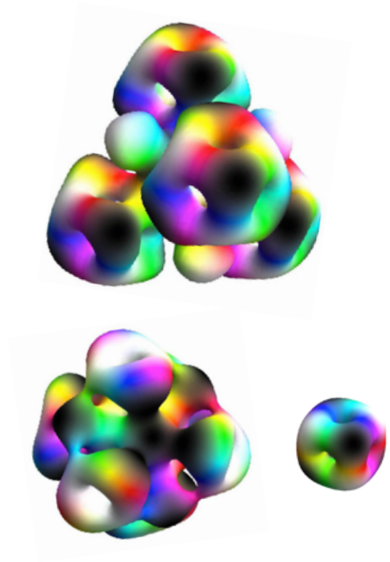
From homogeneous matter straight to finite nuclei with a natural EDF Ansatz

Panagiota Papakonstantinou

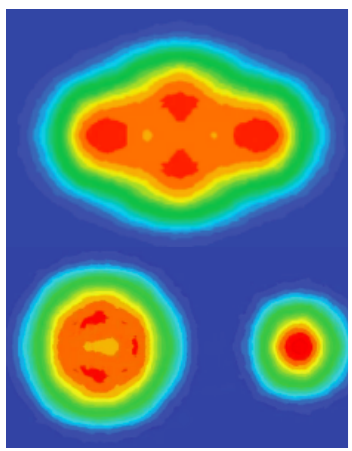
Rare Isotope Science Project – IBS Daejeon, S.Korea

A rendering of the future RAON complex, under construction in Daejeon

Skyrme



N.Manton

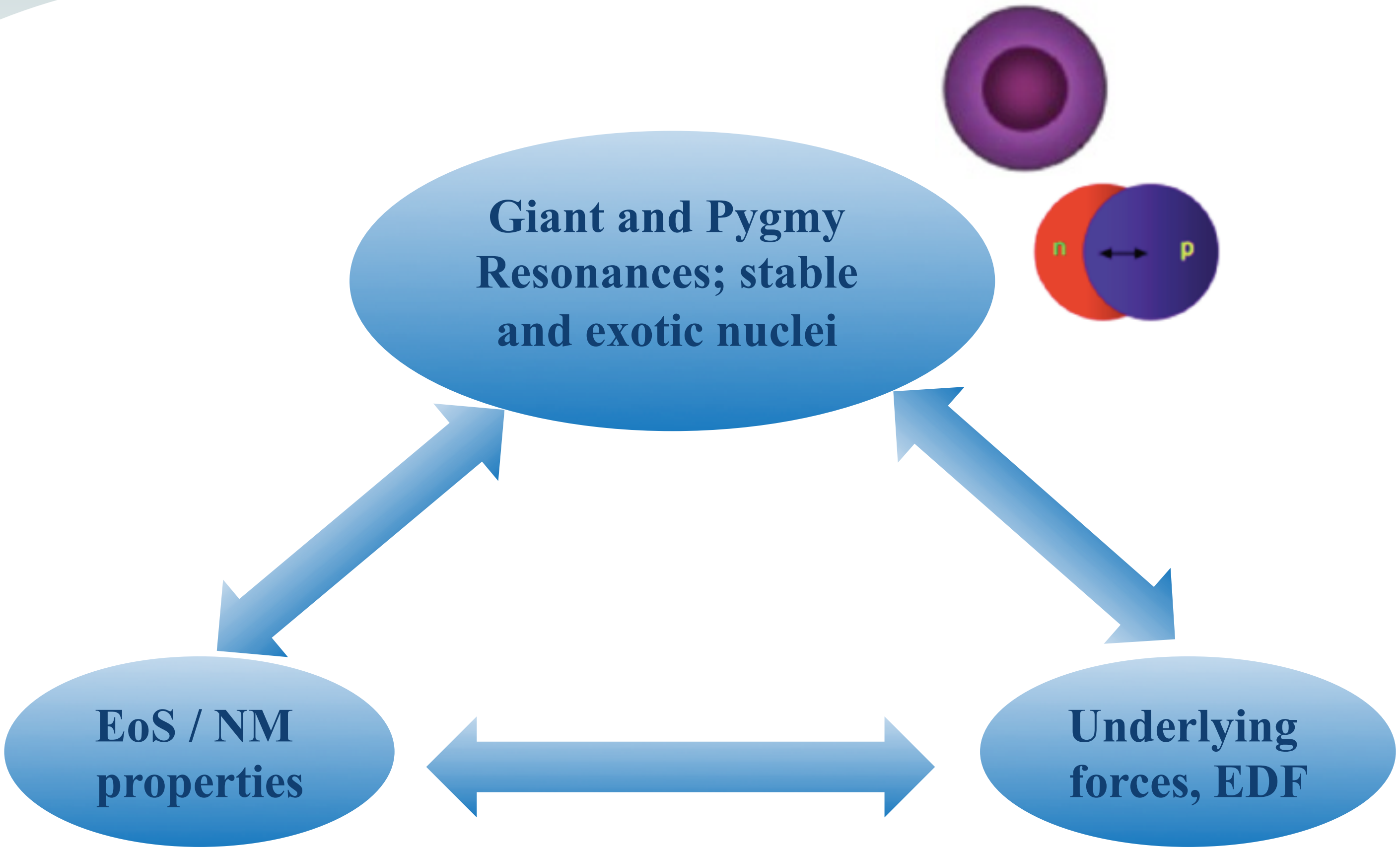


T.Nakatsukasa

Edme Mariotte

Museo Galileo

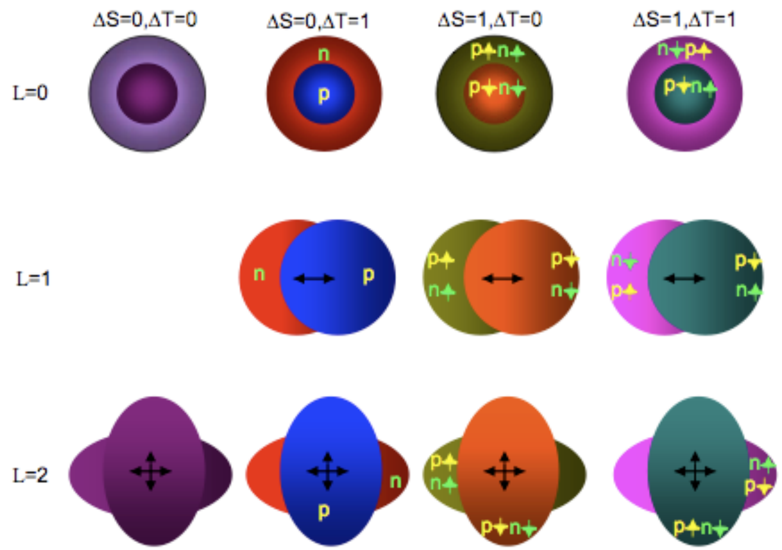
My interests and motivation



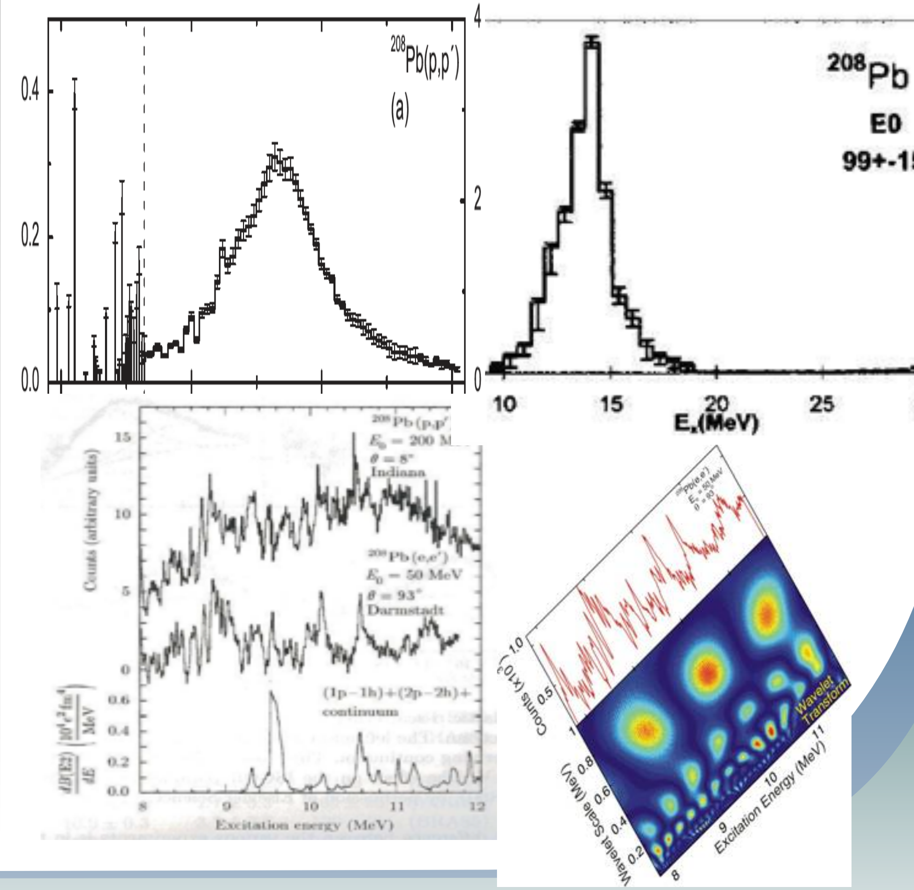
Normal modes: Giant Resonances

Excitation energy ➔Frequency (E=ħω)
Fragmented strength function ➔ Dissipation

The simplified picture



The reality



Overview

- About density-dependent "interactions"
- Motivation for the KIDS Ansatz
  - A textbook example
  - EFT of dilute matter
- Fitting in homogeneous matter
  - APR pseudodata
  - Hierarchy of terms?
  - Naturalness
- Mapping onto a Skyrme functional and applications in nuclei: **Success**!
  - *With no refitting*
- Many prospects ahead

Density-dependent "interaction"

- Original Ansatz by Skyrme [Nucl.Phys.9(1958)615]:

$$T = \sum_{i<j} t_{ij} + \sum_{i<j<k} t_{ijk}$$

$$t_{12} = \delta(\mathbf{r}_1 - \mathbf{r}_2) t(\mathbf{k}', \mathbf{k})$$

$$t_{123} = \delta(\mathbf{r}_1 - \mathbf{r}_2)\delta(\mathbf{r}_3 - \mathbf{r}_1) t_3$$

$$\begin{aligned} t(\mathbf{k}', \mathbf{k}) = & \, t_0(1 + x_0 P^\sigma) + \tfrac{1}{2} t_1 (1 + x_1 P^\sigma)(\mathbf{k}'^2 + \mathbf{k}^2) \\ & + t_2[1 + x_2(P^\sigma - \tfrac{4}{5})]\mathbf{k}' \cdot \mathbf{k} \\ & + \tfrac{1}{2} T[\boldsymbol{\sigma}_1 \cdot \mathbf{k} \boldsymbol{\sigma}_2 \cdot \mathbf{k} - \tfrac{1}{3} \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \mathbf{k}^2 + \text{conj.}] \\ & + \tfrac{1}{2} U[\boldsymbol{\sigma}_1 \cdot \mathbf{k}' \boldsymbol{\sigma}_2 \cdot \mathbf{k} - \tfrac{1}{3} \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \mathbf{k}' \cdot \mathbf{k} + \text{conj.}] \\ & + V[i(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \mathbf{k}' \times \mathbf{k}], \end{aligned}$$

- t_{123} term equivalent to a density-dependent t_{12} term

$$\frac{t}{6}(1 + P_\sigma)\rho[(\mathbf{r}_1 + \mathbf{r}_2)/2]\delta(\mathbf{r}_1 - \mathbf{r}_2)$$ [Vautherin&Brink,PRC5(1972)]

- Extension: fractional-power density dependence

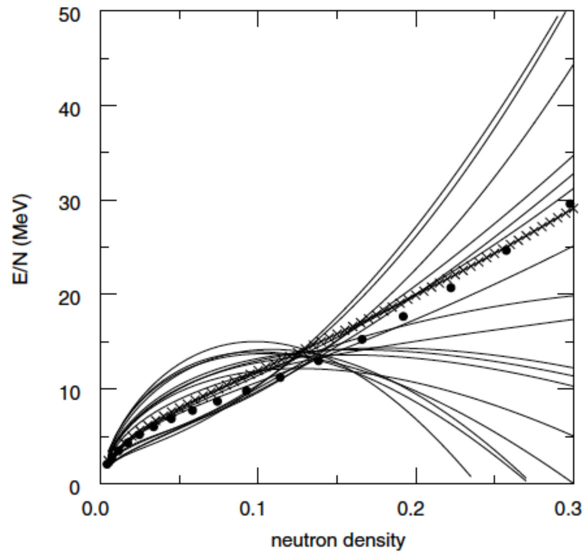
$$\frac{t}{6}(1 + P_\sigma)\rho^\alpha[(\mathbf{r}_1 + \mathbf{r}_2)/2]\delta(\mathbf{r}_1 - \mathbf{r}_2)$$

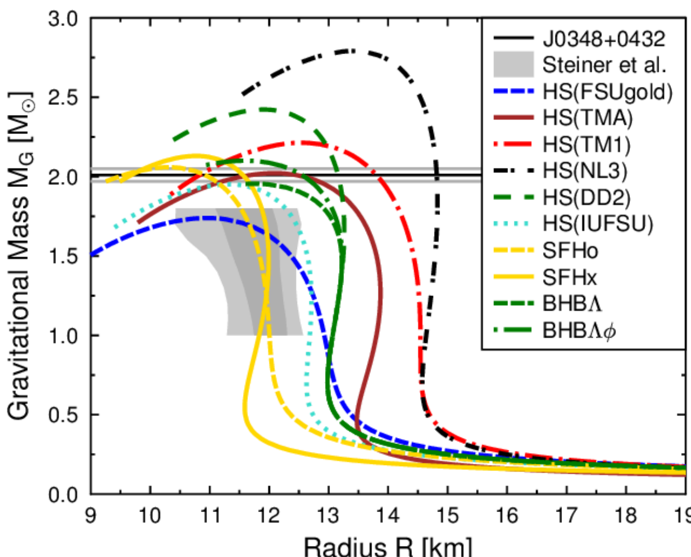
  - Explosion of activity!
  - Gogny-type forces: similar term

Phenomenological energy-density functionals

❖ Hundreds of EDF models for nuclei and nuclear matter
- Typically ~10 parameters fitted to nuclear properties using different data sets and fitting protocols
- Very different predictions below and above ρ_0
- Very different predictions at large isospin asymmetries
  - [cf Dutra et al., PRC85(2012)035201]

B.A.Brown, PRL85 (2000)

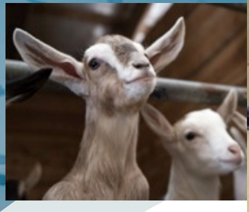




Density-dependent "interaction"

- Many questions:
  - **What should the fraction be?**
    - Precise value often chosen arbitrarily
  - Do we need more than one density-dependent couplings?
  - More terms always provide better fits... but they still risk loss of **predictive power**
  - Is there any guidance *before* we start cumbersome fitting?

Our answer so far:
- Low-order powers of $\boldsymbol{\rho^{1/3}}$
- More than one powers necessary
- **SNM and PNM have different "preferences"**



❖ Chang Ho Hyun, Daegu University
❖ Tae-Sun Park, SKKU
❖ Yeunhwan Lim, IBS (now in Texas)
  - Korea
  - IBS (that's me and YHL)
  - Daegu
  - SKKU

❖ Hana Gil, Kyungpook National University
❖ Yongseok Oh, Kyungpook National University
❖ Gilho Ahn, University of Athens, Greece
❖ Young-Min Kim (UNIST) .....

Interaction vs energy-density functional

RAON

- The elementary entity is the energy density (or energy per particle) as a unique functional of the density $E[\rho(\vec{r}),\delta(\vec{r})]$ [*]
  - Mapping as per Hohenberg-Kohn
  - The function E[ρ,..] is a **black box**
- The "interaction" which, in an orbital basis, yields the correct E[ρ,..] is an **auxiliary entity with no immediate connection to an on-shell interaction**
- Density-dependent couplings in the "interaction" **arise even in the absence of three-nucleon interactions** – fundamental requirement

[*] or $E[\rho(\vec{r}),s(\vec{r}),\delta(\vec{r})]$

Very dilute Fermi system

To our knowledge, the most complete low-density expansion for the ground-state energy per particle of such a system is [13–16]:

$$\frac{E}{N}=\frac{k_{\mathrm{F}}^2}{2M}\left[\frac{3}{5}+(g-1)\left\{\frac{2}{3\pi}(k_{\mathrm{F}}a_s)+\frac{4}{35\pi^2}(11-2\ln 2)(k_{\mathrm{F}}a_s)^2+\frac{1}{10\pi}(k_{\mathrm{F}}r_s)(k_{\mathrm{F}}a_s)^2 + \big(0.076+0.057(g-3)\big)(k_{\mathrm{F}}a_s)^3\right\}+(g+1)\frac{1}{5\pi}(k_{\mathrm{F}}a_p)^3 + (g-1)(g-2)\frac{16}{27\pi^3}\big(4\pi-3\sqrt{3}\big)(k_{\mathrm{F}}a_s)^4\ln(k_{\mathrm{F}}a_s)+\cdots\right]. \tag{1}$$

In Eq. (1), a_s and r_s are the s-wave scattering length and effective range, and a_p is the p-wave scattering length. The spin degeneracy is denoted by g. For a natural system, this is an expansion in Fermi momentum k_{F} over the scale Λ. The mean-field correction of $\mathcal{O}(k_{\mathrm{F}}^3)$ dates from 1929 [17], the $\mathcal{O}(k_{\mathrm{F}}^4)$ correction from the 1950's [18,19], while the $\mathcal{O}(k_{\mathrm{F}}^5)$ corrections and the logarithm were found in the 1960's [20]. The complete expression in Eq. (1) has been derived using the method of correlation functions [13,14], by expanding Goldstone diagrams [15,16], and by expanding Feynman diagrams [16]. Here we rederive and illuminate this result using EFT methods.

H.-W. Hammer, R.J. Furnstahl / Nuclear Physics A 678 (2000) 277–294

Functional vs "interaction"

- Any term of E/A ~ ρ^{1+a} can be generted by a density-dependent zero-range "interaction" ~$\rho^a\delta(r_{12})$
- More generally, any term of E/A ~ f(ρ) can be generted by a density-dependent "interaction" ~ f(ρ)/ρ

- Plus asymmetry depenence: exchange term

We will determine an Ansatz for EDF

We will fix everything in homogeneous matter

- Statistical analysis: how many terms **do** we need?

Nuclei will give us the unconstrained parameters:

- Effective masses and spin-orbit force

Guessing the EDF form part I: Brueckner theory

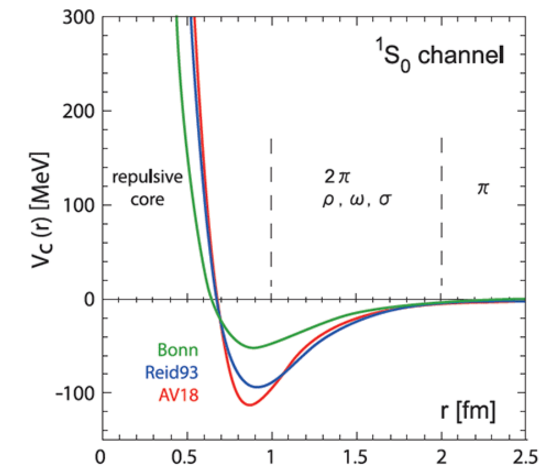
Fetter and Walecka, "Quantum theory of many-particle systems"

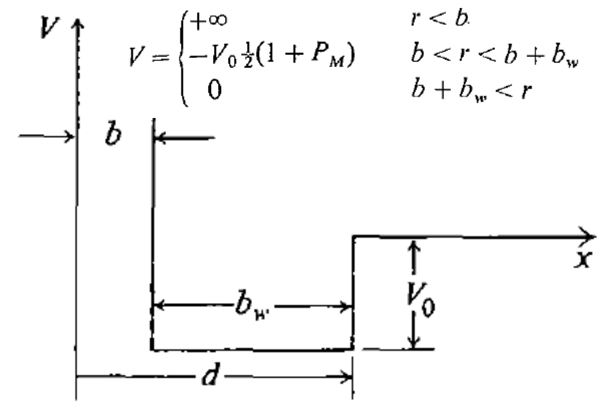
- Realistic potential: strong repulsive core plus attraction at longer range
- Apply Brueckner methodology in the calculation of nuclear matter energy

➔ Result: k_F^2, k_F^3, k_F^4, k_F^5, k_F^6, …

- Even powers: from repulsive part
- Odd powers: from both

➔ The Fermi momentum is the relevant variable : **powers of $\rho^{1/3}$**





Guessing the EDF form part II: effective field theory

- Saturation density is low...
  - with respect to (effective) boson exchange range (?)
    - one-pion exchange: vanishing expectation value
    - next boson: rho with m_ρ~775MeV~4fm^{-1}
  - Effective Lagrangian in powers of $\mathbf{k_F/m_\rho}$
- Expansion of E/A in powers of k_F
  - ... which means, again, powers of $\rho^{1/3}$
  - The Fermi momentum as the relevant variable
  - k_F^3 and k_F^4 (i.e., coupling~$\rho^{1/3}$) known to be important for obtaining saturation [Kaiser et al.,NPA697(2002)]
- Dilute Fermi gas: plus logarithmic terms

Very dilute Fermi system

To our knowledge, the most complete low-density expansion for the ground-state energy per particle of such a system is [13–16]:

$$\frac{E}{N} = \frac{k_F^2}{2M}\Bigg[\frac{3}{5} + (g-1)\Bigg\{\frac{2}{3\pi}(k_F a_s) + \frac{4}{35\pi^2}(11 - 2\ln 2)(k_F a_s)^2 + \frac{1}{10\pi}(k_F r_s)(k_F a_s)^2 + \big(0.076 + 0.057(g-3)\big)(k_F a_s)^3\Bigg\} + (g+1)\frac{1}{5\pi}(k_F a_p)^3 + (g-1)(g-2)\frac{16}{27\pi^3}\big(4\pi - 3\sqrt{3}\big)(k_F a_s)^4 \ln(k_F a_s) + \cdots\Bigg]. \quad (1)$$

In Eq. (1), a_s and r_s are the s-wave scattering length and effective range, and a_p is the p-wave scattering length. The spin degeneracy is denoted by g. For a natural system, this is an expansion in Fermi momentum k_F over the scale Λ. The mean-field correction of $\mathcal{O}(k_F^3)$ dates from 1929 [17], the $\mathcal{O}(k_F^4)$ correction from the 1950's [18,19], while the $\mathcal{O}(k_F^5)$ corrections and the logarithm were found in the 1960's [20]. The complete expression in Eq. (1) has been derived using the method of correlation functions [13,14], by expanding Goldstone diagrams [15,16], and by expanding Feynman diagrams [16]. Here we rederive and illuminate this result using EFT methods.

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NUCLEAR ENERGY DENSITY FUNCTIONAL FOR KIDS

The Ansatz

Explore and fix homogeneous matter first

Map to a Skyrme interaction for nuclei

Initial EDF Ansatz

$$\mathcal{E}(\rho,\delta) = \frac{E(\rho,\delta)}{A} = \mathcal{T}(\rho.\delta) + \sum_{i=0}^{3} c_i(\delta)\rho^{1+i/3} + c_{\ln}(\delta)\rho^2 \ln[\rho/(1\text{fm}^{-3})]$$

kinetic energy: $\mathcal{T} = \mathcal{T}_p + \mathcal{T}_n\,;\; \mathcal{T}_{p,n} = \frac{3}{5}\frac{\hbar^2}{2m_{p,n}} x_{p,n}^{5/3}(3\pi^2\rho)^{2/3}\,;\; x_{p,n} \equiv \rho_{p,n}/\rho$

asymmetry: $\delta = (\rho_n - \rho_p)/\rho$

correspondence with Skyrme

Nuclear potential	Order	KIDS parameter	Skyrme parameter
$\mathcal{E}_0$	k_F^3	$c_0(\delta)$	(t_0, x_0)
$\mathcal{E}_1$	k_F^4	$c_1(\delta)$	$(t_3, x_3), \alpha = 1/3$
$\mathcal{E}_2$	k_F^5	$c_2(\delta)$	$(t_1, x_1), (t_2, x_2), (t_3', x_3'), \alpha' = 2/3$
$\mathcal{E}_3$	k_F^6	$c_3(\delta)$	$(t_3'', x_3''), \alpha'' = 1$

Objectives and procedures

What terms are most important for describing homogeneous matter? Is there a low-order expansion?

- We will fit all possible combinations of 1,2,3,4,5 terms to pseudodata and analyse the fits

Once we choose a robust set, verify:

- Are the parameters natural?

same order of magnitude?

$$\mathcal{E}_i(\rho,\delta) = c_i(\delta)\rho^{1+i/3} = \left[\left(\frac{\nu}{6\pi^2}\right)^{1+i/3} c_i(\delta) m_\rho^{2+i}\right] m_\rho \left(\frac{k_F}{m_\rho}\right)^{3+i}$$

- Can we use them in nuclei without refitting?
  - Under what conditions?

APR pseudodata and cost function

Shown: **(E-T)/A/ρ** [MeVfm^3] from

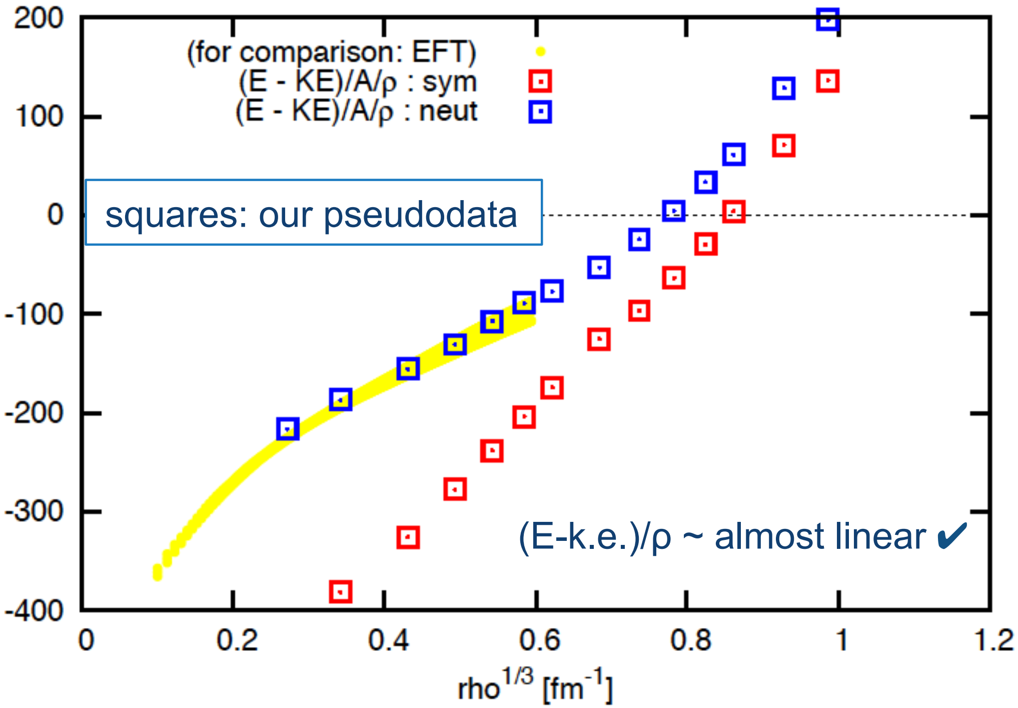
- Akmal,Pandharipande,Ravenhall, Phys Rev. C 58,1804: AV 18+Urbanna
- Drischler, Soma, Schwenk, Phys. Rev. C 89, 025806: chiral EFT (asymmetric matter)

cost function:

$$\chi^2(\delta) = \sum_j \exp\{-\beta\rho_j/\varrho_0\} \left(\frac{\mathcal{E}_j - D_j}{T_j}\right)^2 ; \quad \beta \geq 0$$

normalized:

$$\chi_n^2(\delta) = \chi^2(\delta)\left[\sum_j \exp\{-\beta\rho_j/\varrho_0\}\right]^{-1}$$



31 total combinations of:

- 1 term only
- 2 terms
- ...
- 5 terms

31 fits for PNM and 31 for SNM

Hierarchy of powers ✔

PP,Park,Lim,Hyun, Phys. Rev. C 97,014312

- For an equal number of terms (2,3,...), a combination of lower-power terms gives a better fit than a compination of higher-power terms
- Replacing power 1 with 3 gives higher χ

	$\beta = 0$		$\beta = \frac{1}{2}$		$\beta = 1$	
	SNM	PNM	SNM	PNM	SNM	PNM
$k = 0$	1.595335	0.397036	0.930742	0.171609	0.490650	0.071632
$k = 1$	1.801776	0.346198	1.527834	0.223333	1.089477	0.138133
$k = 0, 1$	0.013044	0.022028	0.003866	0.007482	0.001151	0.001566
$k = 0, 2$	0.009356	0.005804	0.012267	0.001864	0.009435	0.000719
$k = 0, 3$	0.041156	0.002160	0.047771	0.003059	0.035831	0.003220
$k = 1, 2$	0.085297	0.005936	0.108696	0.009991	0.090303	0.010973
$k = 1, 3$	0.175982	0.014031	0.216418	0.022334	0.183405	0.023312
$k = 2, 3$	0.342376	0.031821	0.440564	0.048252	0.398009	0.050970
$k = 0, 1, 2$	0.005009	0.003287	0.002588	0.001781	0.001016	0.000529
$k = 0, 2, 3$	0.006453	0.002055	0.004070	0.001540	0.001284	0.000636
$k = 1, 2, 3$	0.021528	0.005183	0.018591	0.005162	0.008571	0.003018
$k = 0, 1, \ln$	0.007486	0.007088	0.003000	0.002874	0.001025	0.000696
$k = 0, 3, \ln$	0.009117	0.002129	0.006681	0.001878	0.002380	0.000930
$k = 0, 1, 2, 3$	0.001616	0.000163	0.001731	0.000188	0.001015	0.000138
$k = 0, 1, 2, (\frac{7}{3})$	0.001420	0.000115	0.001597	0.000136	0.001016	0.000112
$k = 0, 1, 2, \ln$	0.001314	0.000094	0.001510	0.000107	0.001011	0.000092
$k = 0, 1, 2, (\frac{1}{6})$	0.002277	0.000462	0.002072	0.000415	0.000977	0.000221

Hierarchy of powers ✔

PP,Park,Lim,Hyun, Phys. Rev. C 97,014312

... Though for PNM the linear term k=3 seems even more efficient than k=1

	$\beta=0$	$\beta=\frac{1}{2}$	$\beta=1$
	SNM PNM	SNM PNM	SNM PNM
$k=0$	1.595335 0.397036	0.930742 0.171609	0.490650 0.071632
$k=1$	1.801776 0.346198	1.527834 0.223333	1.089477 0.138133
$k=0,1$	0.013044 0.022028	0.003866 0.007482	0.001151 0.001566
$k=0,2$	0.009356 0.005804	0.012267 0.001864	0.009435 0.000719
$k=0,3$	0.041156 0.002160	0.047771 0.003059	0.035831 0.003220
$k=1,2$	0.085297 0.005936	0.108696 0.009991	0.090303 0.010973
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$k=2,3$	0.342376 0.031821	0.440564 0.048252	0.398009 0.050970
$k=0,1,2$	0.005009 0.003287	0.002588 0.001781	0.001016 0.000529
$k=0,2,3$	0.006453 0.002055	0.004070 0.001540	0.001284 0.000636
$k=1,2,3$	0.021528 0.005183	0.018591 0.005162	0.008571 0.003018
$k=0,1,\ln$	0.007486 0.007088	0.003000 0.002874	0.001025 0.000696
$k=0,3,\ln$	0.009117 0.002129	0.006681 0.001878	0.002380 0.000930
$k=0,1,2,3$	0.001616 0.000163	0.001731 0.000188	0.001015 0.000138
$k=0,1,2,(\frac{7}{3})$	0.001420 0.000115	0.001597 0.000136	0.001016 0.000112
$k=0,1,2,\ln$	0.001314 0.000094	0.001510 0.000107	0.001011 0.000092
$k=0,1,2,(\frac{1}{6})$	0.002277 0.000462	0.002072 0.000415	0.000977 0.000221

Hierarchy of powers ✔

Saturation of fits at 3 terms for SNM; higher for PNM

	$\beta=0$		$\beta=\frac{1}{2}$		$\beta=1$	
	SNM	PNM	SNM	PNM	SNM	PNM
$k=0$	1.595335	0.397036	0.930742	0.171609	0.490650	0.071632
$k=1$	1.801776	0.346198	1.527834	0.223333	1.089477	0.138133
$k=0,1$	0.013044	0.022028	0.003866	0.007482	0.001151	0.001566
$k=0,2$	0.009356	0.005804	0.012267	0.001864	0.009435	0.000719
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$k=1,2$	0.085297	0.005936	0.108696	0.009991	0.090303	0.010973
$k=1,3$	0.175982	0.014031	0.216418	0.022334	0.183405	0.023312
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$k=0,1,2,3$	0.001616	0.000163	0.001731	0.000188	0.001015	0.000138
$k=0,1,2,(\frac{7}{3})$	0.001420	0.000115	0.001597	0.000136	0.001016	0.000112
$k=0,1,2,\ln$	0.001314	0.000094	0.001510	0.000107	0.001011	0.000092
$k=0,1,2,(\frac{1}{6})$	0.002277	0.000462	0.002072	0.000415	0.000977	0.000221

A statistical analysis with two sets of pseudodata (APR,FP) indicated that a higher number of terms would lead to overfitti ng (stiff vs sloppy parameters)

PP,Park,Lim,Hyun, Phys. Rev. C 97,014312

Fitting results

SNM from fits

SNM from ρ_0, E_0, K_{inf}, (ad-1: and m*)

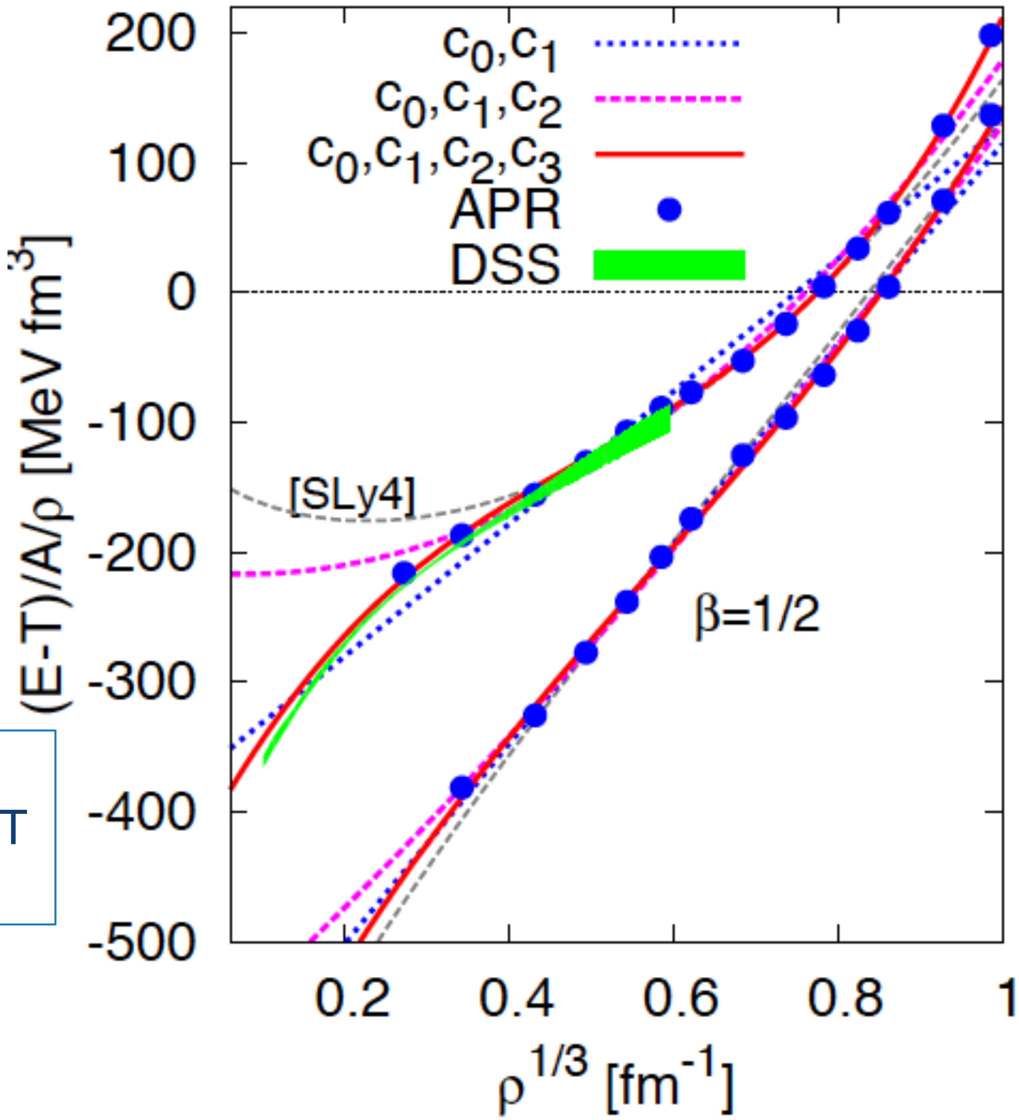
β	Matter	c_0	c_1	c_2	c_3	ϱ_0	$\mathcal{E}_0$ / J	K_∞ / L
0	SNM	-863.36	1945.05	-2060.20	1129.96	0.178	-15.4	215
	PNM	-483.96	1433.54	-2119.68	1385.22		34.2	55.9
$\frac{1}{2}$	SNM	-753.98	1389.20	-1171.03	678.87	0.177	-15.8	234
	PNM	-451.91	1254.32	-1812.62	1221.33		34.4	56.0
1	SNM	-613.13	620.22	154.72	-46.05	0.171	-16.1	247
	PNM	-408.56	991.76	-1323.81	937.96		34.0	54.9
ad-1	SNM	-648.72	676.25	200.92	-98.73	0.160	-16.0	240
	PNM	-451.91	1254.32	-1812.62	1221.33		32.8	47.9
ad-2	SNM	-664.52	763.55	40.13	0.00	0.160	-16.0	240
	PNM	-411.13	1007.78	-1354.64	956.47		33.5	50.5

- c_0,c_1 robust
- For SNM, also c_2, c_3

Fitting results

The data do show a roughly linear dependence on $\rho^{1/3}$

"DSS":
Comparison (not fitting) to χEFT
[Drischler et al., PRC89(2014)]



PP,Park,Lim,Hyun, Phys. Rev. C 97,014312

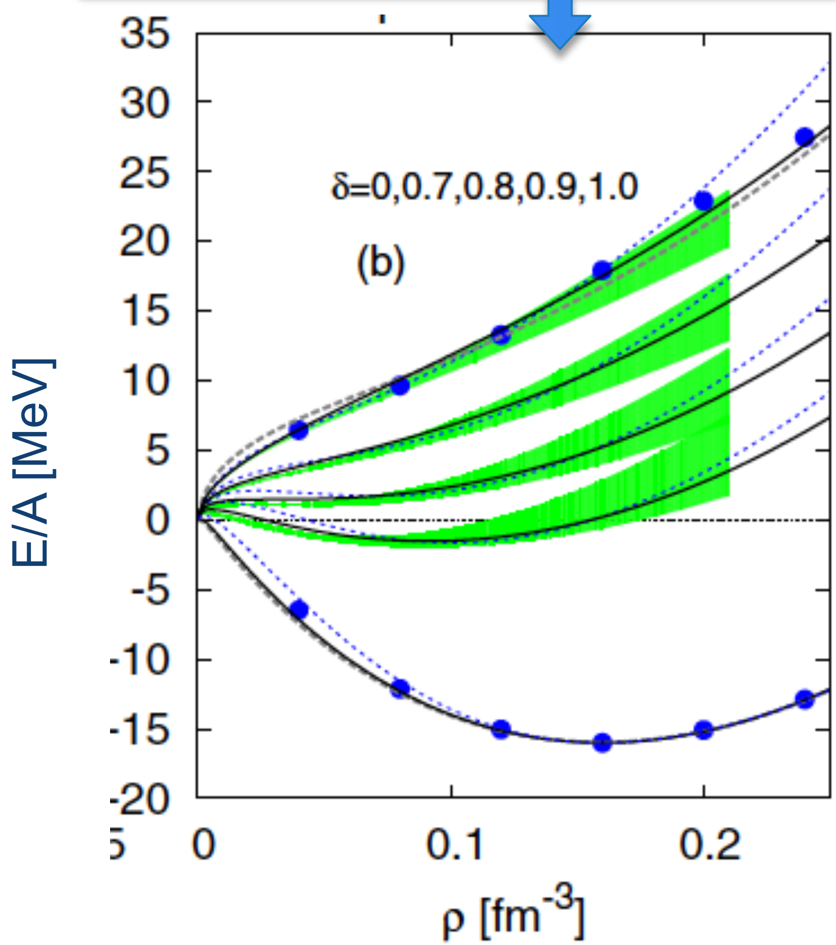
Parameter set KIDS-ad2

- Symmetric nuclear matter:
  - Set ρ_0=0.16 fm -3, E_0=-16MeV, K_0 = 240 MeV
  - Determine $c_{0,1,2}(0)$ (analytical expressions)
  - Leads to Q_0=-373 MeV
- Pure neutron matter:
  - Fit $c_{0,1,2,3}(1)$ to the APR pseudodata for PNM
  - Resulting symmetry-energy parameters:

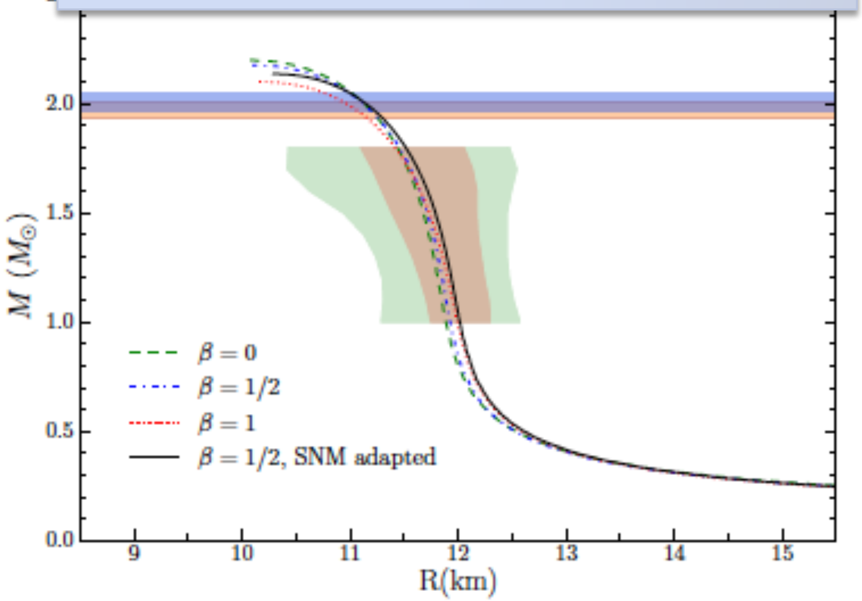
J=33MeV, L=49MeV, K_{sym}=-157MeV, Q_{sym}=586MeV

Interpolations and extrapolations

Calculations with chiral interactions reproduced, although they were not used for fitting

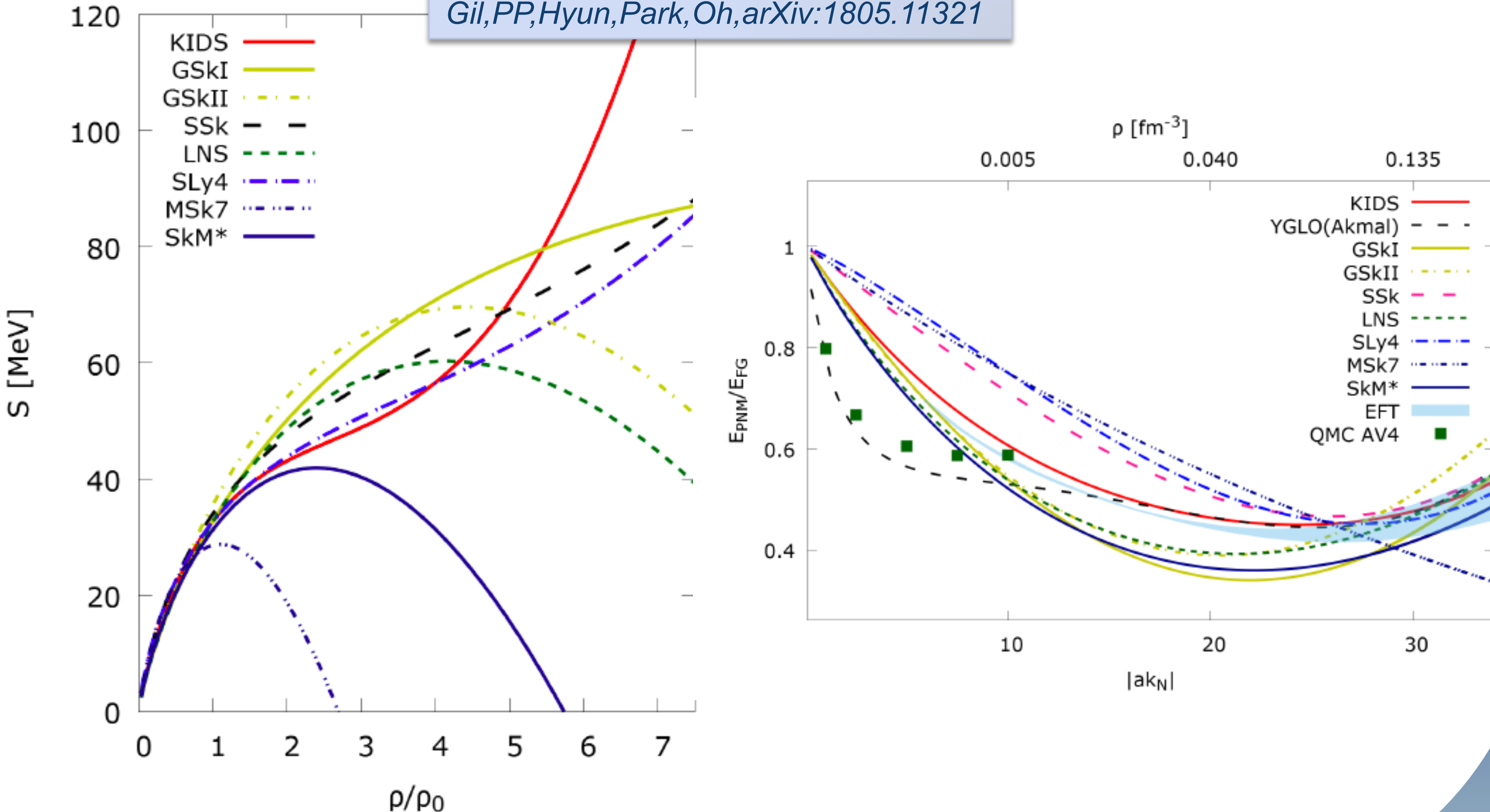


PP,Park,Lim,Hyun,Phys. Rev. C 97,014312



Comparisons with other models

Gil,PP,Hyun,Park,Oh,arXiv:1805.11321



Objectives and procedures

What terms are most important for describing homogeneous matter?

- We will fit all possible combinations of 1,2,3,4,5 terms to the APR pseudodata and analyse the fits

Once we choose a robust set, verify:

- Are the parameters natural?

same order of magnitude?

$$\mathcal{E}_i(\rho,\delta) = c_i(\delta)\rho^{1+i/3} = \left[\left(\frac{\nu}{6\pi^2}\right)^{1+i/3} c_i(\delta) m_\rho^{2+i}\right] m_\rho \left(\frac{k_F}{m_\rho}\right)^{3+i}$$

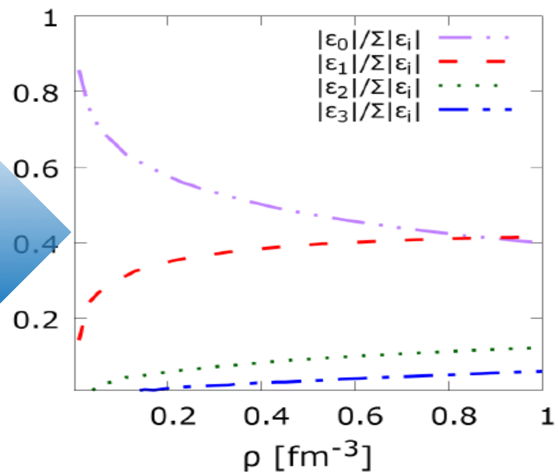
- Can we use them in nuclei without refitting?
  - Under what conditions?

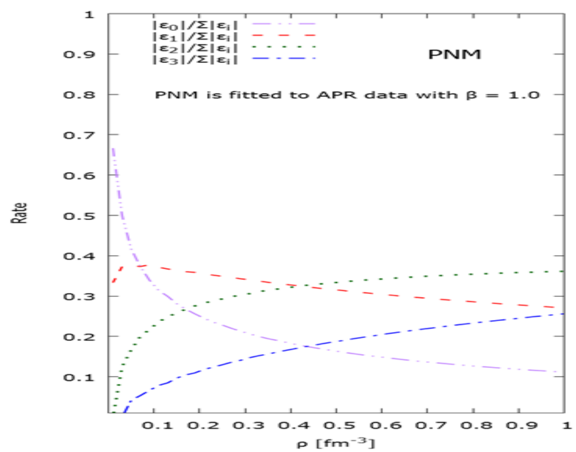
Power hierarchy and naturalness

❖ Fermi momentum calculus and power hierarchy:

- ✓ $|E_0| > |E_1| > |E_2| > |E_3|$ within a large density range
- ✓ **For SNM up to ~1fm^{-3}**, for PNM up to **0.05**fm^{-3}.

E.g. from the β =1 fits to APR





adopted "ad-2" set

❖ Naturalness?

- ❖ SNM: $c_0^{dim} = -3.6, \quad c_1^{dim} = 6.6, \quad c_2^{dim} = 0.6$
- ❖ PNM: $c_0^{dim} = -1.1, \quad c_1^{dim} = 3.4, \quad c_2^{dim} = -5.9, \quad c_3^{dim} = 5.3$

“Natural” Ansatz

- At the very least: reproduce homogeneous matter (to the best of our knowledge)
- Better: based on a power expansion
  - Underlying EFT??
- Best: coefficients showing naturalness

Objectives and procedures

What terms are most important for describing homogeneous matter?

- We will fit all possible combinations of 1,2,3,4,5 terms to the APR pseudodata and analyse the fits

Once we choose a robust set, verify:

- Are the parameters natural?

same order of magnitude?

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- **Can we use them in nuclei without refitting?**

PROOF OF PRINCIPLE: APR TAKEN TO NUCLEI

Skyrme parameters by reverse engineering

$$v_{i,j} = (t_0 + y_0 P_\sigma)\delta(\boldsymbol{r}_{ij}) + \frac{1}{2}(t_1 + y_1 P_\sigma)[\delta(\boldsymbol{r}_{ij})\boldsymbol{k}^2 + \text{h.c.}]$$
$$+(t_2 + y_2 P_\sigma)\boldsymbol{k}' \cdot \delta(\boldsymbol{r}_{ij})\boldsymbol{k} + iW_0\, \boldsymbol{k}' \times \delta(\boldsymbol{r}_{ij})\, \boldsymbol{k} \cdot (\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j)$$
$$+\frac{1}{6}\sum_{n=1}^{3}(t_{3n} + y_{3n} P_\sigma)\rho^{n/3}\delta(\boldsymbol{r}_{ij}), \qquad (3)$$

Minimal Skyrme-type "force"

$$t_0 = \frac{8}{3}c_0(0)\,, \quad y_0 = \frac{8}{3}c_0(0) - 4c_0(1),$$
$$t_{3n} = 16c_n(0)\,, \quad y_{3n} = 16c_n(0) - 24c_n(1), \quad (n \neq 2$$
$$t_{32} = 16c_2(0) - \frac{3}{5}\left(\frac{3}{2}\pi^2\right)^{2/3}\theta_s,$$
$$y_{32} = 16c_2(0) - 24c_2(1) + \frac{3}{5}(3\pi^2)^{2/3}\left(3\theta_\mu - \frac{\theta_s}{2^{2/3}}\right)$$

with

$$\theta_s \equiv 3t_1 + 5t_2 + 4y_2\,, \quad \theta_\mu \equiv t_1 + 3t_2 - y_1 + 3y_2\,.$$

unconstrained from homogenous matter ➔ vary freely

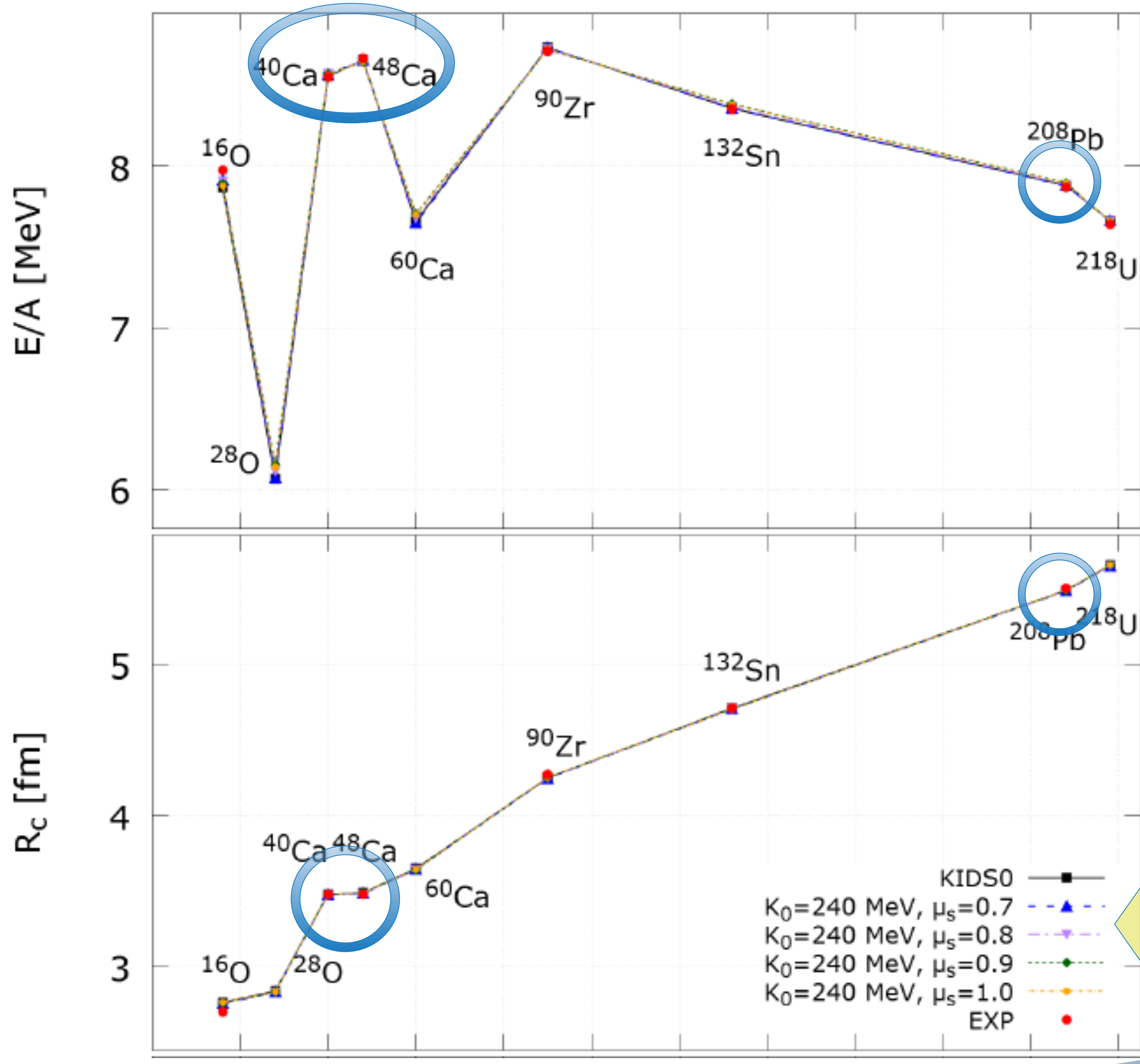
But the total $c_2(0)$, $c_2(1)$ will remain unchanged!

Procedure

For given KIDS functional $c_i(0)$, $c_i(1)$ (i.e., fixed SNM, PNM)

- Chose effective masses (vary at will)
- All t_i, y_i are now known except t_1,t_2,x_1,x_2
- The two combinations θ_s,θ_μ also known (eff. masses)
- **Two independent free parameters plus spin-orbit W_0**
  - Fit only to ^{40}Ca, ^{48}Ca, ^{208}Pb
  - Only bulk properties: E/A, charge radius: 6 data

Binding energy, charge radii

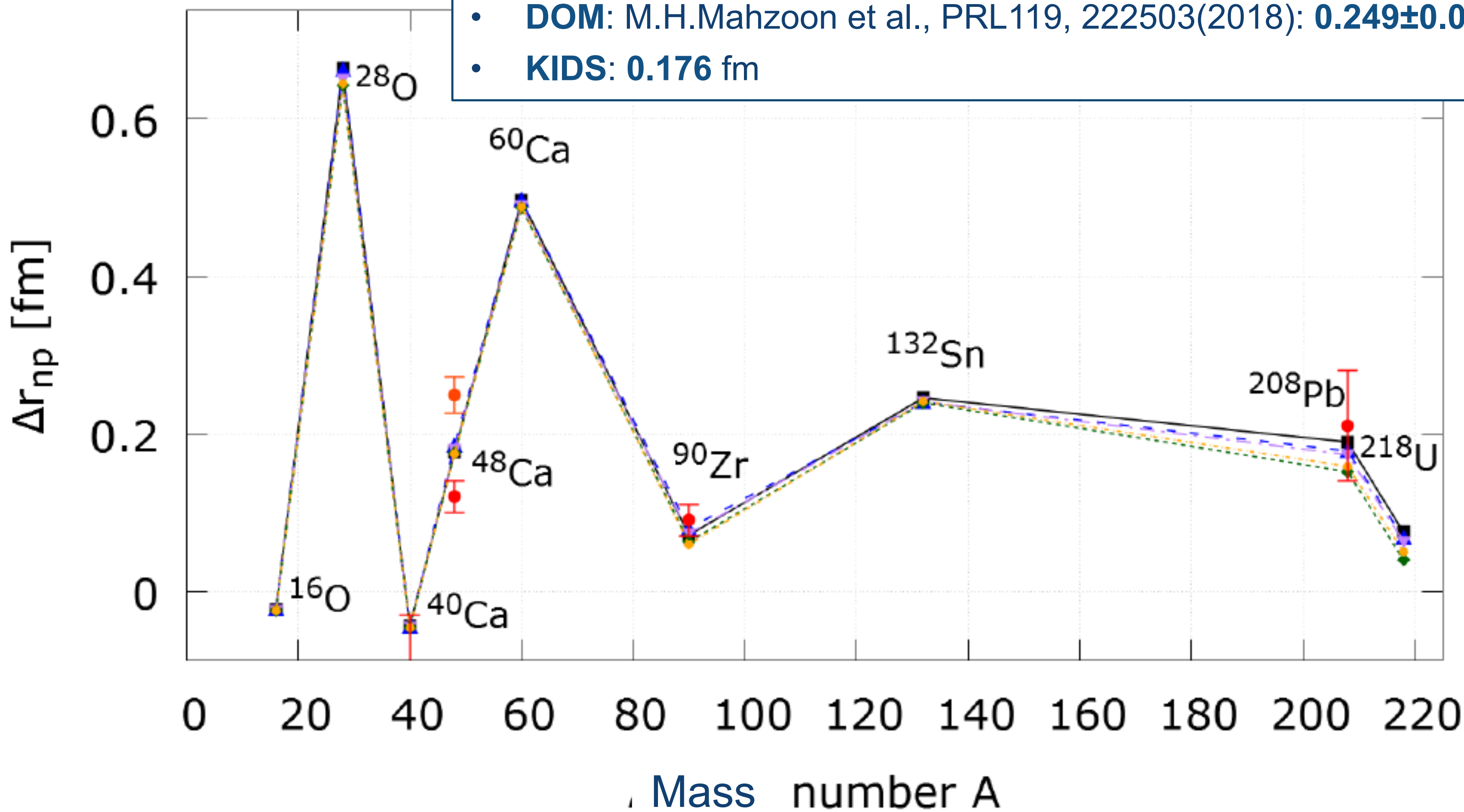


predictions independent of the effective mass assumed

Neutron skin thickness

<u>neutron skin of 48Ca:</u>

- **CCM**: G.Hagen et al., Nature Phys. 12,186(2016): **0.12-0.15** fm
- **DOM**: M.H.Mahzoon et al., PRL119, 222503(2018): **0.249±0.023** fm
- **KIDS**: **0.176** fm



Data: antiprotonic atoms, DOM (^{48}Ca, upper)

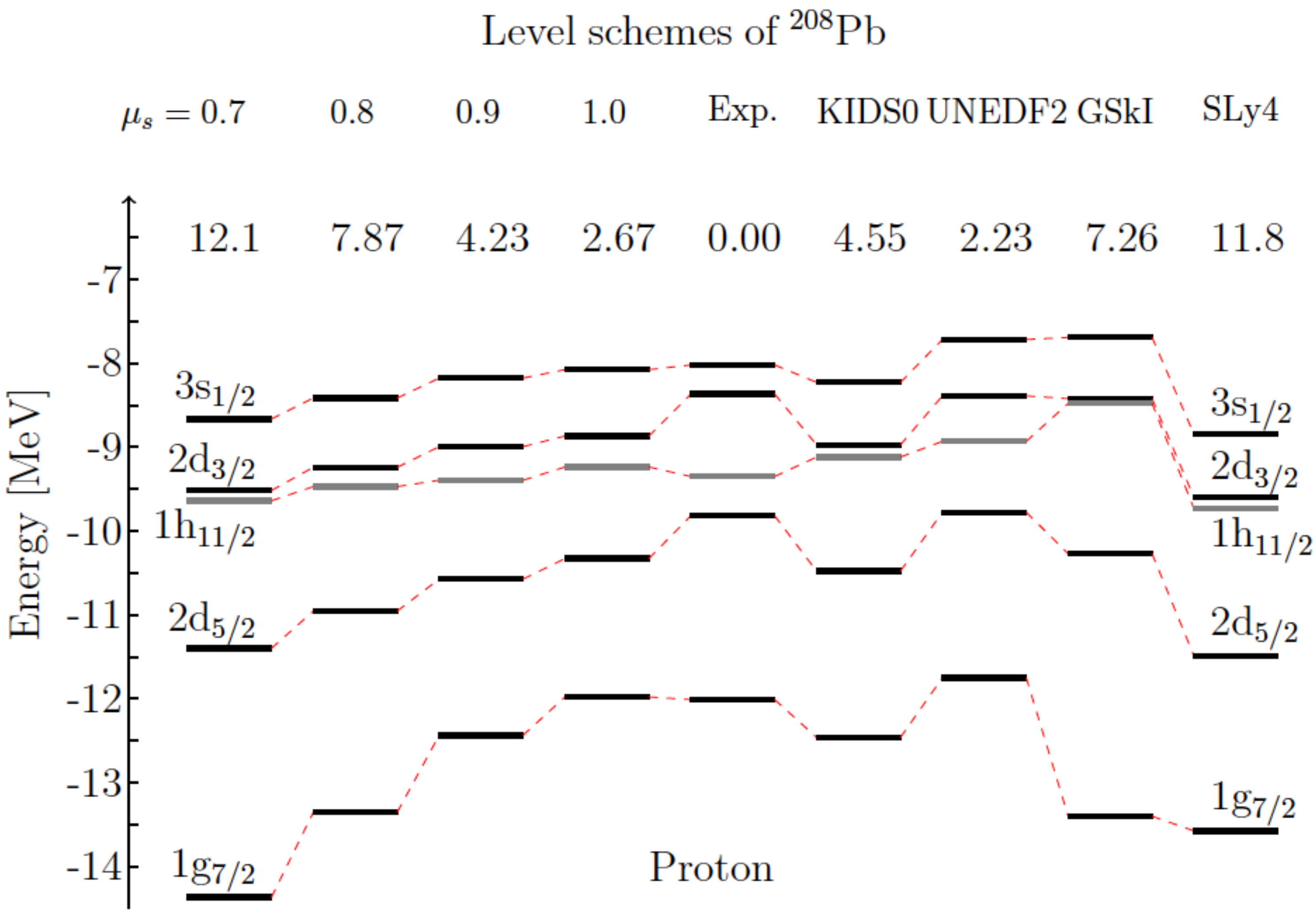
Predictions of APR EoS for the neutron skin thickness!

Precision comparison and ^{60}Ca predictions

Model	t_0 / y_0	t_1 / y_1	t_2 / y_2	t_{31} / y_{31}	t_{32} / y_{32}	t_{33} / y_{33}	(a_1, a_2, a_3)	W_0	$D_E[\%]$ / $D_R[\%]$	^{60}Ca: E/A [MeV] / R_c [fm]
KIDS0	−1772.04	275.72	−161.50	12216.73	571.08	0.00	(1/3, 2/3, 1)	108.35	0.32	7.6561
	−127.52	0.000	0.000	−11969.99	29485.52	−22955.28			0.56	3.6465
K240,1.0,0.82	−1772.04	270.52	−355.95	12216.73	642.12	0.00	(1/3, 2/3, 1)	97.61	0.41	7.6993
	−127.52	235.79	374.20	−11969.99	29224.07	−22955.28			0.56	3.6416
K240,0.7,0.82	−1772.04	448.99	−279.45	12216.73	−2572.65	0.00	(1/3, 2/3, 1)	135.24	0.26	7.6464
	−127.52	−345.72	234.74	−11969.99	41318.69	−22955.28			0.44	3.6494
K240,0.9,1.00	−1772.04	315.97	−527.58	12216.73	−191.34	0.00	(1/3, 2/3, 1)	107.58	0.38	7.6933
	−127.52	−56.87	480.10	−11969.99	36289.12	−22955.28			0.57	3.6370
K220,1.0,0.82	−1938.71	281.04	−479.05	15900.76	−2750.91	0.00	(1/3, 2/3, 1)	88.96	0.52	7.7701
	−294.19	236.07	388.03	−8285.96	25831.04	−22955.28			0.94	3.6524
K220,0.7,0.82	−1938.71	466.23	−439.68	15900.76	−5965.68	0.00	(1/3, 2/3, 1)	133.36	0.44	7.6807
	−294.19	−247.10	422.10	−8285.96	37925.67	−22955.28			0.82	3.6663
GSkI [32]	−1855.45	397.23	264.63	13858.00	−2694.06	−319.87	(1/3, 2/3, 1)	169.57	0.20	7.6294
	−219.02	−698.59	−478.13	1747.29	3200.69	146.94			0.68	3.6640
SLy4 [37]	−2488.91	486.82	−546.39	13777.00			(1/6, −, −)	122.69	0.38	7.7030
	−2075.75	−167.37	546.39	18652.68					0.91	3.6734

$$D_O = \frac{1}{N_{\rm nucl}} \sum_{i=1}^{N_{\rm nucl}} \left| \frac{O_i^{\rm expt} - O_i^{\rm cal}}{O_i^{\rm expt}} \right|$$

Single-particle levels



Remarks

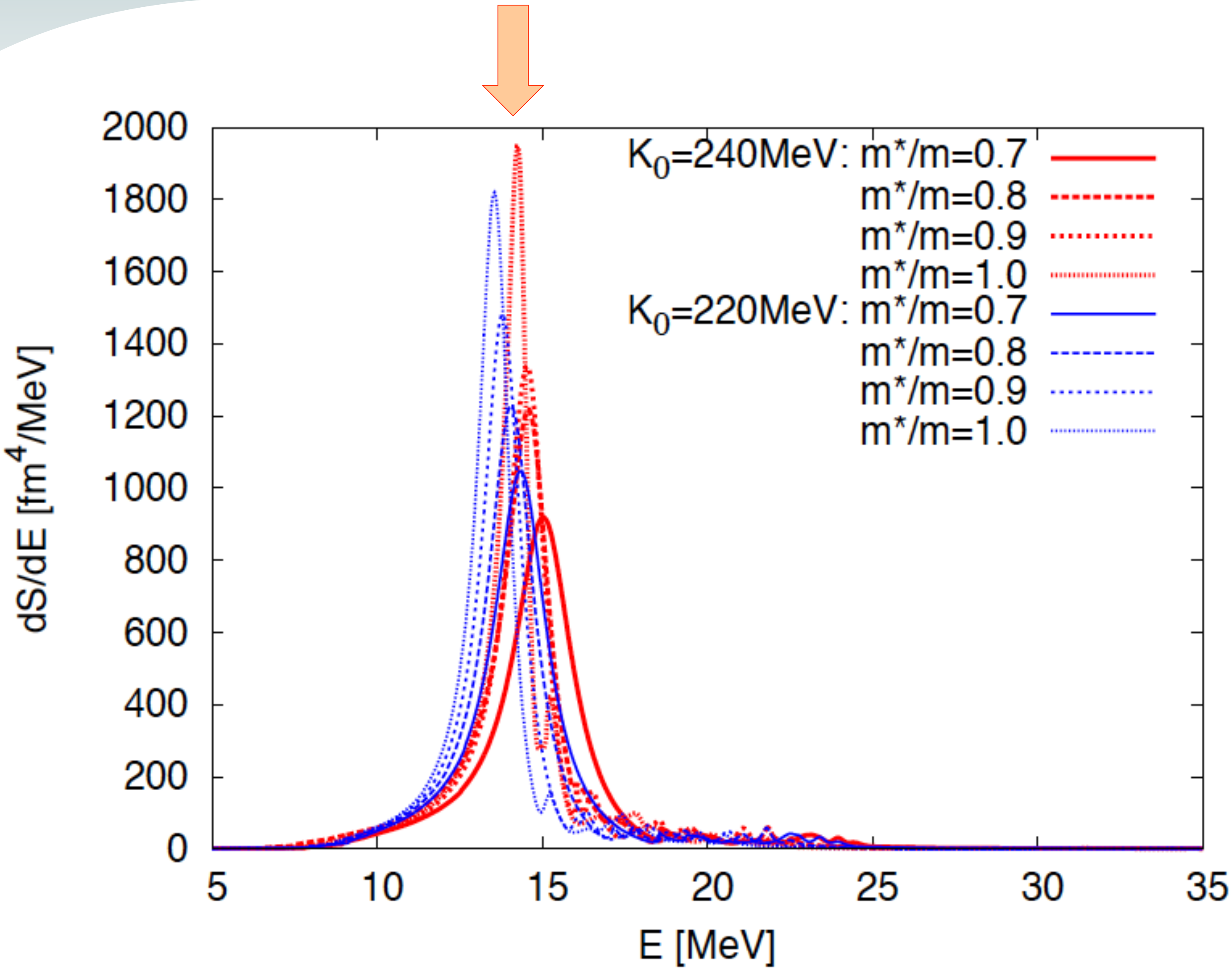
- For given **immutable** EoS (no refitting), a Skyrme-type functional can easily be reverse-engineered
- Bulk, static properties: practically independent of the effective mass!
  - We can vary EoS parameters and m* independently and examine effect on observables
- *Effective mass: relevant for dynamics*

preliminary

GIANT RESONANCES

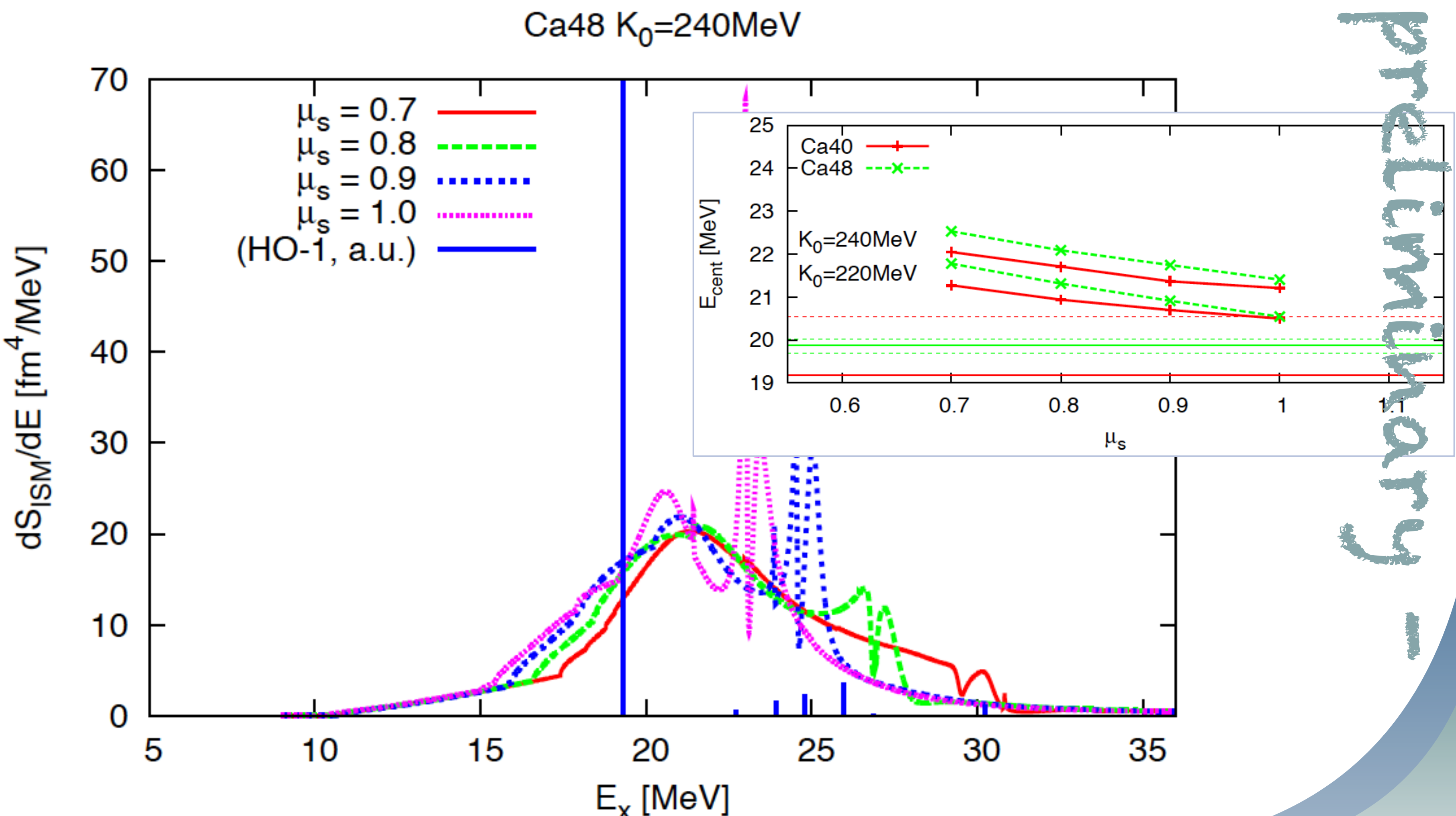
- You see it first: breathing mode
- Giant dipole resonance of ^{68}Ni

Breathing mode of ^{208}Pb



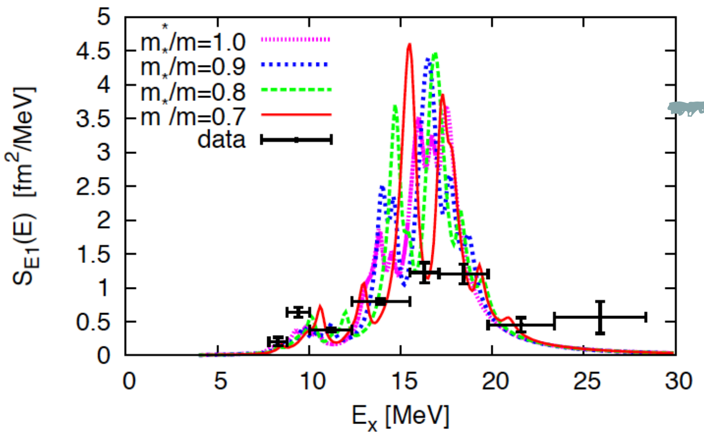
- preliminary -

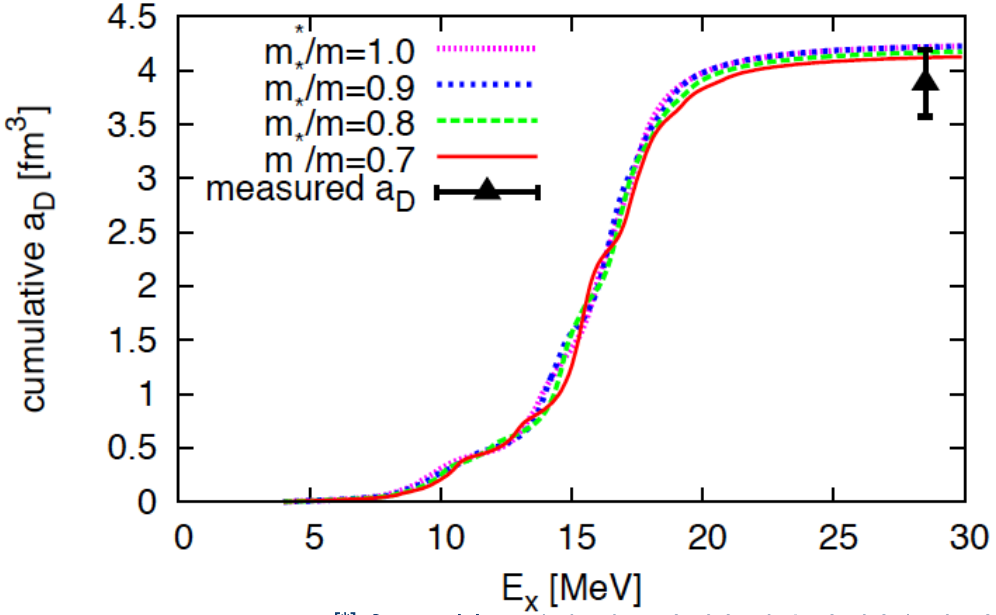
Breathing mode of ^{48}Ca



KIDS-ad2: Predictions for ^{68}Ni (not fitted)

- Binding energy per particle:
  - KIDS-ad2: **8.68~8.69** MeV [*]
  - AME2016: **8.68247(4)** MeV
- Centroid: **16.1-16.6** MeV
- Dipole polarizability:





- preliminary -

[*] *for m*/m=1.0~0.7:8.68794; 8.68176; 8.68838; 8.68912 MeV*
[**] *a_D measurementT.Aumann and D.Rossi, private communication*

Summary

RAON

- Natural Ansatz + Skyrme formalism: **KIDS functional**
  - 3 terms in expansion sufficient for SNM: $\{\rho_0, E_0, K_0\}$
  - **4 terms necessary for neutron matter and symmetry energy**: $\{J, L, K_{sym}, Q_{sym}\}$
- **From fixed EoS straight to nuclei**
- APR: static, bulk nuclear properties insensitive to
  - Effective-mass parameters
  - High-order parameters of symmetry energy

Skyrme- type "interaction" by reverse engineering

- **Flexibility to choose parameter values at will** for sensitivity studies **or adjust them to**
  - Dynamical observables (e.g., giant resonances)
  - **Ab initio pseudodata (polarized matter, neutron drops...**
  - Astrophysical and HIC constraints

Thank you!