TRIUMF

Ab initio calculations for exotic nuclei

Understanding Nuclei from Different Theoretical Approaches

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Matteo Vorabbi | TRIUMF

Collaborators:

Michael Gennari, Petr Navratil (TRIUMF)

Sofia Quaglioni (LLNL)

Guillaume Hupin (CNRS)

Outline

1. No-core shell model with continuum
    a. Structure of the exotic ^{9}He system
    b. Study of A=7 systems

2. Microscopic optical potentials for intermediate energies with nonlocal *ab initio* densities
    - Results for stable nuclei
    - Results for ^{6}He and ^{8}He

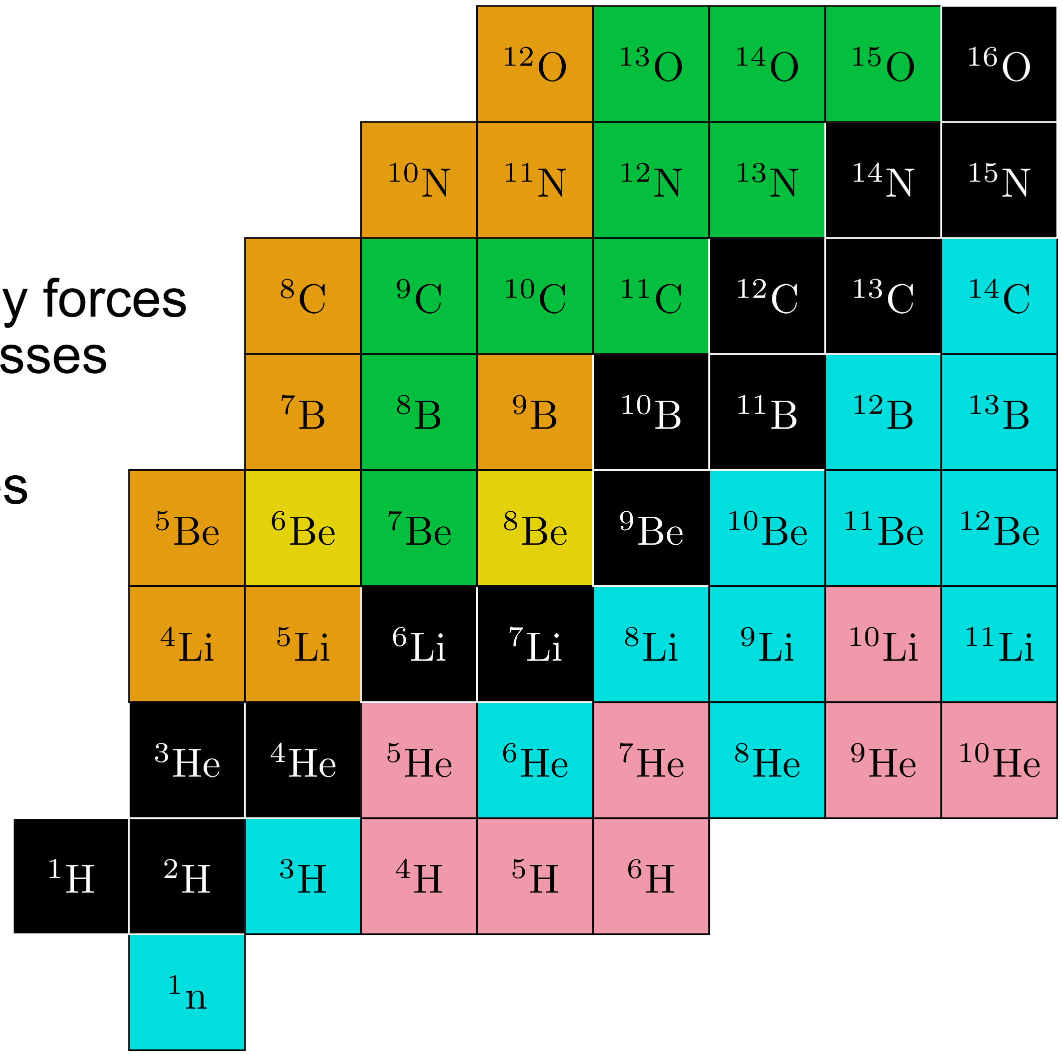
1a.

Structure of the exotic ^{9}He

Motivations

Neutron-rich nuclei

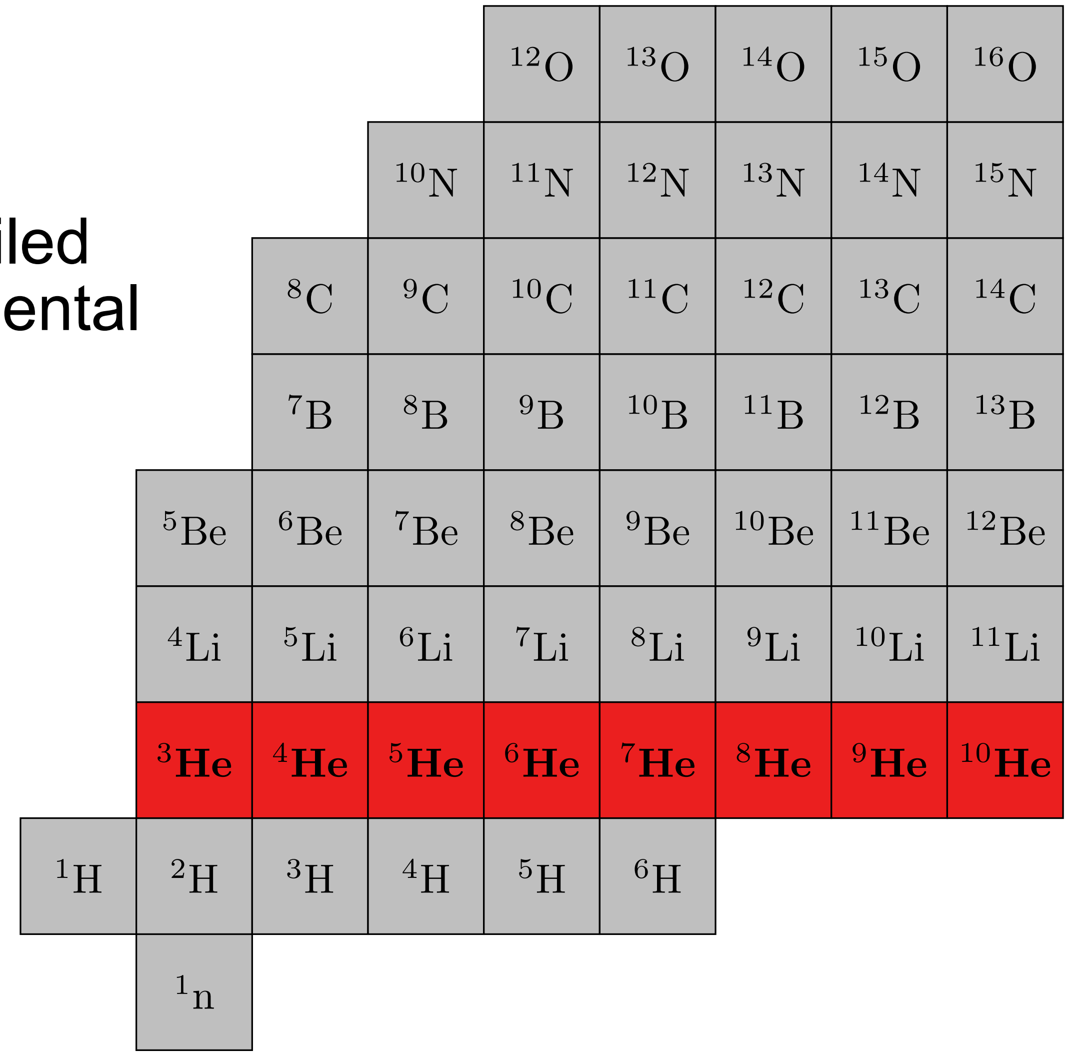
- Theory
  - Importance of many-body forces at extreme neutron excesses
  - Challenge to our current computational techniques
- Experiment
  - Difficult to produce in sufficient quantities
  - Challenging to analyze



Motivations

The He isotopic chain

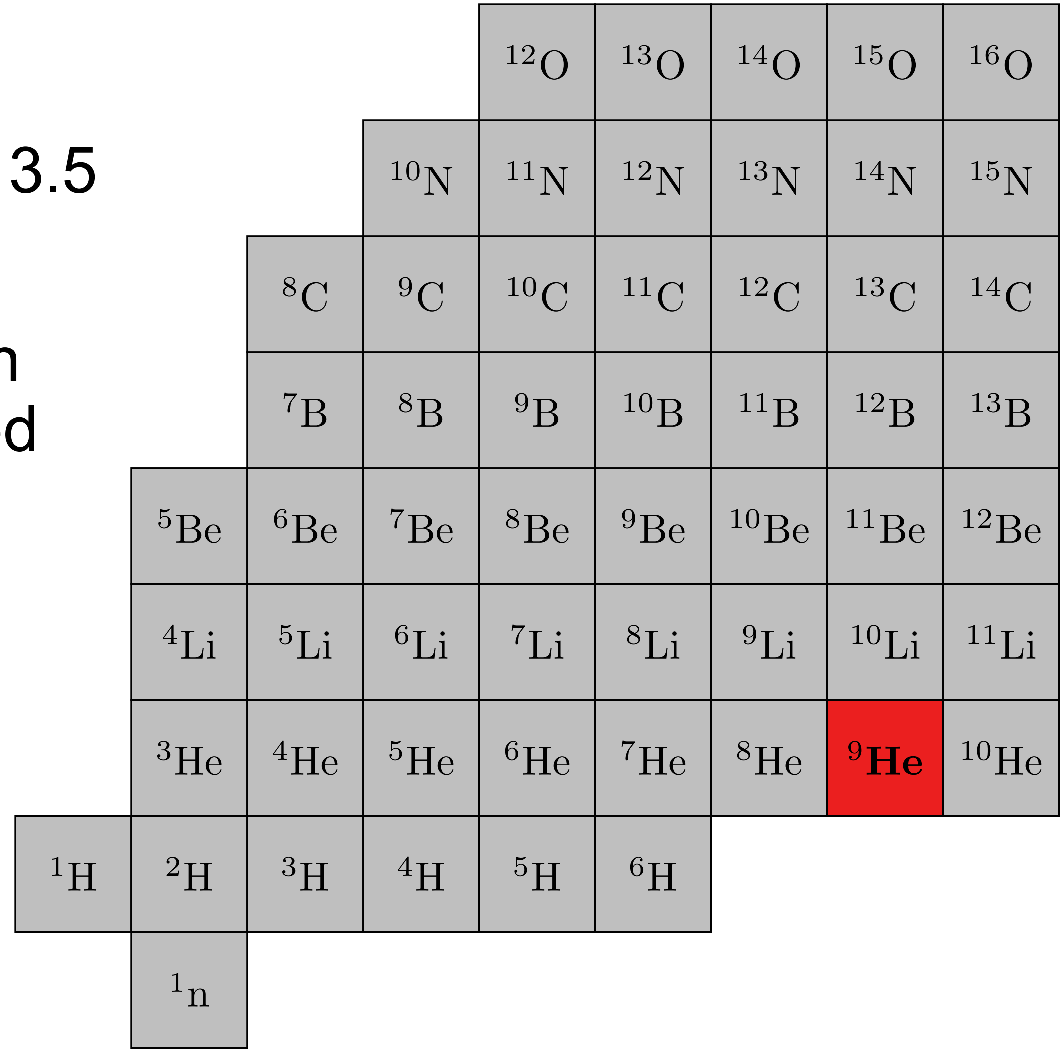
- One of the few chains accessible to both detailed theoretical and experimental studies



Motivations

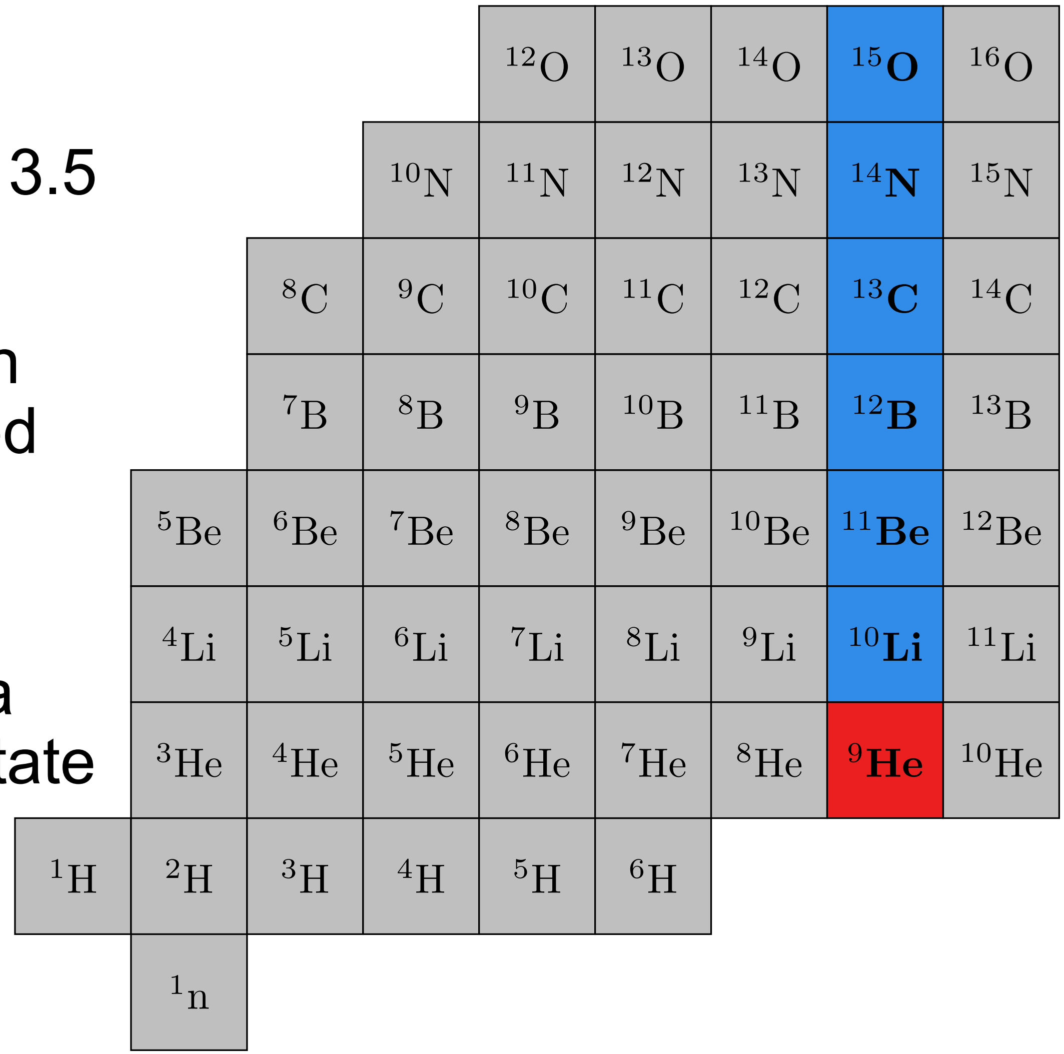
The ^{9}He system

- Characterized by N/Z = 3.5
- One of the most neutron extreme systems studied so far

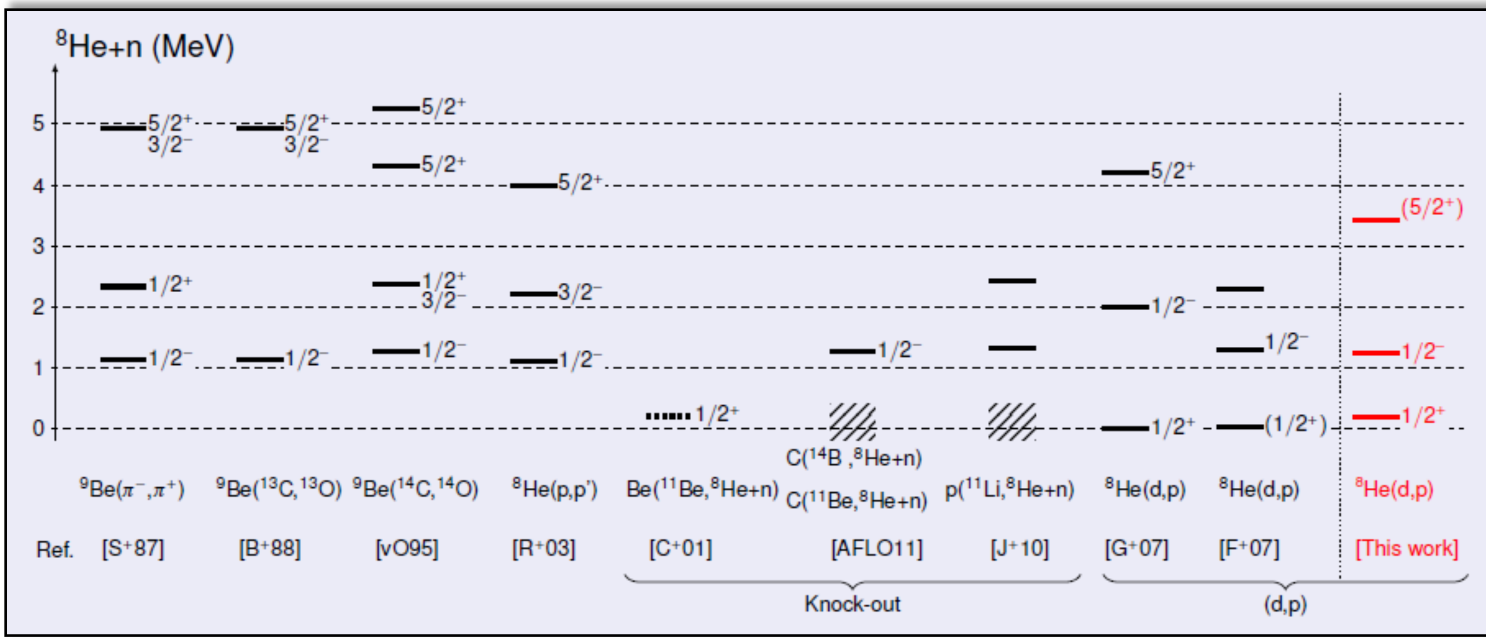


The ^{9}He system

- Characterized by $N/Z = 3.5$
- One of the most neutron extreme systems studied so far
- Possible candidate for a positive parity ground state
  Famous example: ^{11}Be



Experimental history of ^{9}He

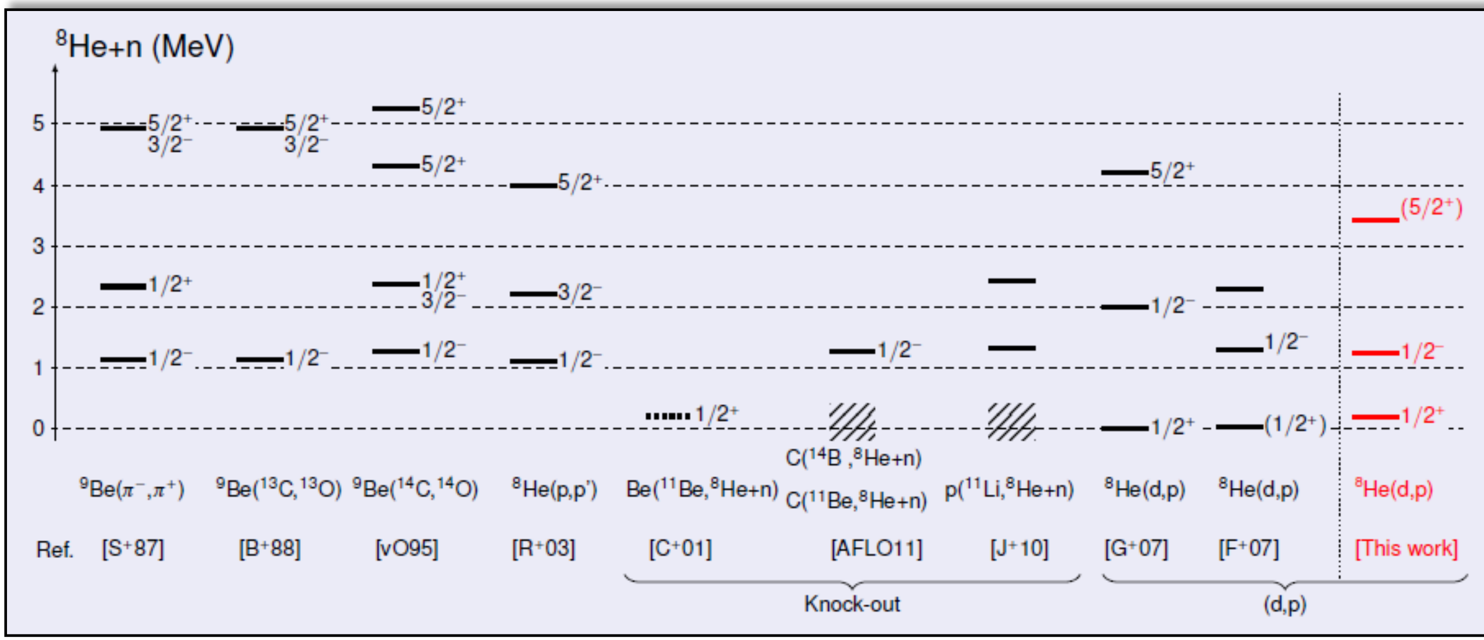


From talk by Nigel Orr at ECT* (2013)

Controversial experimental situation

- No bound state
- Most experiments see a 1/2$^-$ resonance at ~ 1 MeV
- Is there a 1/2$^+$ resonance? Is the ground state 1/2$^+$ or 1/2$^-$?
  - a_0 ~ -10 fm (Chen et al.)
  - a_0 ~ -3 fm (Al Falou et al.)
- Any higher-lying resonances?
- Recent ^{8}He(p,p) measurement at TRIUMF by Rogachev
  Found no T= 5/2 resonances [PLB **754** (2016) 323]

Experimental history of ^{9}He



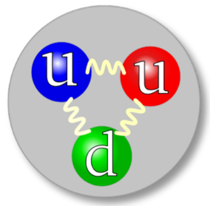
From talk by Nigel Orr at ECT* (2013)

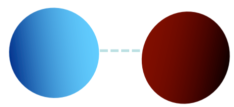
Two longstanding problems affect the physics of the ^{9}He system

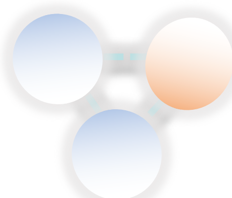
1. The existence of the $1/2^{+}$ resonance

2. The width of the $1/2^{-}$ resonance
   - Experimentally ~ 0.1 MeV
   - Theoretically ~ 1 MeV

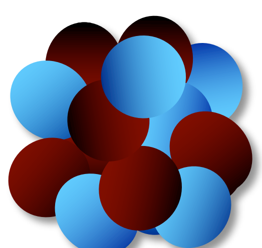
From QCD to nuclei

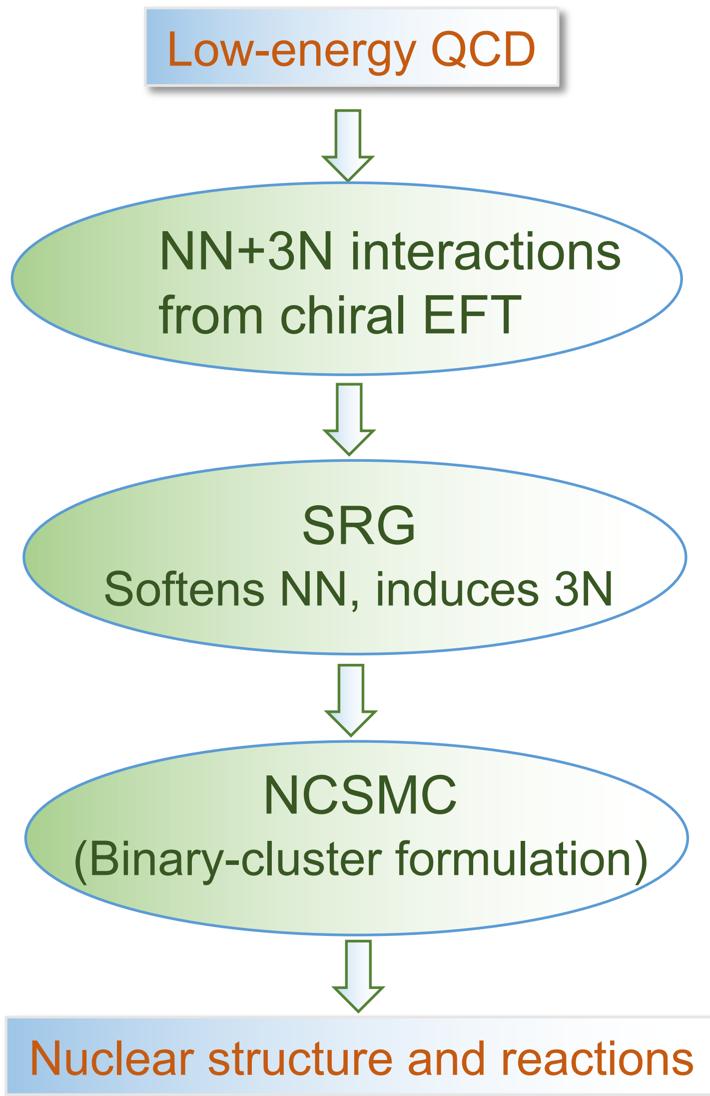






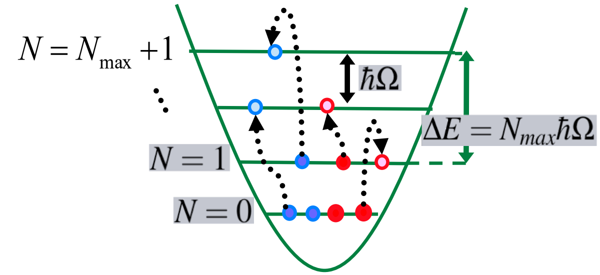
$$H|\Psi\rangle = E|\Psi\rangle$$

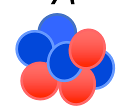




No-core shell model

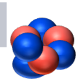
- No-core shell model (NCSM)
  - A-nucleon wave function expansion in the harmonic oscillator (HO) basis
  - Short- and medium-range correlations
  - Bound-states, narrow resonances





$$\Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni} \Phi^A_{Ni}$$

$$\Psi^{(A)} = \sum_\lambda c_\lambda \left| {}^{(A)} \,\cdot\,, \lambda \right\rangle$$

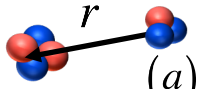


Unknowns

No-core shell model with RGM

- NCSM with Resonating Group Method (NCSM/RGM)
  - Cluster expansion, clusters described by NCSM
  - Proper asymptotic behavior
  - Long-range correlations

$$\Psi^{(A)} = \sum_{\nu} \int d\vec{r}\; \gamma_{\nu}(\vec{r}\,)\; \hat{A}_{\nu} \left| \underset{(A-a)}{} \quad \overset{\vec{r}}{} \quad \underset{(a)}{} , \nu \right\rangle$$

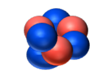


Unknowns

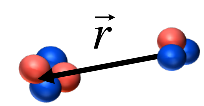
Unified approach to bound & continuum states; to nuclear structure and reactions

- No-core shell model (NCSM)
  - A-nucleon wave function expansion in the harmonic oscillator (HO) basis
  - Short- and medium-range correlations
  - Bound-states, narrow resonances
- NCSM with Resonating Group Method (NCSM/RGM)
  - Cluster expansion, clusters described by NCSM
  - Proper asymptotic behavior
  - Long-range correlations
- Most efficient: *ab initio* no-core shell model with continuum (NCSMC)

NCSM



NCSM/RGM



NCSMC

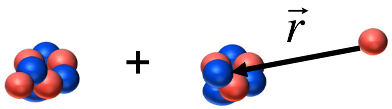
S. Baroni, P. Navratil, and S. Quaglioni,
PRL **110**, 022505 (2013); PRC **87**, 034326 (2013).

$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| {}^{(A)} \, , \lambda \right\rangle + \sum_{\nu} \int d\vec{r} \; \gamma_{\nu}(\vec{r}\,) \; \hat{A}_{\nu} \left|_{(A-a)} \; \overset{\vec{r}}{\longleftarrow} \; (a) \, , \nu \right\rangle$$

Unknowns

NCSMC ^{9}He calculations

- NCSMC calculations with several interactions
  - N^2LO_{sat} NN + 3N
  - NN N^3LO + 3N N^2LO
  - **NN SRG-N^4LO500**
- Calculations with NN SRG-N^4LO500
  - ^{9}He ~ $(^9\text{He})_{NCSM}$ + $(n\text{-}^8\text{He})_{NCSM/RGM}$

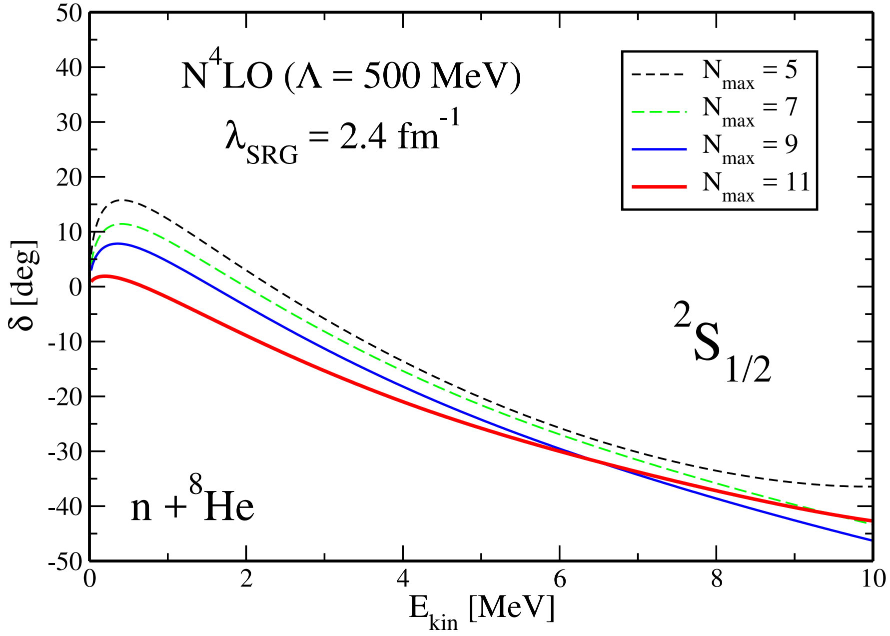


    - ^{8}He: 0^+ and 2^+ NCSM eigenstates
    - ^{9}He: 4 negative-parity NCSM eigenstates
      6 positive-parity NCSM eigenstates
  - Importance of large N_{max} basis:
    - NN SRG-N^4LO500 with λ = 2.4 fm^{-1}
      up to N_{max} = 11 with ^{9}He NCSM (m-scheme basis of 350 million)

Structure of unbound ^{9}He

Phase shift convergence with NN SRG-N^{4}LO500 λ=2.4 fm^{-1}

Vorabbi, Calci, Navratil, Kruse, Quaglioni, Hupin, PRC **97**, 034314 (2018)



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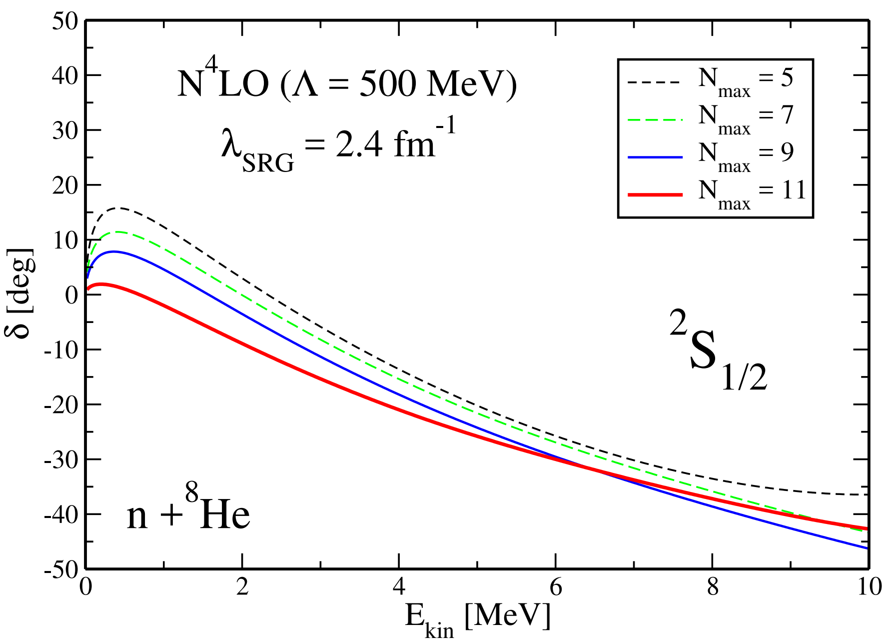

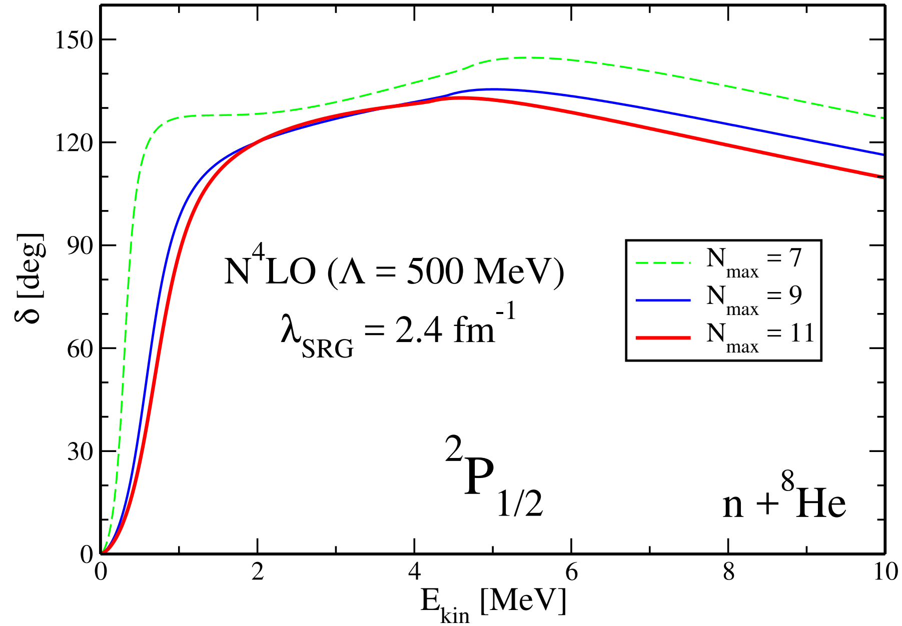

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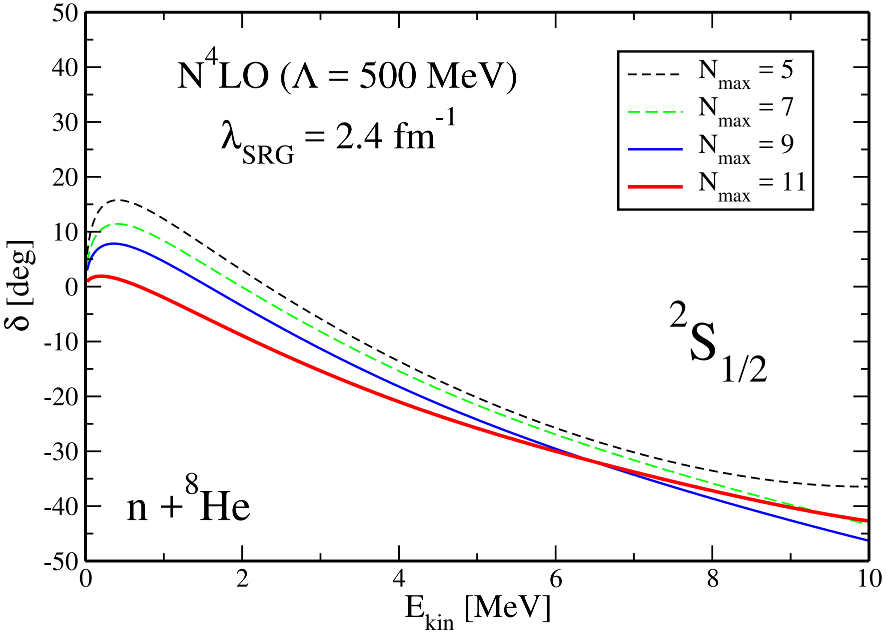
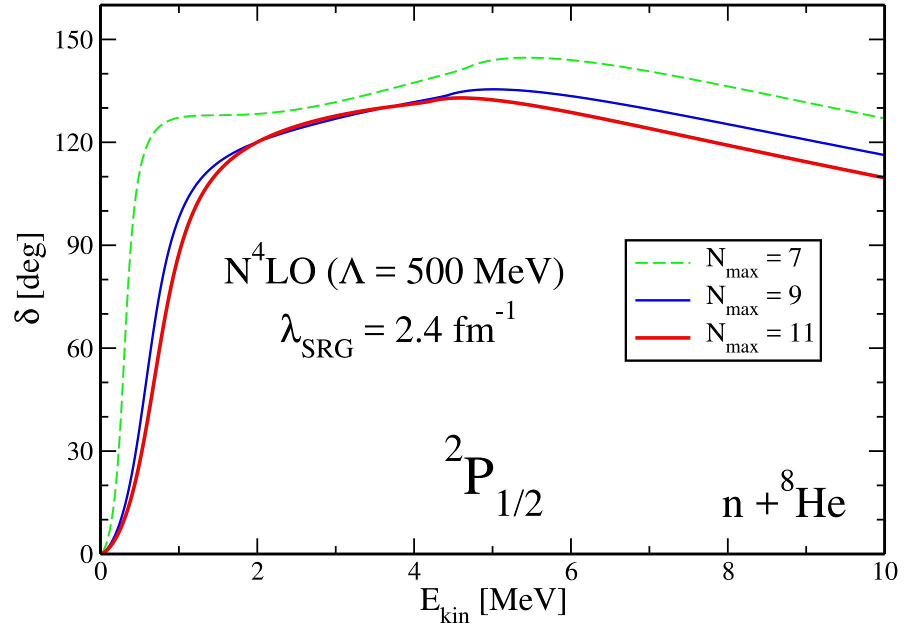
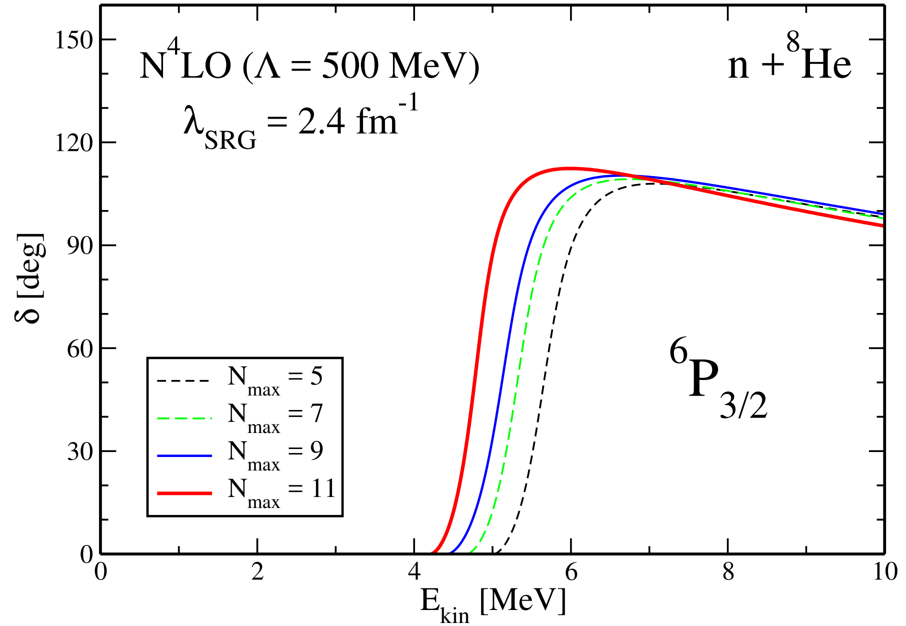






Structure of unbound ^{9}He

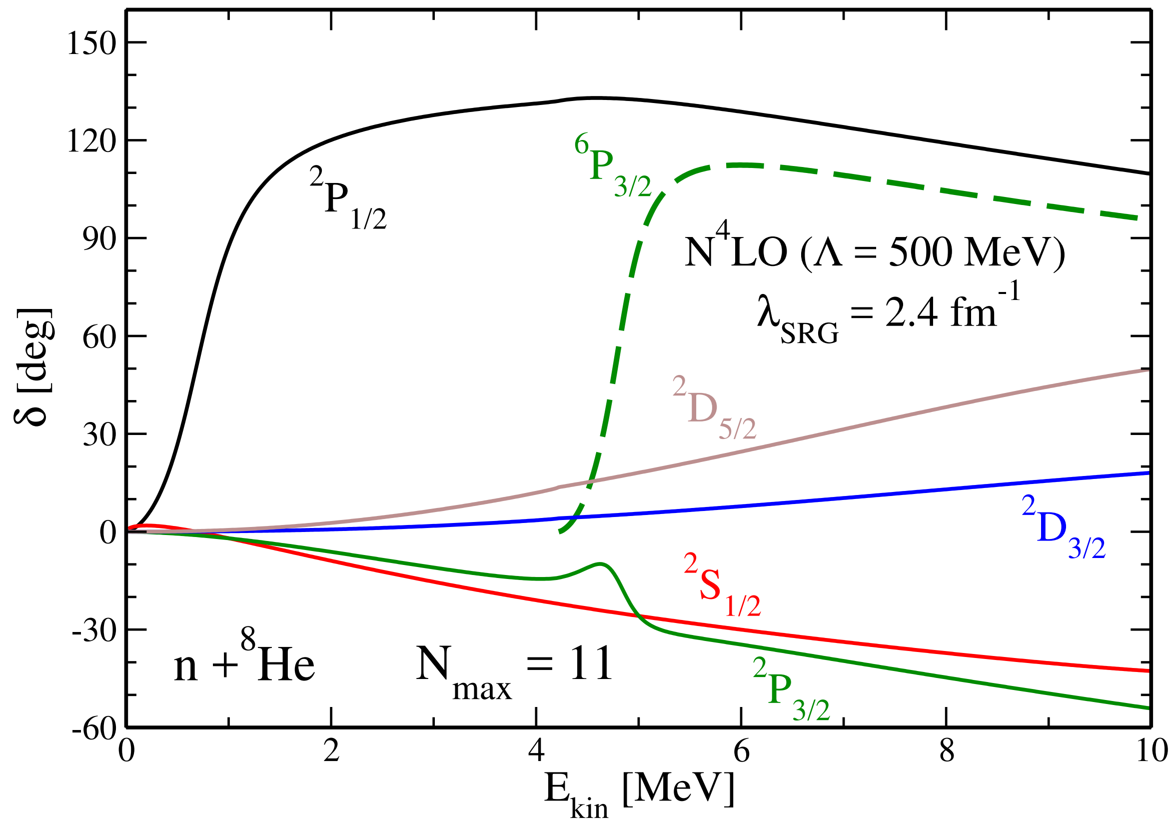
Phase shifts with NN SRG-N^{4}LO500 λ=2.4 fm^{-1}

Energy spectrum

No bound state

Two resonances in the $2P_{1/2}$ and $^6P_{3/2}$ channels

No resonance in the $^2S_{1/2}$ state



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Structure of unbound ^{9}He

Eigenphase shifts with NN SRG-N^{4}LO500 λ=2.4 fm^{-1}

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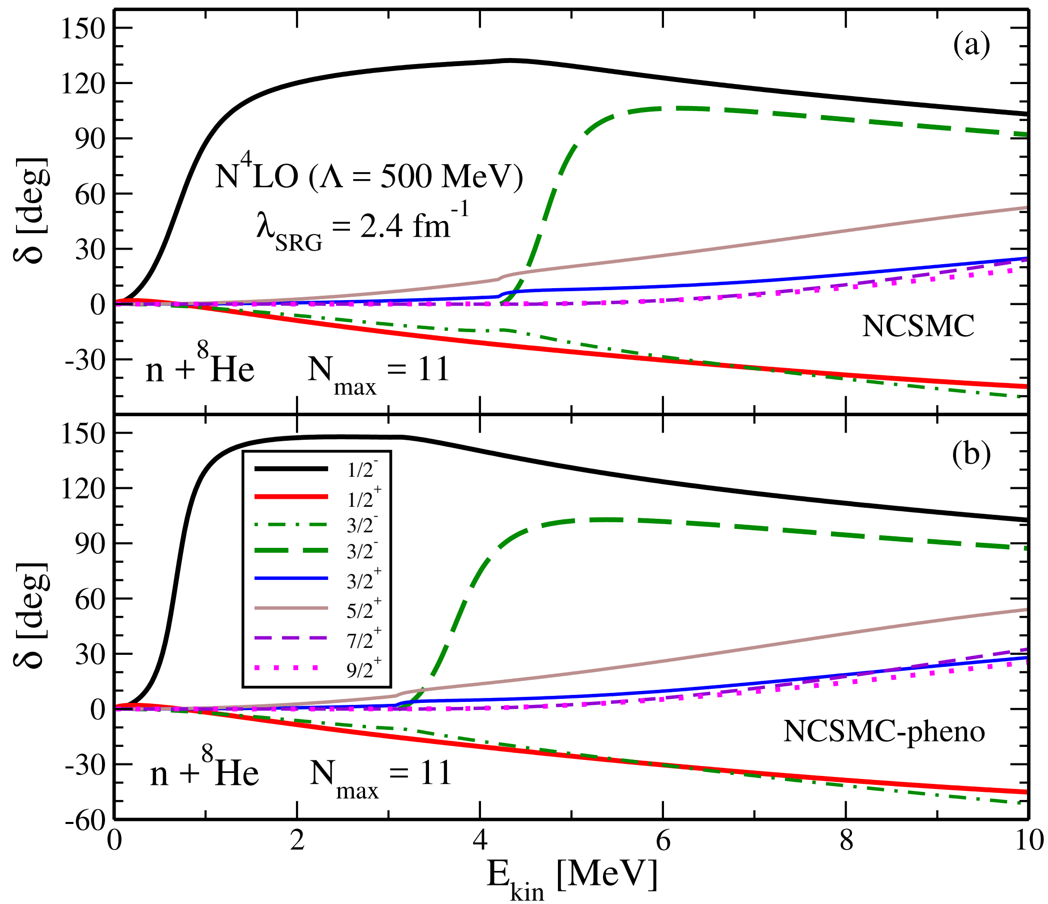


Summary

Robust results for **1/2$^-$** (~ 1MeV) and **3/2$^-$** (~4 MeV) **P-wave** resonances (3/2$^-$ resonance in n-^{8}He(2$^+$) channel)

1/2$^+$ S-wave with vanishing scattering length: a_s= 0 ~ -1 fm

No evidence for other higher lying resonances

J^π	NCSMC		NCSMC-pheno	
$1/2^-$	$E_R = 0.69$	$\Gamma = 0.83$	$E_R = 0.68$	$\Gamma = 0.37$
$3/2^-$	$E_R = 4.70$	$\Gamma = 0.74$	$E_R = 3.72$	$\Gamma = 0.95$

1b.

Study of A=7 systems

^{7}Be system

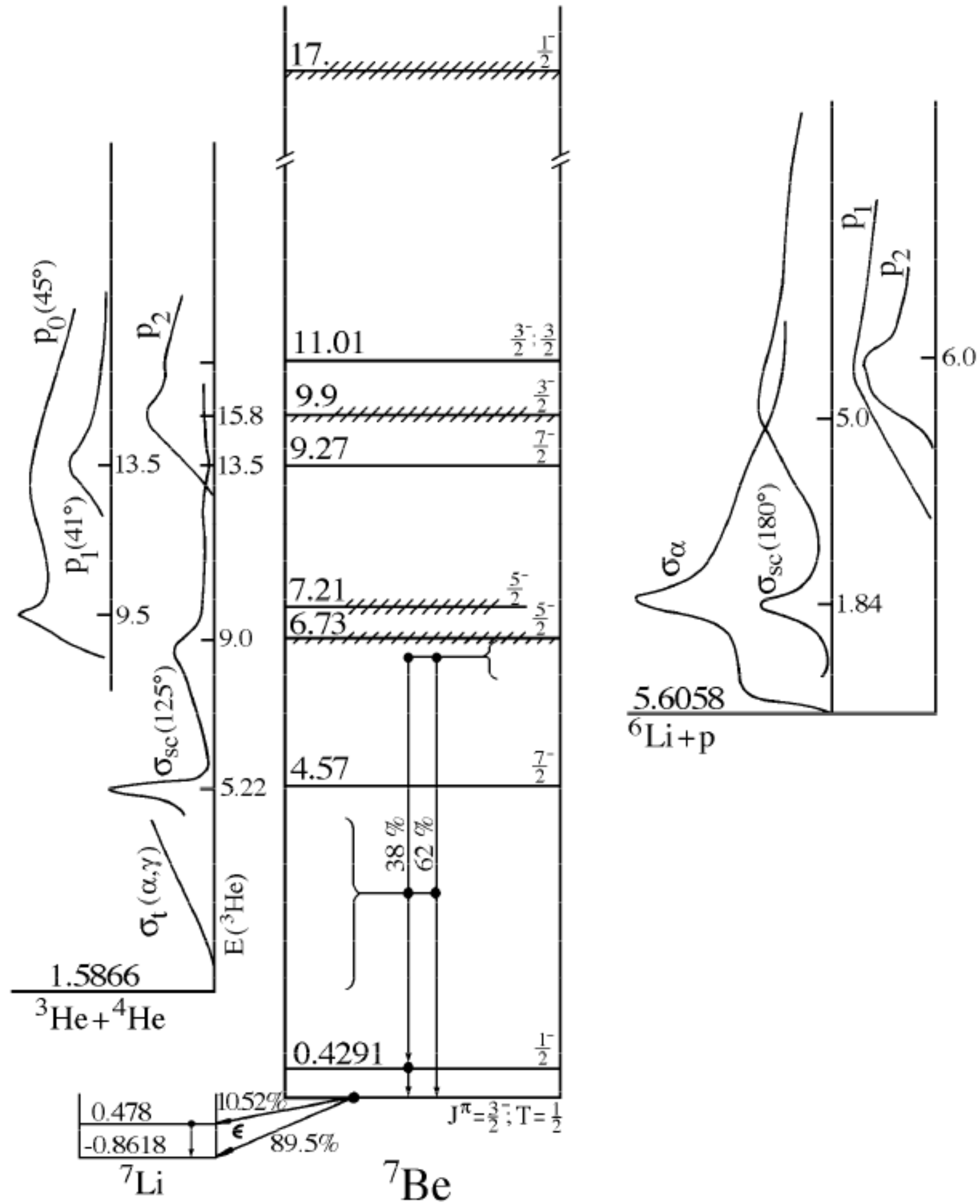
Analyzed mass partitions

- ^{3}He + ^{4}He
- p + ^{6}Li

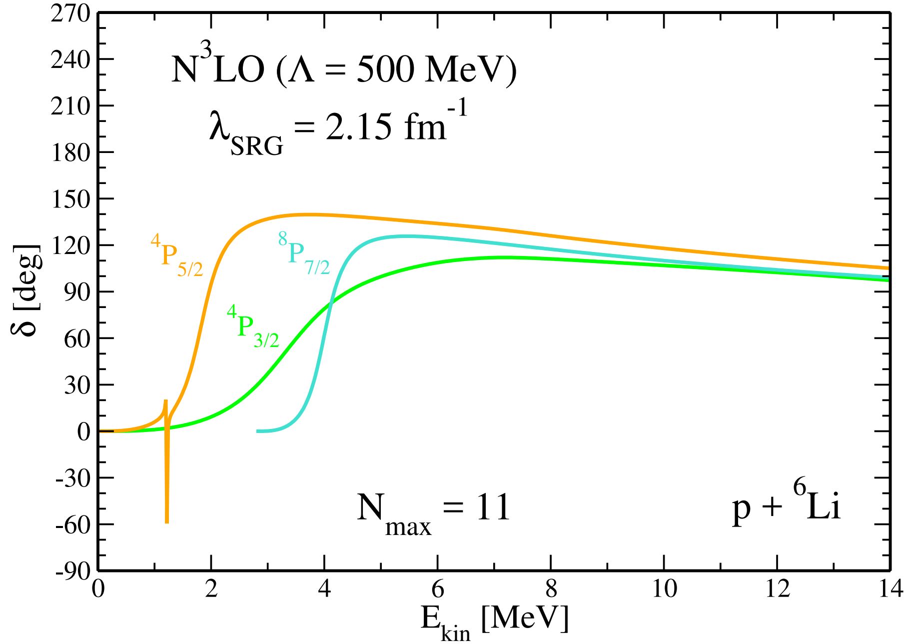
Exp.	Jπ = 3/2-
E [MeV]	-37.60

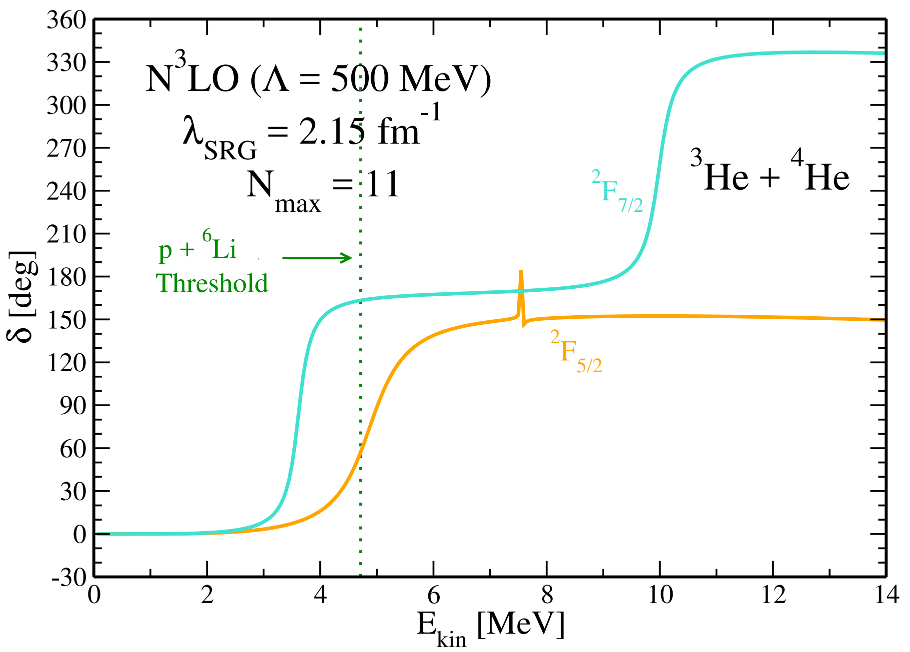
^{3}He + ^{4}He	Jπ = 3/2-	Jπ = 1/2-
E_{bound}	-1.519	-1.256
E [MeV]	-36.98	-36.71

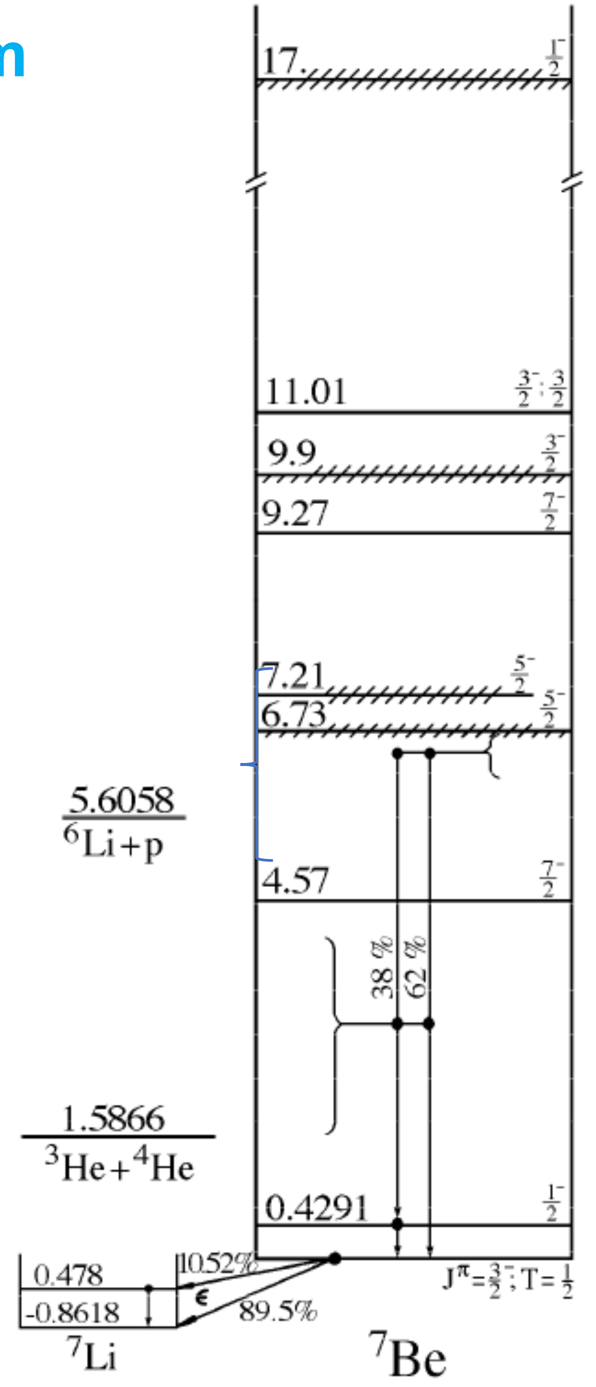
p + ^{6}Li	Jπ = 3/2-	Jπ = 1/2-
E_{bound}	-5.729	-5.389
E [MeV]	-36.47	-36.13



^{7}Be – Reproducing the energy spectrum







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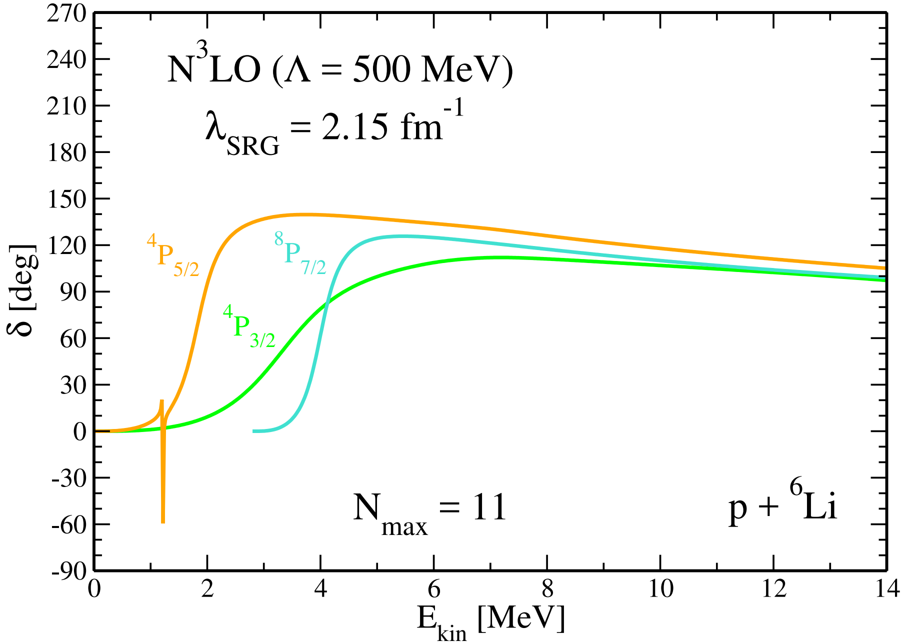
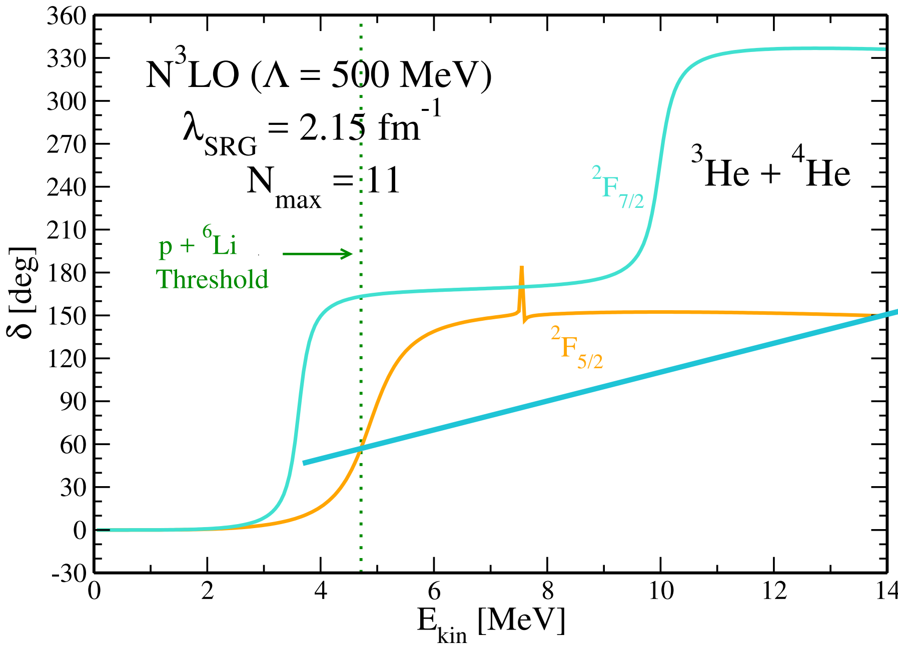
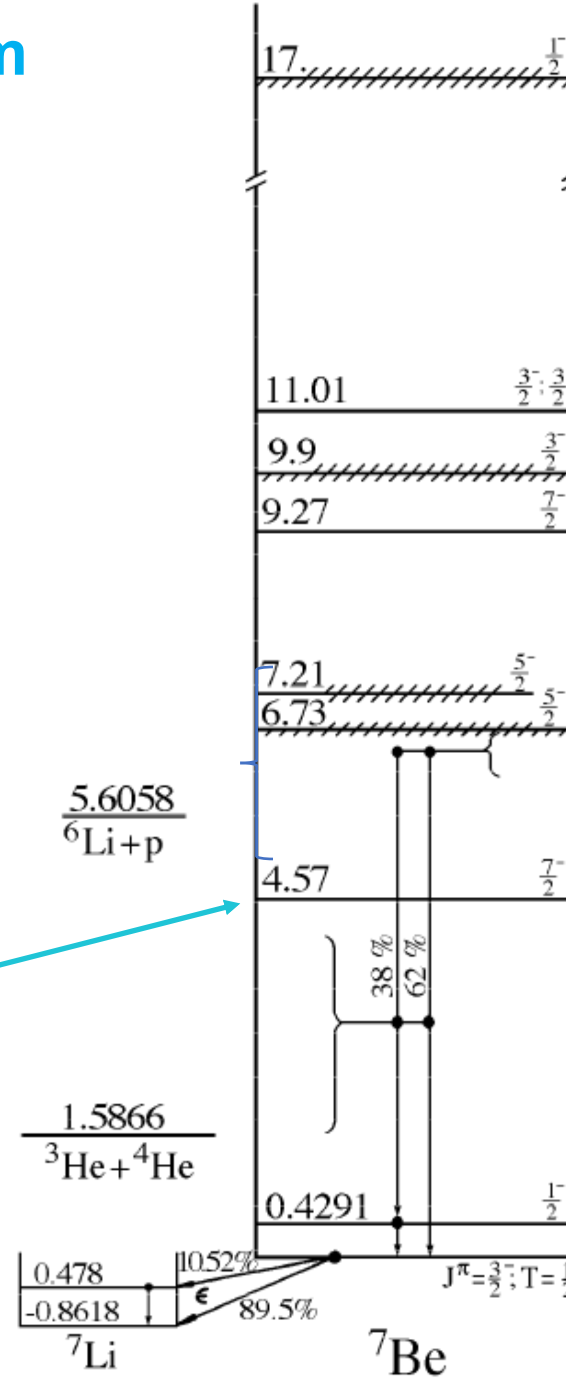
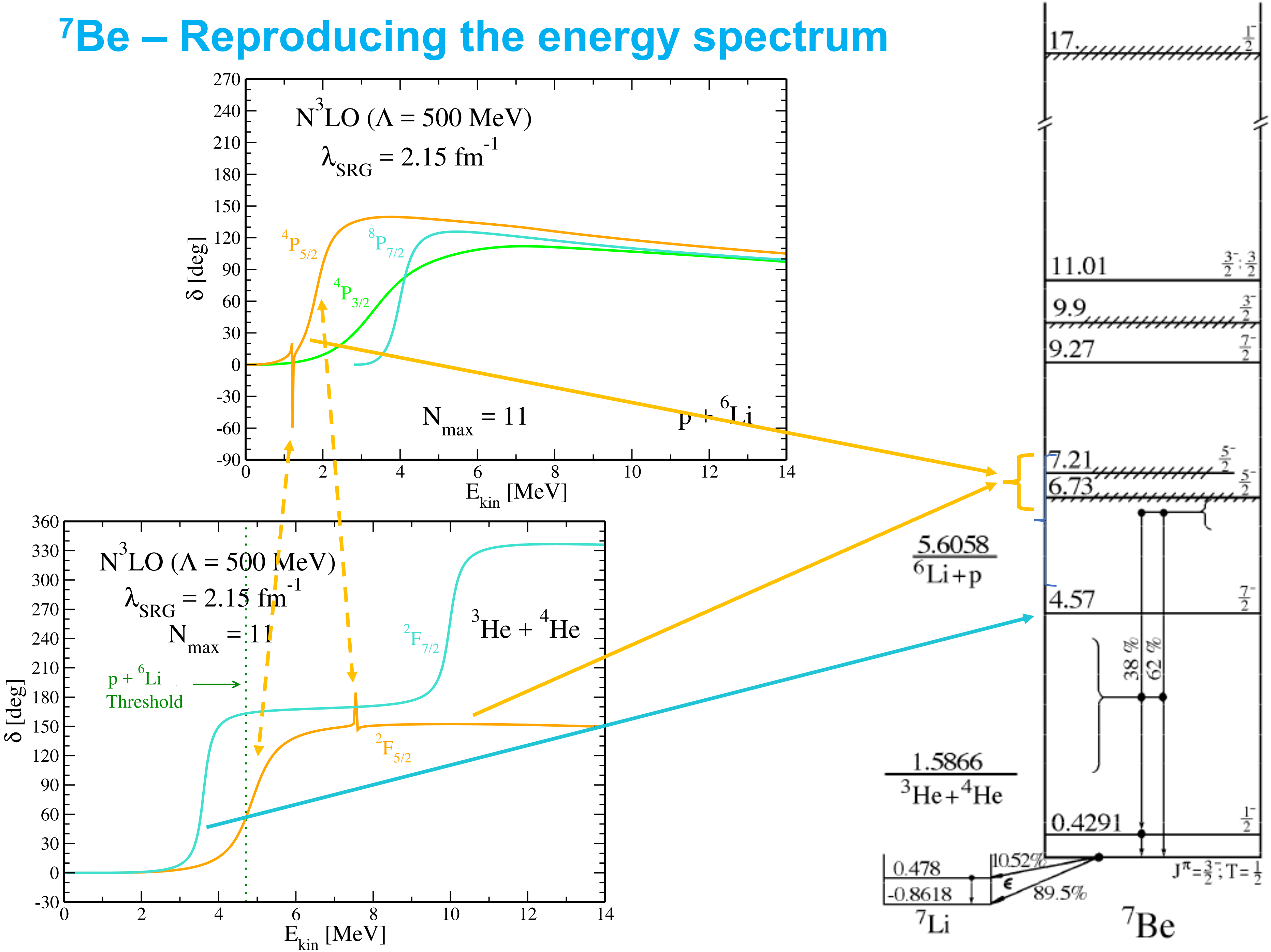






^{7}Be – Reproducing the energy spectrum



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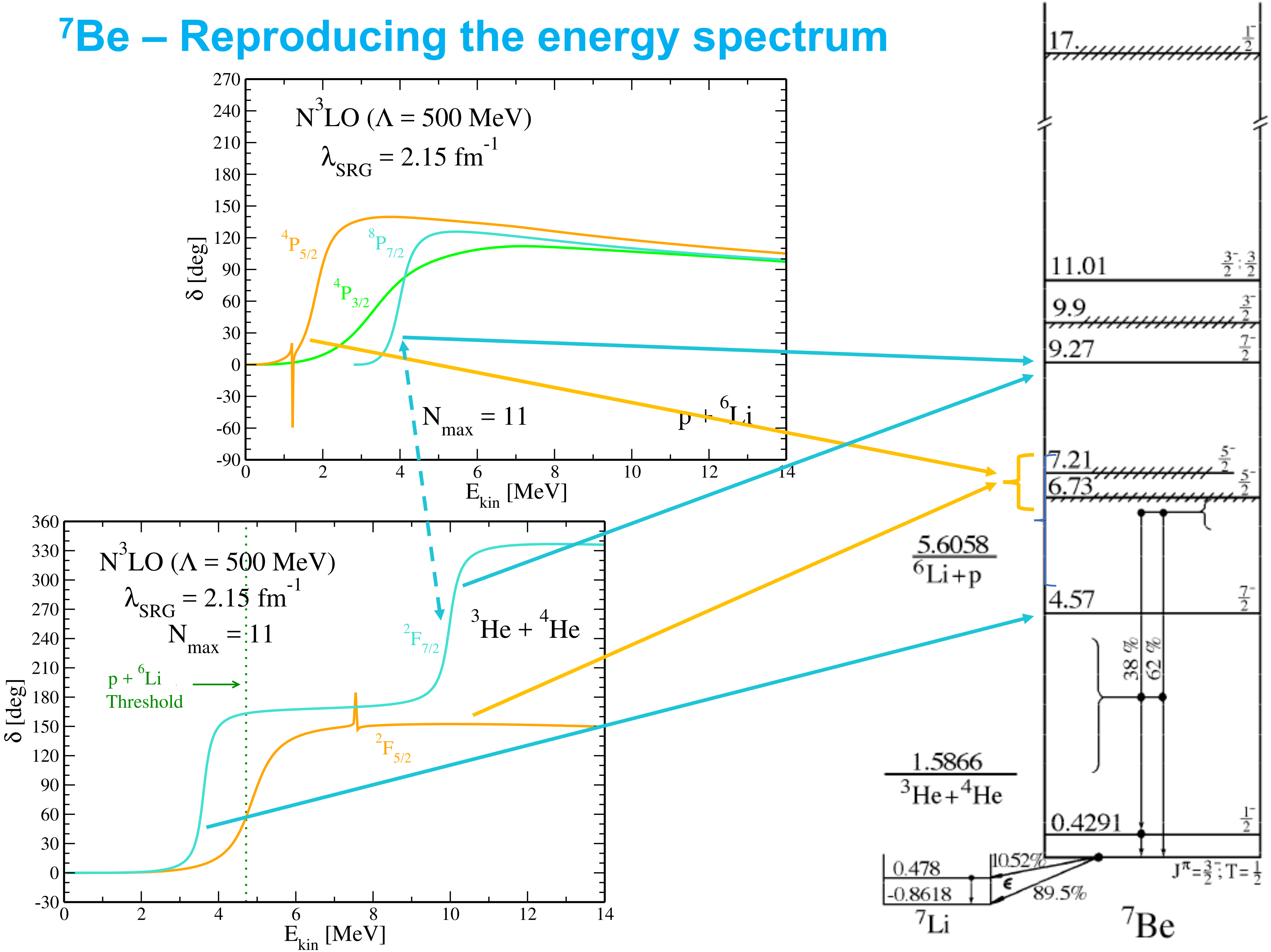

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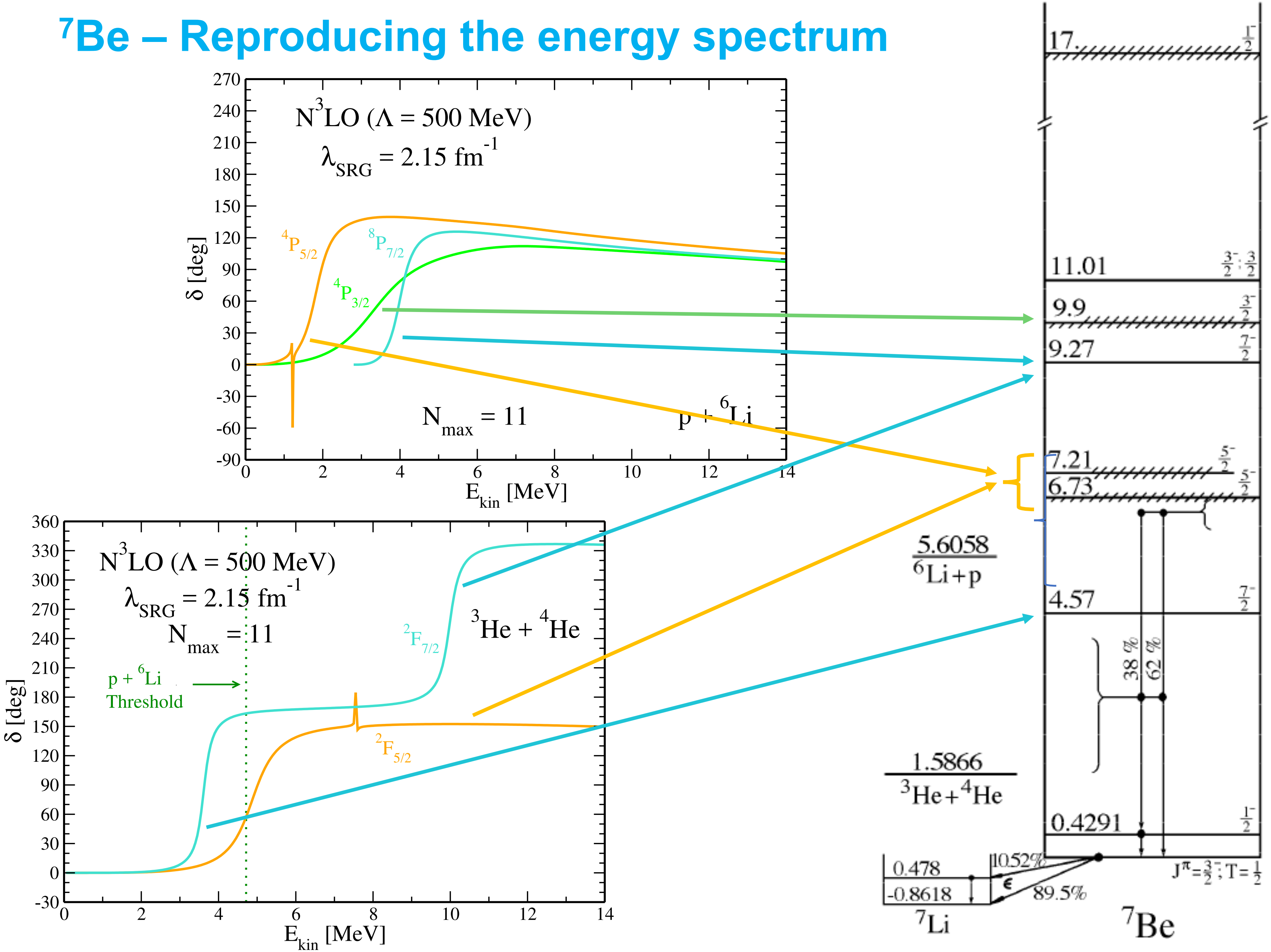
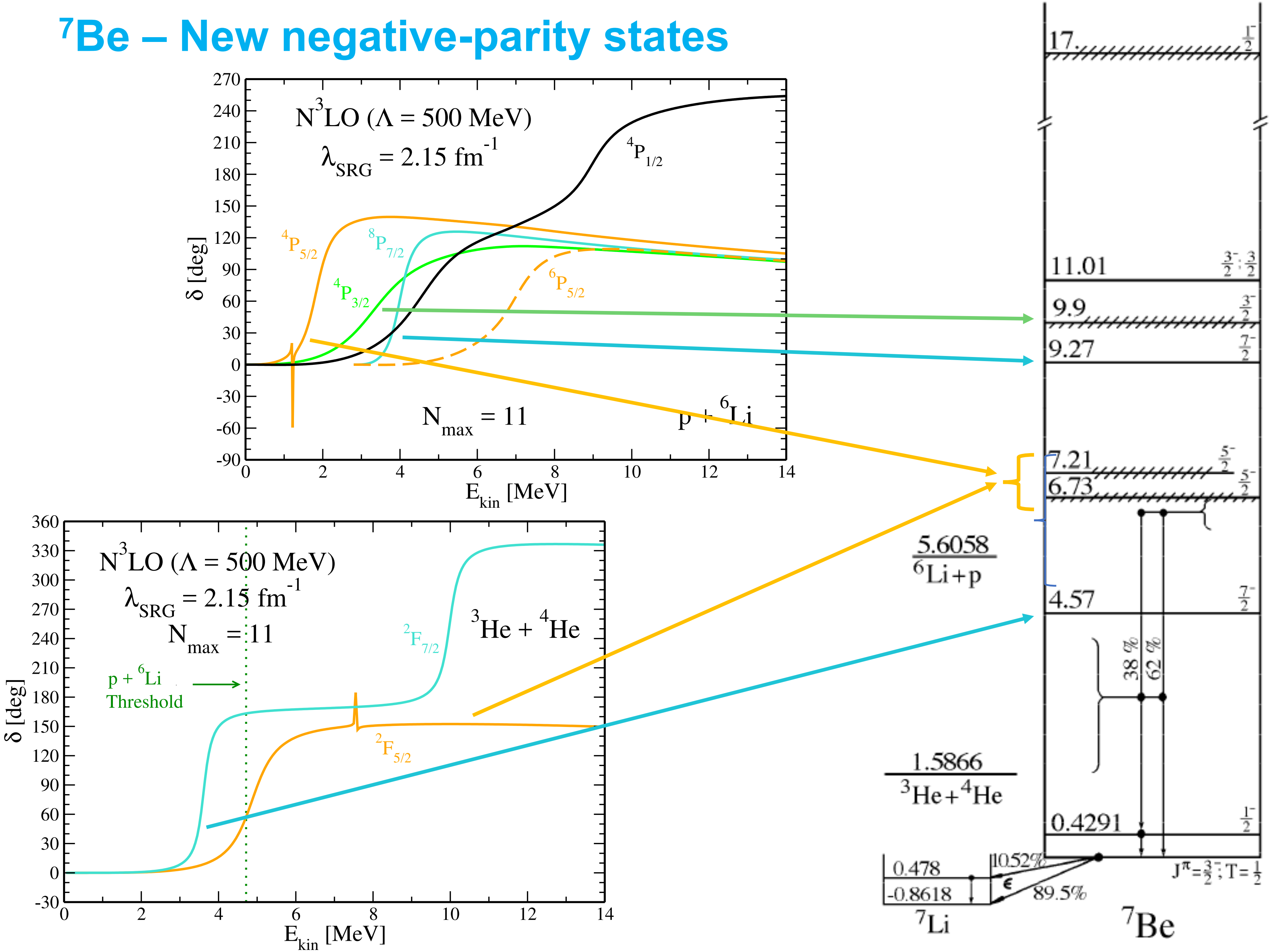
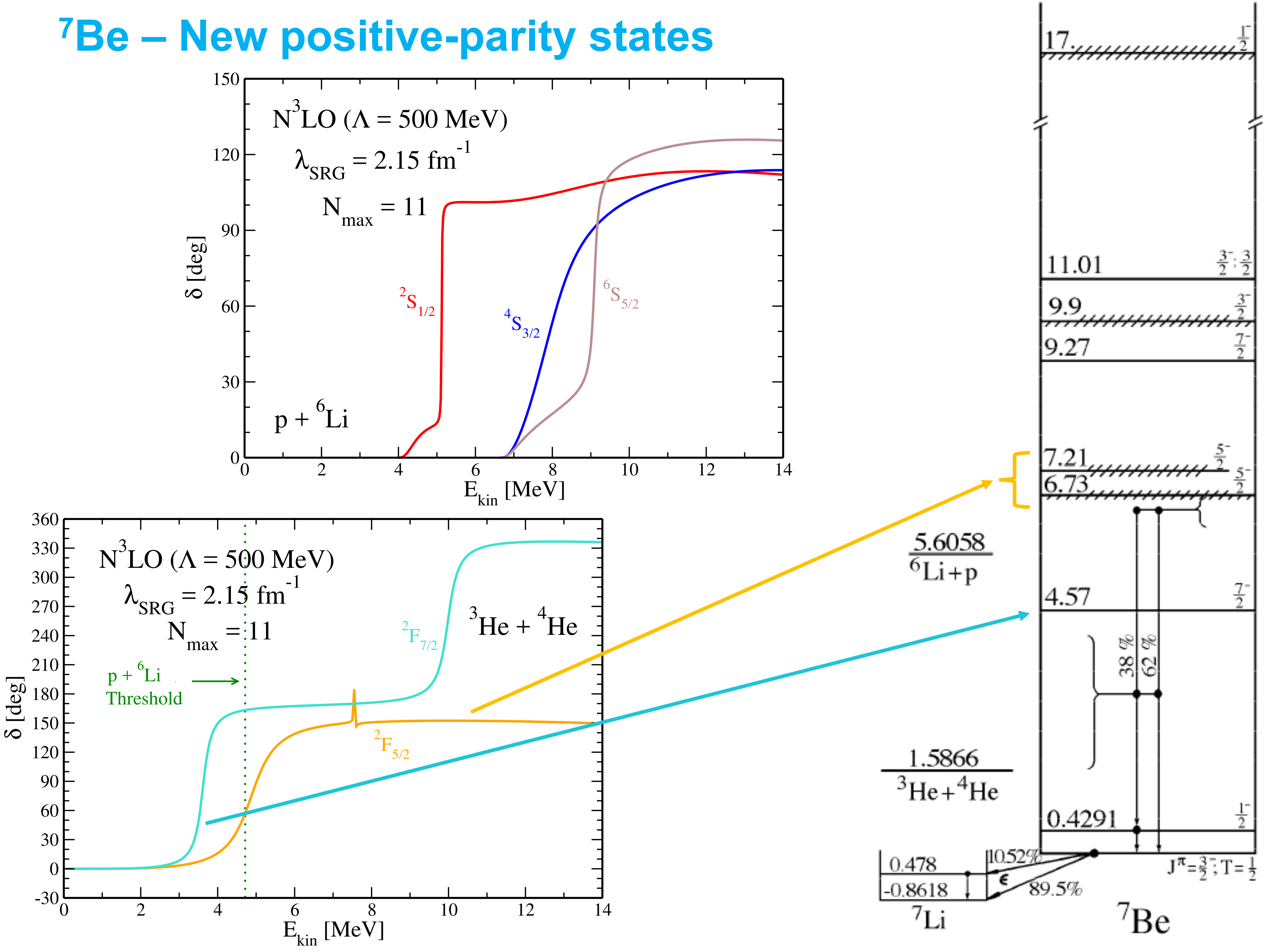


^{7}Be – New negative-parity states



^{7}Be – New positive-parity states



^{7}Li system

Analyzed mass partitions

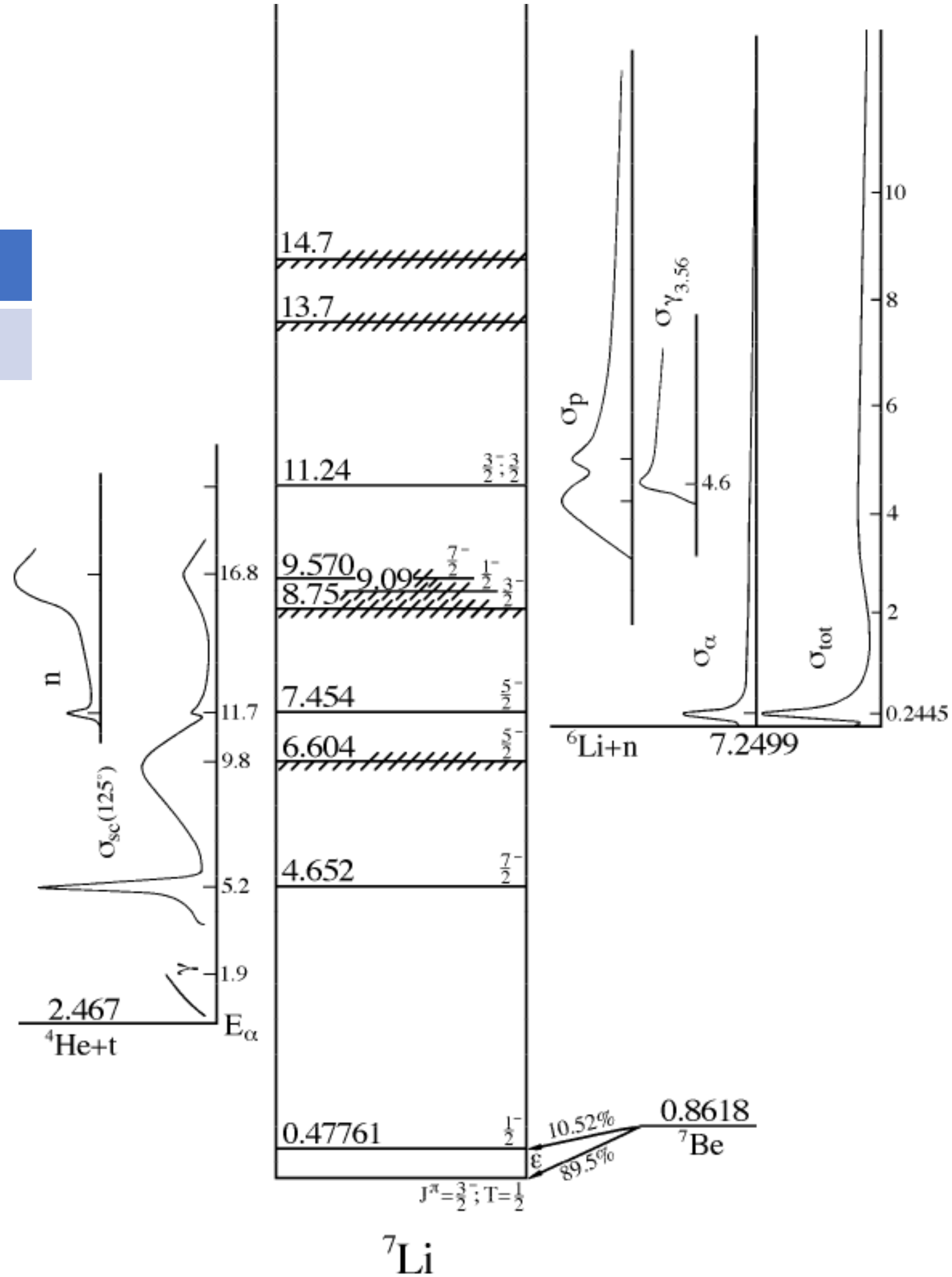
- ^{3}H + ^{4}He
- n + ^{6}Li
- p + ^{6}He

Exp.	$J^\pi = 3/2^-$
E [MeV]	-39.245

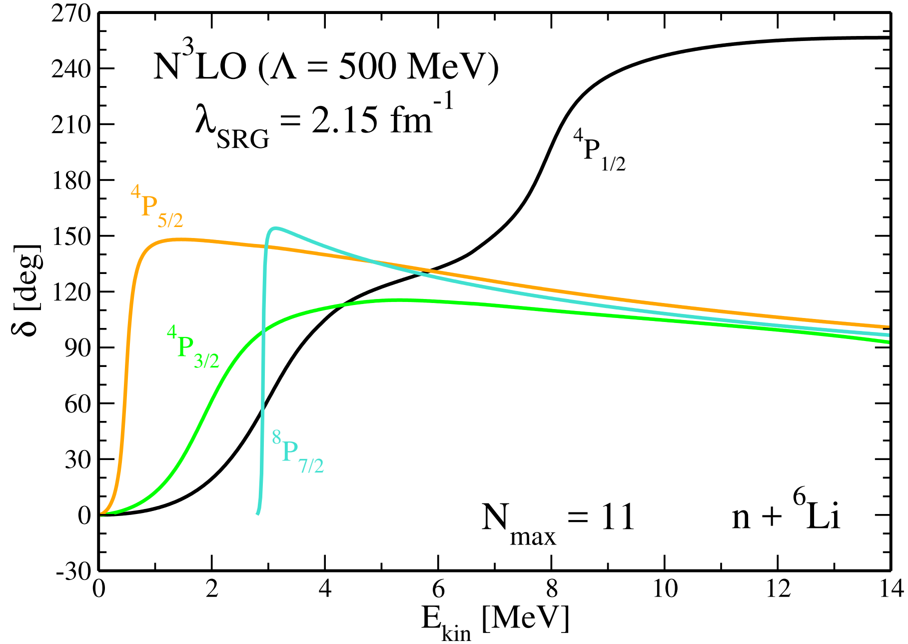
^{3}H + ^{4}He	$J^\pi = 3/2^-$	$J^\pi = 1/2^-$
E_{bound}	-2.432	-2.153
E [MeV]	-38.65	-38.37

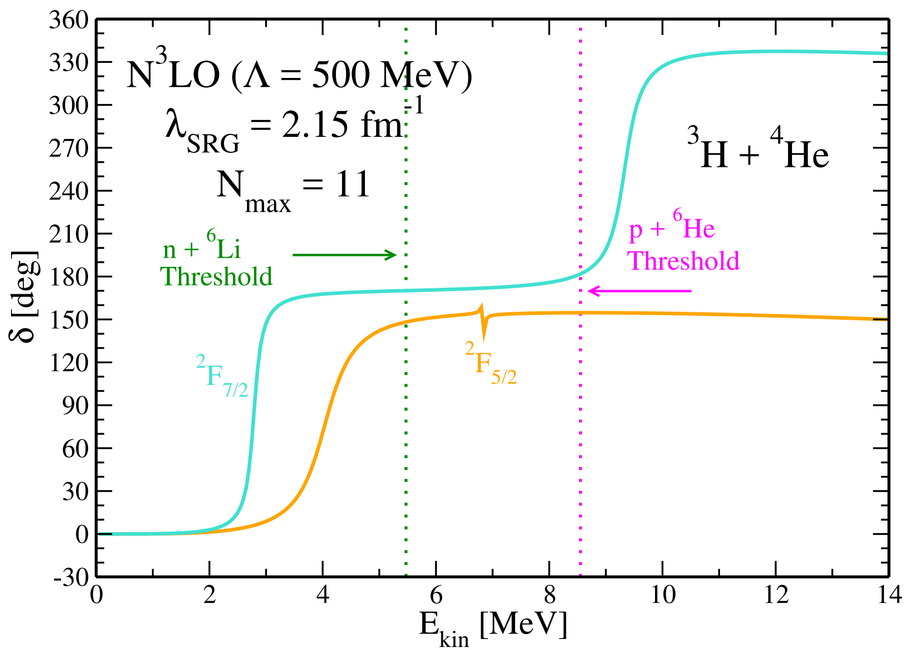
n + ^{6}Li	$J^\pi = 3/2^-$	$J^\pi = 1/2^-$
E_{bound}	-7.381	-7.048
E [MeV]	-38.13	-37.79

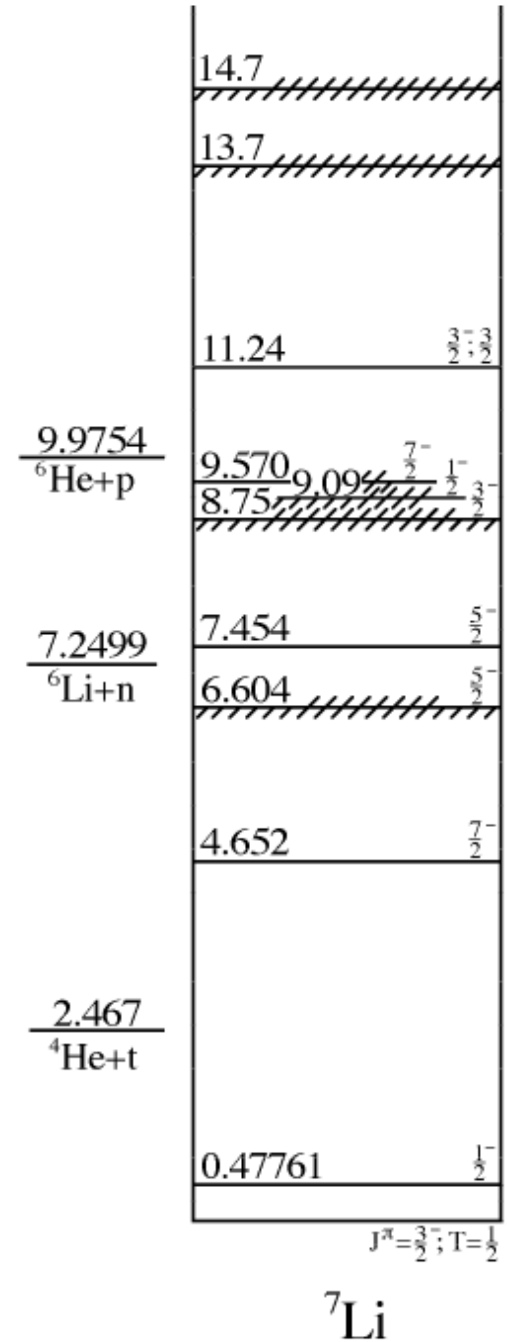
p + ^{6}He	$J^\pi = 3/2^-$	$J^\pi = 1/2^-$
E_{bound}	-10.40	-10.06
E [MeV]	-38.06	-37.73



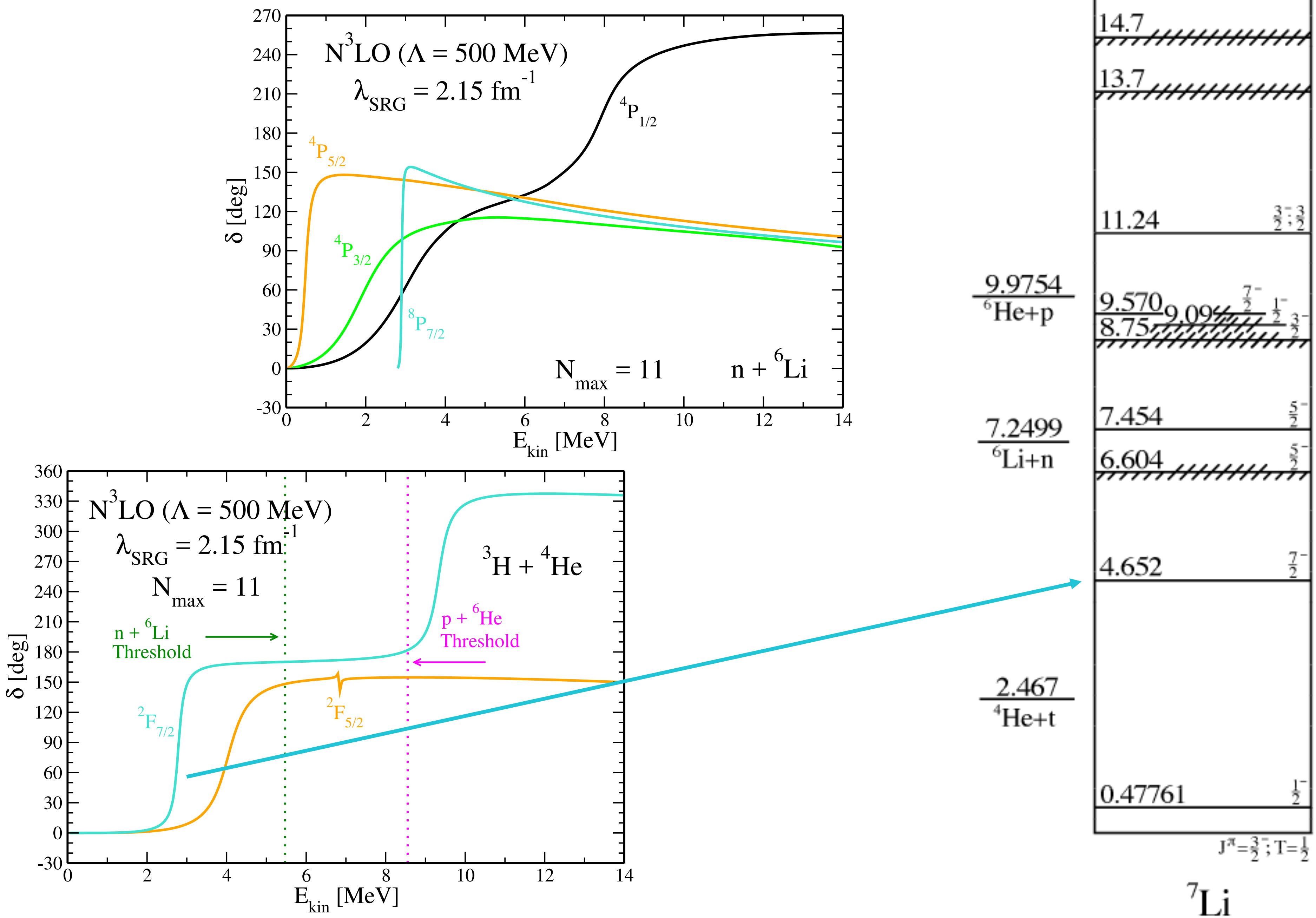
^{7}Li – Reproducing the energy spectrum







^{7}Li – Reproducing the energy spectrum



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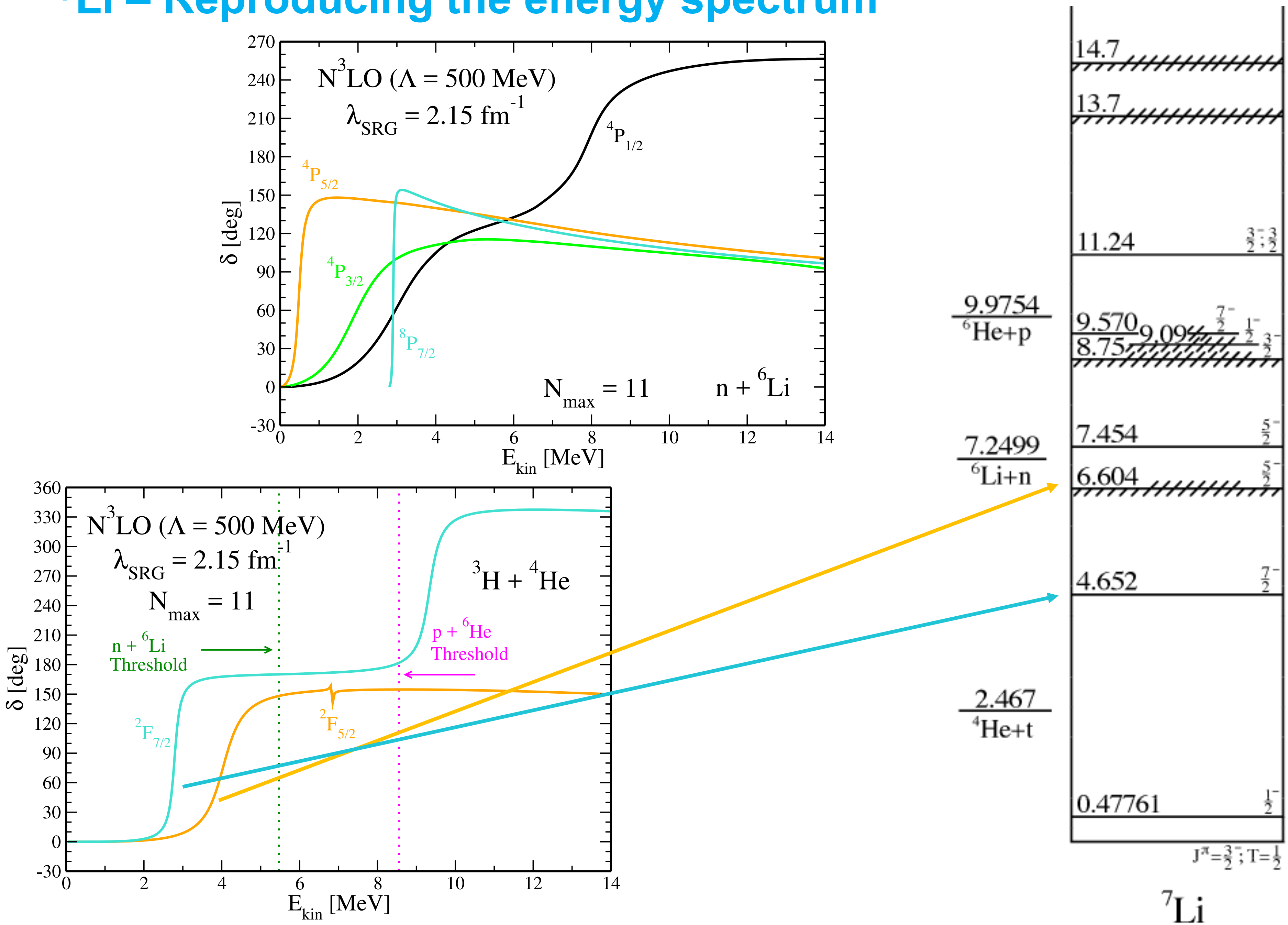

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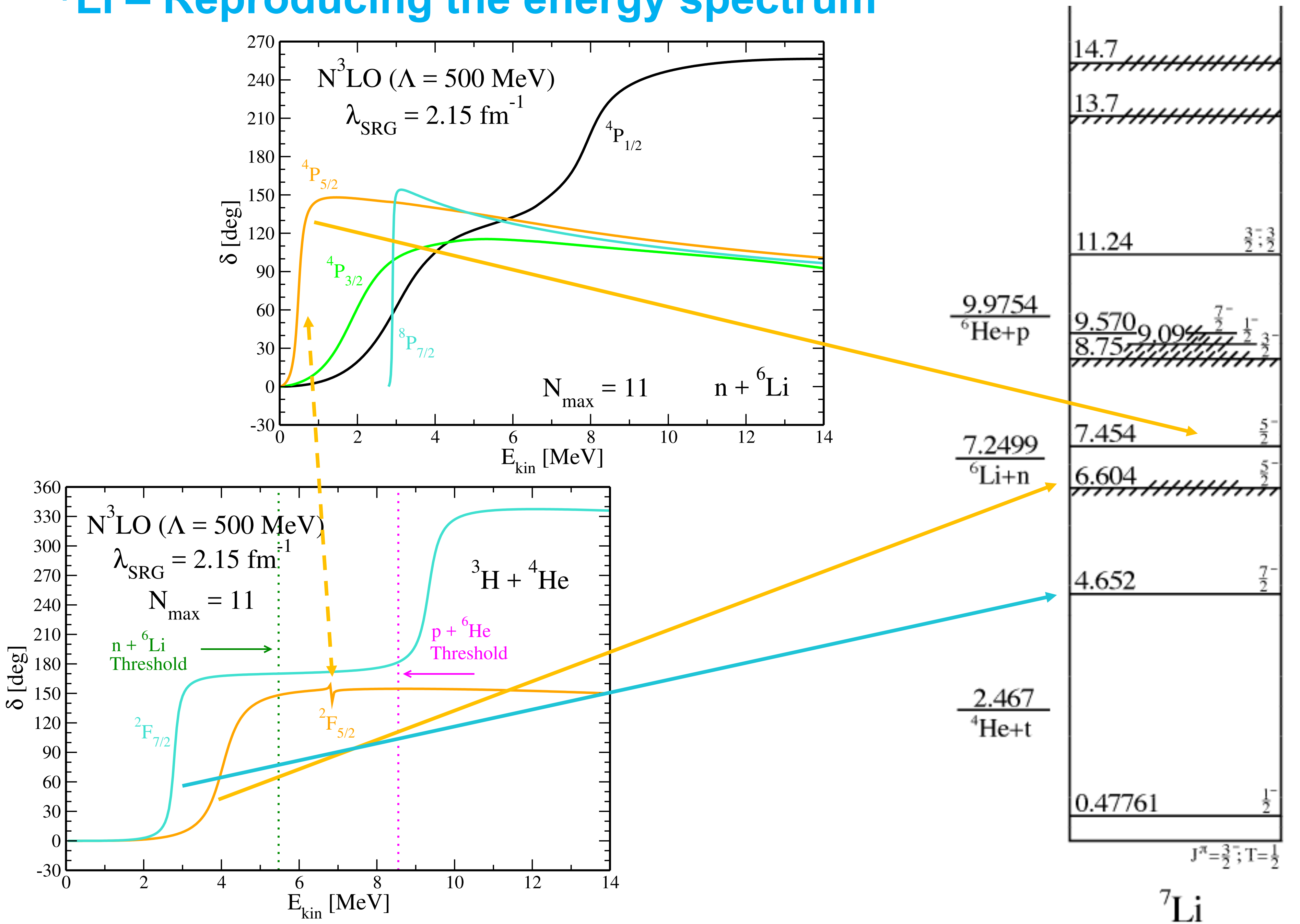

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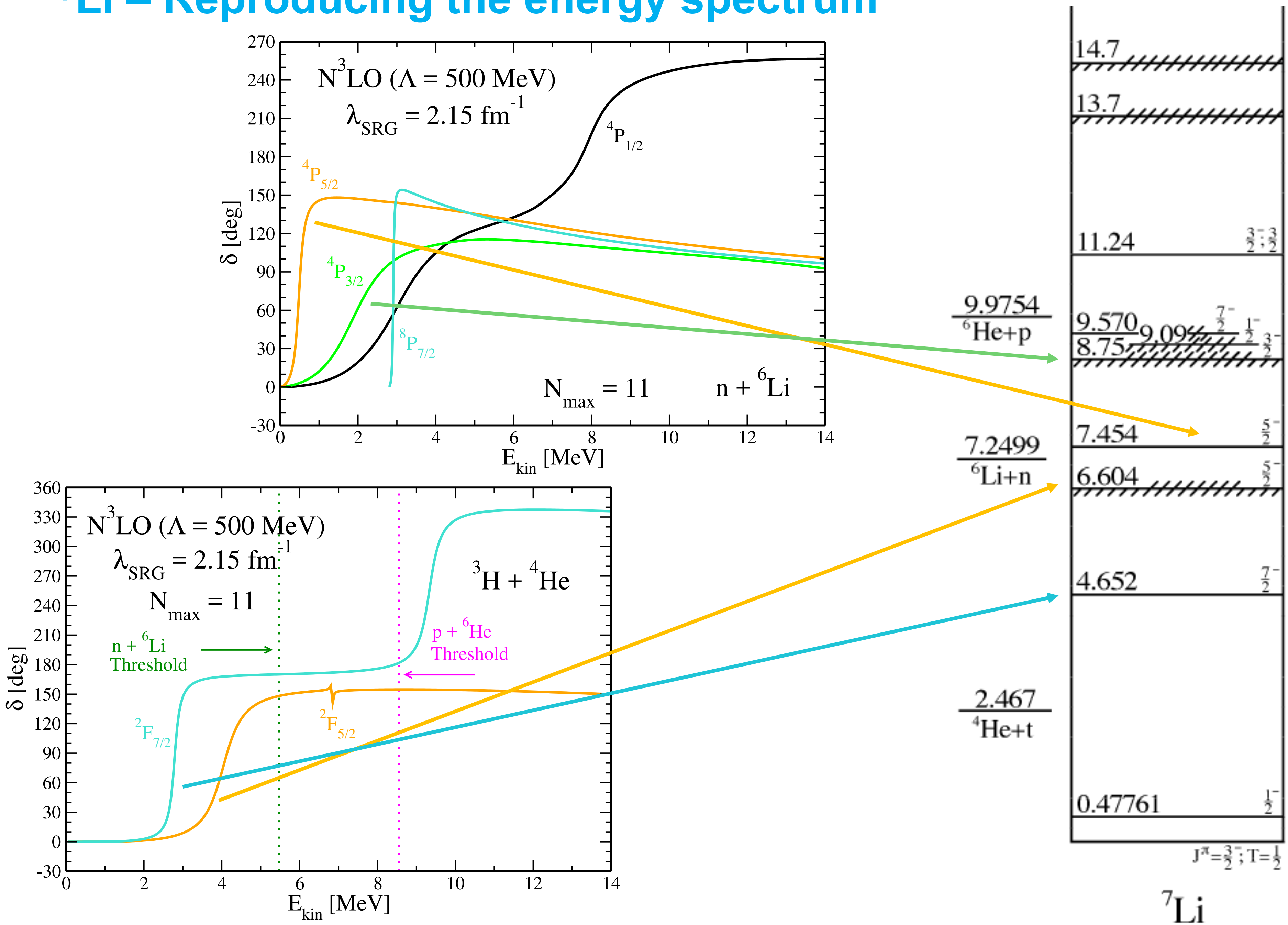

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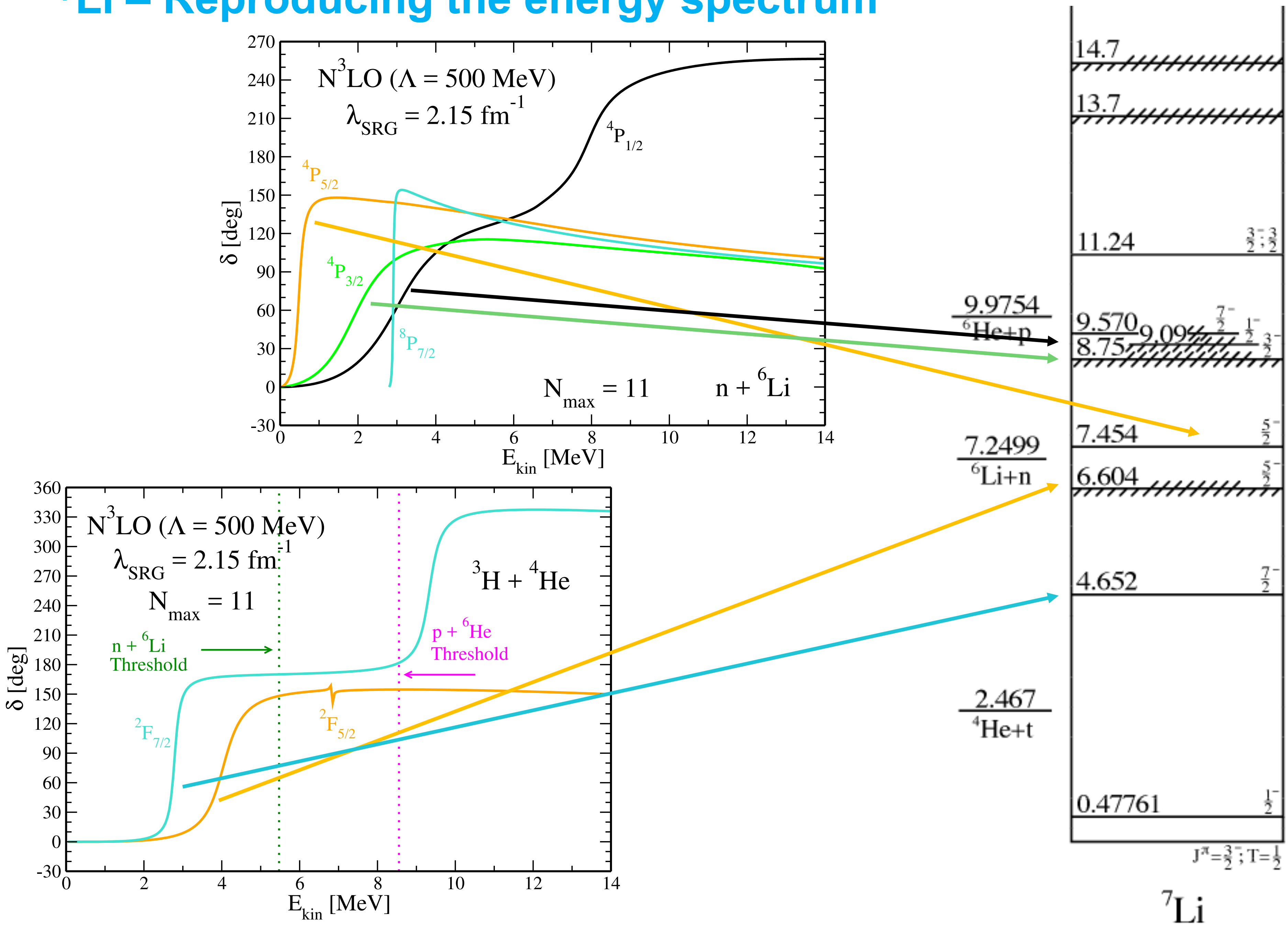

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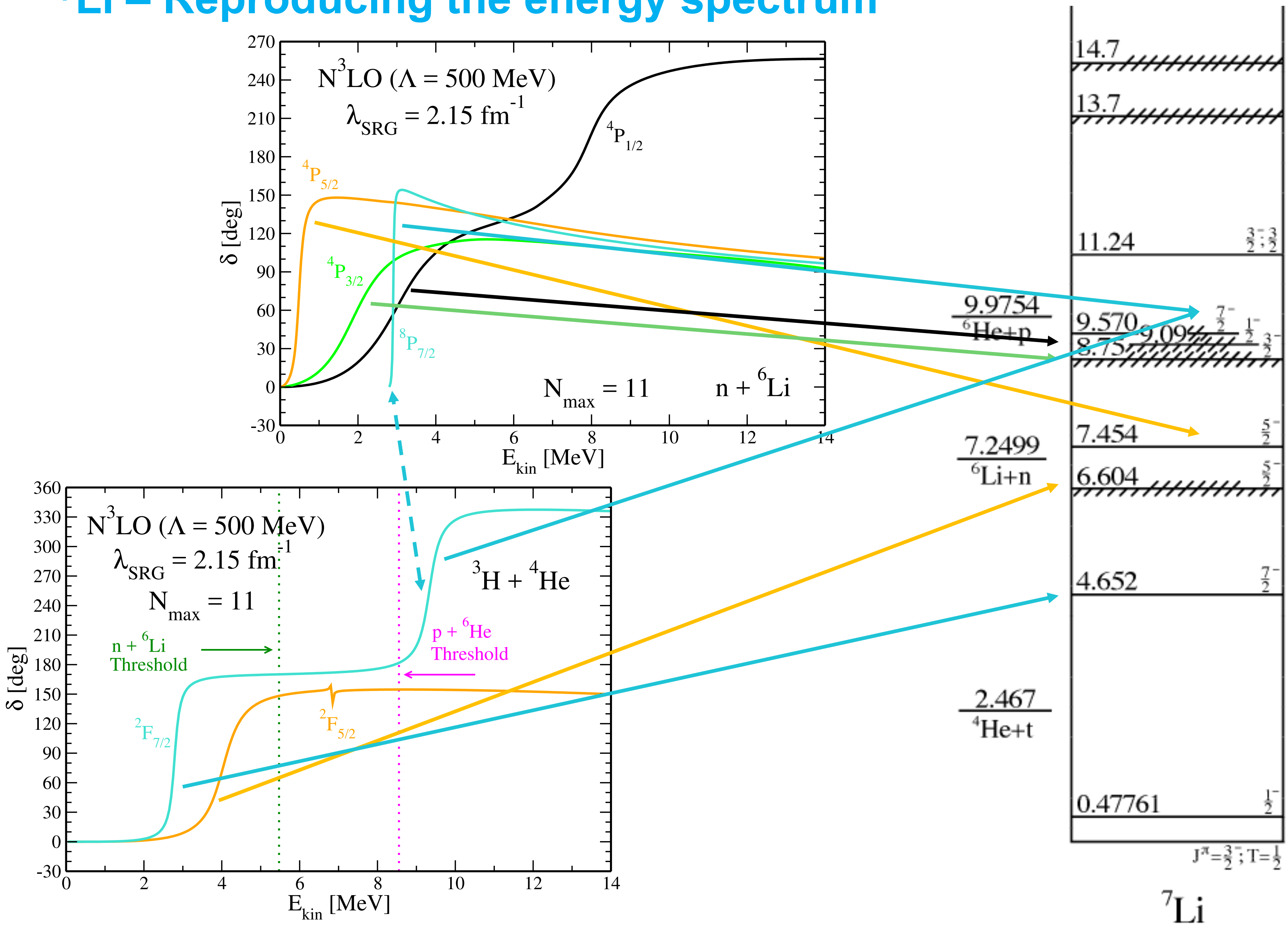

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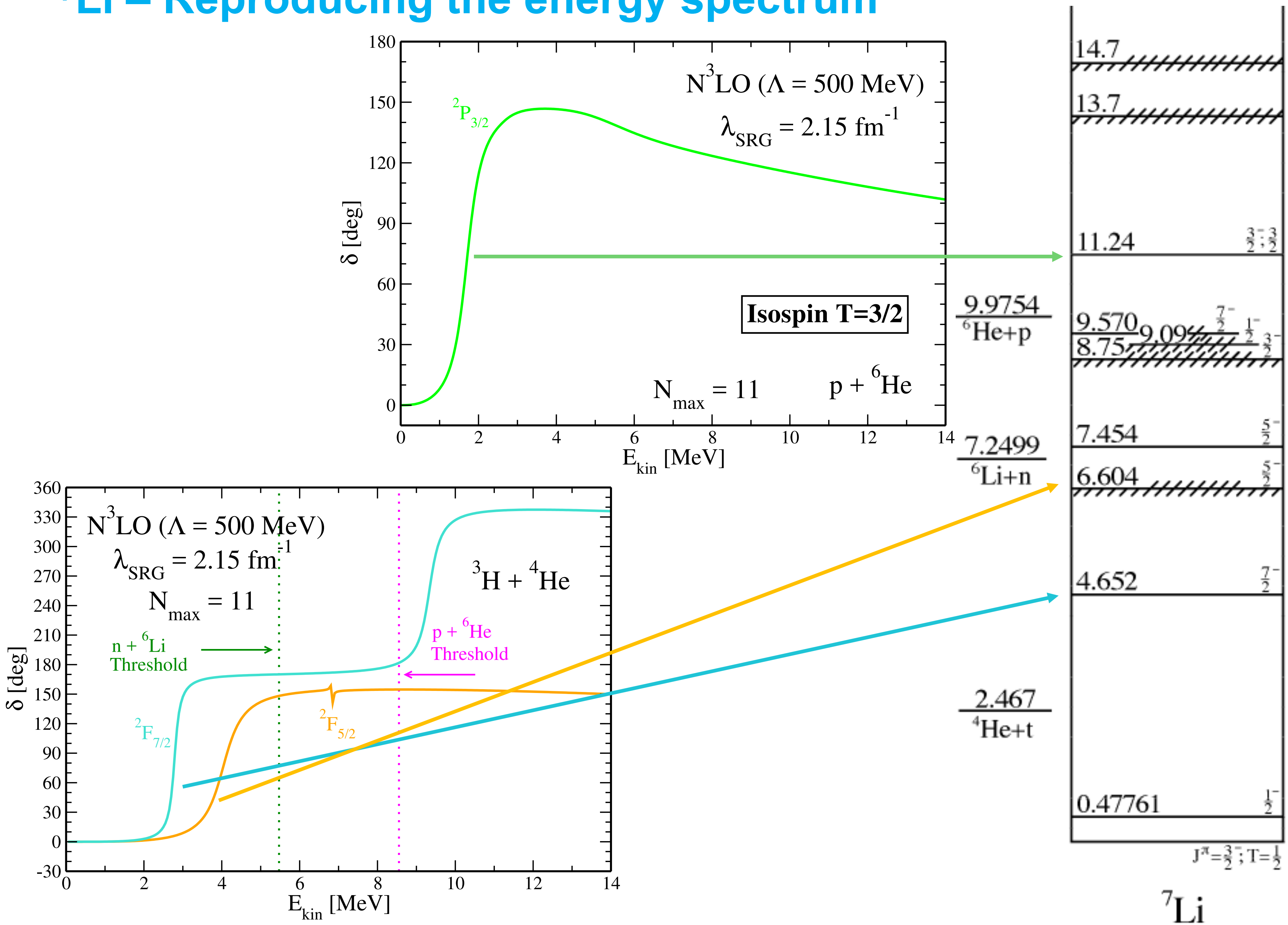
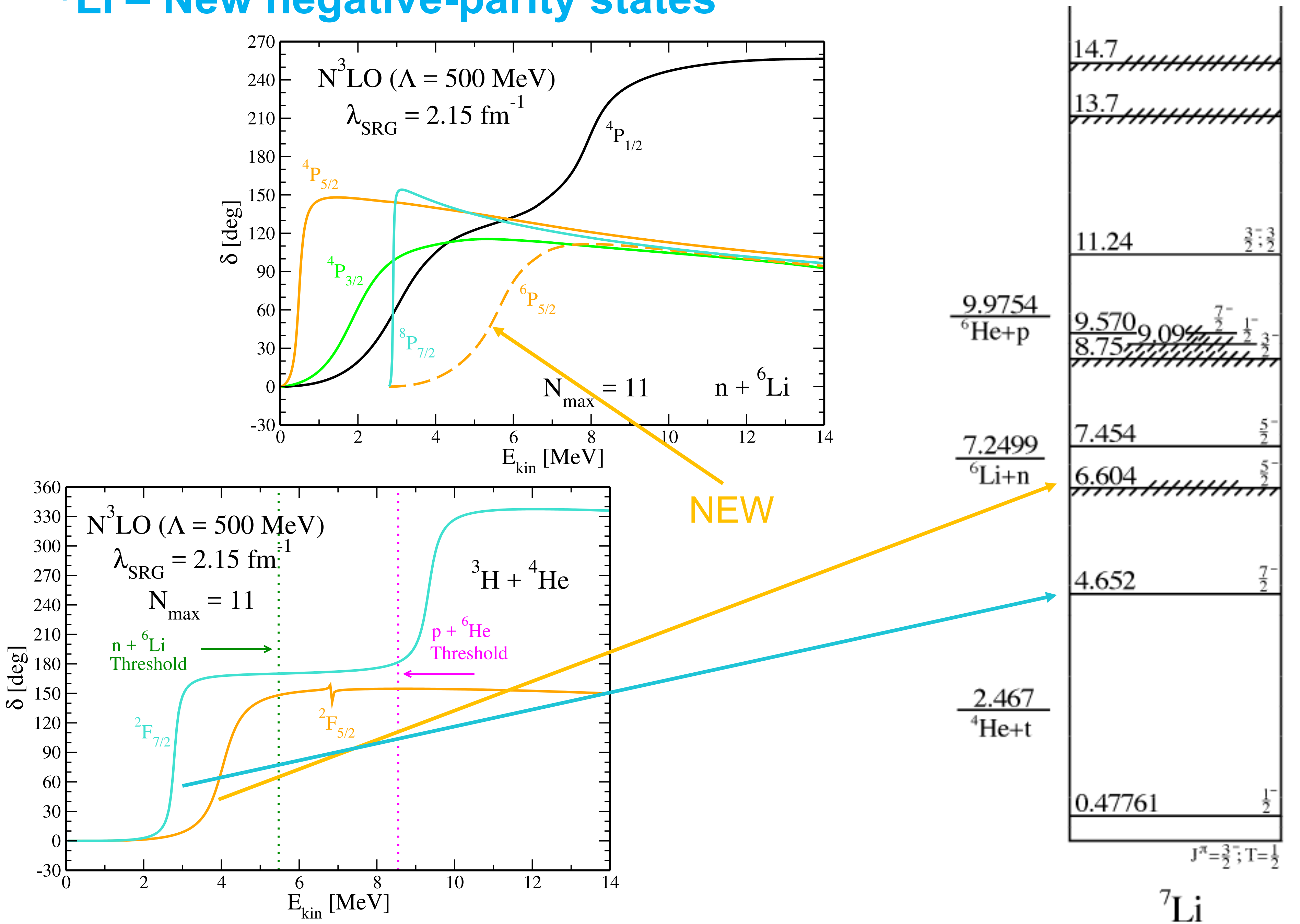
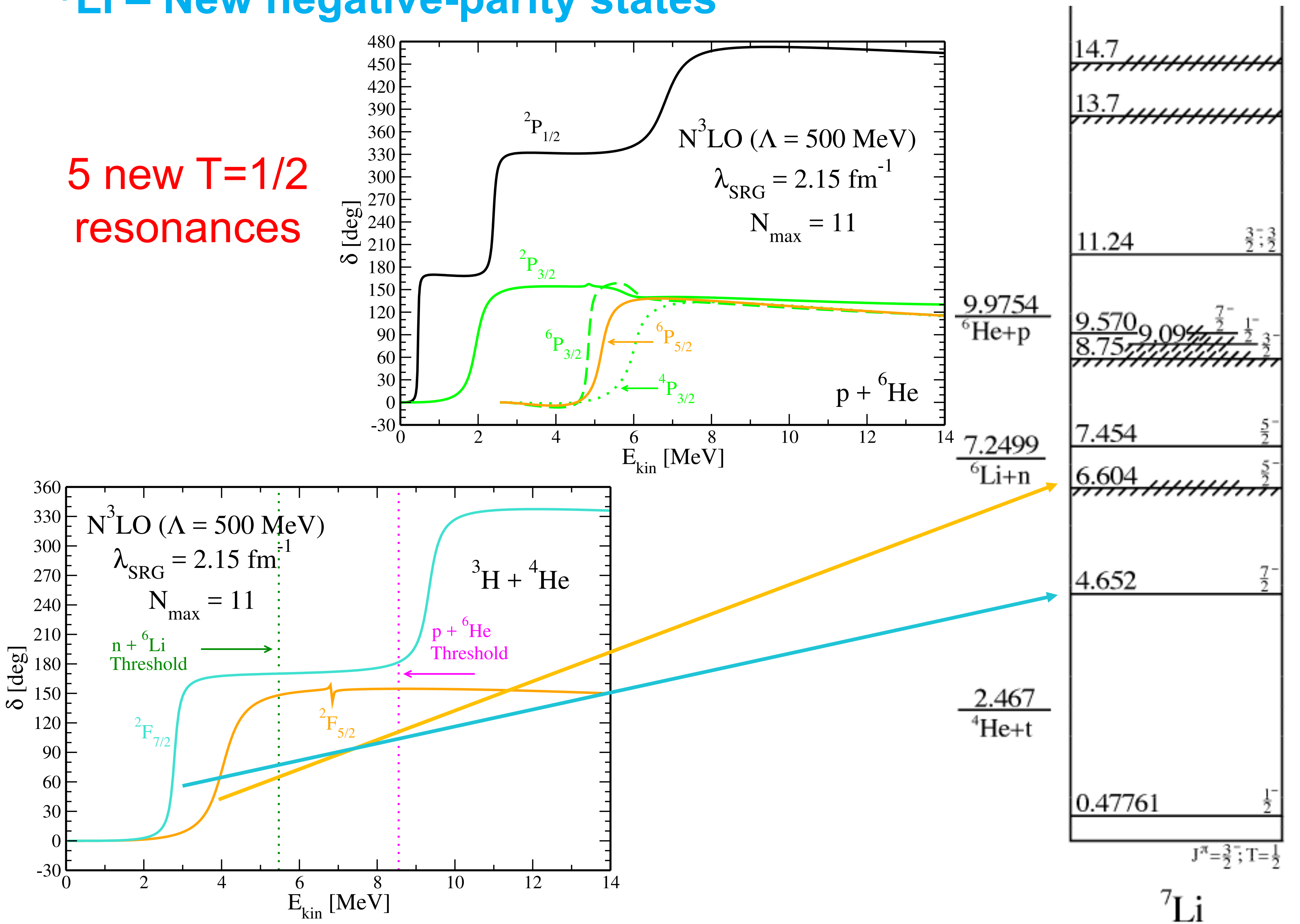


^{7}Li – New negative-parity states



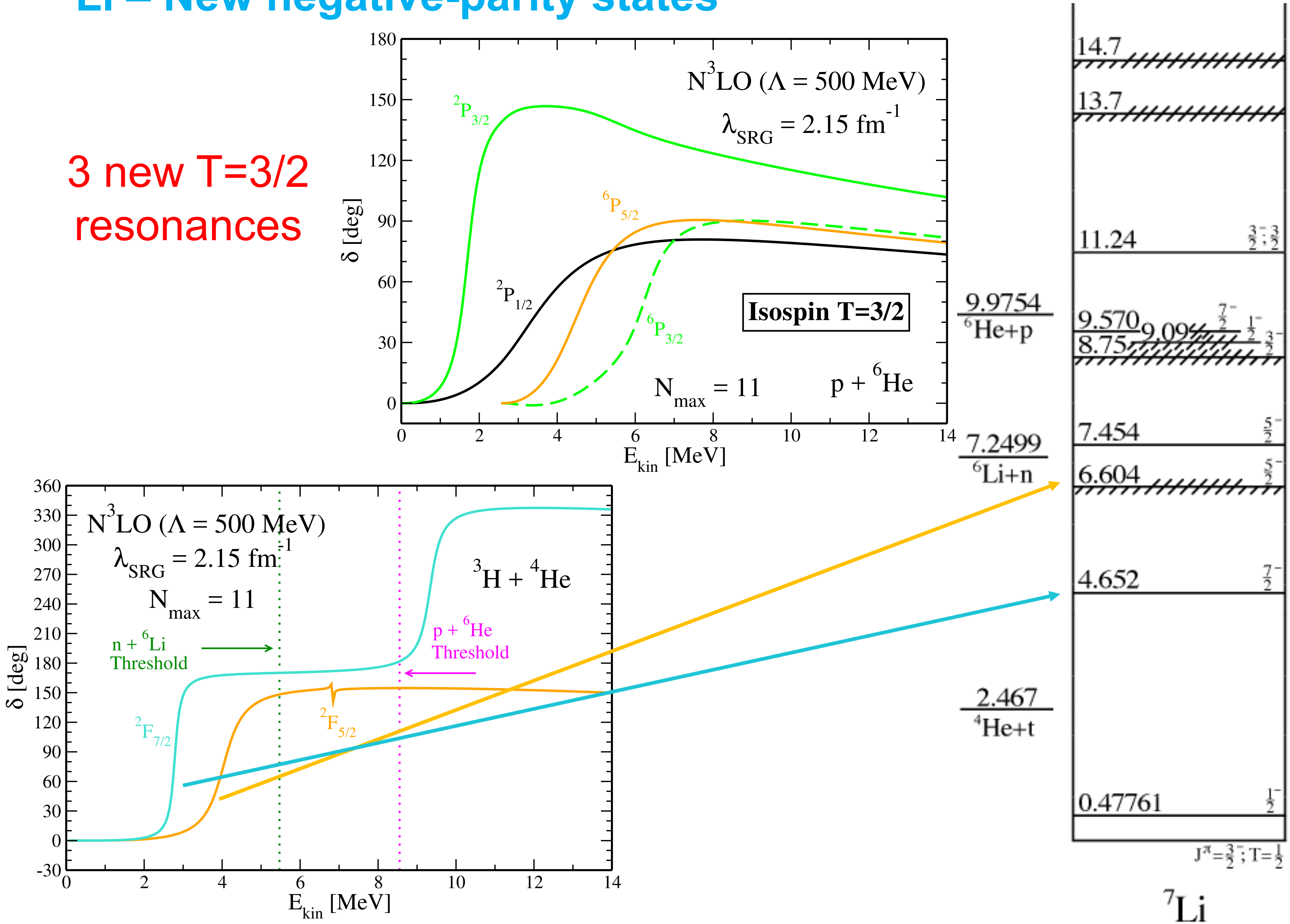
^{7}Li – New negative-parity states

5 new T=1/2 resonances

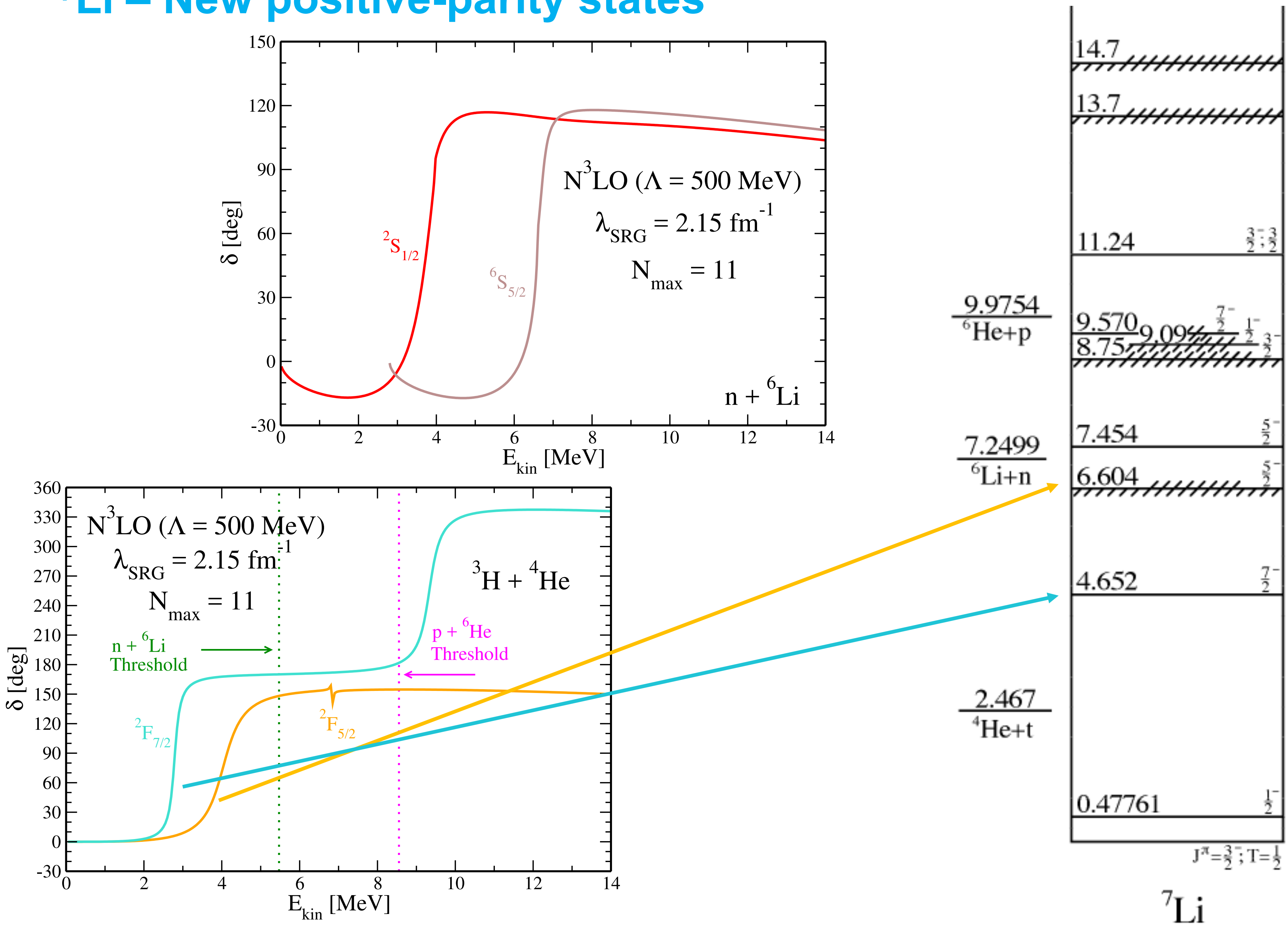


^{7}Li – New negative-parity states

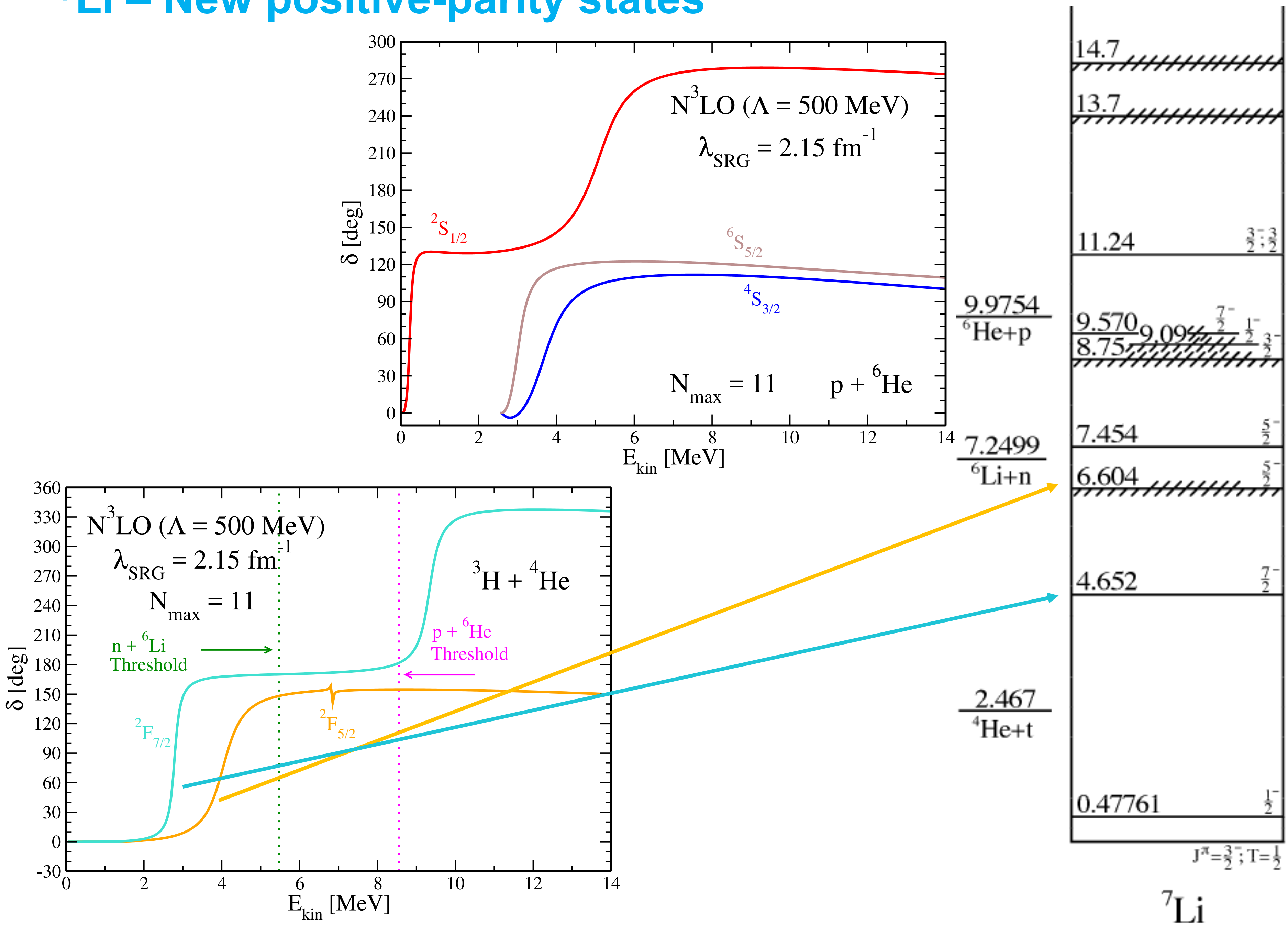
3 new T=3/2 resonances



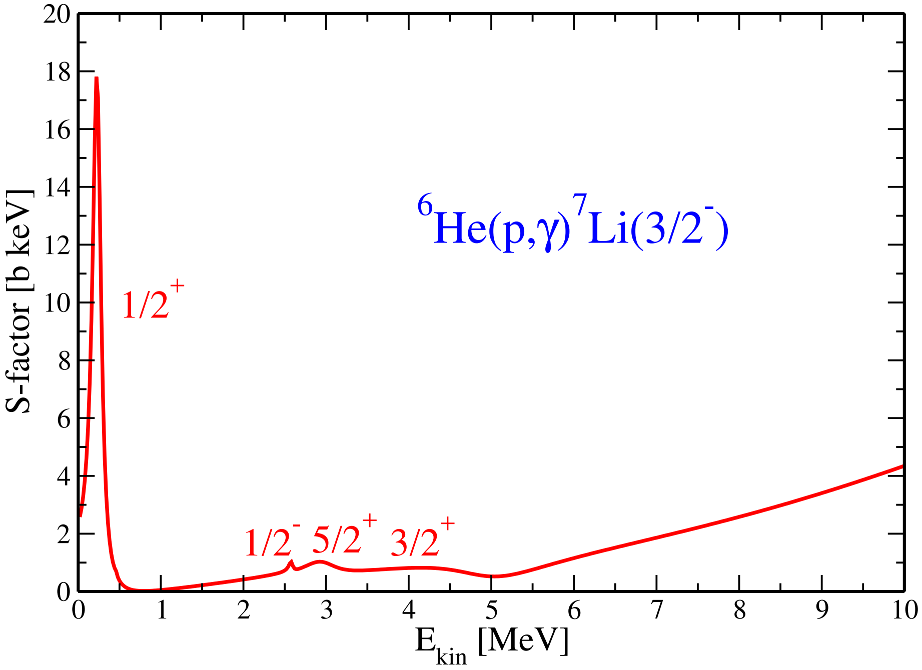
^{7}Li – New positive-parity states



^{7}Li – New positive-parity states



S-factor for $^6He(p,\gamma)^7Li$ reaction

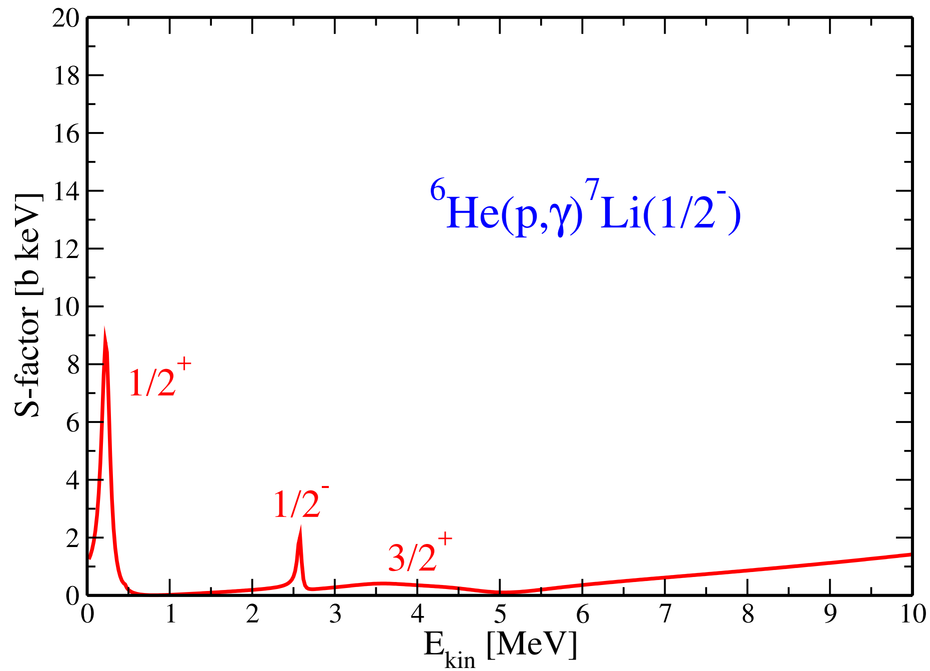


Cross section and S-factor

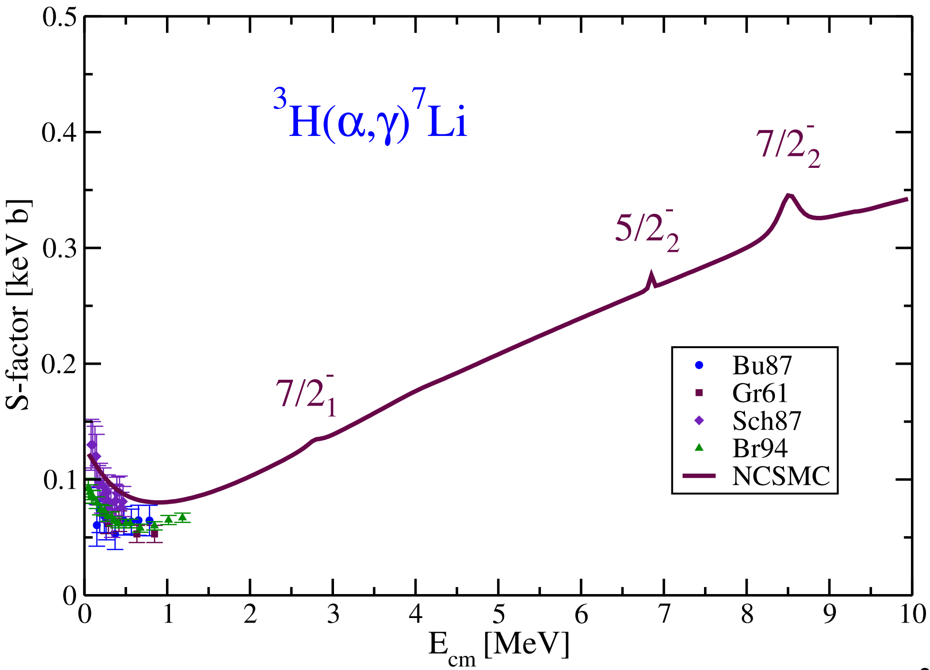
$$\sigma(E) = S(E)E^{-1}\exp[-2\pi\eta(E)]$$

Sommerfeld parameter

$$\eta(E) = \frac{Z_1 Z_2 e^2}{\hbar v}$$



S-factor for $^{3}H(\alpha,\gamma)^{7}Li$ and $^{3}He(\alpha,\gamma)^{7}Be$ reactions



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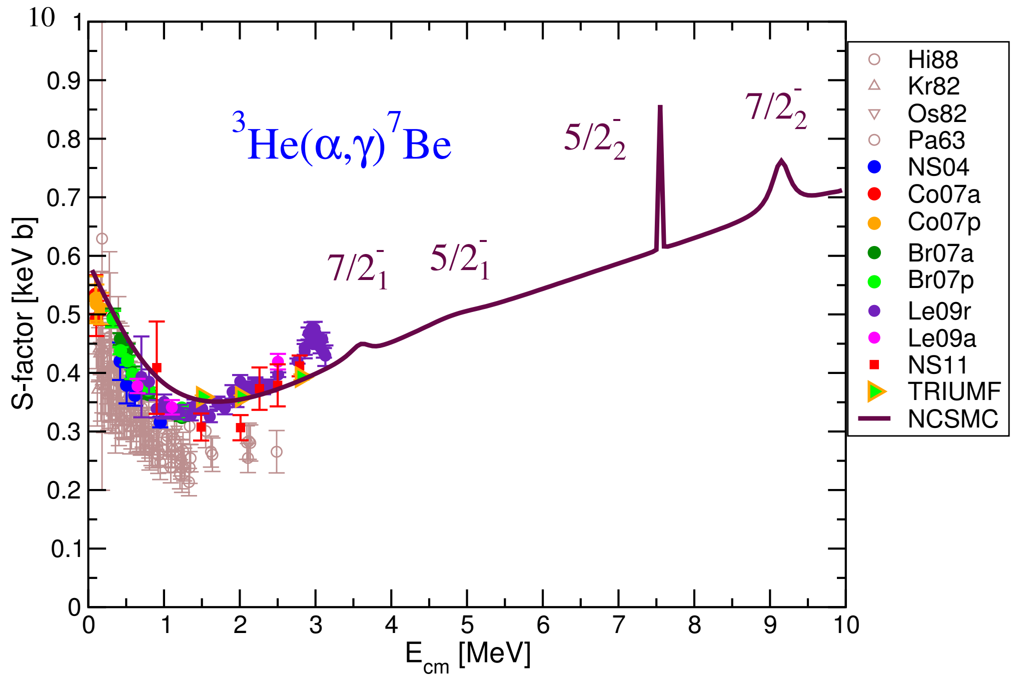


2.

Microscopic optical potentials for intermediate energies

Motivation

Nuclear reaction theory relies on reducing the many-body problem to a problem with few degrees of freedom: **optical potentials**.

Phenomenological	Microscopical
Unfortunately, currently used optical potentials for low-energy reactions are phenomenological, and primarily constrained by elastic scattering. **Unreliable when extrapolated beyond their fitted range in energy and nuclei**	Existing microscopic optical potentials are *usually* developed in an high-energy regime (≥ 100 MeV) and not applicable for lower energy reactions. **No fitting**

Statement of the problem

Lippmann-Schwinger (LS) equation for the nucleon-nucleus transition amplitude

$$T = V + VG_0(E)T$$

All two nucleon interactions

$$V = \sum_{i=1}^{A} v_{0i}$$

Green function propagator

$$G_0(E) = (E - H_0 + i\epsilon)^{-1}$$

where

$$H_0 = h_0 + H_A$$

h_0 kinetic term of the projectile

$$H_A |\Phi_A\rangle = E_A |\Phi_A\rangle$$

target Hamiltonian

Statement of the problem

Lippmann-Schwinger (LS) equation for the nucleon-nucleus transition amplitude

$$T = V + VG_0(E)T$$

Let's introduce the optical potential U

$$T = U + UG_0(E)PT$$

$$U = V + VG_0(E)QU$$

Projection operators

$$P + Q = 1$$

$$[G_0, P] = 0$$

In the case of elastic scattering,

P projects onto the elastic channel

$$P = \frac{|\Phi_A\rangle\langle\Phi_A|}{\langle\Phi_A|\Phi_A\rangle}$$

Statement of the problem

Lippmann-Schwinger (LS) equation for the nucleon-nucleus transition amplitude

$$T = V + VG_0(E)T$$

Transition amplitude *T* for elastic scattering

$$T_{\rm el} \equiv PTP = PUP + PUPG_0(E)T_{\rm el}$$

we need to calculate *PUP*

Statement of the problem

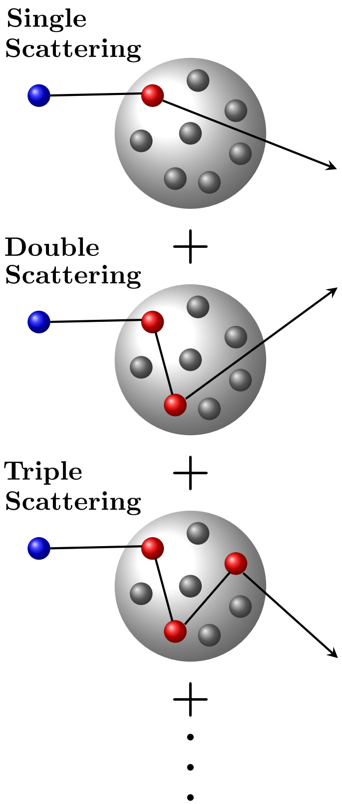
Lippmann-Schwinger (LS) equation for the nucleon-nucleus elastic transition amplitude

$$T_{\text{el}} = PUP + PUPG_0(E)T_{\text{el}}$$

Spectator expansion for U

Chinn, Elster, Thaler, Weppner, PRC **52,** 1992 (1995)

$$U = \sum_{i=1}^{A} \tau_i + \sum_{i,j\neq i}^{A} \tau_{ij} + \sum_{i,j\neq i,k\neq i,j}^{A} \tau_{ijk} + \cdots$$



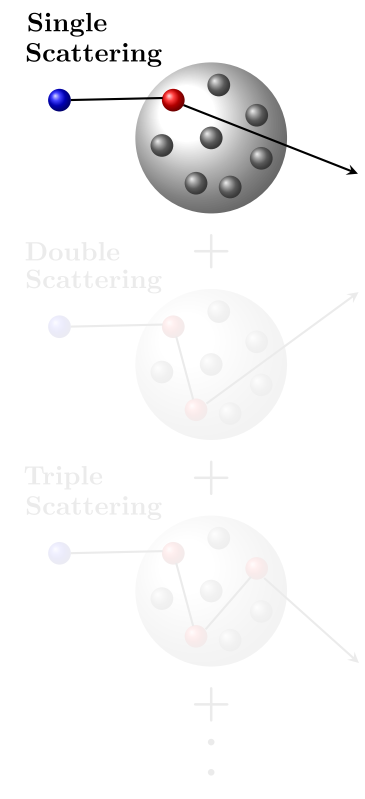
Single scattering approximation

Lippmann-Schwinger (LS) equation for the nucleon-nucleus <u>elastic</u> transition amplitude

$$T_{\rm el} = PUP + PUPG_0(E)T_{\rm el}$$

Spectator expansion for U

Chinn, Elster, Thaler, Weppner, PRC **52,** 1992 (1995)



Single scattering approximation

$$U = \sum_{i=1}^{A} \tau_i + \sum_{i,j\neq i}^{A} \tau_{ij} + \sum_{i,j\neq i,k\neq i,j}^{A} \tau_{ijk} + \cdots$$

Impulse approximation

Lippmann-Schwinger (LS) equation for the nucleon-nucleus elastic transition amplitude

$$T_{\rm el} = PUP + PUPG_0(E)T_{\rm el}$$

Spectator expansion for U

Chinn, Elster, Thaler, Weppner, PRC **52,** 1992 (1995)

$$\tau_i \approx t_{0i}$$

Impulse approximation

$$U = \sum_{i=1}^{A} t_{0i}$$

$$\begin{cases} t_{0i} = v_{0i} + v_{0i} g_i t_{0i} \\ g_i = \dfrac{1}{E - h_0 - h_i + i\epsilon} \end{cases}$$

The interaction between the projectile and the target nucleon is considered as free

The first-order optical potential

$$U(\boldsymbol{q},\boldsymbol{K};E) = \sum_{\alpha=n,p} \int d^3\boldsymbol{P}\,\eta(\boldsymbol{P},\boldsymbol{q},\boldsymbol{K})\,t_{p\alpha}\left[\boldsymbol{q},\frac{1}{2}\left(\frac{A+1}{A}\boldsymbol{K}-\boldsymbol{P}\right);E\right]$$

$$\times\,\rho_\alpha\left(\boldsymbol{P}-\frac{A-1}{2A}\boldsymbol{q},\boldsymbol{P}+\frac{A-1}{2A}\boldsymbol{q}\right)$$

$$\boldsymbol{q} = \boldsymbol{k}' - \boldsymbol{k}$$

$$\boldsymbol{K} = \frac{1}{2}(\boldsymbol{k}' + \boldsymbol{k})$$

Basic ingredients

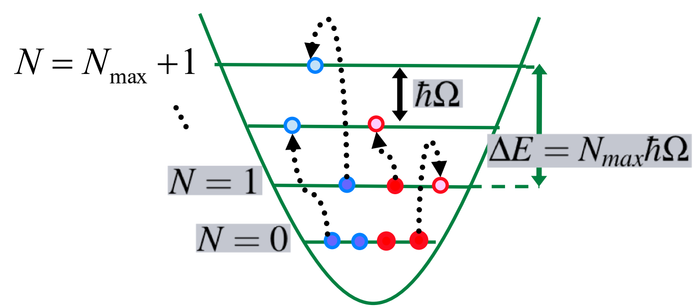
- Nucleon-nucleon scattering matrix t_{NN}
- Non-local nuclear densities

$$\rho_{\mathrm{op}} = \sum_{i=1}^{A} \delta(\boldsymbol{r}-\boldsymbol{r}_i)\delta(\boldsymbol{r}'-\boldsymbol{r}'_i)$$

The matrix elements between a general initial and final state are obtained from the NCSM
Extension of: Navratil, PRC **70**, 014317 (2004)

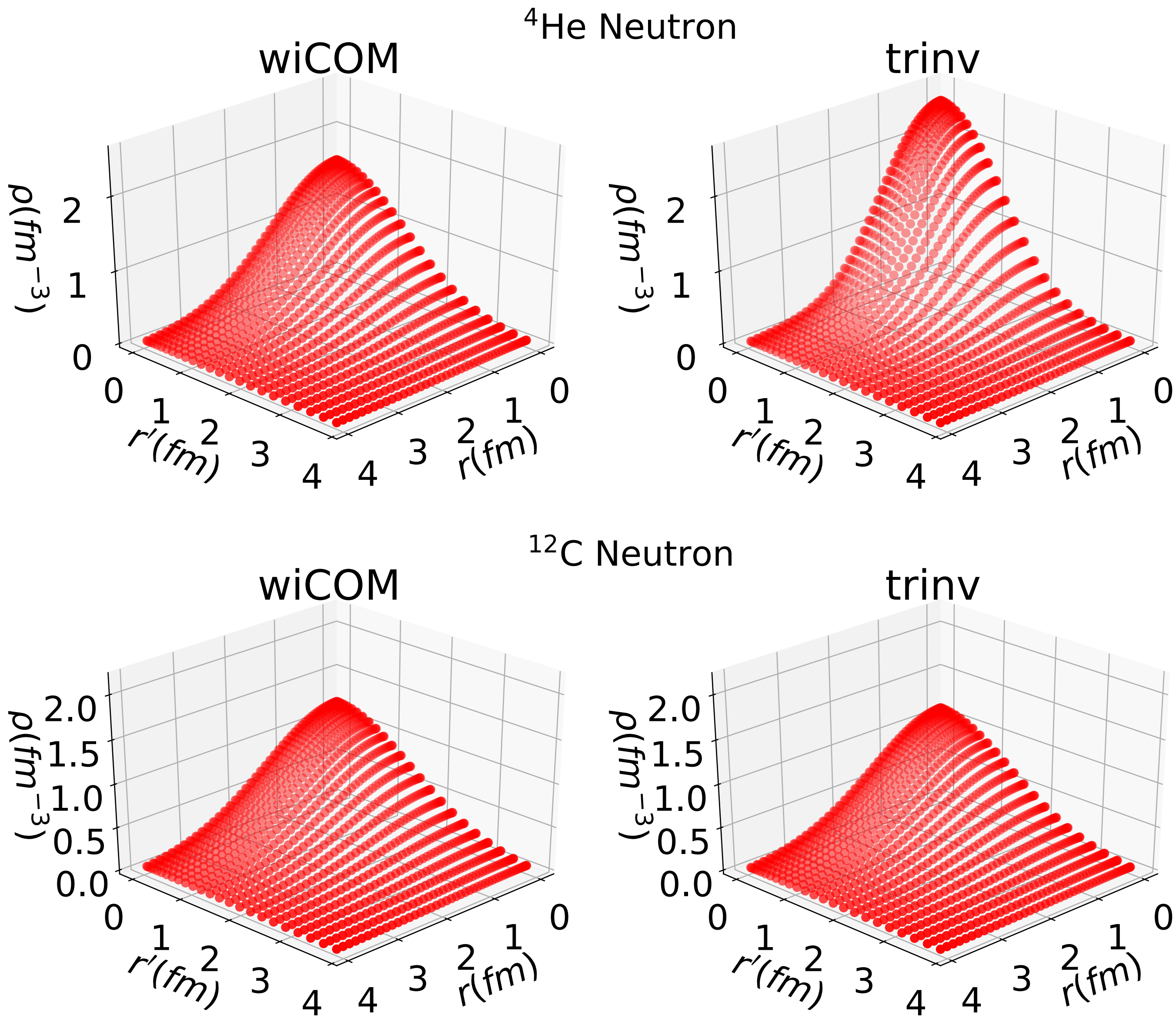
NCSM

$$|\Psi_A^{J^\pi T}\rangle = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni}^{J^\pi T} |ANiJ^\pi T\rangle$$



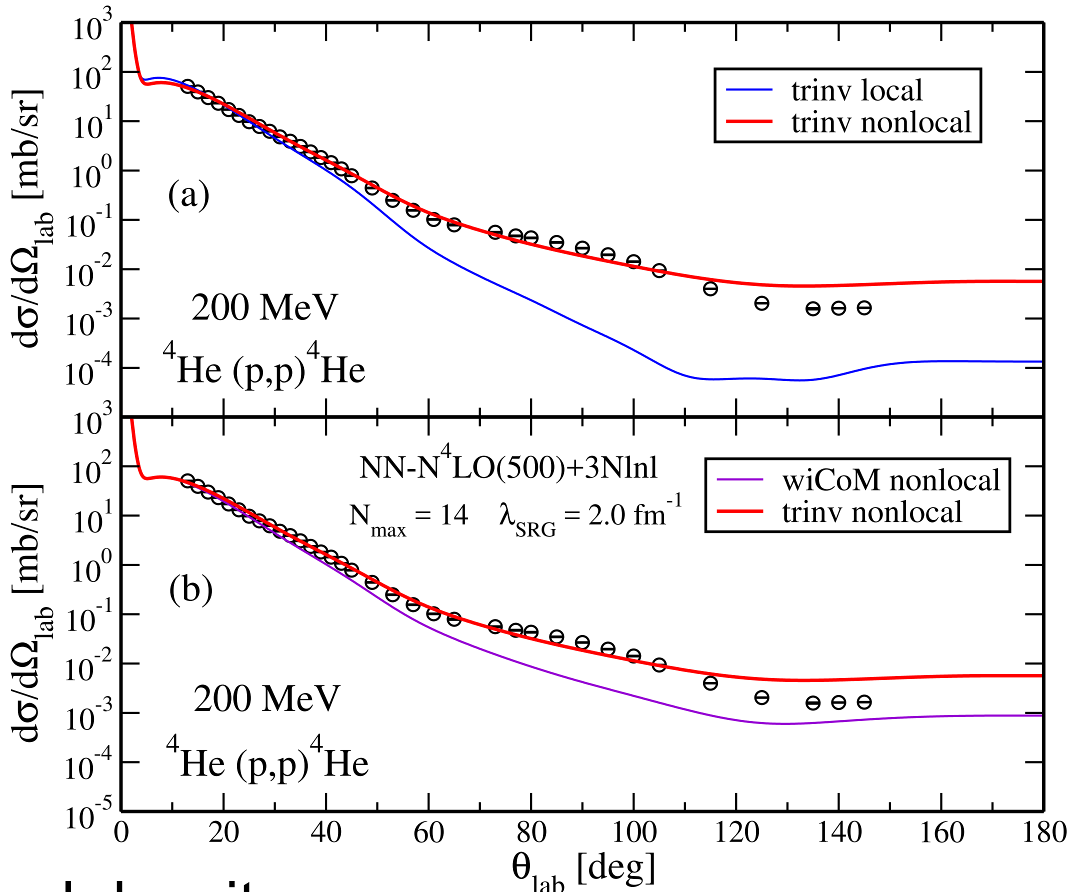
Non-local densities

Gennari, Vorabbi, Calci, Navratil, PRC **97**, 034619 (2018)



Scattering observables – Stable nuclei

Gennari, Vorabbi, Calci, Navratil, PRC **97**, 034619 (2018)



FF from local density

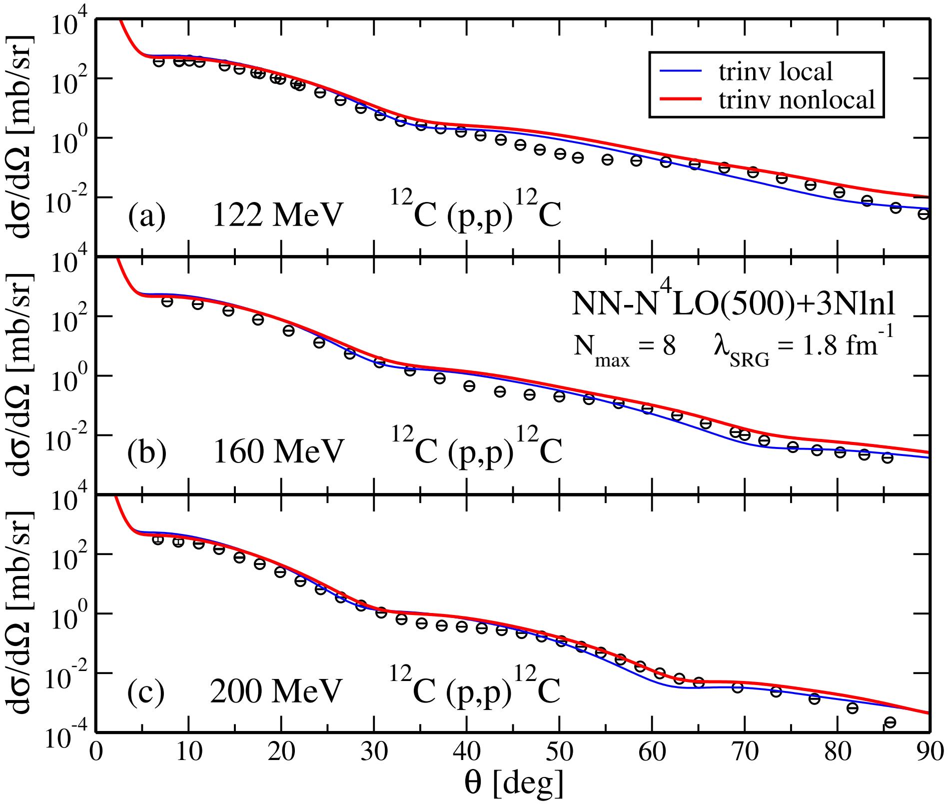
$$\rho_\alpha(q) = 4\pi \int_0^\infty dr r^2 j_0(qr) \rho_\alpha(r)$$

Navratil, PRC **70**, 014317 (2004)

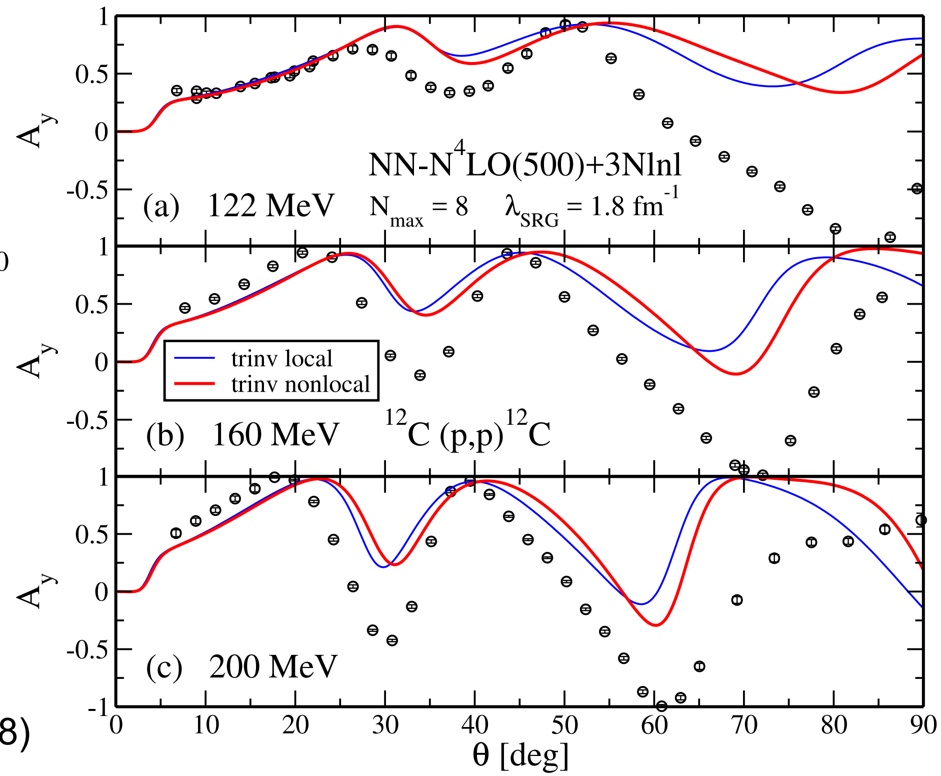
Factorized optical potential

$$U(\boldsymbol{q}, \boldsymbol{K}; E) \sim \sum_{\alpha=n,p} t_{p\alpha} \left[\boldsymbol{q}, \frac{A+1}{2A} \boldsymbol{K}; E \right] \rho_\alpha(q)$$

Scattering observables – Stable nuclei



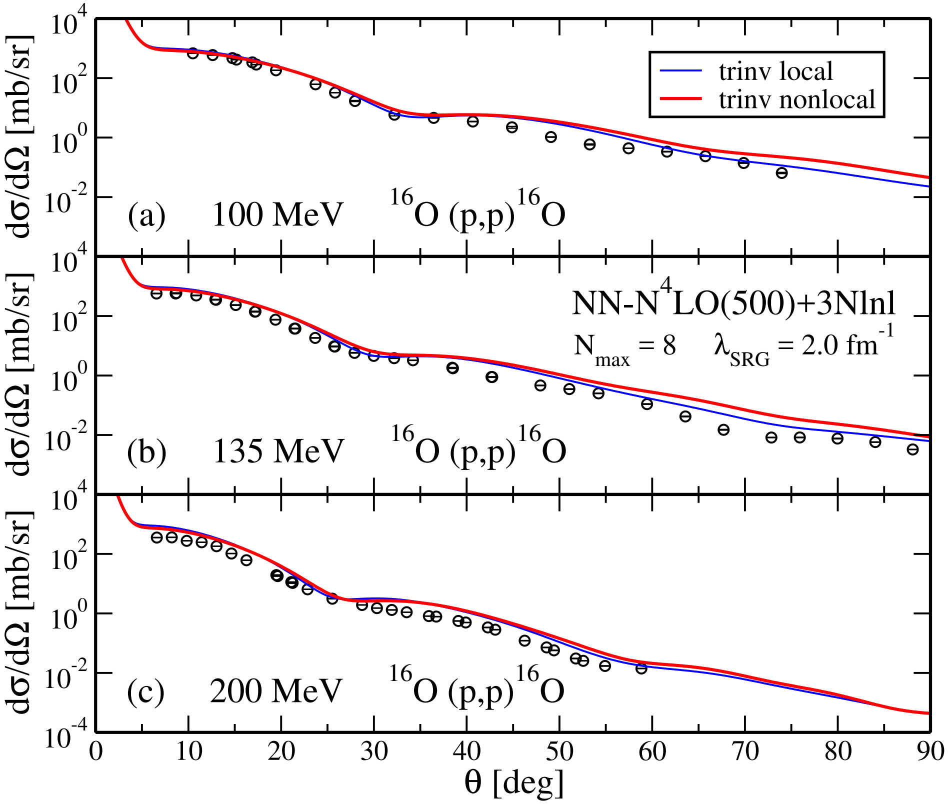
Good description of differential cross sections

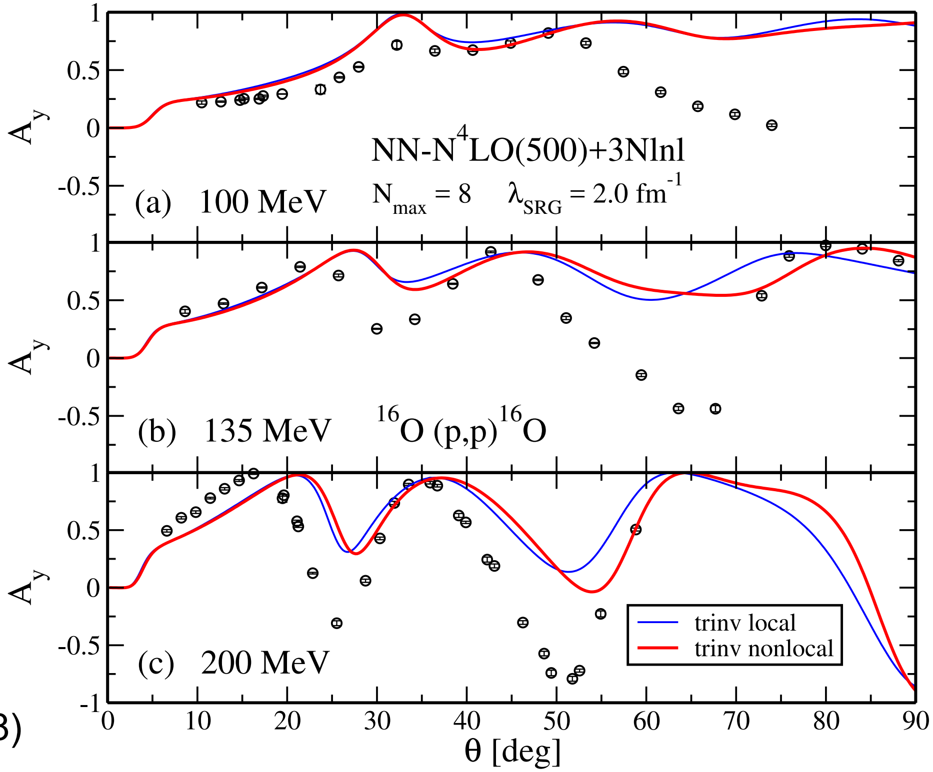


Reproduction of the general trend of the A_y

Gennari, Vorabbi, Calci, Navratil, PRC **97**, 034619 (2018)

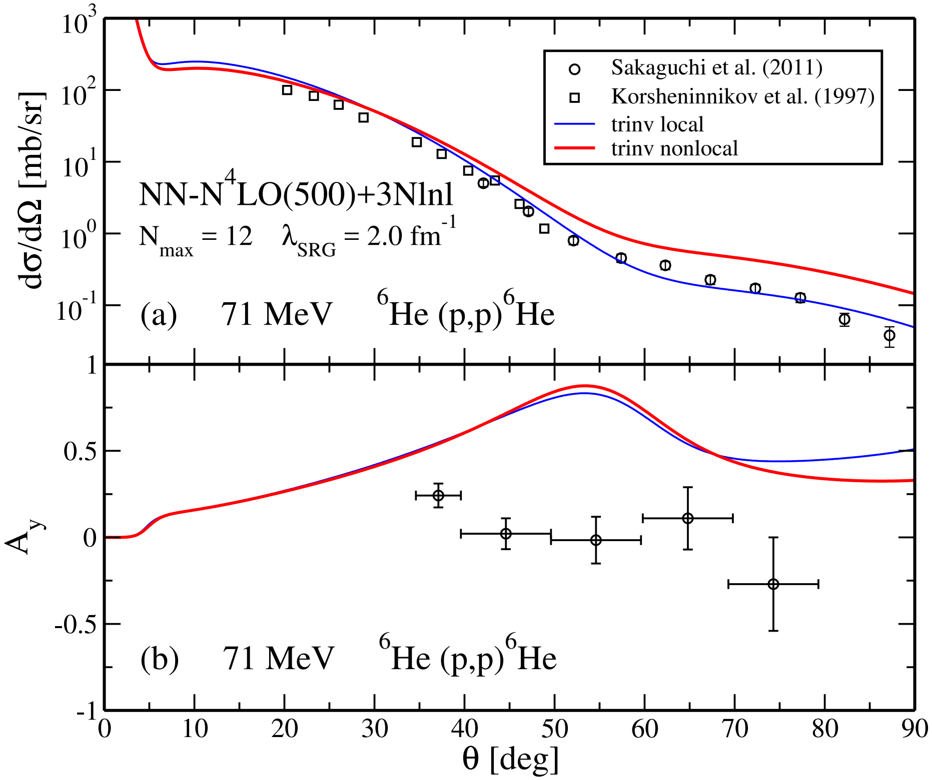
Scattering observables – Stable nuclei

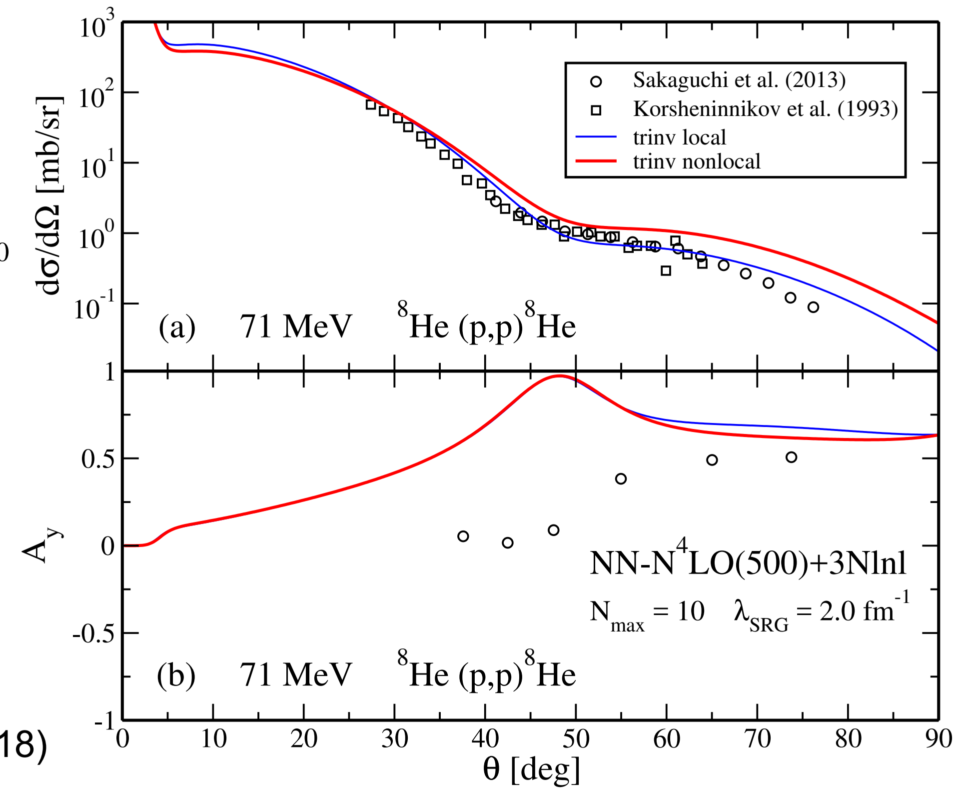




Gennari, Vorabbi, Calci, Navratil, PRC **97**, 034619 (2018)

Scattering observables – Halo nuclei

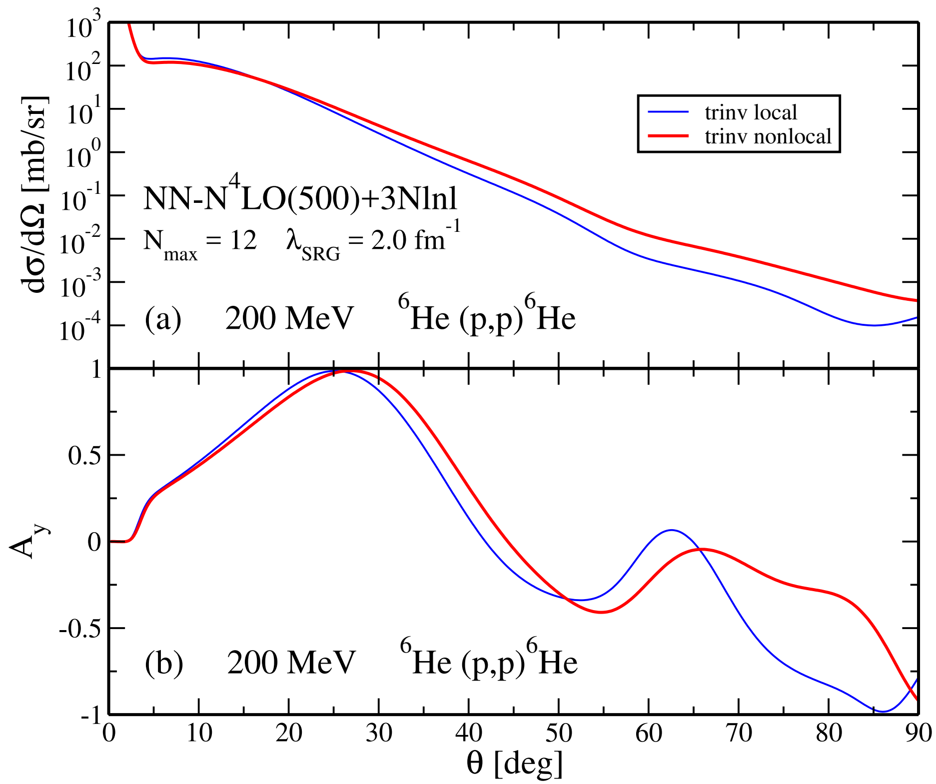


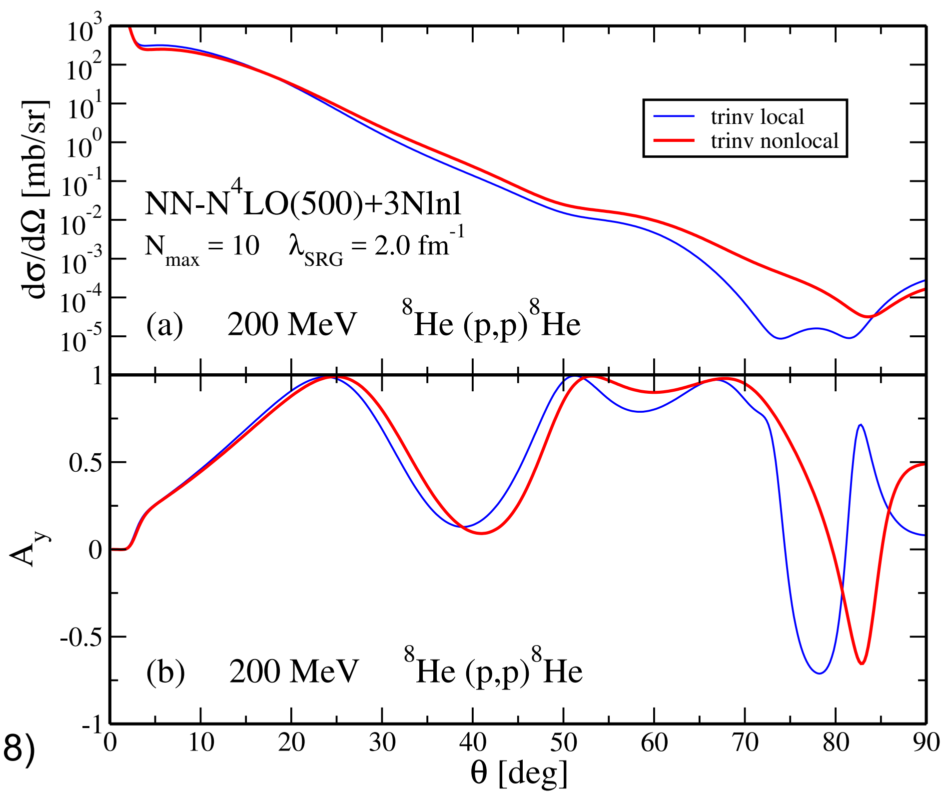


Reasonable description of the differential cross section

Gennari, Vorabbi, Calci, Navratil, PRC **97**, 034619 (2018)

Scattering observables – Halo nuclei





Different behavior of A_y after 40 degrees

Gennari, Vorabbi, Calci, Navratil, PRC **97**, 034619 (2018)

Outlook

- Investigation of the ^{9}He structure with the inclusion of the three-nucleon interaction
  - Introducing a controlled approximation for the 3N terms
- Calculation of the p+^{8}He scattering process
  - New TRIUMF experiment for $^{8}He(p,p)$ reaction
- Coupling between different mass partitions for A=7 systems
- Improvement of the optical potential
  - Inclusion of three-nucleon interaction
  - Inclusion of medium effects