189	8.311	0.464	0.842	11.992	1.403	1.887	3.334
SLy230a [21]	1.15	2.69	10.25	2.10	2.99	7.07	0.508	0.925	11.8	1.4	1.85	2.60
SLy230b [21]	1.21	2.85	9.99	2.05	2.91	6.79	0.538	0.985	11.7	1.4	1.85	2.61

Nucleon in Finite Nuclei

Nucleon in finite nuclei

The nucleon in a nucleus will include

- Local density approach for environment
- R dependence of a results
- Deformations
  - In particular, axially symmetric case allows the deformations in polar direction
- Polar deformations can be represented
  - in the isotopic vector and
  - in the profile function



$$\boldsymbol{N}(\boldsymbol{r}-\boldsymbol{R}) = \begin{pmatrix} \sin\Theta(\boldsymbol{r}-\boldsymbol{R})\cos\varphi \\ \sin\Theta(\boldsymbol{r}-\boldsymbol{R})\sin\varphi \\ \cos\Theta(\boldsymbol{r}-\boldsymbol{R}) \end{pmatrix}$$

$$P = P(|\boldsymbol{r}-\boldsymbol{R}|,\theta), \qquad \Theta = \Theta(|\boldsymbol{r}-\boldsymbol{R}|,\theta)$$

$$U(\boldsymbol{r}-\boldsymbol{R}) = \exp\left[i\boldsymbol{\tau}\cdot\boldsymbol{N}(\boldsymbol{r}-\boldsymbol{R})P(\boldsymbol{r}-\boldsymbol{R})\right]$$

Nucleon in finite nuclei

The Equations of Motion

- The coupled partial differential equations (not an easy problem)

$$f(F_{\tilde{r}\tilde{r}}, F_{\theta\theta}, F_{\tilde{r}}, F_{\theta}, \Theta_{\theta}, F, \Theta) = 0,$$

$$g(\Theta_{\theta\theta}, \Theta_{\theta}, F_{\tilde{r}}, F_{\theta}, \Theta, F) = 0,$$

- A numerical variational method can be applied

$$P(r,\theta) = 2\arctan\left\{\frac{r_0^2}{r^2}(1+m_\pi r)\big(1+u(\theta)\big)\right\} e^{-f(r)r}$$

$$\Theta(r,\theta) = \theta + \zeta(r,\theta),$$

$$F(r) = 2\arctan\left\{\frac{r_0^2}{r^2}(1+m_\pi r)\right\} e^{-f(r)r}, \qquad u(\theta) = \sum_{n=1}^{\infty} \gamma_n \cos^n\theta$$

$$f(r) = \beta_0 + \beta_1 e^{\beta_2 r^2}.$$

$$\zeta(r,\theta) = r e^{-\delta_0^2 r^2} \sum_{n=1}^{\infty} \delta_n \sin 2n\theta,$$

$$\lim_{r\to 0} F(r) = \pi - Cr,$$

$$\lim_{r\to\infty} F(r) = D\,(1+m_\pi r)\,\frac{e^{-m_\pi r}}{r^2},$$

Nucleon in finite nuclei

Accuracy of the variational method

- In spherically symmetric approximation (e.g. nucleon in the centre of the spherical nucleus) one can explicitly solve Equations of Motion and compare with results of variational method
- Skyrme term is not modified in nuclear matter (table below)

Element		r_0 [fm]	$10\beta_0$ $[m_\pi]$	β_1 $[m_\pi]$	β_2 $[m_\pi^2]$	m_{p}^* [MeV]	Δm_{np}^* [MeV]	$\Delta m_{\mathrm{np}}^{*(\mathrm{EM})}$ [MeV]	μ_{p}^* [n.m.]	μ_{n}^* [n.m.]	$\langle r^2\rangle_{\mathrm{E,S}}^{*1/2}$ [fm]	$\langle r^2\rangle_{\mathrm{E,V}}^{*1/2}$ [fm]
	i)	–	–	–	–	938.268	1.291	−0.686	1.963	−1.236	0.481	0.739
free space	ii)	0.954	0.075	1.311	−0.009	938.809	1.313	−0.687	1.966	−1.241	0.481	0.739
	i)	–	–	–	–	593.285	1.668	−0.526	2.355	−1.276	0.656	0.850
^{14}N	ii)	1.393	0.076	0.920	0.226	598.505	1.655	−0.536	2.230	−1.209	0.648	0.810
	i)	–	–	–	–	585.487	1.697	−0.517	2.393	−1.297	0.667	0.863
^{16}O	ii)	1.426	0.076	0.907	0.219	590.175	1.685	−0.527	2.341	−1.232	0.660	0.825
	i)	–	–	–	–	558.088	1.804	−0.480	2.584	−1.422	0.722	0.942
^{38}K	ii)	1.493	0.076	0.841	0.153	559.957	1.802	−0.485	2.550	−1.377	0.718	0.910
	i)	–	–	–	–	557.621	1.804	−0.478	2.569	−1.428	0.724	0.947
^{40}Ca	ii)	1.489	0.076	0.839	0.149	559.378	1.802	−0.483	2.557	−1.383	0.720	0.914

The Hamiltonian of the model

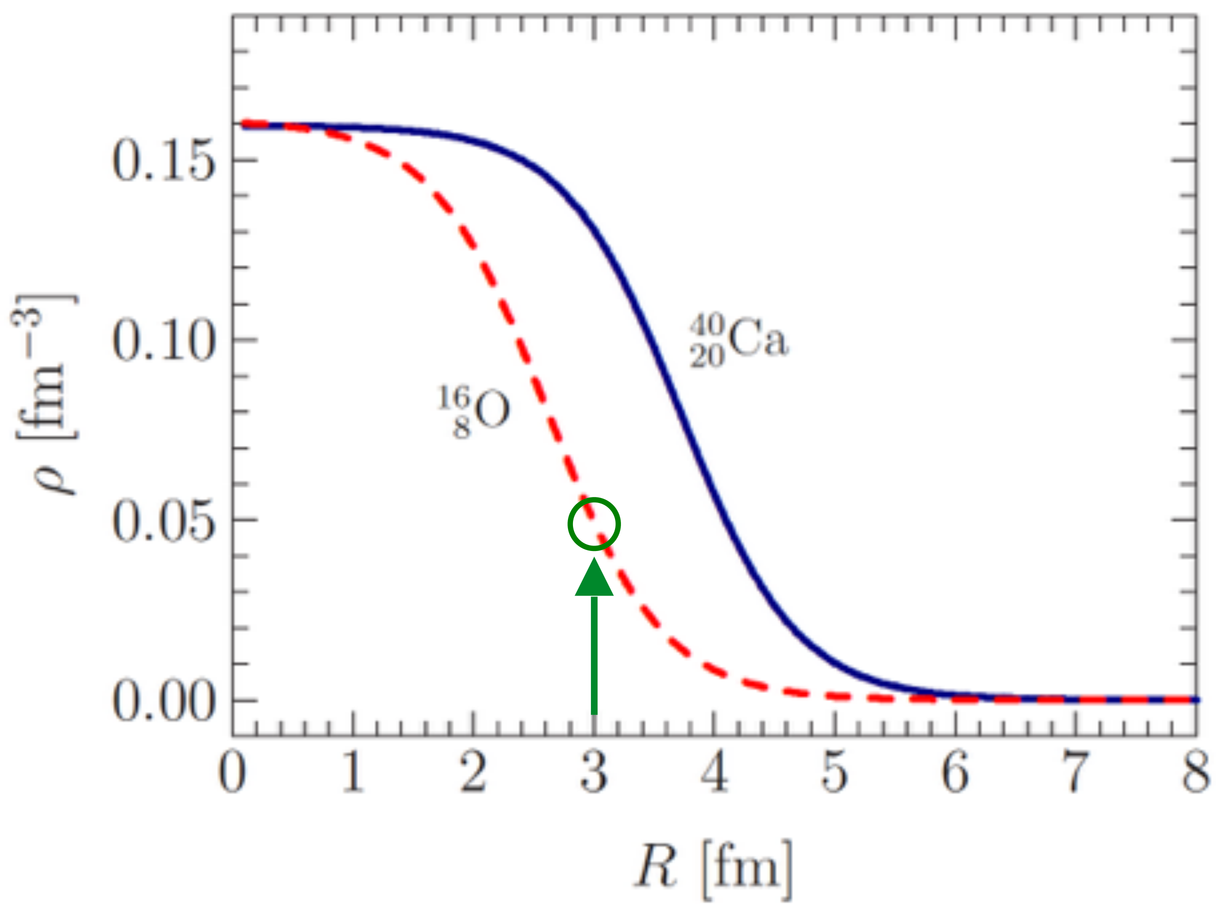
- Has the form as in the case of symmetric top

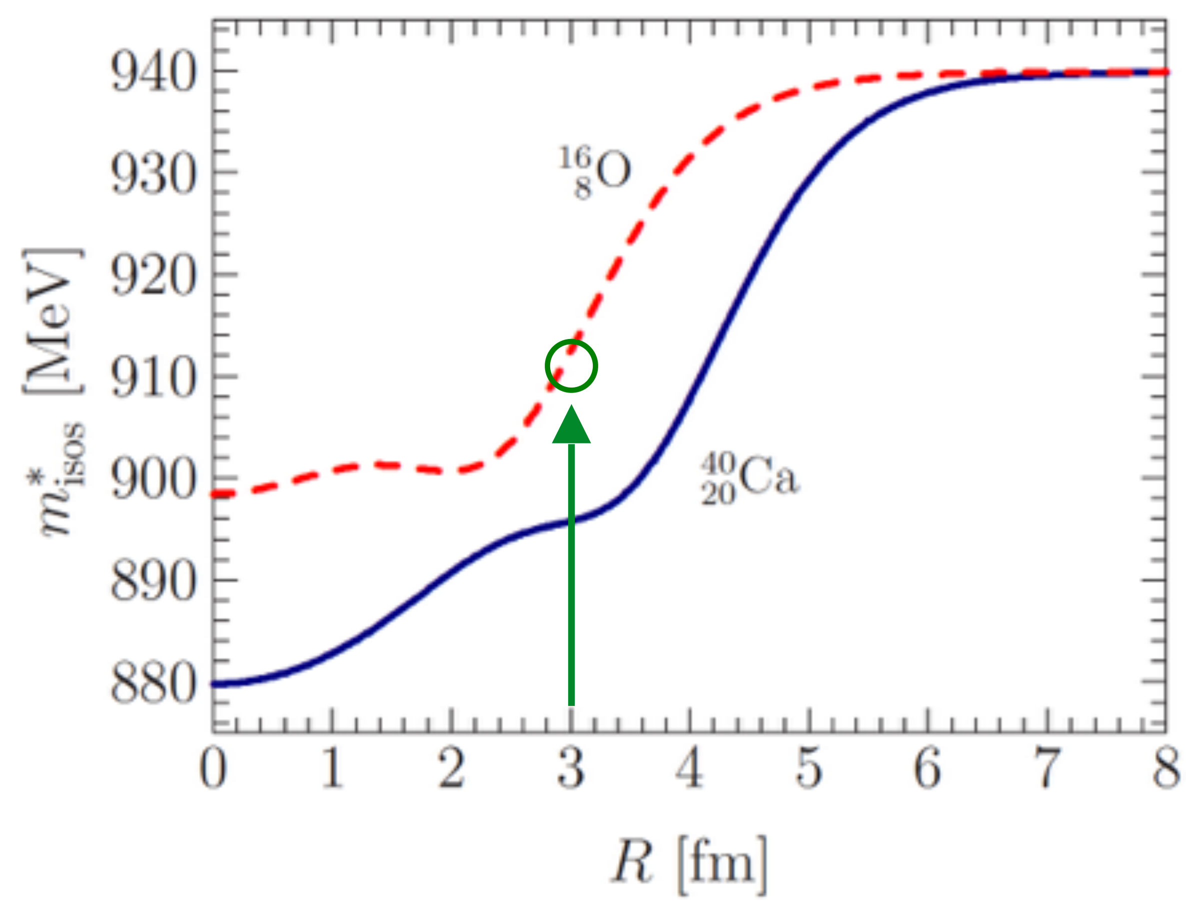
$$\hat{H} = M^*_{\mathrm{NP}} + \mathcal{M}^2_{-}\Lambda_{\mathrm{mes}} + \frac{\Lambda^{*2}_{\mathrm{env}}}{2\Lambda^*_{\omega\Omega,33}} + \frac{(\hat{T}^2_1 + \hat{T}^2_2)\Lambda^*_{\Omega\Omega,12} + (\hat{J}^2_1 + \hat{J}^2_2)\Lambda^*_{\omega\omega,12}}{2(\Lambda^*_{\omega\omega,12}\Lambda^*_{\Omega\Omega,12} - \Lambda^{*2}_{\omega\Omega,12})}$$
$$+ \frac{(\hat{T}_1\hat{J}_1 + \hat{T}_2\hat{J}_2)\Lambda^*_{\omega\Omega,12}}{\Lambda^*_{\omega\omega,12}\Lambda^*_{\Omega\Omega,12} - \Lambda^{*2}_{\omega\Omega,12}} + \frac{\hat{T}^2_3}{2\Lambda^*_{\omega\Omega,33}} - \left(a^* + \frac{\Lambda^*_{\mathrm{env}}}{\Lambda^*_{\omega\Omega,33}}\right)\hat{T}_3\,.$$

Neutron-proton mass difference in finite nuclei

Nucleon in finite nuclei

The densities of nuclei (left) and the isoscalar mass in nuclei (right)

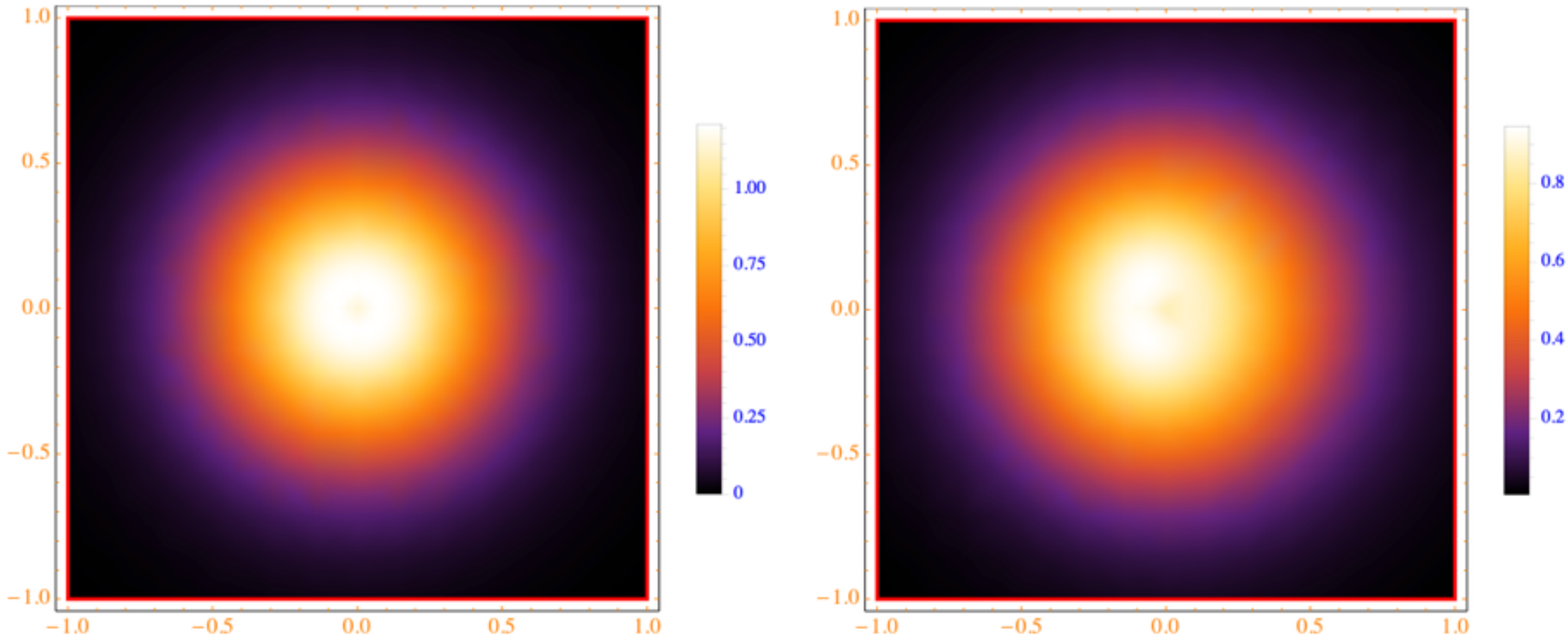




On the right panel R is a distance between the geometrical centres of nucleus and nucleon

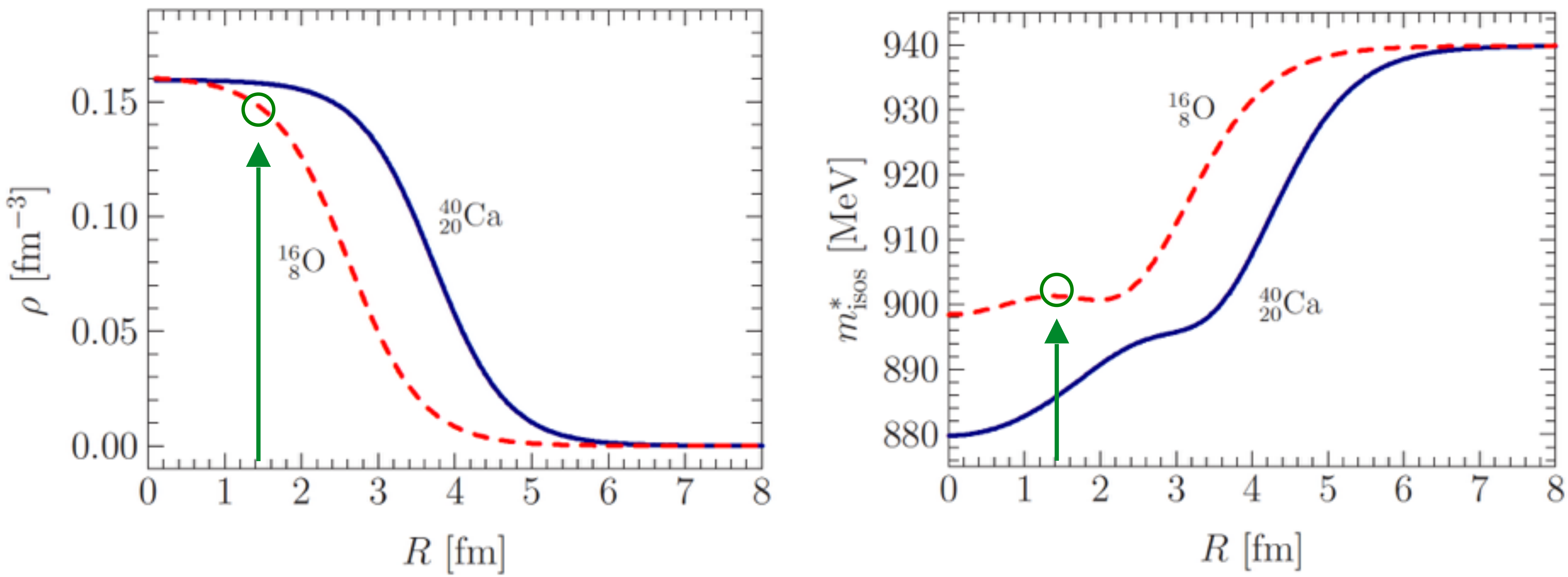
Nucleon in finite nuclei

Baryon charge distribution inside the nucleon under the consideration



In free space (left) and in O16 (right), R = 3fm

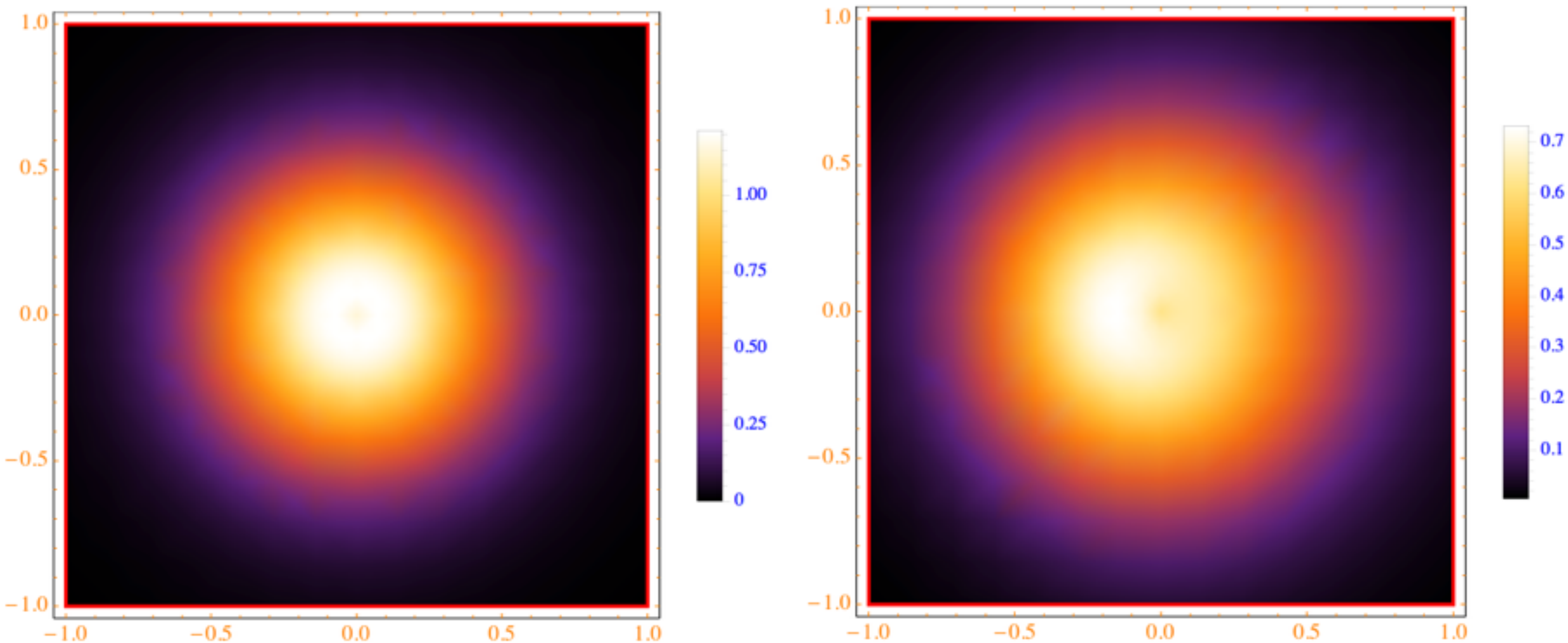
The densities of nuclei (left) and the isoscalar mass in nuclei (right)



On the right panel R is a distance between the geometrical centres of nucleus and nucleon

Nucleon in finite nuclei

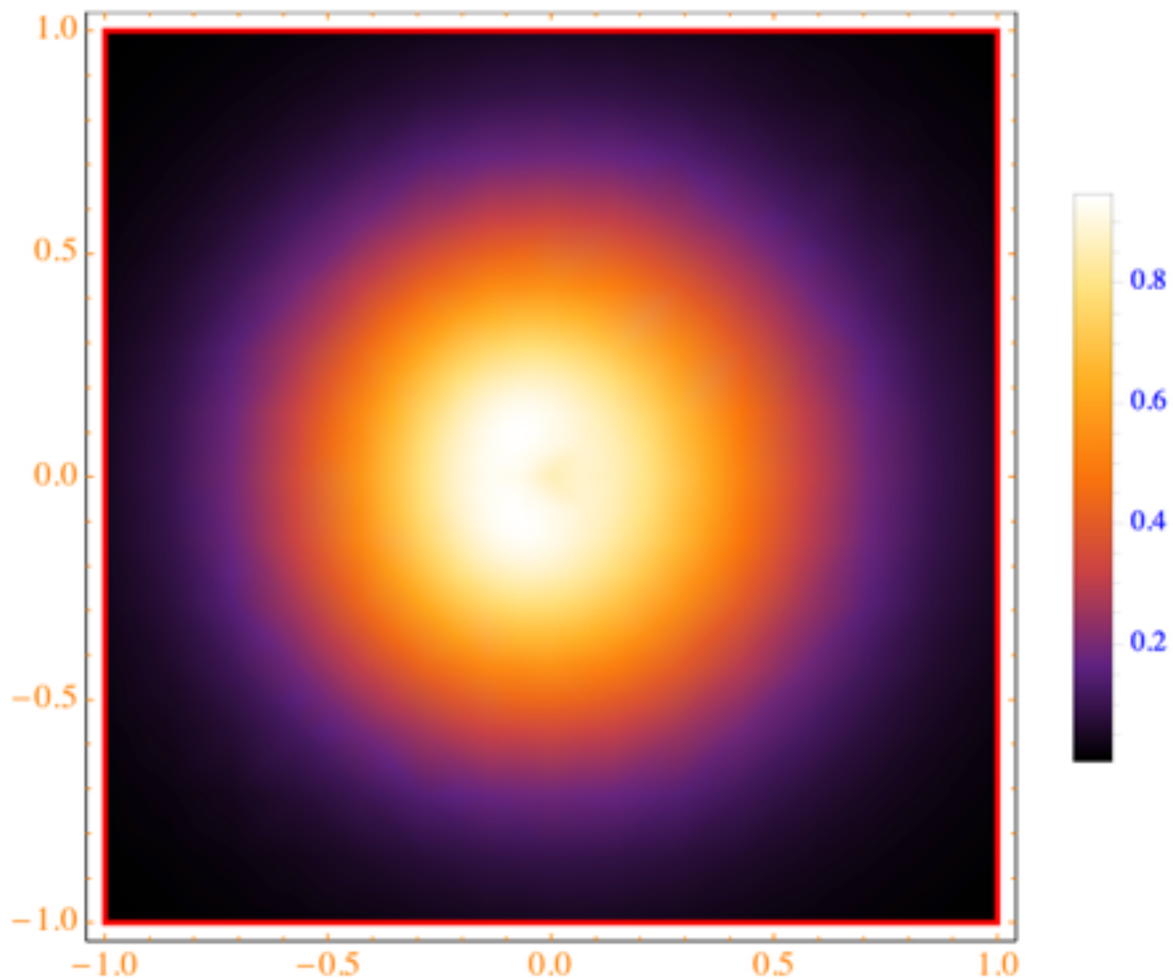
Baryon charge distribution inside the nucleon under the consideration

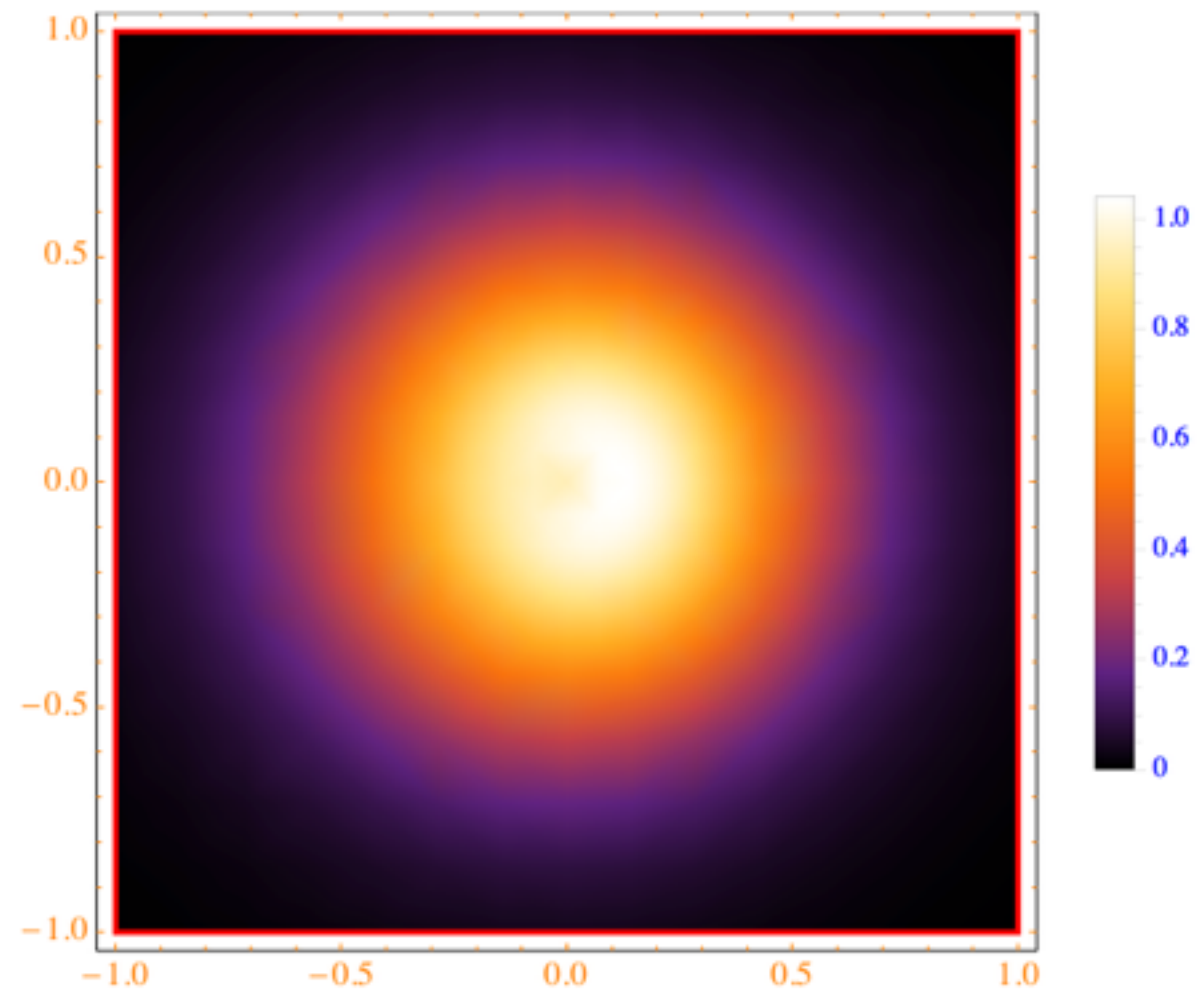


In free space (left) and in O16 (right), R = 1.5fm

Nucleon in finite nuclei

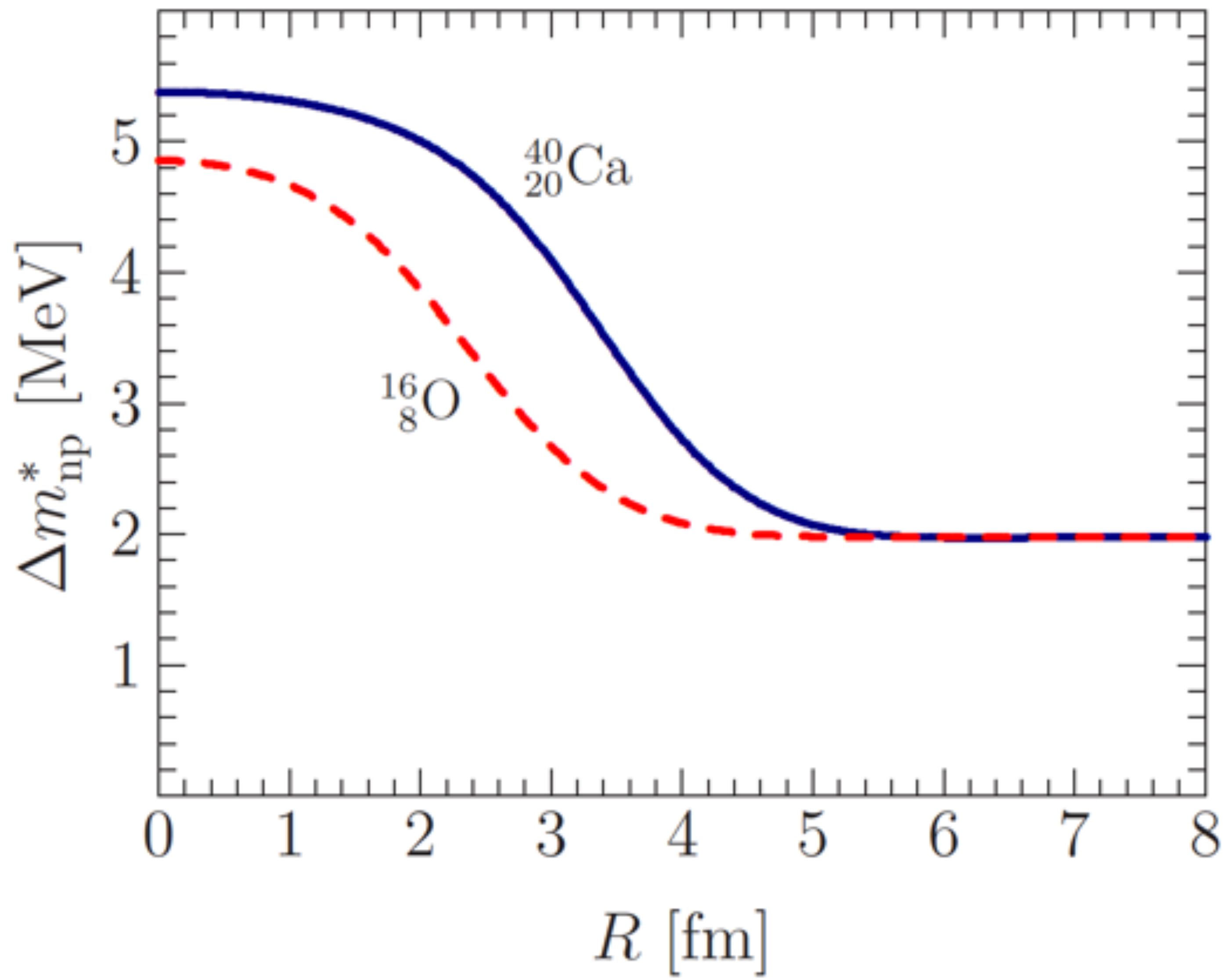
Baryon charge distribution inside the nucleon under the consideration





In O16 (left), R = 3fm and in Ca40 (right), R = 4.5fm

The neutron-proton mass difference in finite nuclei



R is a distance between the geometrical centres of nucleus and nucleon

Properties of Finite Nuclei

Mirror nuclei

The Nolen-Schiffer anomaly (NSA)

- The mass difference of mirror nuclei

$$\Delta M \equiv {}^{A}_{Z+1}\mathbf{M}_{N} - {}^{A}_{Z}\mathbf{M}_{N+1} = \Delta E_{\mathrm{EM}} - \Delta m^{*}_{\mathrm{np}}$$

- EM part was calculated with high accuracy (within 1% error) in very detailed form (e.g., the exchange term, the center-of-mass motion, finite-size effects of the proton and neutron charges, magnetic interactions, vacuum effects, the dynamical effect of the neutron-proton mass difference, and short-range two-body correlations, etc.)
- If neutron-proton mass difference is not changed in nuclear matter then the above formula cannot be satisfied.

$$\overline{\Delta}_{\mathrm{NSA}} = \Delta m_{\mathrm{np}} - \left(\Delta\overline{m}^{*(1)}_{\mathrm{np}} + \Delta\overline{m}^{*(2)}_{\mathrm{np}}\right)$$

The Nolen-Schiffer anomaly (NSA)

- Is defined as ("bar" means averaging over the R)

$$\overline{\Delta}_{\rm NSA} = \Delta m_{\rm np} - \left(\Delta\overline{m}^{*(1)}_{\rm np} + \Delta\overline{m}^{*(2)}_{\rm np}\right)$$

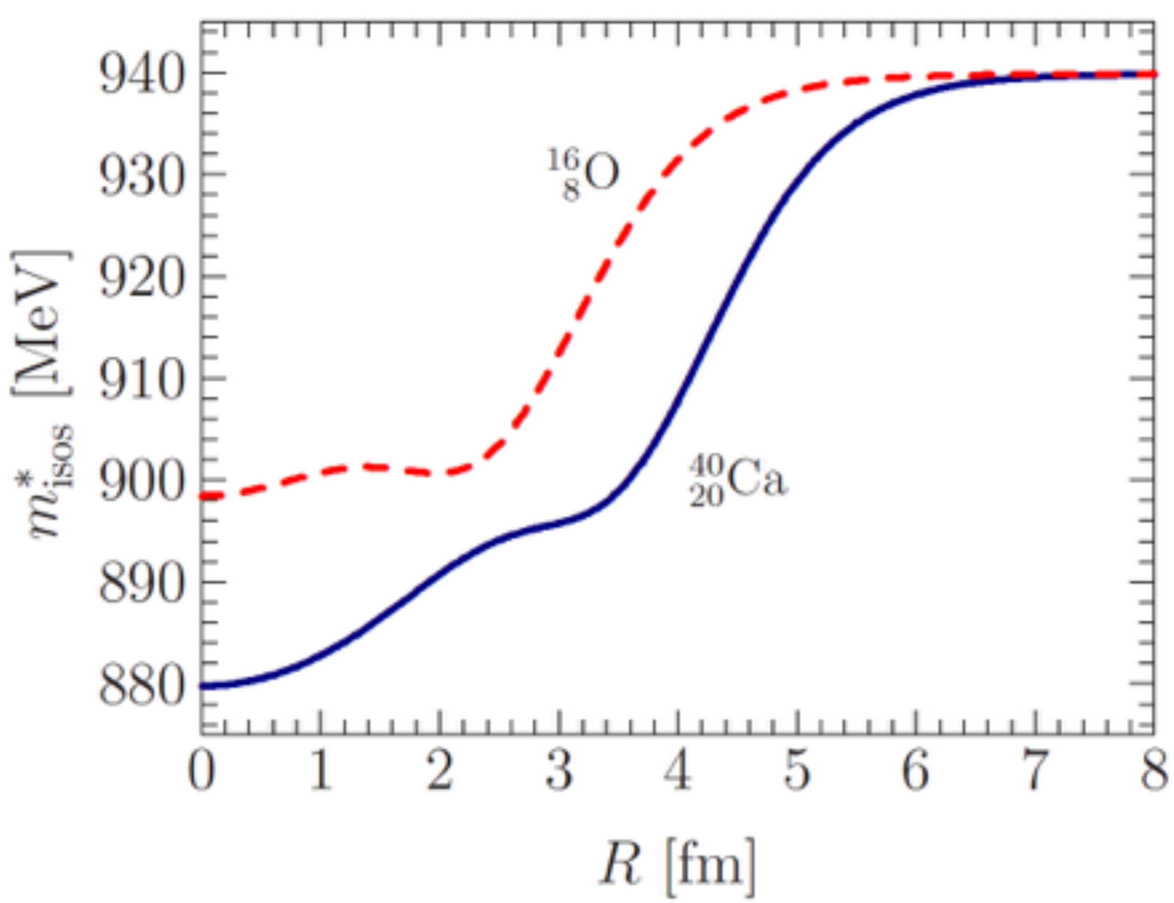
- where

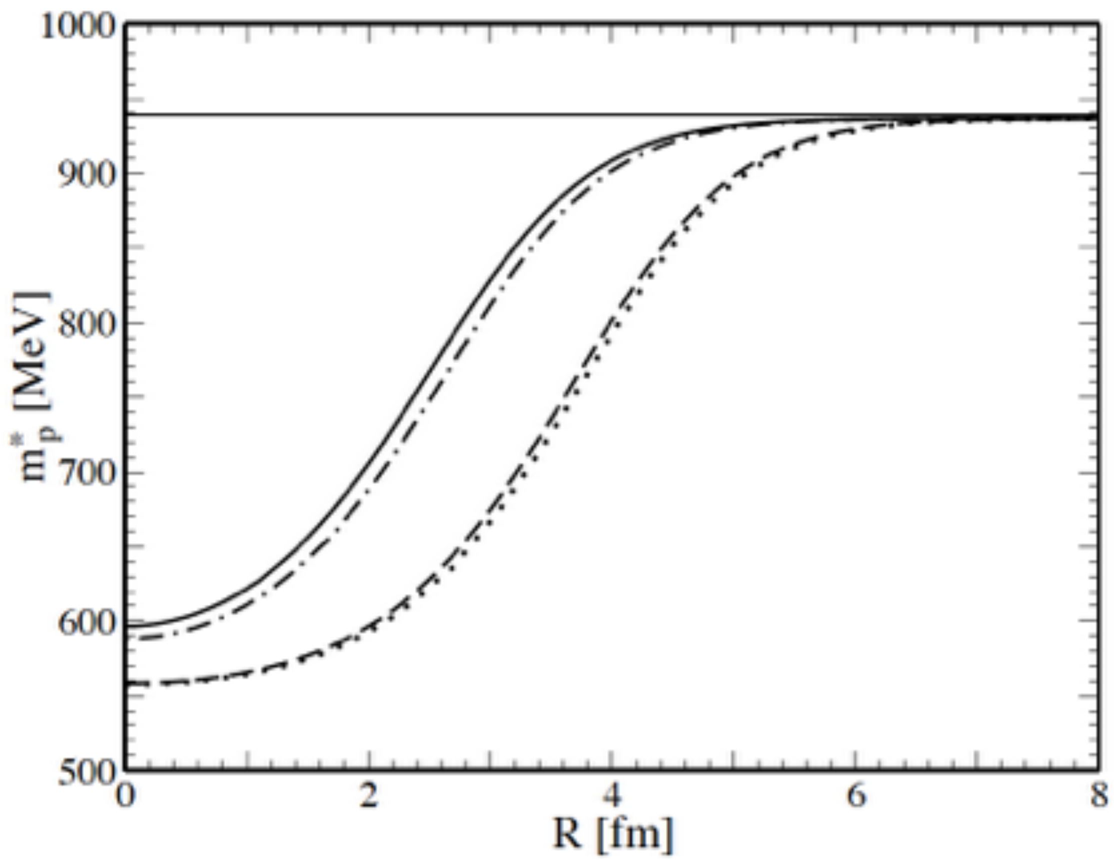
$$\Delta\overline{m}^{*}_{\rm np} \approx \int \left(\Delta\psi^{(2)}_{\rm np} m^{*}_{\rm p} + \left(\psi^{(p)}\right)^2 \Delta m^{*}_{\rm np}\right) {\rm d}^3 R$$
$$\equiv \Delta\overline{m}^{*(1)}_{\rm np} + \Delta\overline{m}^{*(2)}_{\rm np} \,,$$

Nuclei	$\overline{m}^*_{\rm p}$		Present approach						$\overline{\Delta}_{\rm NSA}$	$\overline{\Delta}_{\rm NSA}$
			$\alpha_{\rm ren}=0$			$\alpha_{\rm ren}=0.95$				
	$\alpha_{\rm ren}=0$	$\alpha_{\rm ren}=0.95$	$\Delta\overline{m}^{*(1)}_{\rm np}$	$\Delta\overline{m}^{*(2)}_{\rm np}$	$\overline{\Delta}_{\rm NSA}$	$\Delta\overline{m}^{*(1)}_{\rm np}$	$\Delta\overline{m}^{*(2)}_{\rm np}$	$\overline{\Delta}_{\rm NSA}$	ref. [16]	ref. [17]
^{15}O-^{15}N	767.45	928.30	−4.27	1.56	4.02	−0.21	1.33	0.20	–	0.16 ± 0.04
^{17}F-^{17}O	812.35	930.54	−5.53	1.52	5.33	−0.28	1.32	0.27	0.31	0.31 ± 0.04
^{39}Ca-^{39}K	724.78	926.16	−8.11	1.67	7.75	−0.41	1.33	0.37	–	0.22 ± 0.08
^{41}Sc-^{41}Ca	771.71	928.51	−9.74	1.62	9.44	−0.49	1.33	0.47	0.62	0.59 ± 0.08

U.Meissner, A.Wirzba, A.Rakhimov, UY [EPJ A36(2008)]

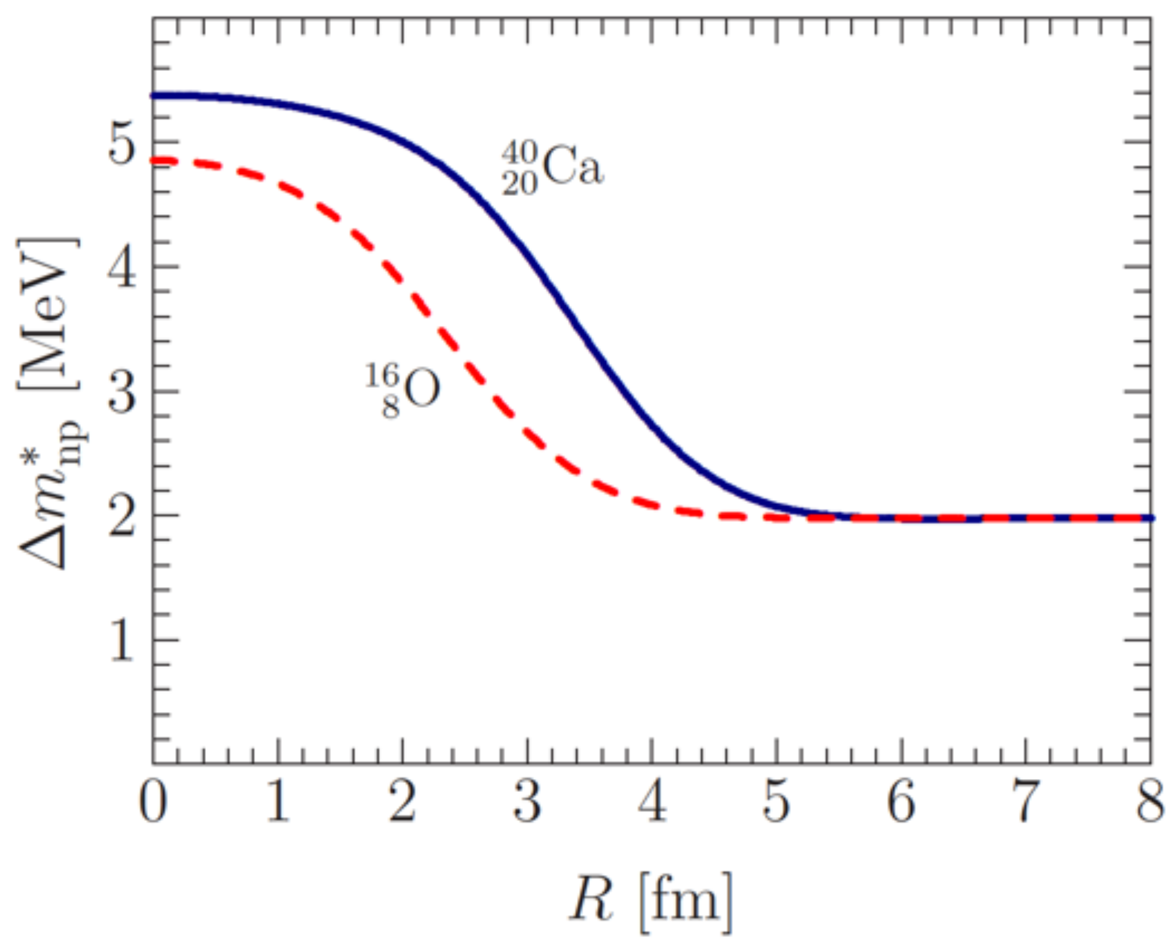
The nucleon mass in nuclei

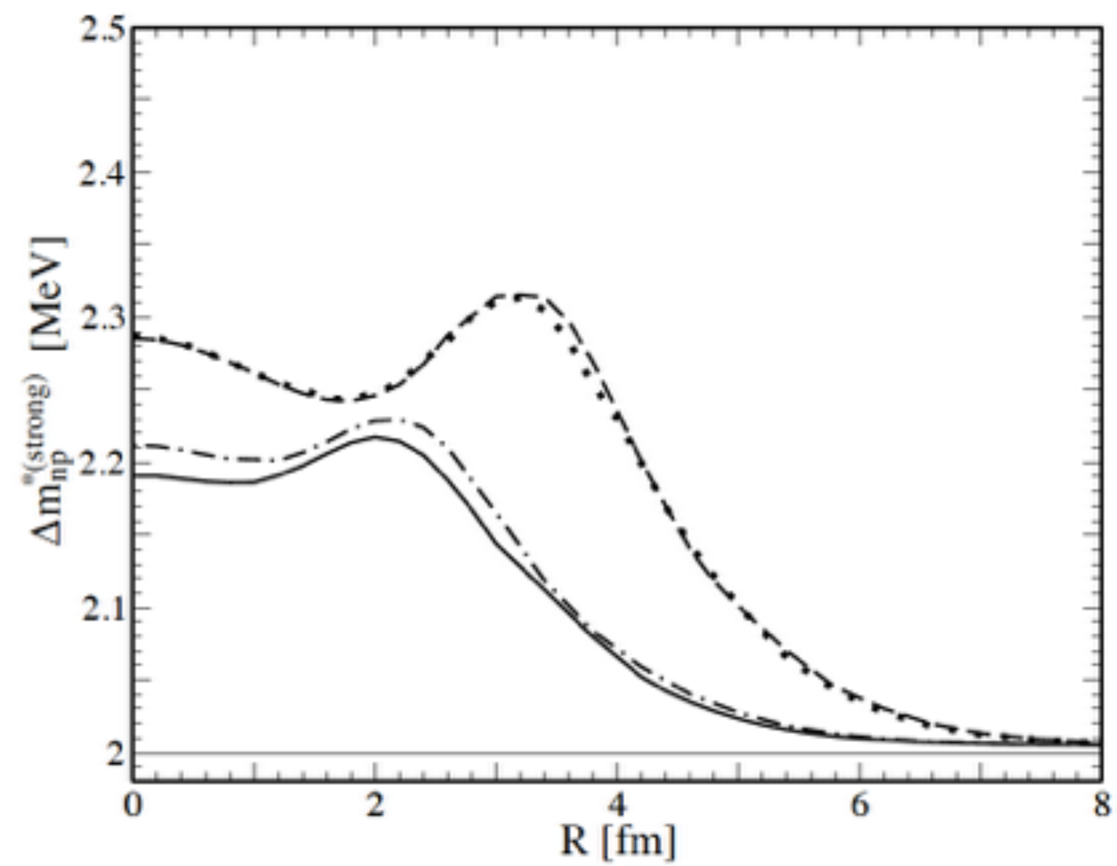




UY [PPNL 15 (2018)] (left) and U.Meissner, A.Wirzba, A.Rakhimov, UY [EPJ A36(2008)] (right)

The neutron-proton mass difference in finite nuclei





UY [PPNL 15 (2018)] (left) and U.Meissner, A.Wirzba, A.Rakhimov, UY [EPJ A36(2008)] (right)

Consistency (difference) with (from) other approaches

Consistency (difference) with (from) other approaches

One can find density functionals from the reparametrization scheme
[UY, PRC88 (2013)]

Shell modifications

$$F_{\pi,\tau} \to F^*_{\pi,\tau}, \quad F_{\pi,s} \to F^*_{\pi,s},$$

Core modifications

$$e_\tau \to e^*_\tau, \quad e_s \to e^*_s$$

$$m_\pi \to m^*_\pi,$$

- Five density dependent parameters

- Rearrangment (technical simplification)

$$1 + C_1 \frac{\rho}{\rho_0} = f_1\left(\frac{\rho}{\rho_0}\right) \equiv \sqrt{\frac{\alpha_p^0}{\gamma_s}}$$

$$1 + C_2 \frac{\rho}{\rho_0} = f_2\left(\frac{\rho}{\rho_0}\right) \equiv \frac{\alpha_s^{00}}{(\alpha_p^0)^2 \gamma_s}$$

$$1 + C_3 \frac{\rho}{\rho_0} = f_3\left(\frac{\rho}{\rho_0}\right) \equiv \frac{(\alpha_p^0 \gamma_s)^{3/2}}{\alpha_s^{02}}$$

$$\frac{\alpha_e}{\gamma_s} = f_4\left(\frac{\rho}{\rho_0}\right) \frac{\rho_n - \rho_p}{\rho_0} = \frac{C_4 \dfrac{\rho}{\rho_0}}{1 + C_5 \dfrac{\rho}{\rho_0}} \frac{\rho_n - \rho_p}{\rho_0}$$

Consistency (difference) with (from) other approaches

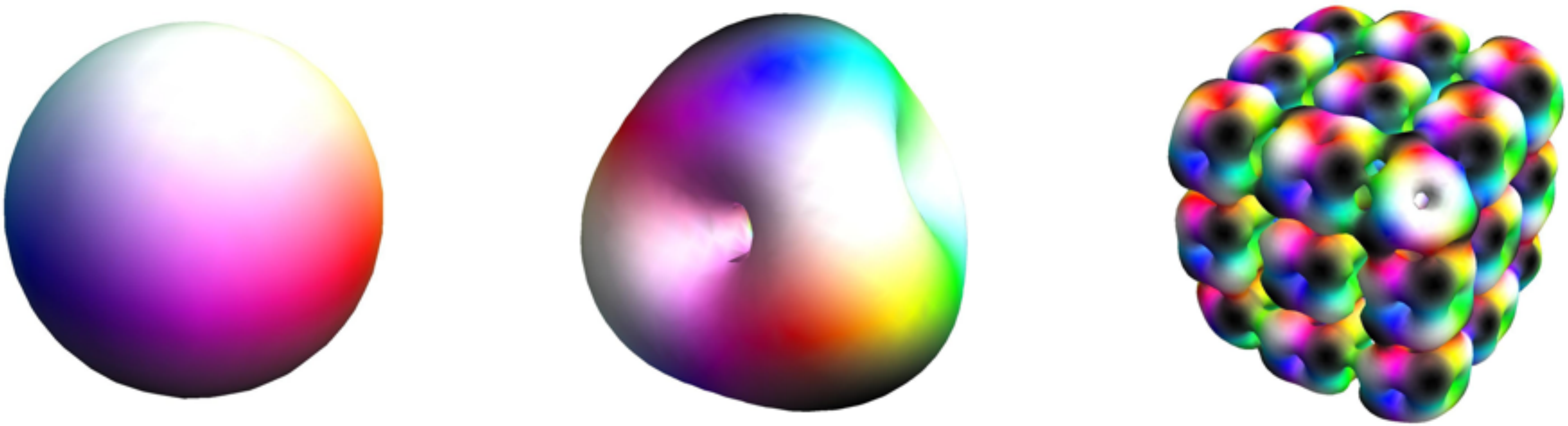
Low energy constants in nuclear at normal nuclear matter density ρ_0

	Present model	ChPT [1]	QCD sum rules [2]
$F^*_{\pi,t} / F_\pi$	0.37	0.74	0.79
$F^*_{\pi,s} / F_\pi$	0.72	< 0	0.78

[1] U. Meissner, J. Oller, A. Wirzba, Annals Phys. 297 (2002) 27.
[2] H. Kim, M. Oka, NPA720 (2003) 368.

Consistency (difference) with (from) other approaches

Surface of constant baryon density skyrmions [Feist, D.T.J. *et al.* Phys.Rev. D87 (2013)]



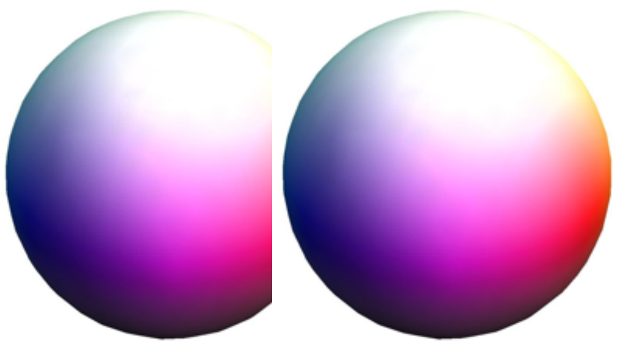
$B = 1$ $B = 3$ $B = 104$

$$\mathcal{L} = \frac{F_\pi^2}{16} \mathrm{Tr} \left(\partial_\mu U \partial^\mu U^\dagger \right) - \frac{1}{16 e^2} \mathrm{Tr} \left[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U \right]^2$$

Changes from a nucleus to a nucleus (“calibration”)

Consistency (difference) with (from) other approaches

Physically consistent picture (ansatz product)



$$B = 2$$

Well defined $\Rightarrow$

$$B = 1$$

$$B = 1$$

Overlapping at small distances

Well separated at large distances

One can reproduce SS potential and project it into NN potential.

$$U_{\text{system}} = U(\vec{r}_1)U(\vec{r}_2)$$

$$U = \exp\{i\vec{\tau}\vec{n}F(r)\}$$

$$F(0) = \pi, \qquad F(\infty) = 0$$

Consistency (difference) with (from) other approaches

Other approaches

- Classical crystalline structures
  - Cubic structure
    [M. Kutschera *et al*. Phys. Rev. Lett. **53** (1984)]
    [I. R. Klebanov, Nucl. Phys. B **262** (1985)]

  - Phase structure analysis using FCC crystal
    [H.-J. Lee *et al.* Nucl. Phys. A **723** (2003)]

- Skyrmions in hypersphere
  - System properties from the single skyrmion in hypersphere
    [N. S. Manton and P. J. Ruback, Phys. Lett. B **181** (1986)]

Summary and outlook

Summary and Outlook

The present model describes at same footing (the corresponding phenomenology always qualitatively and in several cases quantitatively too)

the single nucleon properties

- **in free space considering it as a structure-full system**
- **in nuclear matter (EM and EMT form factors)**

as well as the properties of the whole nucleonic systems

- **infinite nuclear matter properties (volume and symmetry energy properties)**
- **matter under extreme conditions (e.g. neutron stars)**
- **few/many nucleon systems (symmetric nuclei, mirror nuclei, rare isotopes, halo nuclei,…)**
- **nucleon knock-out reactions (lepton-nucleus scattering experiments)**
- **possible changes in in-medium NN interactions**
- **etc**

Thank you very much for your attention!