APCTP-BLTP@2018.08.20

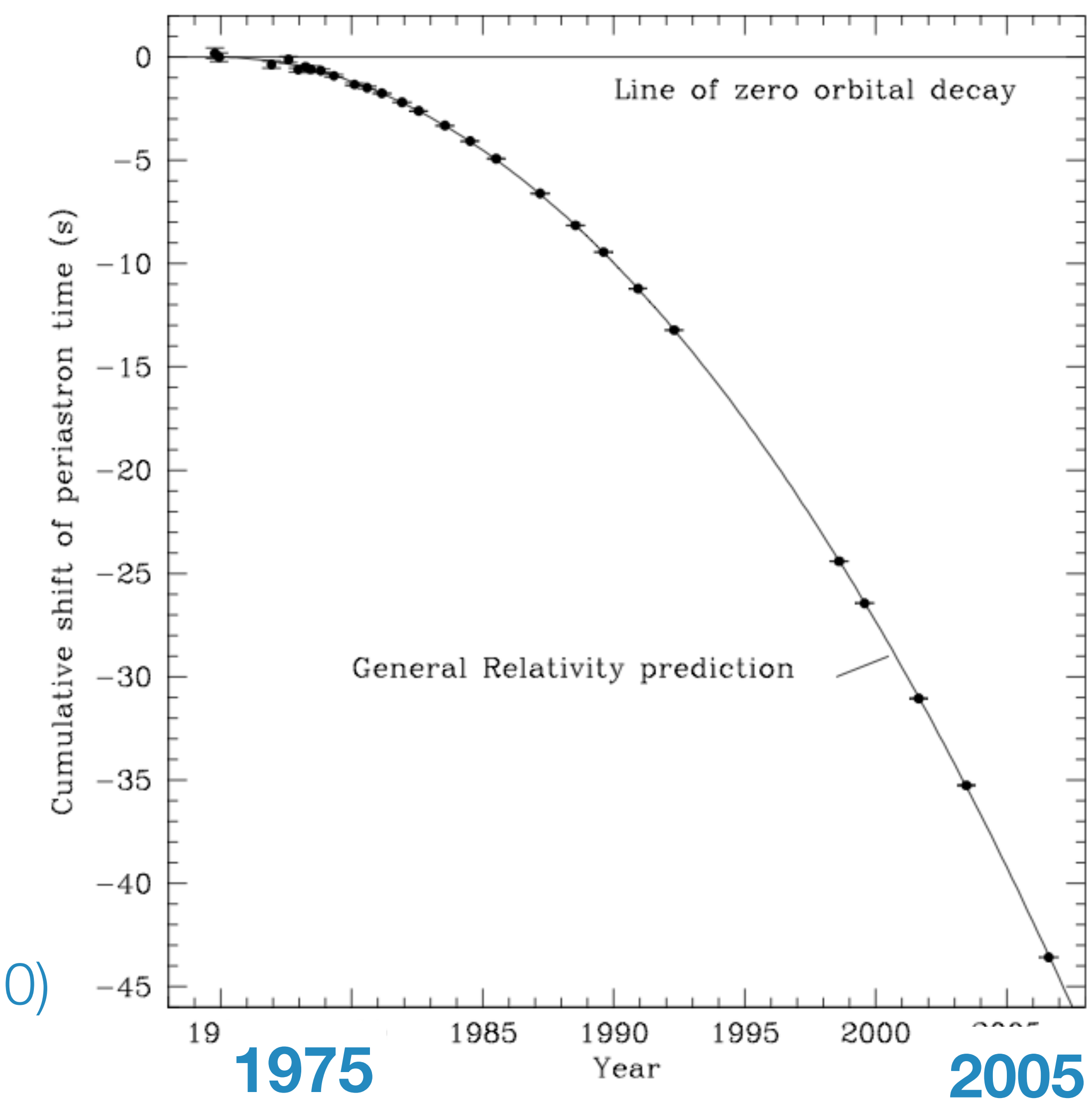
Gravitational Waves and Tidal Deformability of Neutron Stars

Chang-Hwan Lee / Pusan National University

In collaboration with
Young-Min Kim, Kyujin Kwak (UNIST)
Y. Lim (Texas A&M), C.H. Hyun (Daegu)

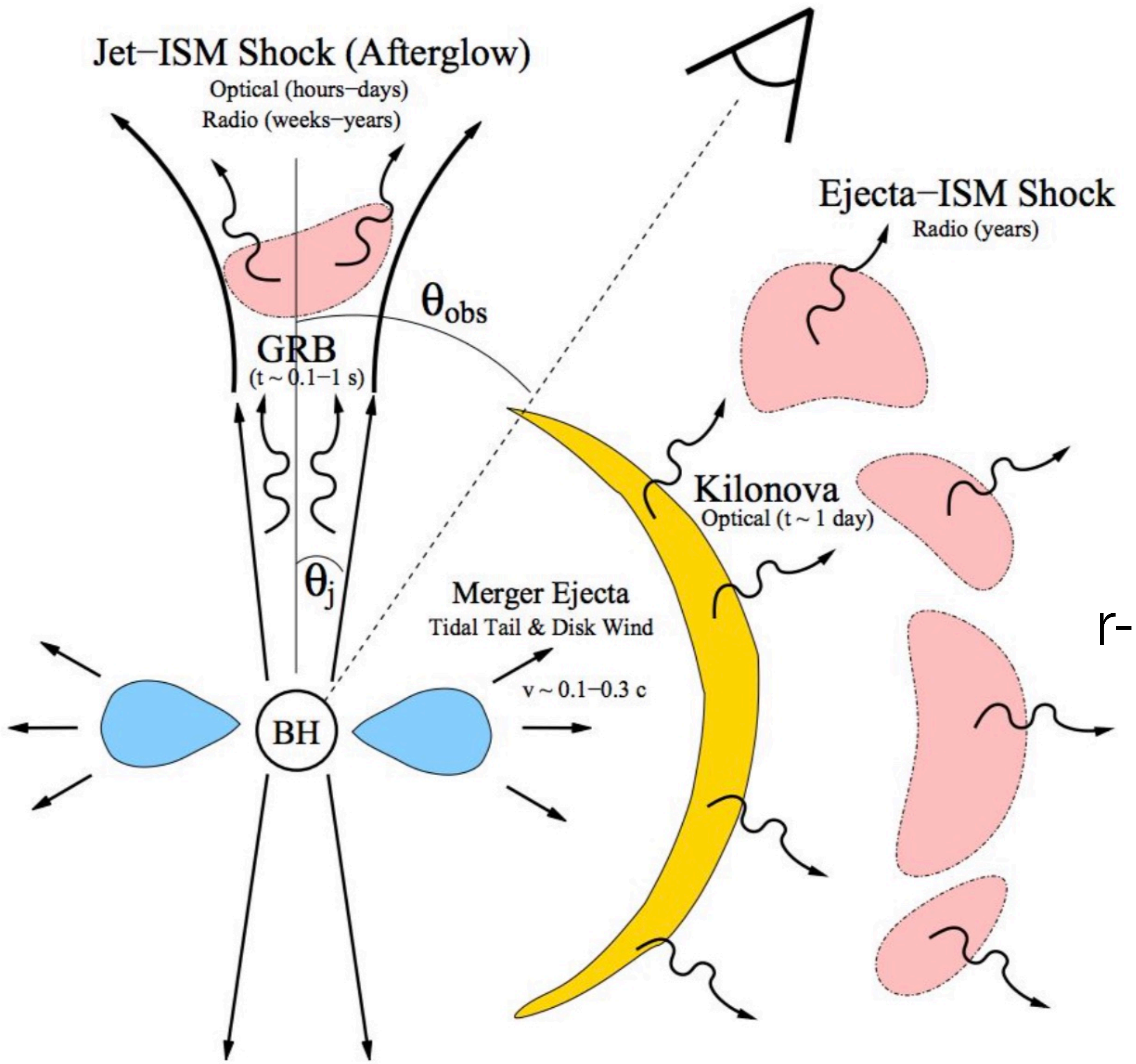
Gravitational waves from neutron star binaries

- B1913+16 / Hulse & Taylor (1975)
- change in the orbital period due to GW radiation
- 1993 Nobel Prize
- LIGO is based on
  NS binary mergers
- GW expected in **2019**
  d = O(100 Mpc)



Weisberg, Nice, Taylor, ApJ (2010)

GRB and Kilonova from NS binaries



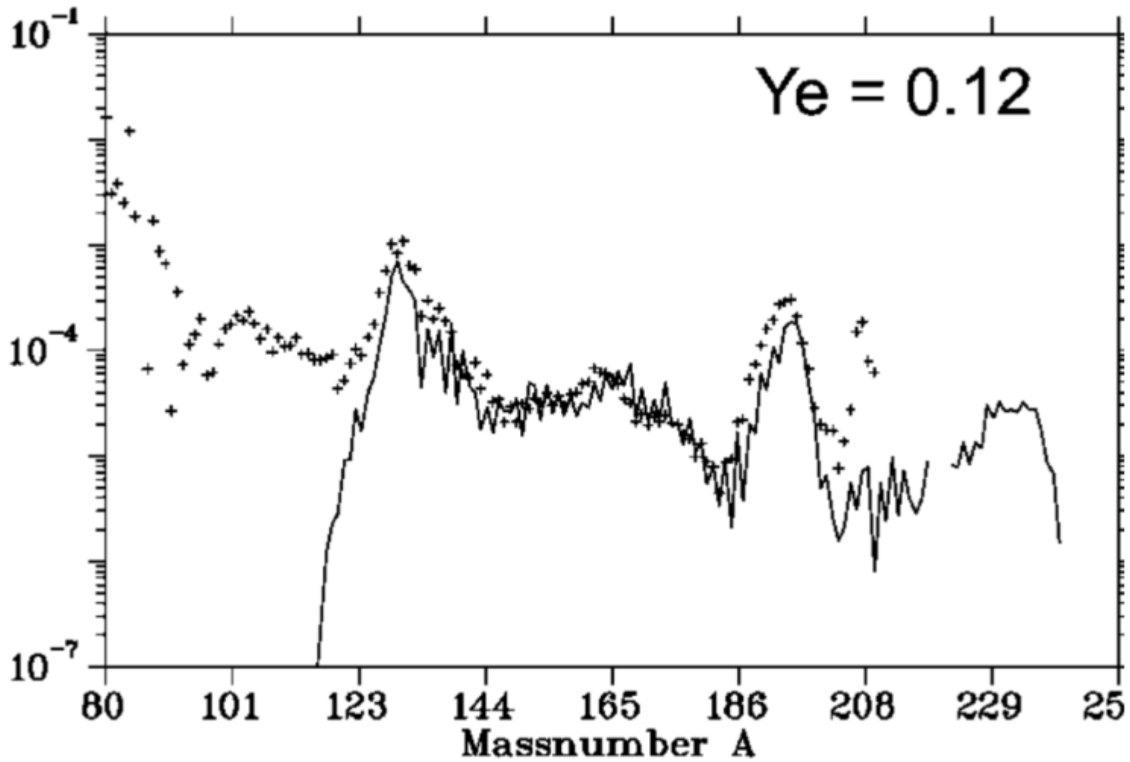
r-process **peak at A~195**

Metzger and Berger 2012

Heavy Elements from NS mergers

Sources of Heavy Elements

S Rosswog 2015



solar pattern vs NS-merger

- **Supernovae:**
  neutrino-driven wind
  r-process **peak at A~130**

- **NS mergers:**
  r-process **peak at A~195**

Press Release Oct 16, 2017

GW from Binary NS Mergers

*GW 170817 (**d=40 Mpc**)*
GRB 170817A by Fermi-GBM
Kilonova/X-ray/Optical Afterglows

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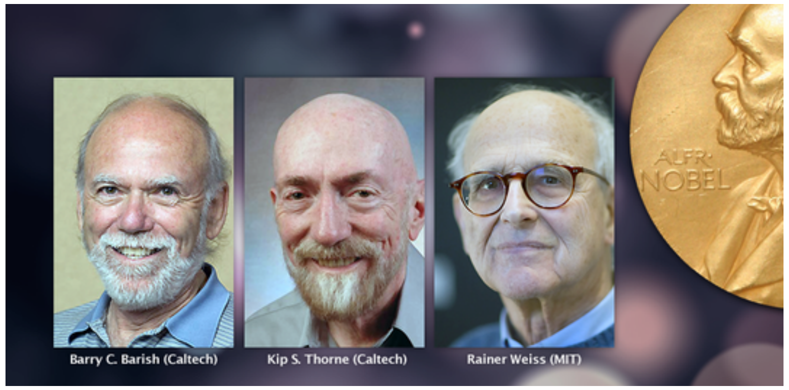
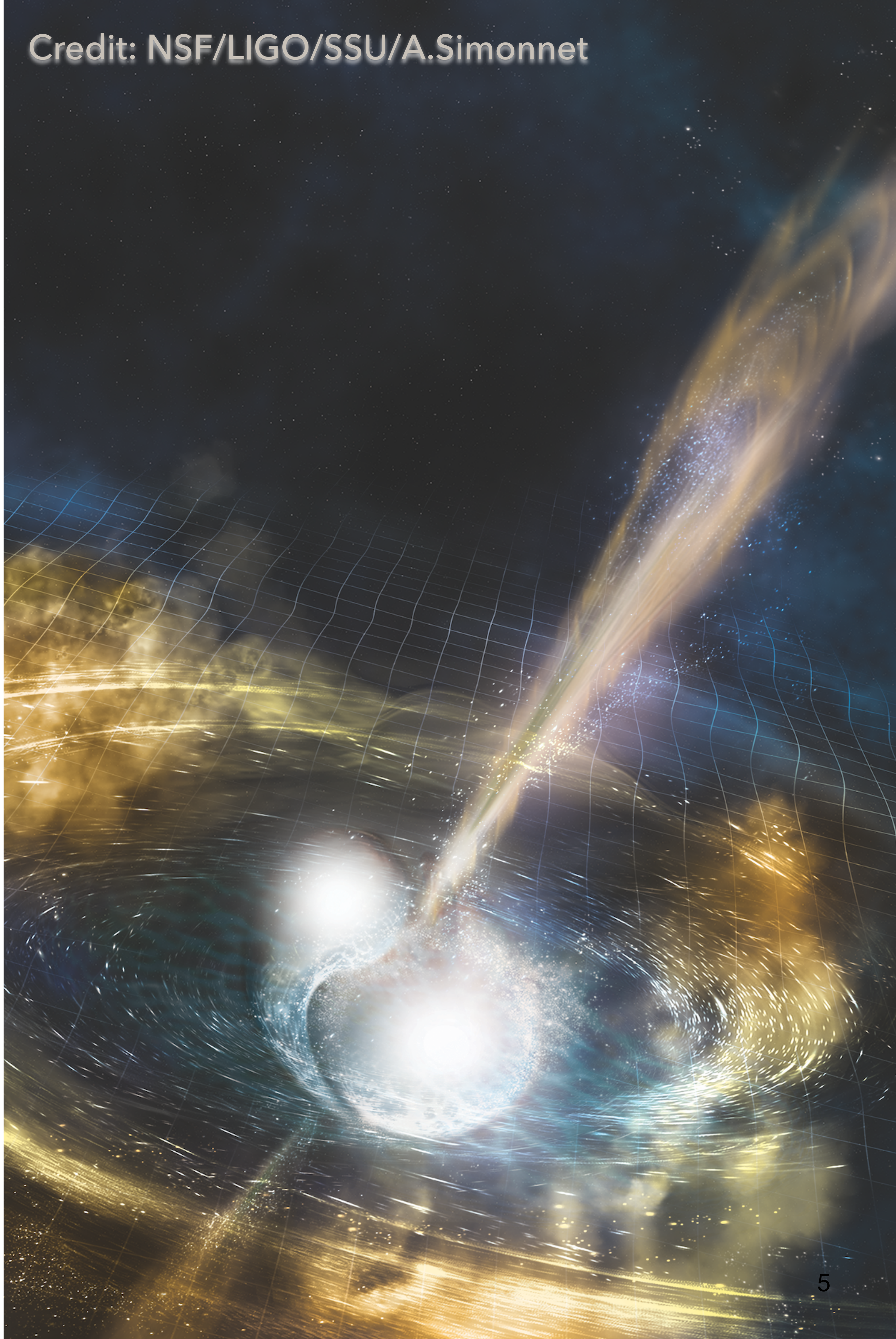
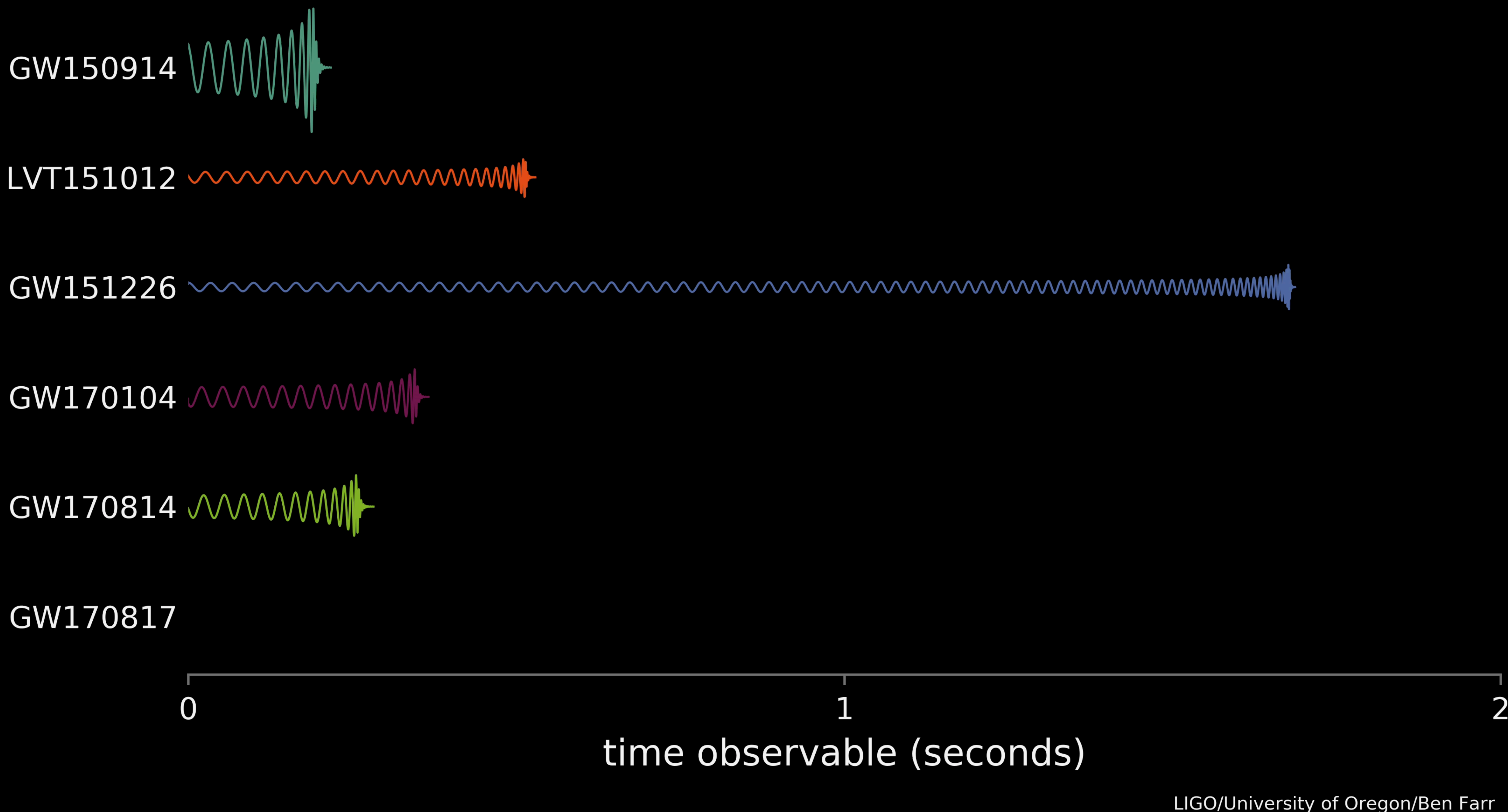
soon after the announcement of 2017 Nobel Prize





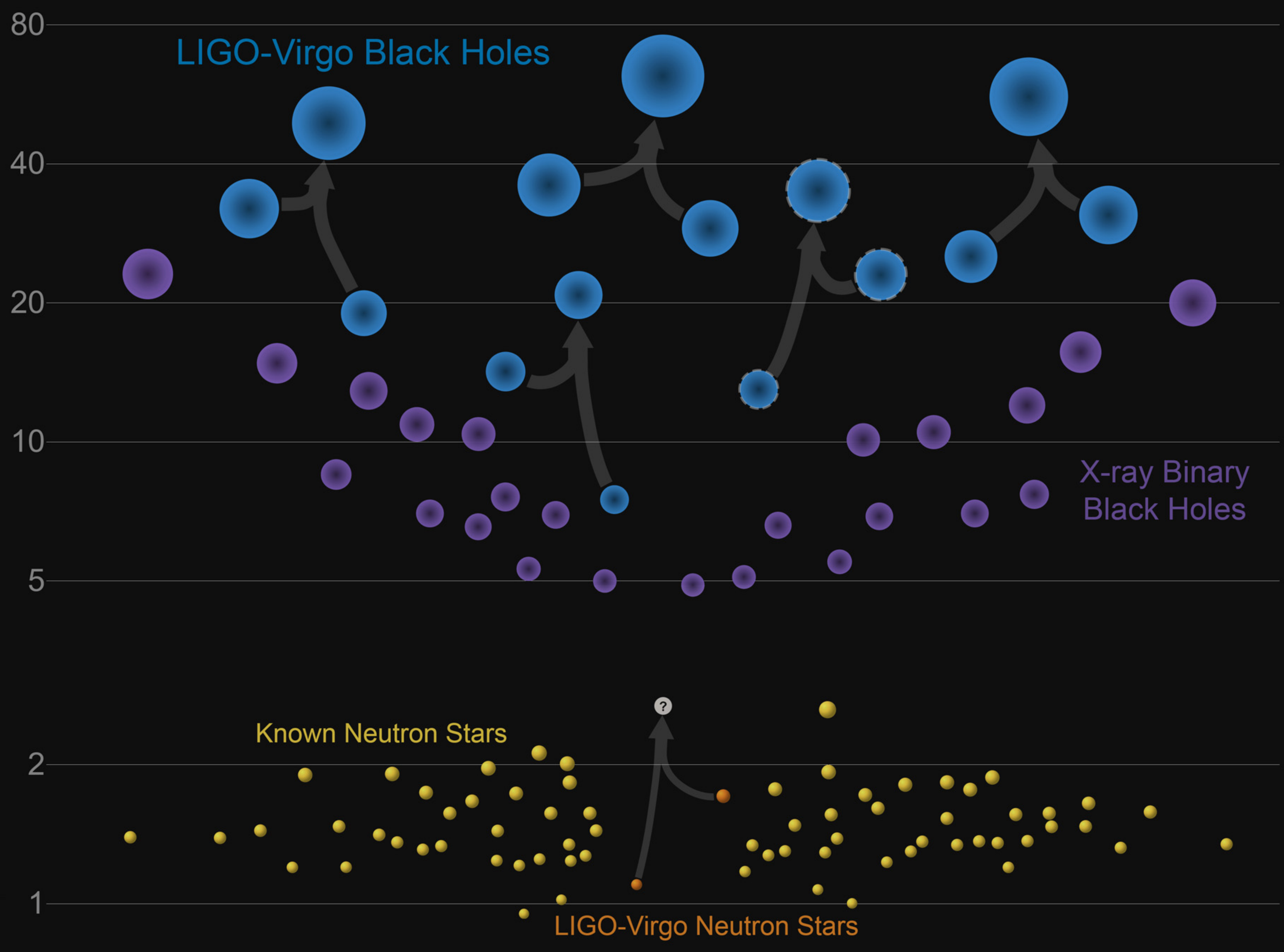


GW170817

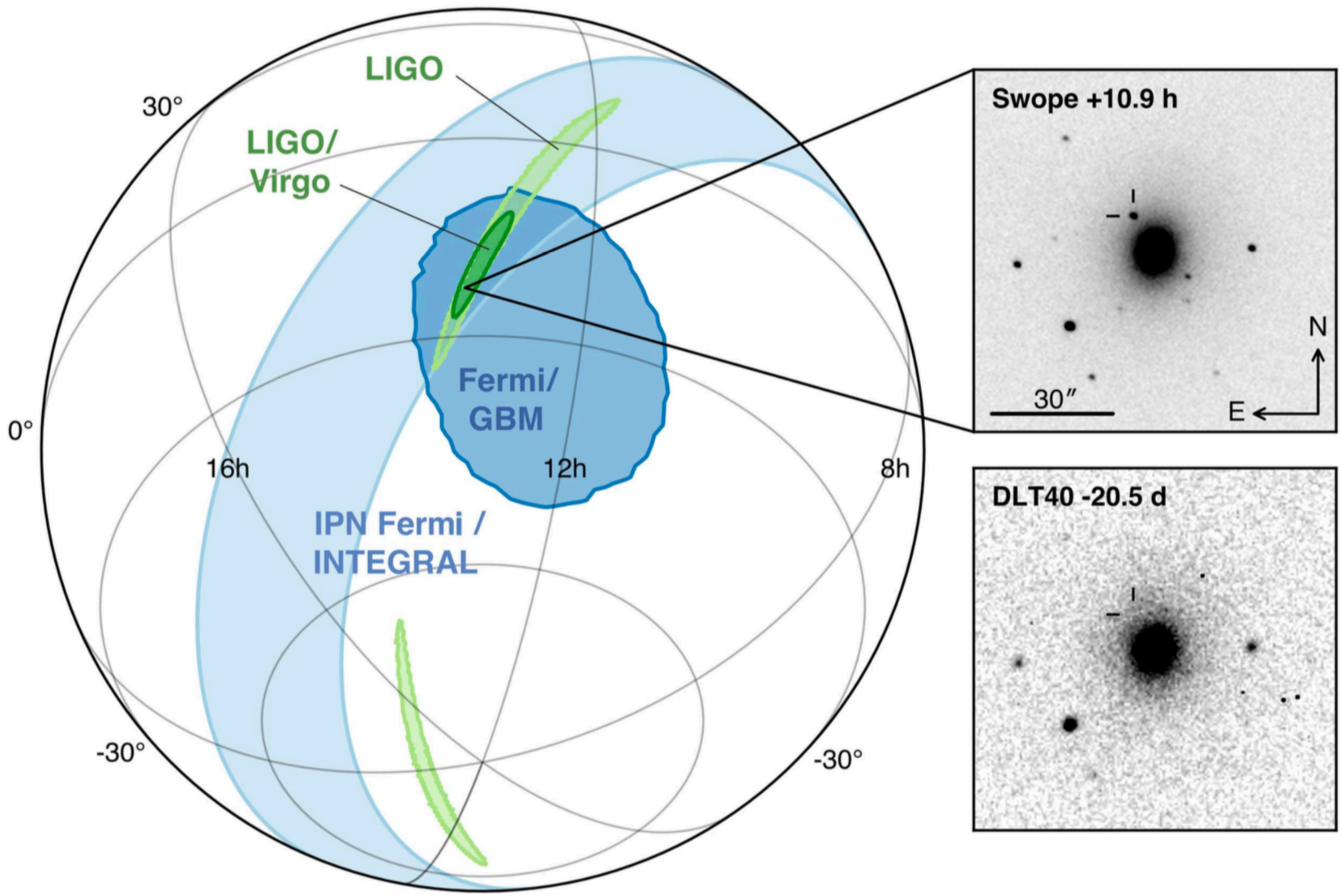
Binary Neutron Star Mergers
Oct 16, 2017

Masses in the Stellar Graveyard

in Solar Masses



GW170817 / GRB170817A

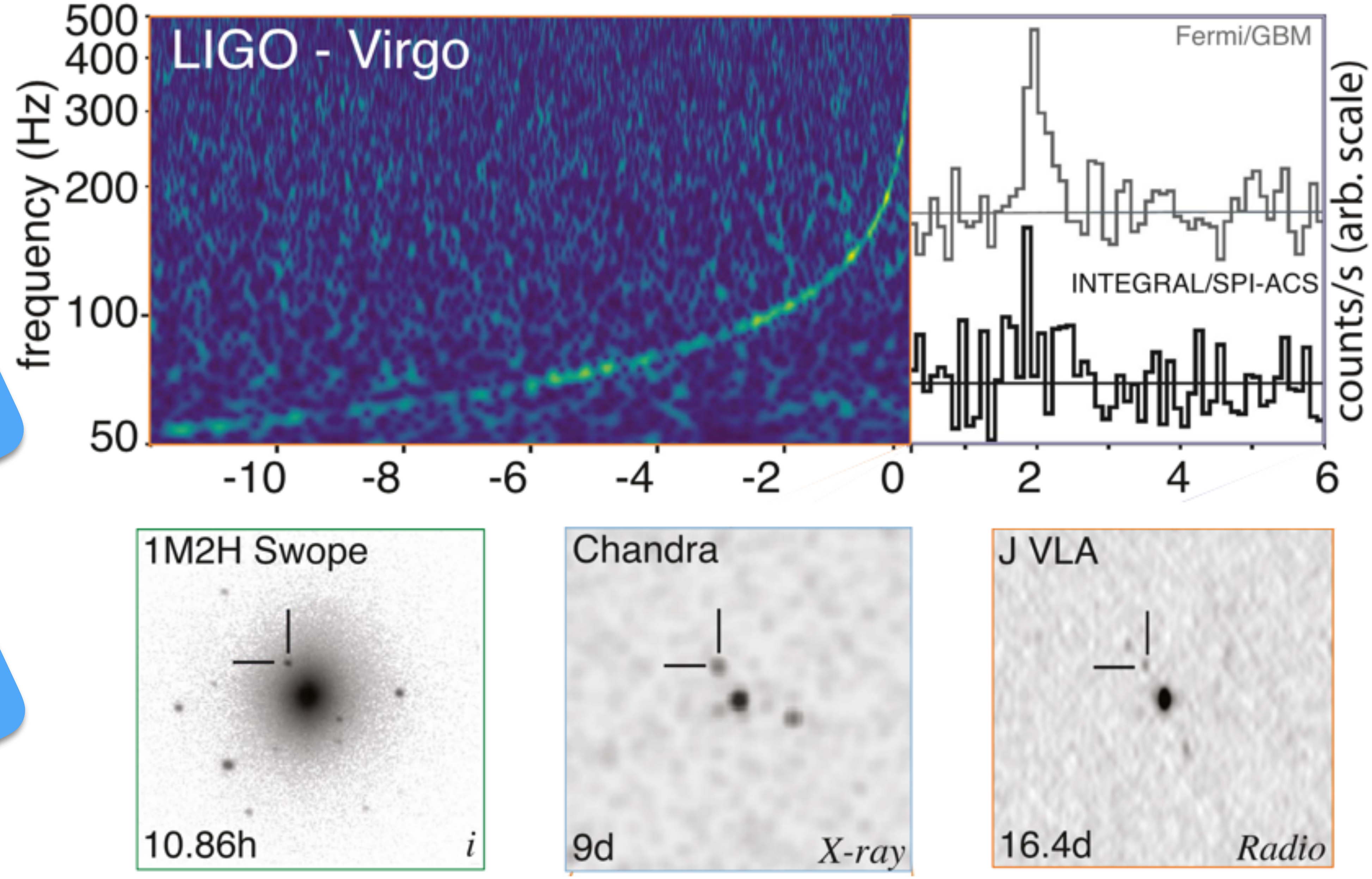


First event of Multi-messenger Astronomy

GW170817

GRB170817A

SSS17a/
AT2017gfo



ApJL.848.L12(2017)

TIMELINE

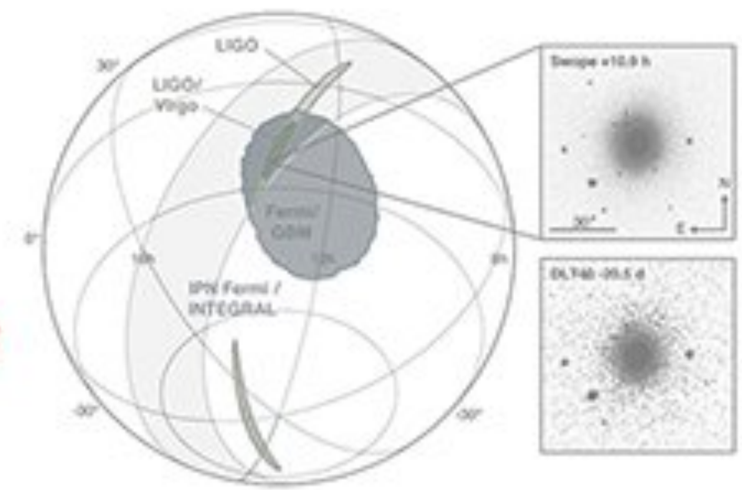
중성자별 충돌에서 발생한 중력파, 감마선, 가시광선, 엑스선 및 전파 관측

2017.08.17.
12:41:04 UTC

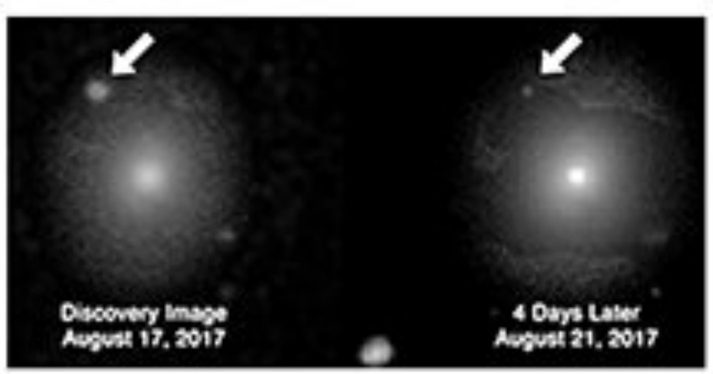
라이고 및 비르고
중력파 신호
포착

+2
seconds

페르미 및 인티그랄
감마선 신호
포착



Fermi/Integral gamma-ray



+11
hours

칠레 천문대
망원경들이
가시광선 신호
포착

Telescopes in Chile

http://horizon.kias.re.kr

+21 hours

국내연구진 호주 이상각망원경으로 추적 관측 시작 이후 약 4주간 추적 관측 (KMTNet, BOOTES-5 망원경 등)

Korean Telescopes
Nature 551, 71 (2017)

Chandra X-ray

+9 Days

찬드라 우주망원경 X선 신호 포착

VLA radio

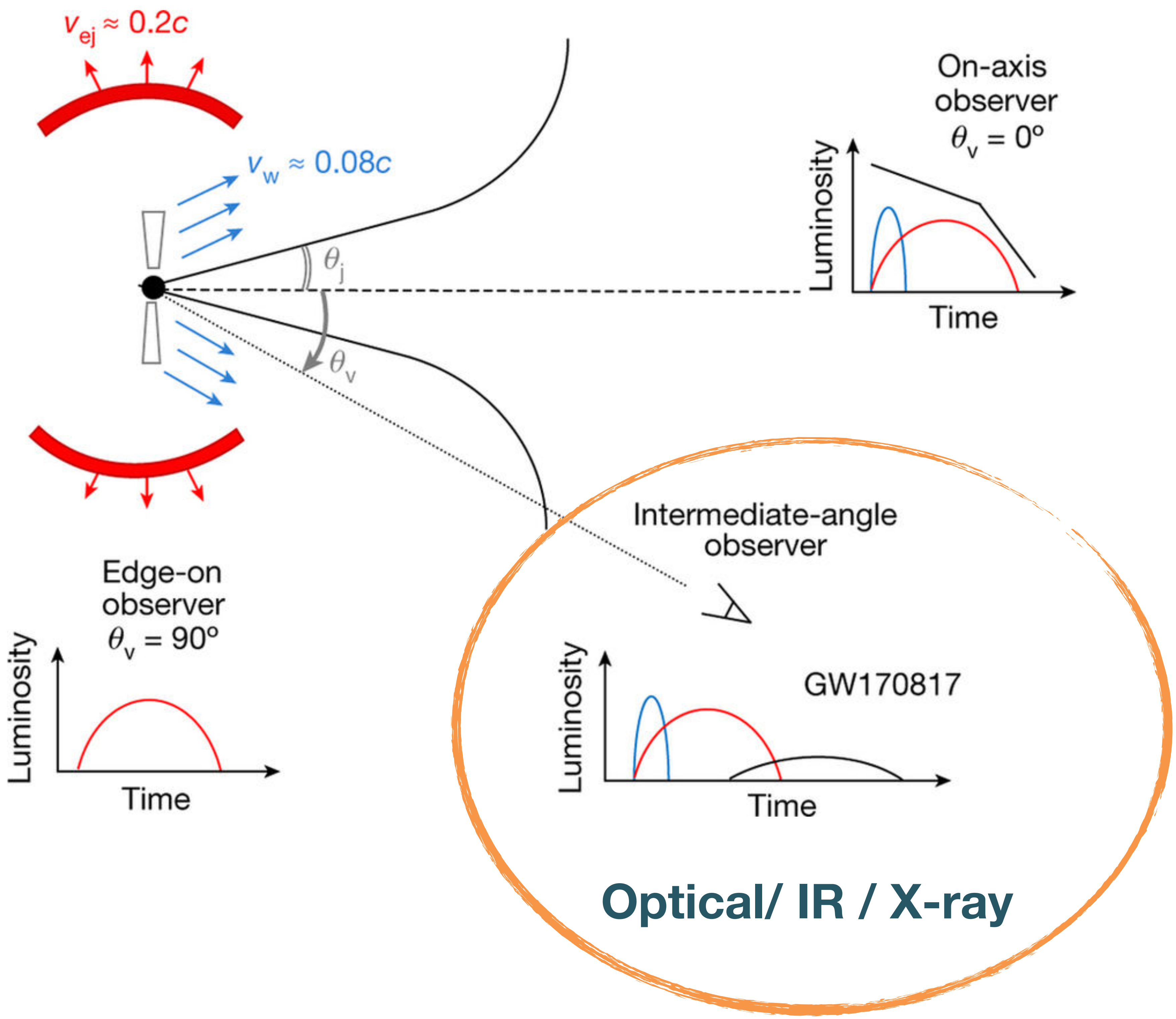
+16 Days

지상 전파 망원경 전파 신호 포착

http://horizon.kias.re.kr

Kilonova / Geometry of GW170817

Nature 551, 71 (2017)



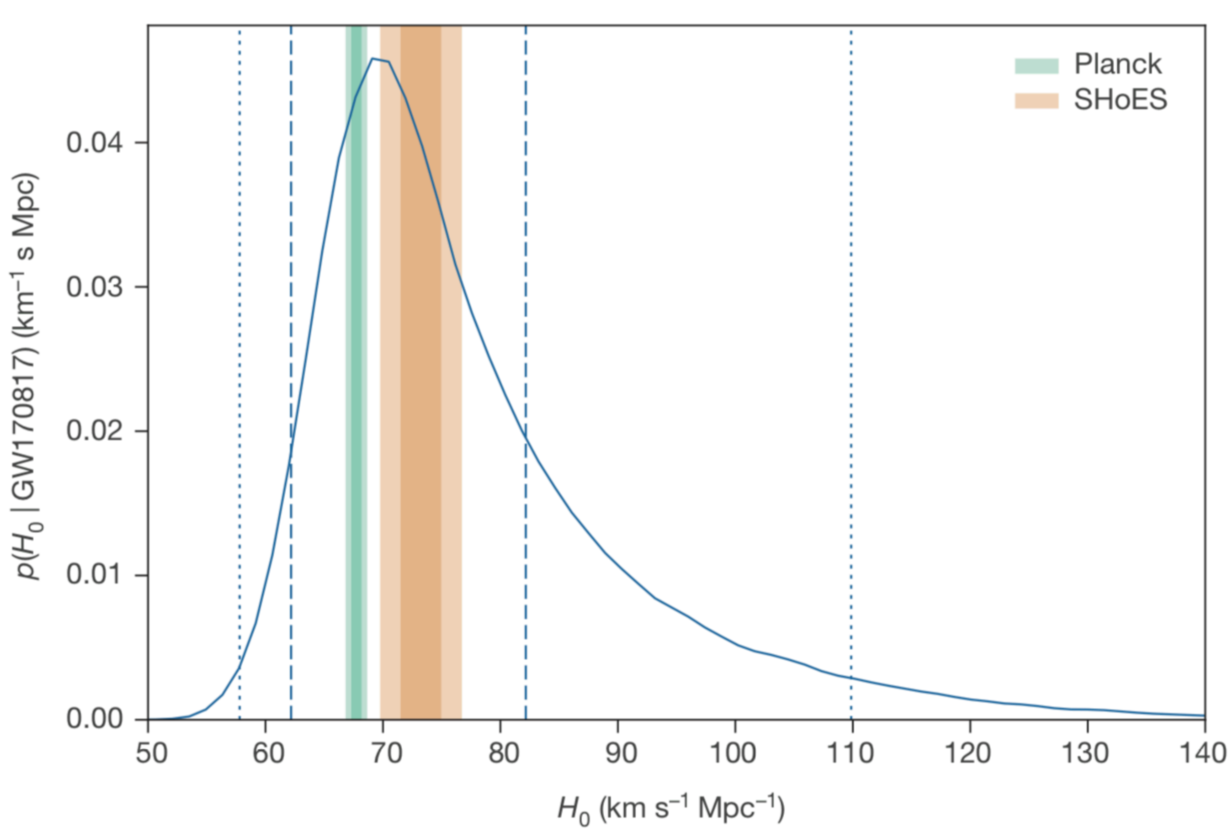
LETTER

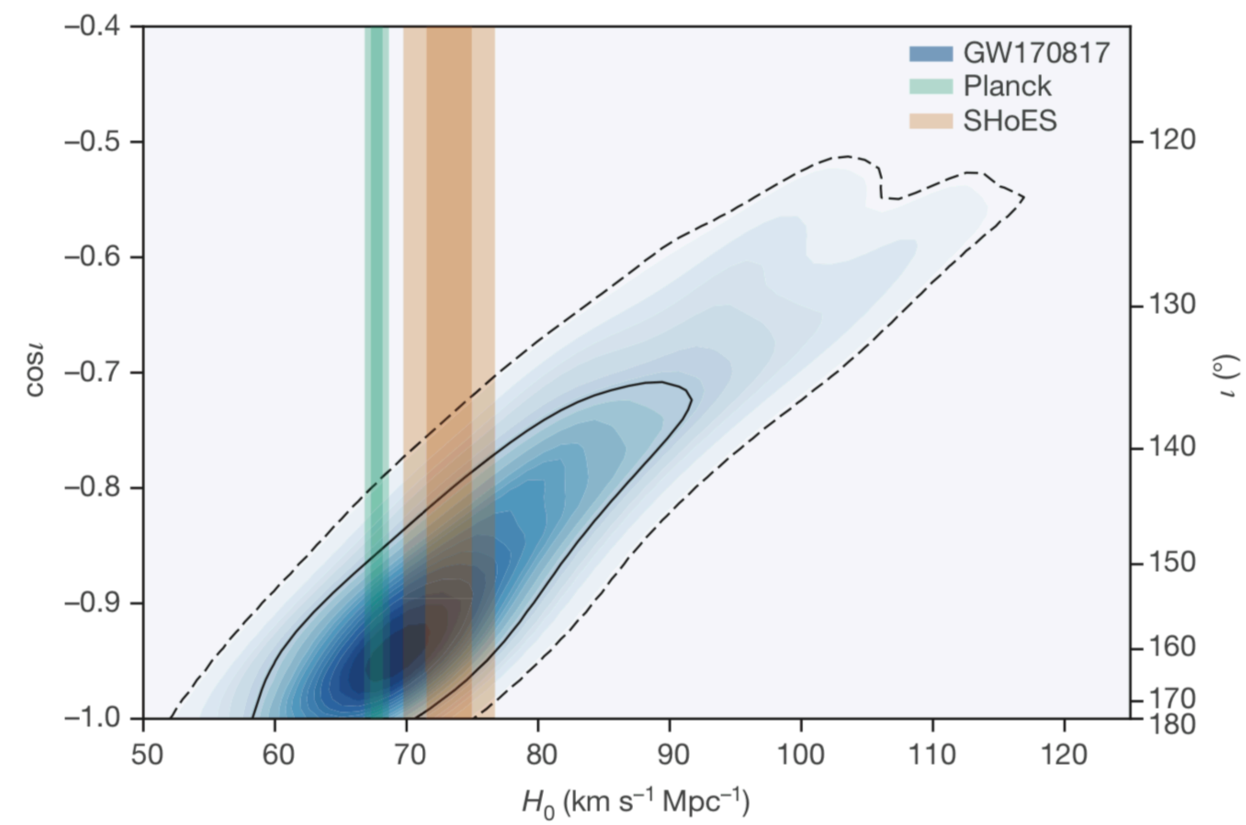
doi:10.1038/nature24471

A gravitational-wave standard siren measurement of the Hubble constant

The LIGO Scientific Collaboration and The Virgo Collaboration*, The 1M2H Collaboration*, The Dark Energy Camera GW-EM Collaboration and the DES Collaboration*, The DLT40 Collaboration*, The Las Cumbres Observatory Collaboration*, The VINROUGE Collaboration* & The MASTER Collaboration*

$$H_0 = 70.0^{+12.0}_{-8.0}\,\mathrm{km\ s^{-1}\,Mpc^{-1}}$$

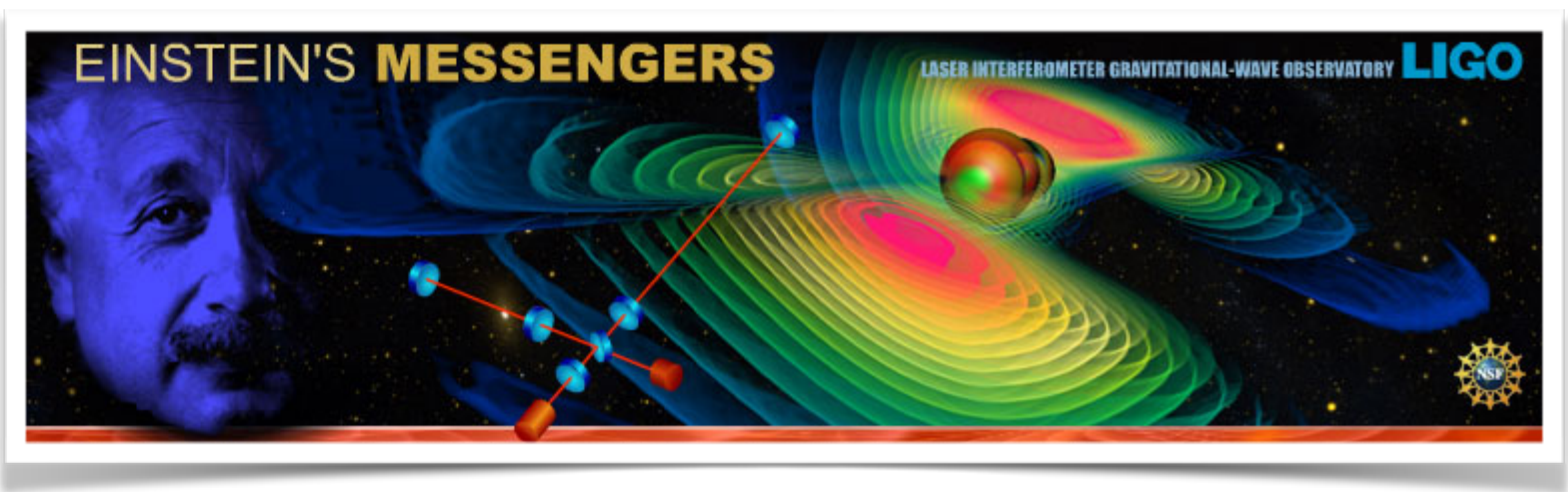




2 NOVEMBER 2017 | VOL 551 | NATURE | 85

Gravitational-Wave & Multi-Messenger Astronomy

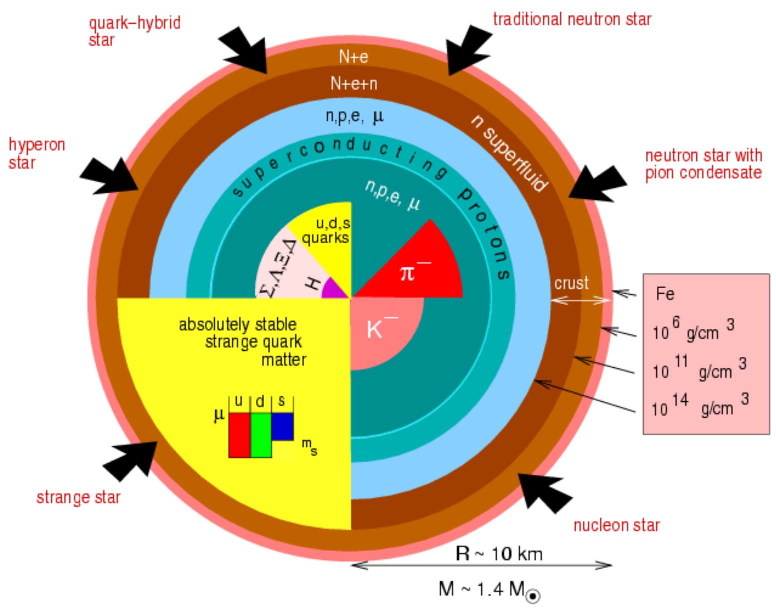
- First direct detection of GW in 2015
- First detection of BHs with masses 30 ~ 60 solar mass
- **GW, Gamma-ray, Optical, X-ray, Radio from NS mergers**
- New era for GW Astronomy & **Multi-Messenger Astronomy**



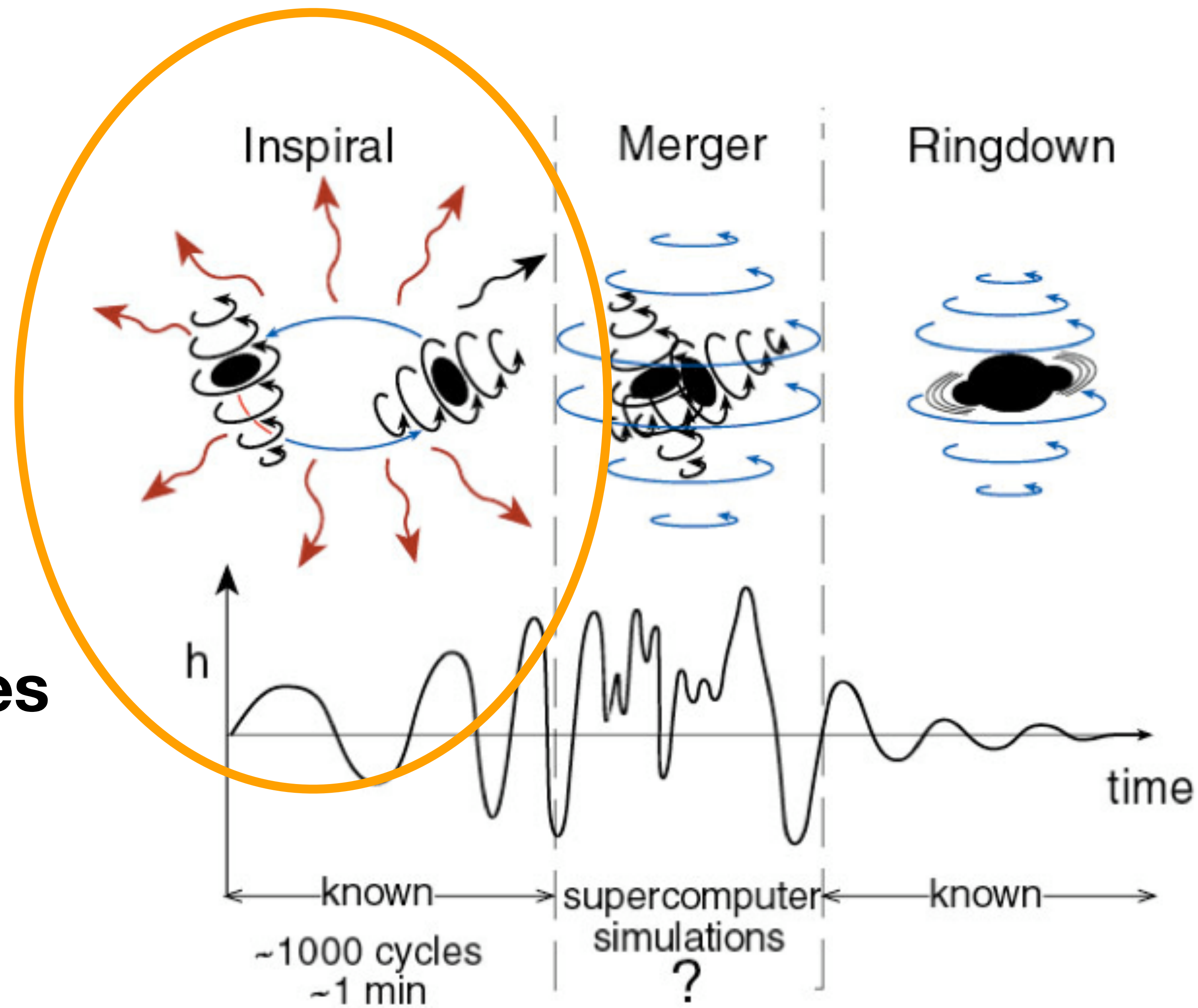
NS in new era of GW & multi-messenger astronomy

Tidal Love number & Deformability

Response of NS to GW during Inspiral



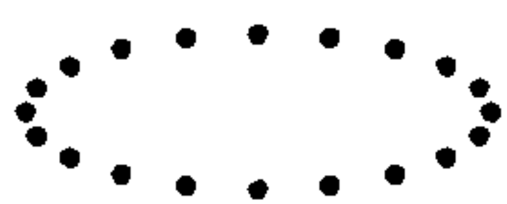
perturbative approaches

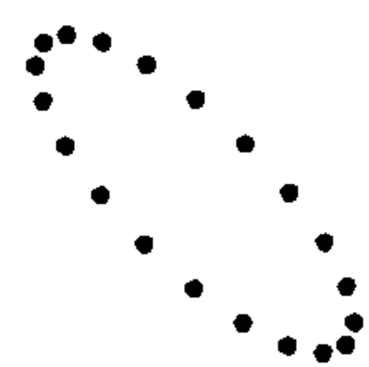


Tidal deformability & Love number

Selected references

- **A.E.H. Love** (1909) - The Yielding of the Earth to Disturbing Forces
- **K.S. Thorne** & A. Campolattaro (1967) - non-radial pulsation of NS
- J.B. Hartle & **K.S. Thorne** (1969) - stability of rotating NS
- … …
- **K.S. Thorne** (1998) - Tidal stabilization of rigid rotating, fully relativistic neutron star
- … …





Tidal deformability & Love number

$$-\frac{(1+g_{tt})}{2} = -\frac{m}{r} - \frac{3Q_{ij}}{2r^3}\left(n^i n^j - \frac{1}{3}\delta^{ij}\right) + \mathcal{O}\left(\frac{1}{r^3}\right) + \frac{\mathcal{E}_{ij}}{2} r^2 n^i n^j + \mathcal{O}(r^3)$$

$\mathcal{E}_{ij}$: external quadrupole tidal field

Q_{ij} : quadrupole moment of NS

λ : Tidal deformability

$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$

$$Q_{ij} = \int d^3x \delta\rho(x) \left(x_i x_j - \frac{1}{3} r^2 \delta_{ij}\right)$$

$$n^i = \frac{x^i}{r}$$

dimensionless parameter

k_2 : $l = 2$ Tidal Love number

$$k_2 = \frac{3}{2} G \lambda R^{-5}$$

Hinderer et al. PRD 81 (2010)

Systematic Parameter Errors in Inspiraling Neutron Star Binaries

Marc Favata*

$$\tilde{h}_T(f) = \mathcal{A} f^{-7/6} e^{i\Psi_T}(f)$$

$$\Psi_T(f) = \varphi_c + 2\pi f t_c + \frac{3}{128\eta v^5}(\Delta\Psi^{pp}_{3.5PN} + \Delta\Psi^{spin}_{3PN} + \Delta\Psi^{ecc.}_{2PN} + \Delta\Psi^{tidal}_{6PN} + \Delta\Psi^{tm}_{6PN})$$

$$v = (\pi f M)^{1/3}$$

$$v/c = (GM\pi f/c^3)^{1/3}$$

$$\Delta\Psi^{tidal}_{6PN} = -\frac{39}{2}\tilde{\Lambda} v^{10} + v^{12}\left(\frac{6595}{364}\delta\tilde{\Lambda} - \frac{3115}{64}\tilde{\Lambda}\right)$$

$$\tilde{\Lambda} \equiv 32\frac{\tilde{\lambda}}{M^5} = \frac{8}{13}[(1+7\eta-31\eta^2)(\hat{\lambda}_1+\hat{\lambda}_2) - \sqrt{1-4\eta}(1+9\eta-11\eta^2)(\hat{\lambda}_1-\hat{\lambda}_2)].$$

PRL **112**, 101101 (2014) PHYSICAL REVIEW LETTERS week ending 14 MARCH 2014

Systematic Parameter Errors in Inspiraling Neutron Star Binaries

Marc Favata[*]

phase shift vs deformability

$$\left.\frac{d\Phi}{dx}\right|_{\mathrm{T}} = -\frac{195}{8}\frac{x^{3/2}}{\eta}\frac{\tilde{\lambda}}{M^5} \propto \frac{\tilde{\lambda}}{M^5}$$

$$x = (\omega M)^{2/3} \Rightarrow \left(\omega \frac{GM}{c^3}\right)^{2/3}$$

$$\eta = m_1 m_2 / M^2$$

dimensionless

$$\Lambda = \frac{\lambda}{m^5} \Rightarrow G\lambda \left(\frac{c^2}{Gm}\right)^5 \approx 950.5 \left(\frac{m_\odot}{m}\right)^5 \left(\frac{\lambda}{10^{36}\ \mathrm{g\ cm^2\ s^2}}\right)$$

$$\Lambda = G\left(\frac{c^2}{Gm}\right)^5 \times \frac{2}{3}\frac{R^5}{G}k_2 = \frac{2}{3}\left(\frac{Rc^2}{Gm}\right)^5 k_2 \approx 9495 \left(\frac{R_{10\mathrm{km}}}{m_{M_\odot}}\right)^5 k_2$$

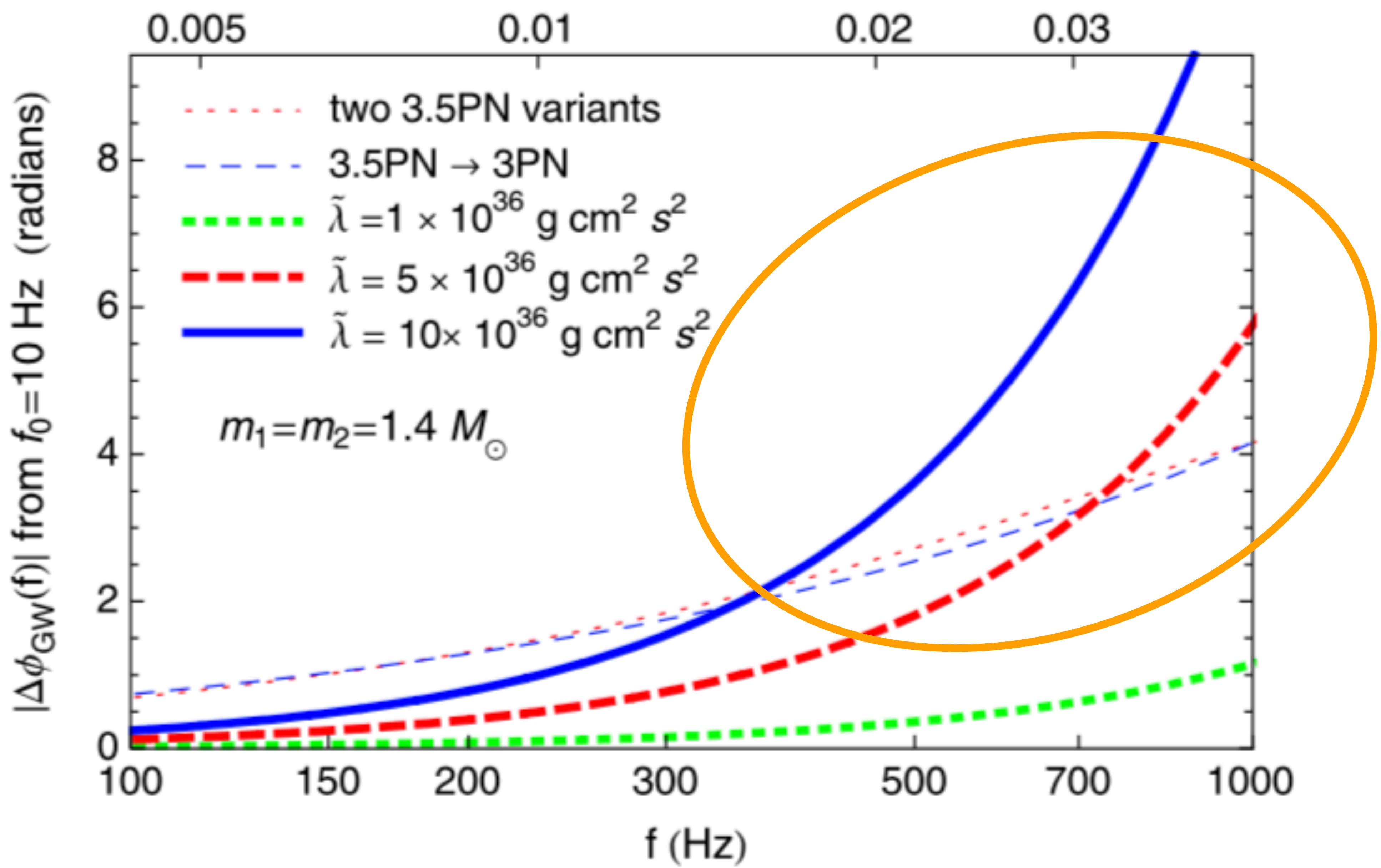
accumulated GW phase

PHYSICAL REVIEW D **81,** 123016 (2010)

Tidal deformability of neutron stars with realistic equations of state and their gravitational wave signatures in binary inspiral

Tanja Hinderer,[1] Benjamin D. Lackey,[2] Ryan N. Lang,[3,4] and Jocelyn S. Read[5]

$$|\Delta\phi_{\rm GW}(f)| = |\Phi_{3.5,\rm pp}(f_{\rm GW}) - \Phi_{3.5,\lambda}(f_{\rm GW})|$$



Accumulated GW phase

the number of wave cycles in frequency domain

$$\Delta N_{\mathrm{cyc},\Psi} = \frac{1}{2\pi}\left[\Psi(f_2) - \Psi(f_1) + (f_1 - f_2)\frac{d\Psi}{df_1}\right]$$

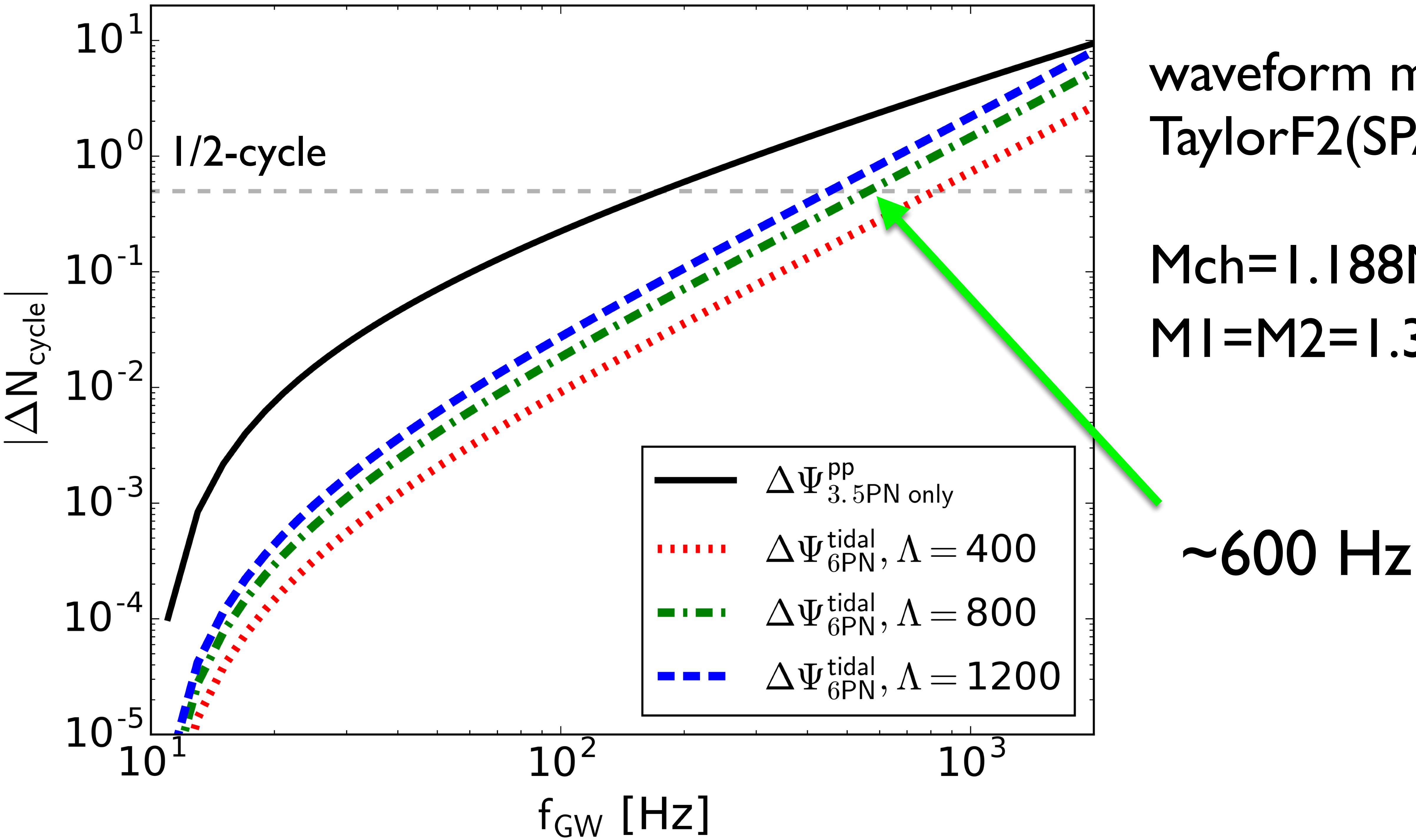
f_1 = 10 Hz,
the low frequency cutoff for Advanced LIGO due to seismic noises

Waveform models:
TaylorT2 for ΔN_{cyc}
TaylorF2(SPA) $\Delta N_{cyc,\Psi}$

Moore et al., PRD.93.124061(2016)

	$1.4M_\odot + 1.4M_\odot$, $f_2 = 1000$ Hz		
PN order	ΔN_{cyc}	$\Delta N_{\mathrm{cyc},\Psi}$	$\Delta N_{\mathrm{useful}}^{\mathrm{norm}}$
0PN(circ)	16 031	986 372	1821
0PN(ecc)	−463	−36 137	−6.37
1PN(circ)	439	21 743	125
1PN(ecc)	−15.8	−1193	−0.332
1.5PN(circ)	−208	−8520	−94.8
1.5PN(ecc)	1.67	103	0.113
2PN(circ)	9.54	294	6.70
2PN(ecc)	−0.215	−15.4	−0.008 17
2.5PN(circ)	−10.6	−218	−10.6
2.5PN(ecc)	0.0443	2.61	0.004 73
3PN(circ)	2.02	18.2	2.80
3PN(ecc)	0.002 00	0.119	−0.000 238
3.5PN(circ)	−0.662	−4.39	−0.977
Total	15 785	962 445	1843

accumulated GW phase



waveform model:
TaylorF2(SPA)

$Mch=1.188M_\odot$
$M1=M2=1.365M_\odot$

Measurement error vs. source distance

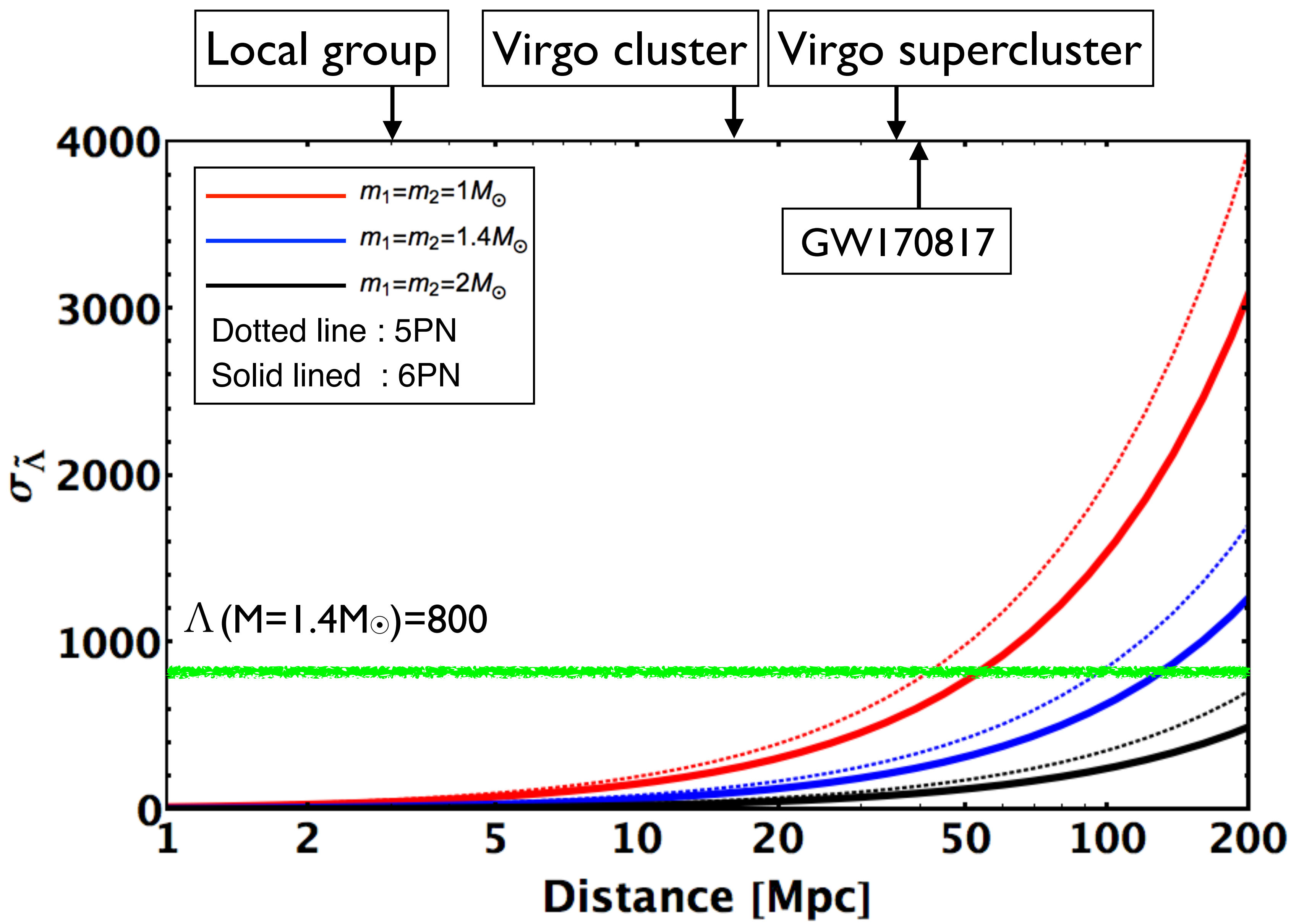


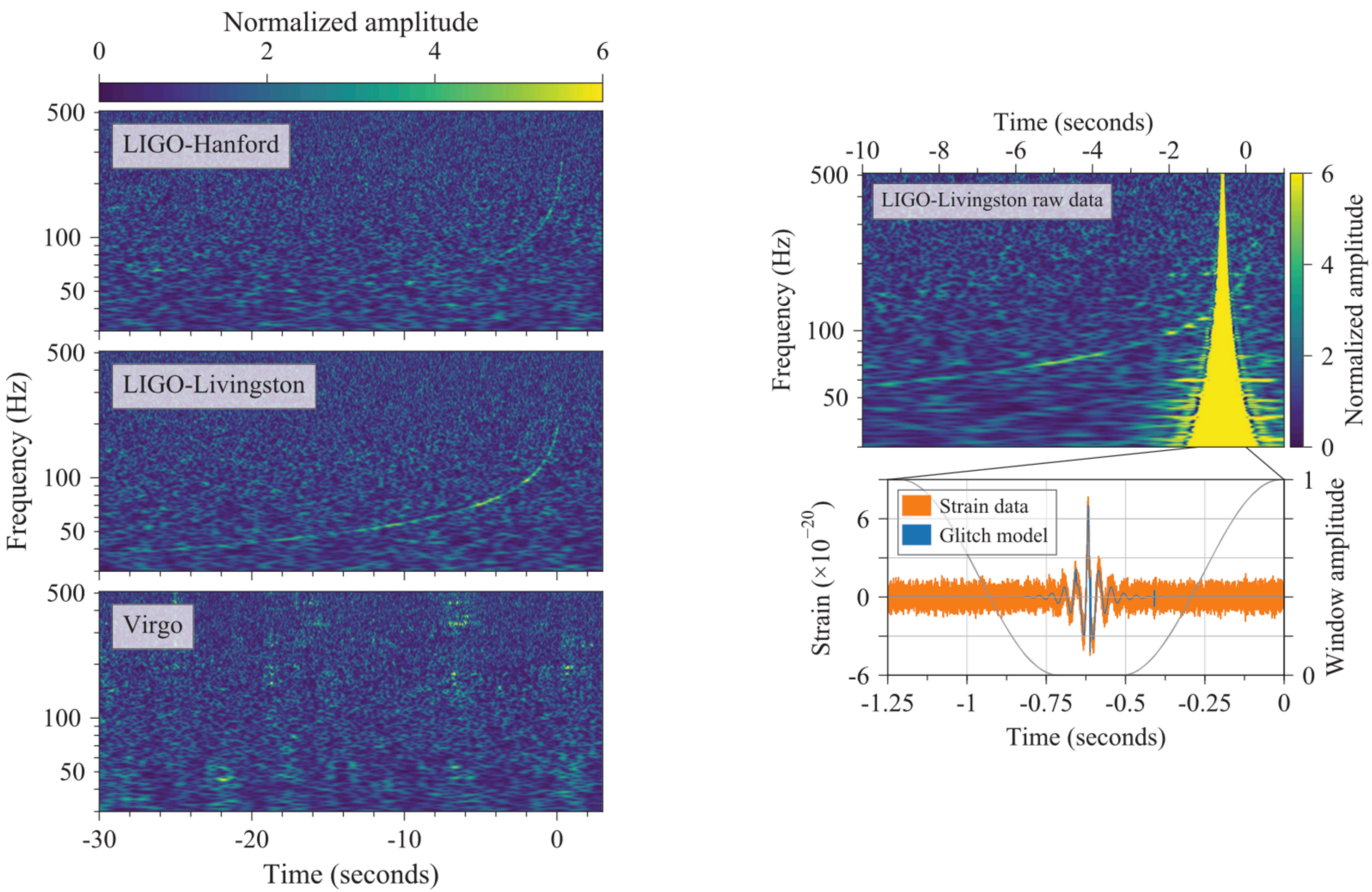
Fig. 1: Tidal deformability measurement error vs distance to the source. distances to galaxy clusters and GW20170817 distance are marked.

Y.B. Choi, H. S. Cho, C.-H. Lee

Selected for a Viewpoint in *Physics*
PRL **119,** 161101 (2017) PHYSICAL REVIEW LETTERS week ending 20 OCTOBER 2017

GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral

B. P. Abbott *et al.**
(LIGO Scientific Collaboration and Virgo Collaboration)
(Received 26 September 2017; revised manuscript received 2 October 2017; published 16 October 2017)



Selected for a Viewpoint in Physics
PRL 119, 161101 (2017) PHYSICAL REVIEW LETTERS week ending 20 OCTOBER 2017

GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral

TABLE I. Source properties for GW170817: we give ranges encompassing the 90% credible intervals for different assumptions of the waveform model to bound systematic uncertainty. The mass values are quoted in the frame of the source, accounting for uncertainty in the source redshift.

	Low-spin priors ($\|\chi\| \leq 0.05$)	High-spin priors ($\|\chi\| \leq 0.89$)
Primary mass m_1	1.36–1.60 $M_\odot$	1.36–2.26 $M_\odot$
Secondary mass m_2	1.17–1.36 $M_\odot$	0.86–1.36 $M_\odot$
Chirp mass $\mathcal{M}$	$1.188^{+0.004}_{-0.002} M_\odot$	$1.188^{+0.004}_{-0.002} M_\odot$
Mass ratio m_2/m_1	0.7–1.0	0.4–1.0
Total mass $m_{\rm tot}$	$2.74^{+0.04}_{-0.01} M_\odot$	$2.82^{+0.47}_{-0.09} M_\odot$
Radiated energy $E_{\rm rad}$	$> 0.025 M_\odot c^2$	$> 0.025 M_\odot c^2$
Luminosity distance $D_{\rm L}$	40^{+8}_{-14} Mpc	40^{+8}_{-14} Mpc
Viewing angle Θ	$\leq 55°$	$\leq 56°$
Using NGC 4993 location	$\leq 28°$	$\leq 28°$
Combined dimensionless tidal deformability $\tilde{\Lambda}$	≤ 800	≤ 700
Dimensionless tidal deformability $\Lambda(1.4 M_\odot)$	≤ 800	≤ 1400

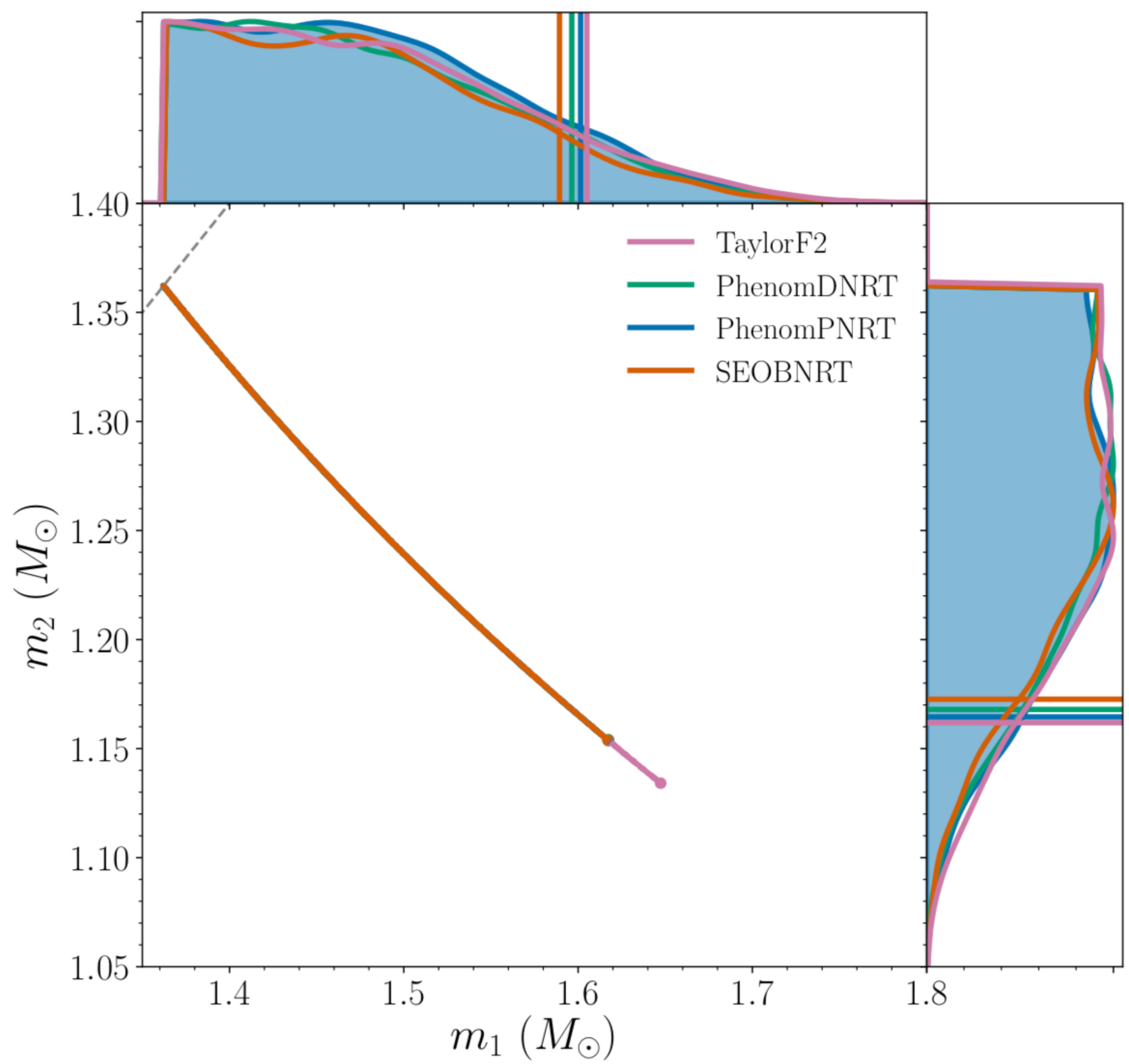
GW170817
Information of Neutron Star Structure
has been revealed by Gravitational Waves

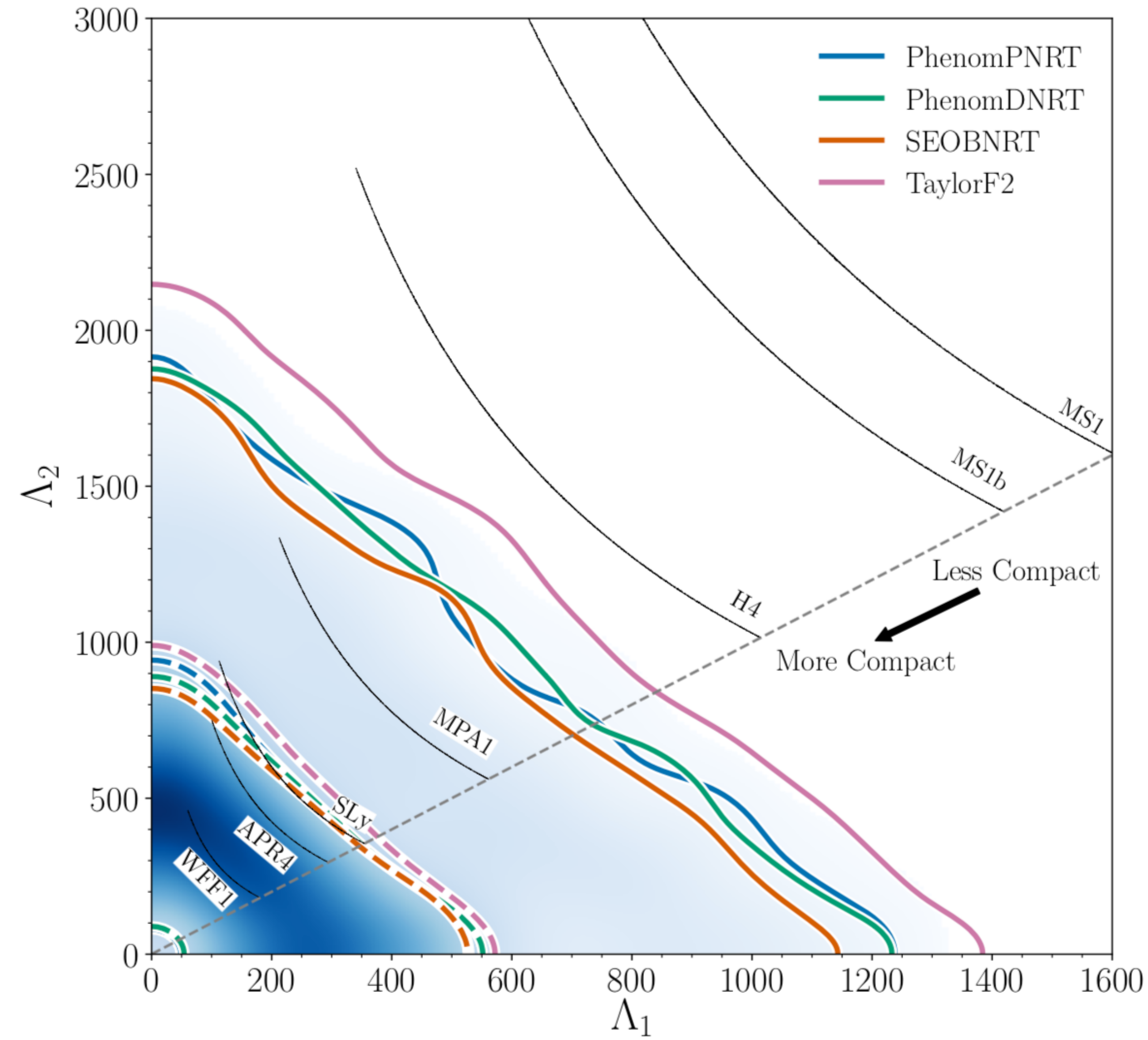
(revised) properties of GW170817

Abbott et al. (LSC and Virgo), arxiv:1805.11579

	Low-spin prior ($\chi \leq 0.05$)	High-spin prior ($\chi \leq 0.89$)
Binary inclination $\theta_{\rm JN}$	146^{+25}_{-27} deg	152^{+21}_{-27} deg
Binary inclination $\theta_{\rm JN}$ using EM distance constraint [104]	151^{+15}_{-11} deg	153^{+15}_{-11} deg
Detector frame chirp mass $\mathcal{M}^{\rm det}$	$1.1975^{+0.0001}_{-0.0001}{\rm M}_\odot$	$1.1976^{+0.0004}_{-0.0002}{\rm M}_\odot$
Chirp mass $\mathcal{M}$	$1.186^{+0.001}_{-0.001}{\rm M}_\odot$	$1.186^{+0.001}_{-0.001}{\rm M}_\odot$
Primary mass m_1	$(1.36, 1.60)$ ${\rm M}_\odot$	$(1.36, 1.89)$ ${\rm M}_\odot$
Secondary mass m_2	$(1.16, 1.36)$ ${\rm M}_\odot$	$(1.00, 1.36)$ ${\rm M}_\odot$
Total mass m	$2.73^{+0.04}_{-0.01}{\rm M}_\odot$	$2.77^{+0.22}_{-0.05}{\rm M}_\odot$
Mass ratio q	$(0.73, 1.00)$	$(0.53, 1.00)$
Effective spin $\chi_{\rm eff}$	$0.00^{+0.02}_{-0.01}$	$0.02^{+0.08}_{-0.02}$
Primary dimensionless spin χ_1	$(0.00, 0.04)$	$(0.00, 0.50)$
Secondary dimensionless spin χ_2	$(0.00, 0.04)$	$(0.00, 0.61)$
Tidal deformability $\tilde{\Lambda}$ with flat prior	300^{+500}_{-190}(symmetric)/ 300^{+420}_{-230}(HPD)	$(0, 630)$

A new constraint by GW Observation





Abbott et al. (LSC and Virgo), arxiv:1805.11579

Low-spin prior : χ <= 0.05

What we have done

arXiv:1805.00219v1 [nucl-th] 1 May 2018

Tidal Deformability of Neutron Stars with Realistic Nuclear Energy Density Functionals

Young-Min Kim,[1] Yeunhwan Lim,[2] Kyujin Kwak,[1] Chang Ho Hyun,[3] and Chang-Hwan Lee[4]

[1]*School of Natural Science, Ulsan National Institute of Science and Technology (UNIST), Ulsan 44919, Korea*

[2]*Cyclotron Institute, Texas A&M University, College Station, TX 77843, USA*

[3]*Department of Physics Education, Daegu University, Gyeongsan 38453, Korea*

[4]*Department of Physics, Pusan National University, Busan 46241, Korea*

Constraints on Nuclear EoS

- Nuclear data: hundreds of models (Skyrme force, RMF, …)
- Neutron star maximum mass

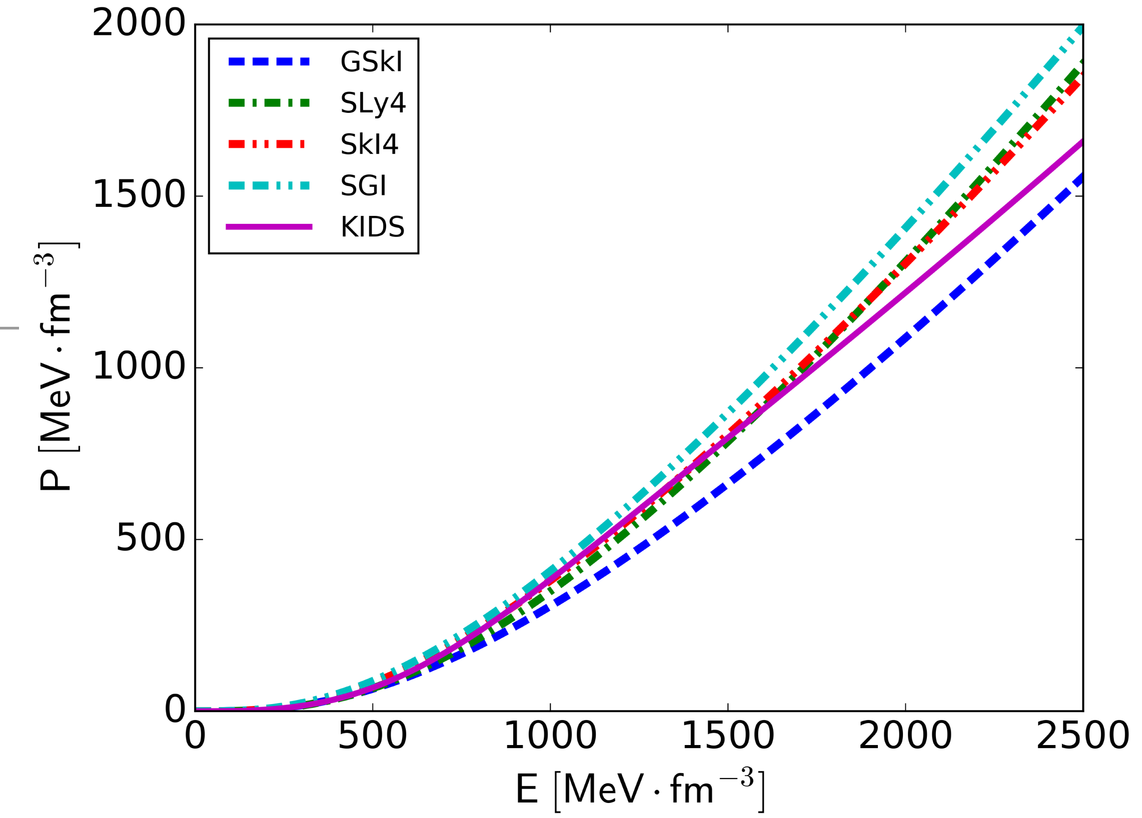
  1.97 ± 0.04 M⊙ [Nature 467, 1081 (2010)]

  2.01 ± 0.04 M⊙ [Science 340, 448 (2013)]
- 11 experimental/empirical data for nuclear matter around saturation density [Phys.Rev. C 85, 035201 (2012)]

Constraint	Quantity	Eq.	Density Region	Range of constraint exp/emp	Range of constraint from CSkP	Ref.
SM1	K_o	(7),(15)	ρ_o (fm^{-3})	200 – 260 MeV	202.0 – 240.3 MeV	[64]
SM2	$K' = -Q_o$	(8),(16)	ρ_o (fm^{-3})	200 – 1200 MeV	362.5 – 425.6 MeV	[65]
SM3	$P(\rho)$	(6)	$2 < \frac{\rho}{\rho_o} < 3$	Band Region	see Fig. 1	[78]
SM4	$P(\rho)$	(6)	$1.2 < \frac{\rho}{\rho_o} < 2.2$	Band Region	see Fig. 2	[80]
PNM1	$\frac{E_{PNM}}{E^o_{PNM}}$	(31)	$0.014 < \frac{\rho}{\rho_o} < 0.106$	Band Region	see Fig. 3	[39, 40]
PNM2	$P(\rho)$	(6)	$2 < \frac{\rho}{\rho_o} < 3$	Band Region	see Fig. 5	[78]
MIX1	J	(9)	ρ_o (fm^{-3})	30 – 35 MeV	30.0 – 35.5 MeV	[44]
MIX2	L	(10)	ρ_o (fm^{-3})	40 – 76 MeV	48.6 – 67.1 MeV	[101]
MIX3	$K_{\tau,v}$	(21)	ρ_o (fm^{-3})	-760 – -372 MeV	-407.1 – -360.1 MeV	[107]
MIX4	$\frac{S(\rho_o/2)}{J}$	-	ρ_o (fm^{-3})	0.57 – 0.86	0.61 – 0.67	[110]
MIX5	$\frac{3P_{PNM}}{L\rho_o}$	(41)	ρ_o (fm^{-3})	0.90 – 1.10	1.02 – 1.10	[112]

Selected EoSs

- Skyrme force models
- Basically fitted to properties of well-known nuclei
- Good saturation properties
- Mmax more than 2Msun



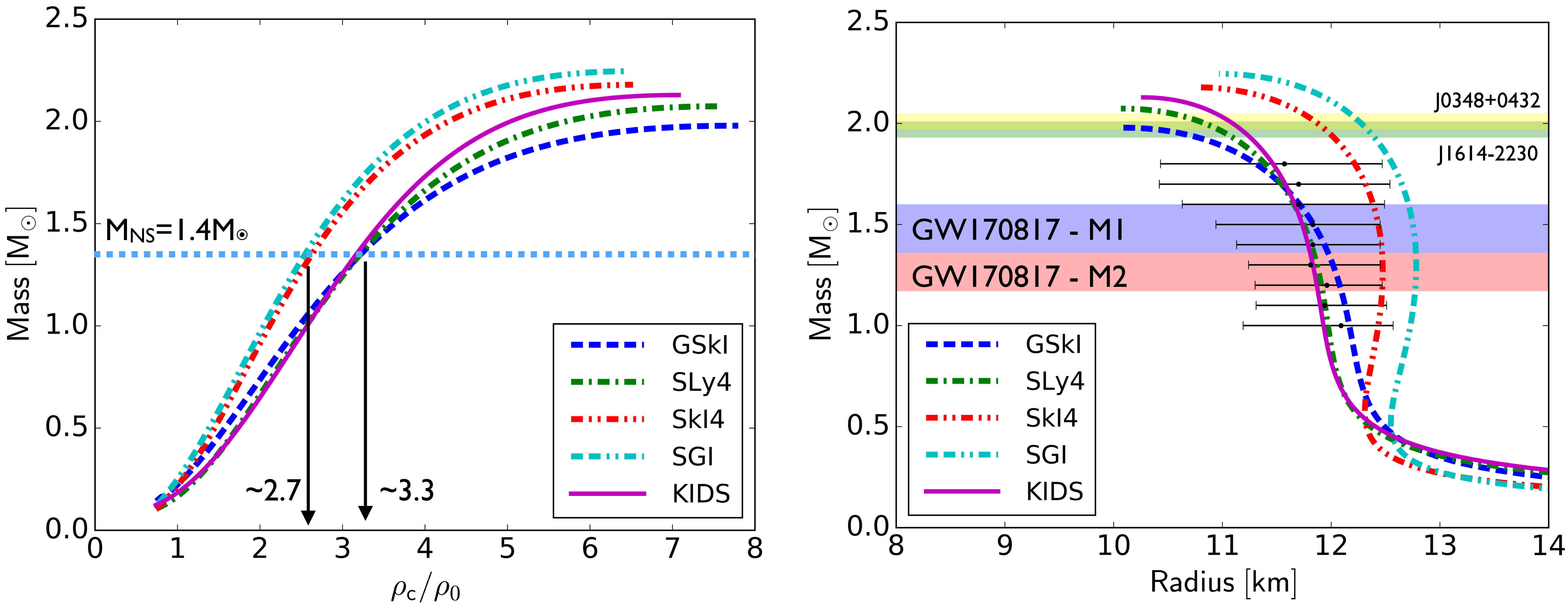
Model	ρ_0	E_0	K_0	$-Q_0$	J	L	$-K_\tau$	$M_{\max}$
Exp/Emp	$\simeq 0.16$	$\simeq 16.0$	$200 \sim 260$	$200 \sim 1200$	$30 \sim 35$	$40 \sim 76$	$372 \sim 760$	$\geq 1.93 \sim 2.05$
CSkP	-	-	$202.0 \sim 240.3$	$362.5 \sim 425.6$	$30.0 \sim 35.5$	$48.6 \sim 67.1$	$360.1 \sim 407.1$	-
GSkI	0.159	16.02	230.2	405.6	32.0	63.5	364.2	1.98
SLy4	0.160	15.97	229.9	363.1	32.0	45.9	322.8	2.07
SkI4	0.160	15.95	248.0	331.2	29.5	60.4	322.2	2.19
SGI	0.154	15.89	261.8	297.9	28.3	63.9	362.5	2.25
KIDS	0.160	16.00	240.0	372.7	32.8	49.1	375.1	2.14

KIDS (Korea: IBS-Daegu-Sungkyunkwan): A new systematic expansion scheme for nuclear EDF [Phys. Rev. C 97, 014312 (2018)]

Kim et al., arxiv:1805.00219

Mass-Radius relations

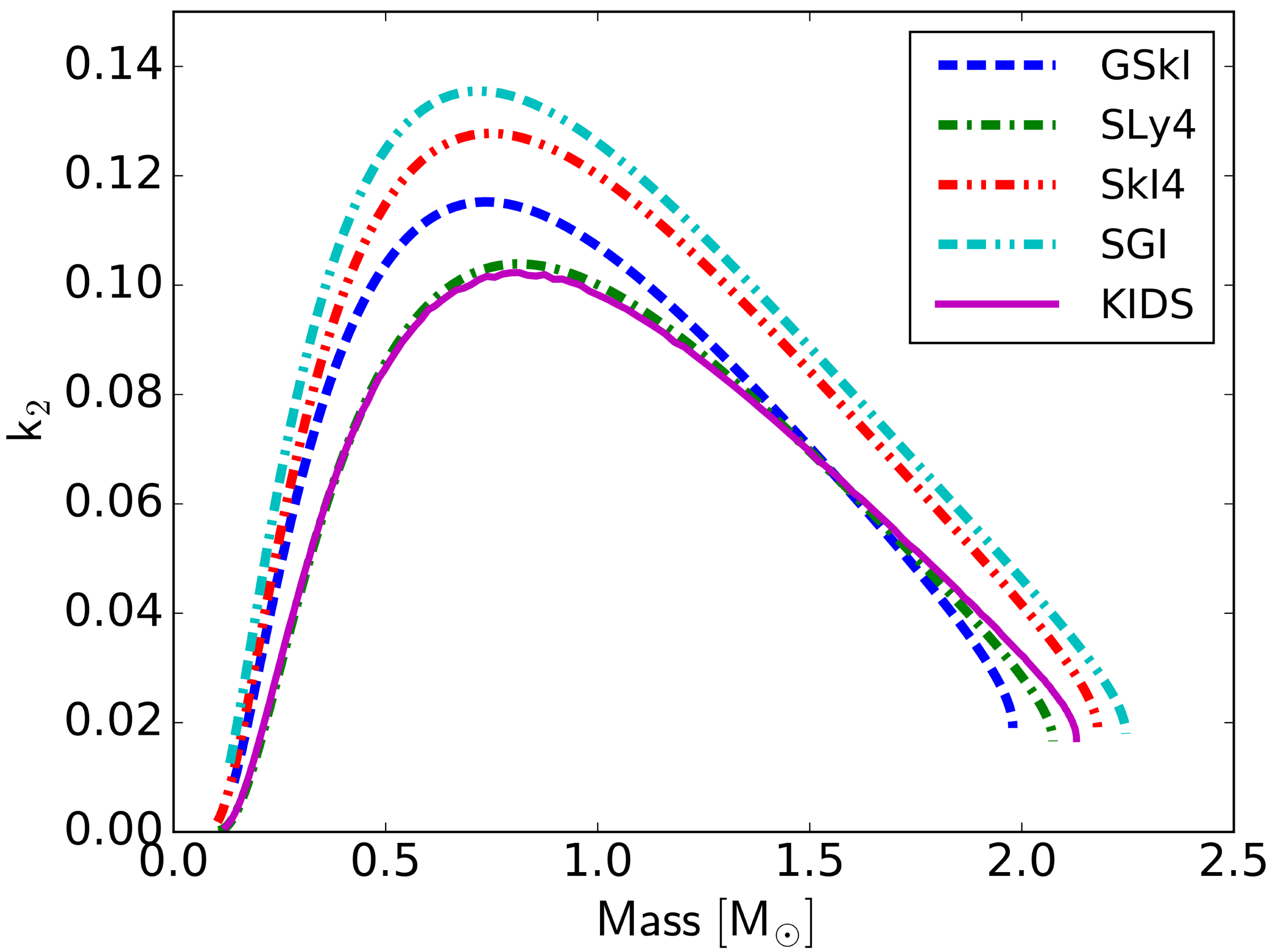
Kim et al., arxiv:1805.00219

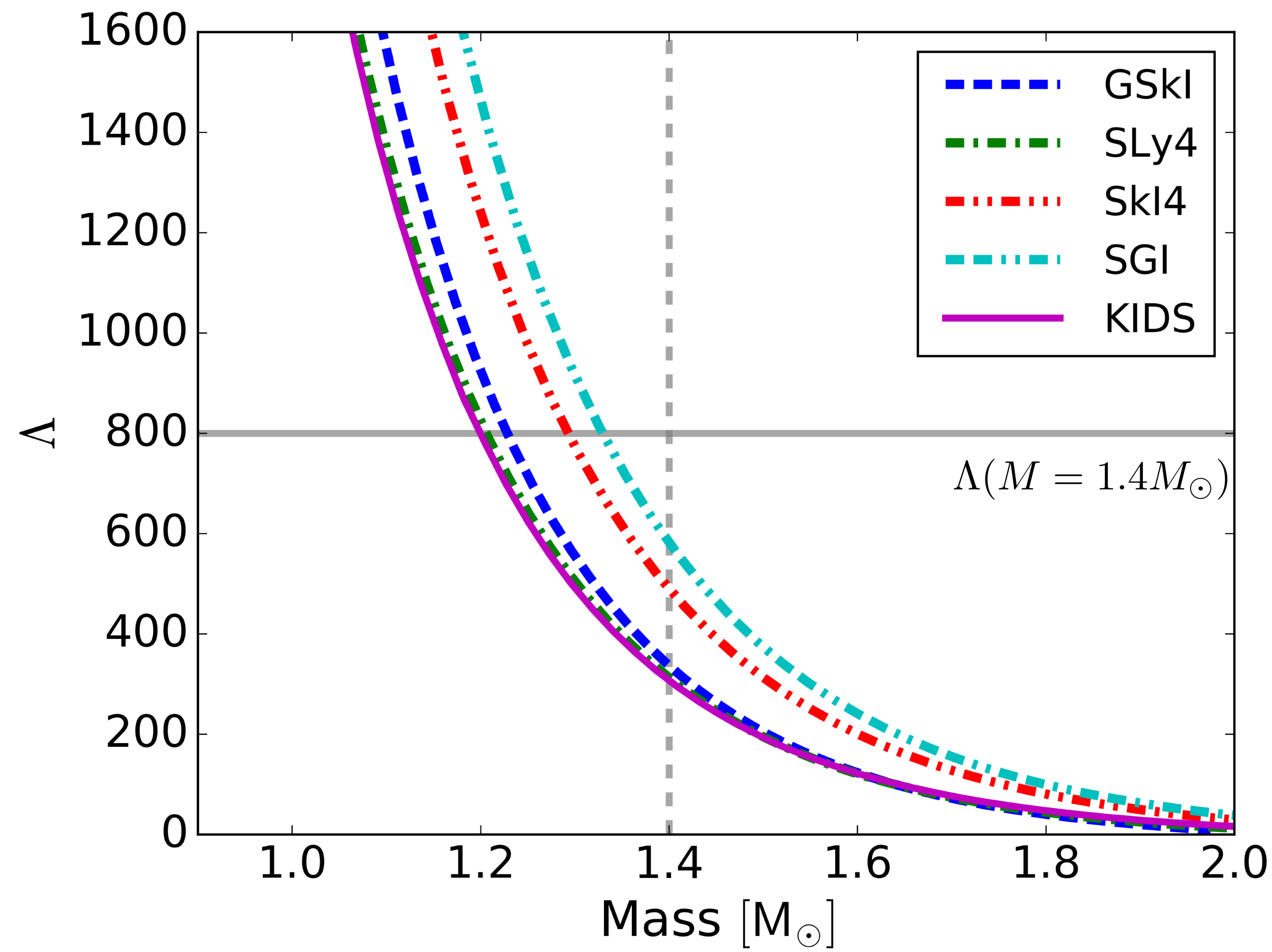


GW170817
- Mchirp = 1.188 M⊙
- low spin prior : M1 = 1.36 ~ 1.60 M⊙ , M2 = 1.17 ~ 1.36 M⊙
- high spin prior : M1 = 1.36 ~ 2.26 M⊙ , M2 = 0.86 ~ 1.36 M⊙

Tidal deformability of a NS

Kim et al., arxiv:1805.00219

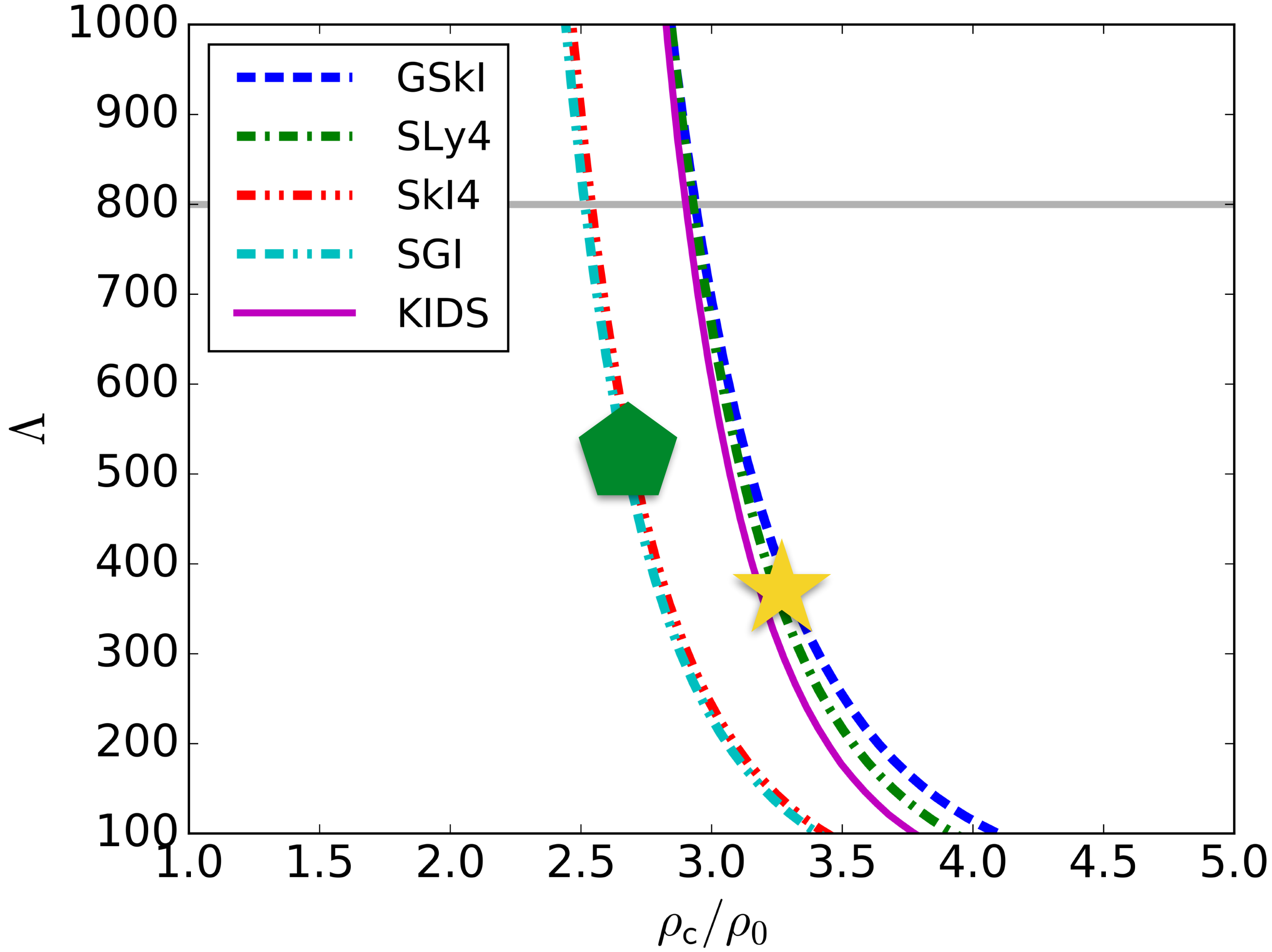


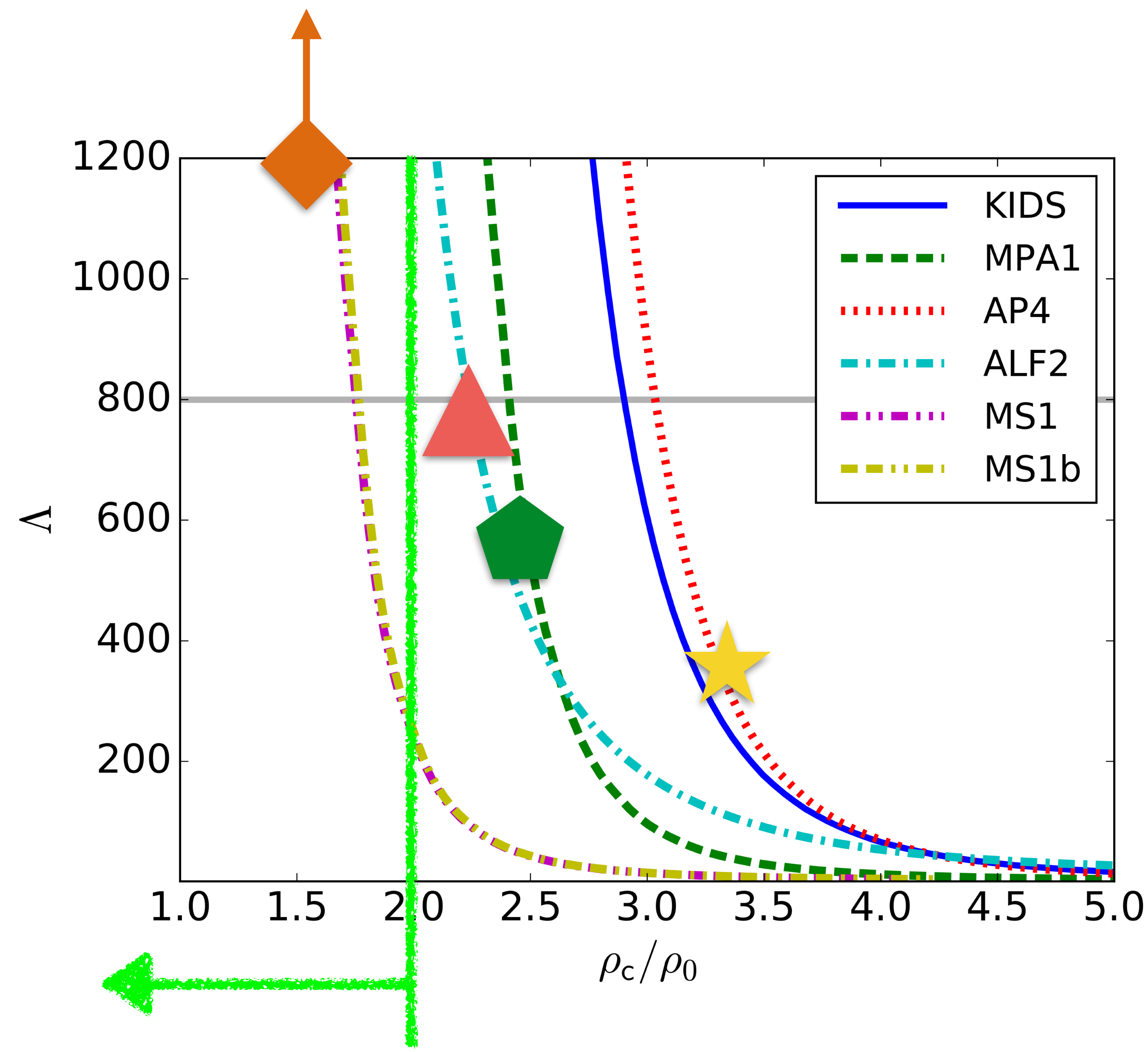


GW170817, Mchirp = 1.188 $M_\odot$
- low spin prior : Lambda(1.4 $M_\odot$) < 800
- high spin prior : Lambda(1.4 $M_\odot$) < 1400

Central Density at M_{NS}=1.4 M⊙

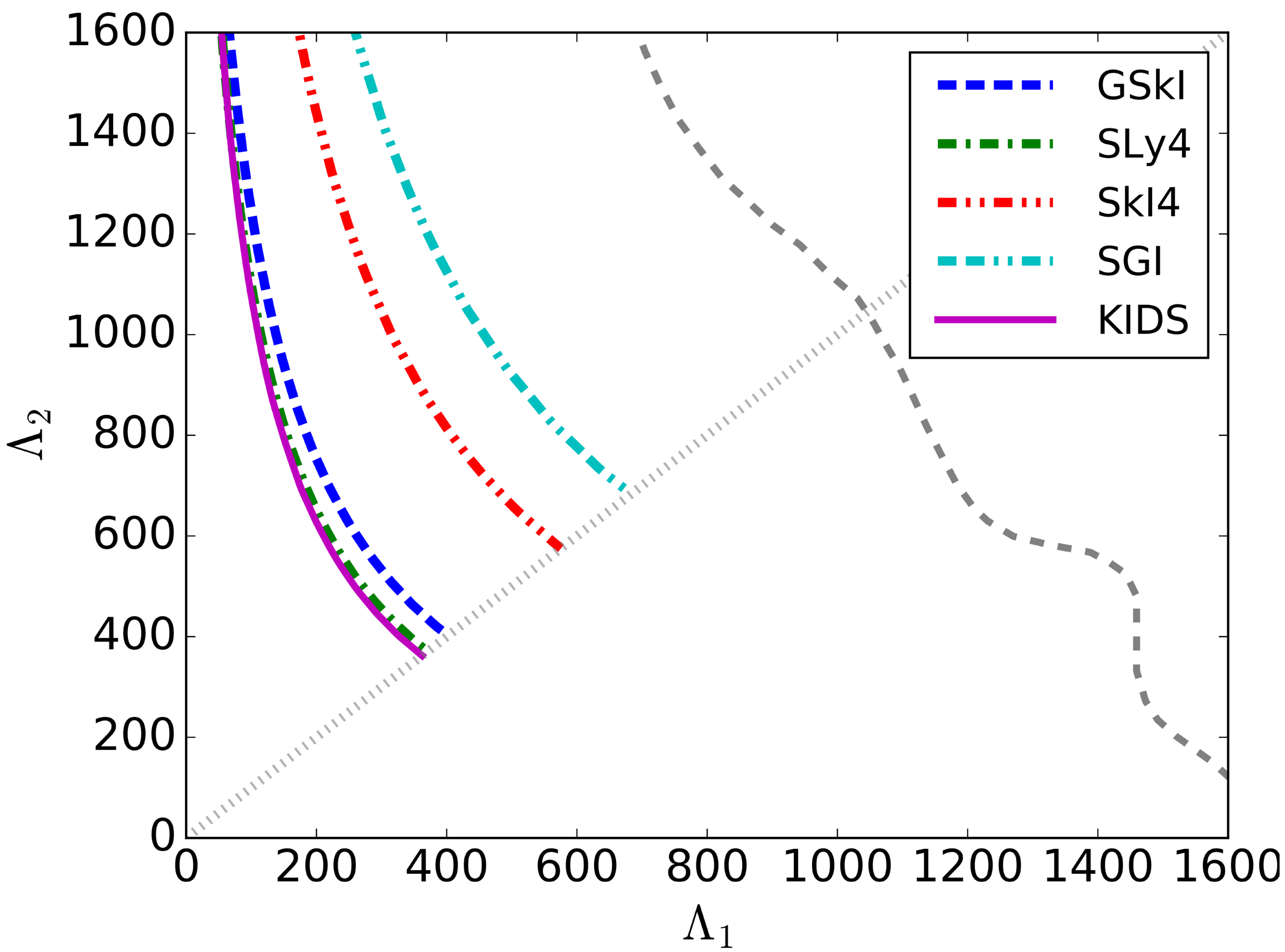
well-selected EoSs

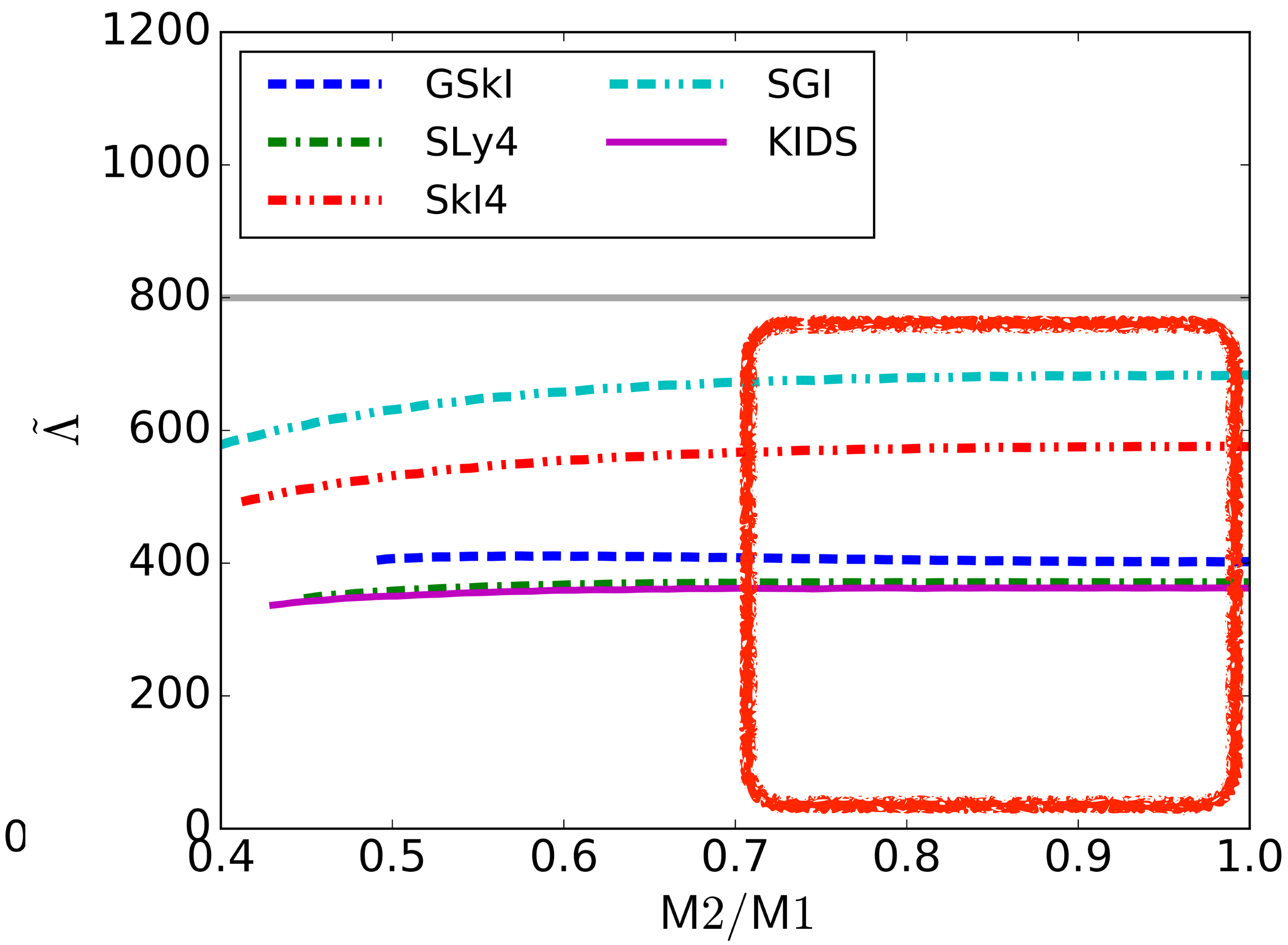




Tidal deformability in BNS

Kim et al., arxiv:1805.00219



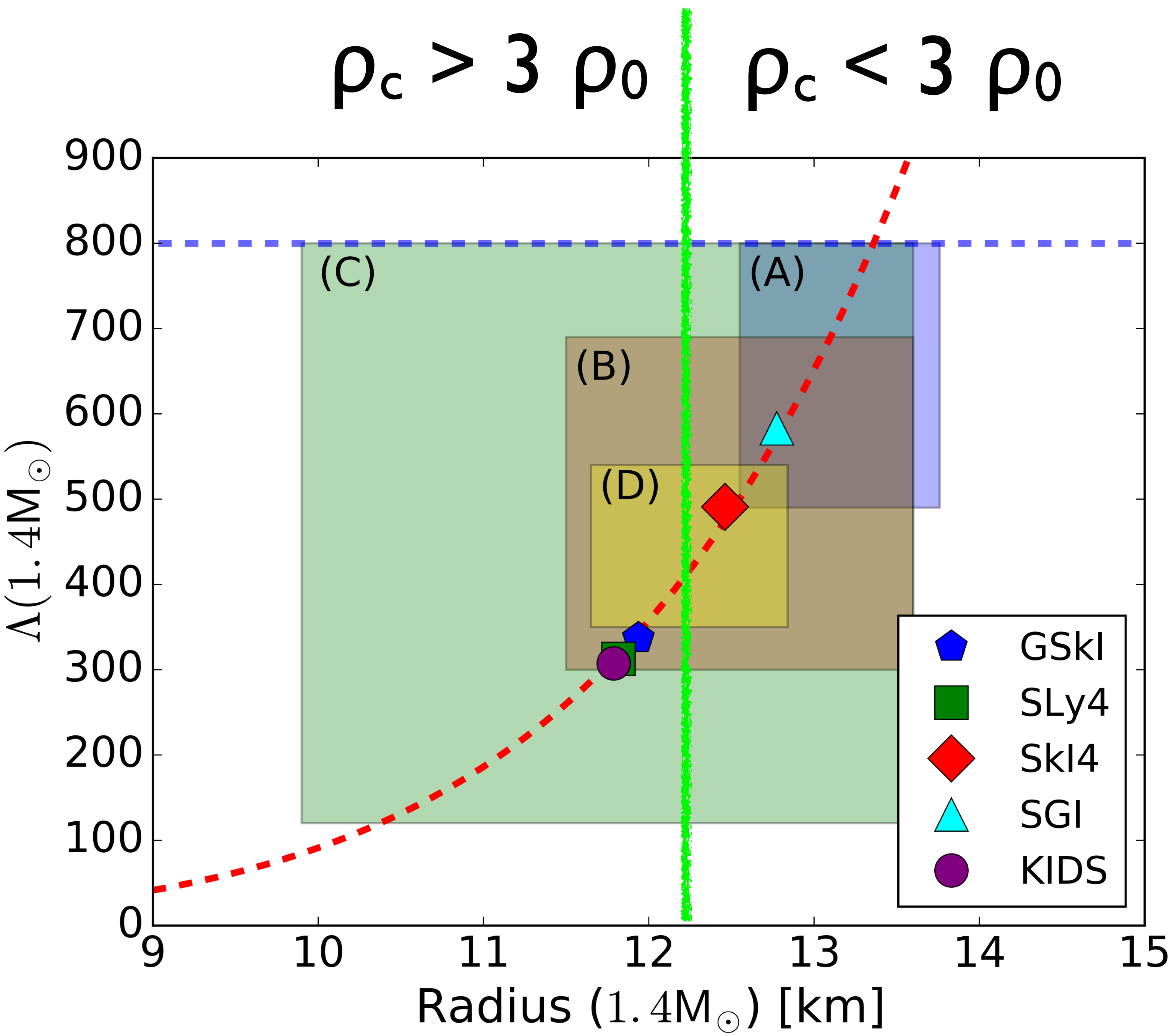


$$\tilde{\Lambda} = \frac{16}{13}\frac{(m_1 + 12m_2)m_1^4\Lambda_1 + (m_2 + 12m_1)m_2^4\Lambda_2}{(m_1+m_2)^5}$$

GW170817, Mchirp = 1.188 M☉

- low spin prior : reduced Lambda < 800
- high spin prior : reduced Lambda < 700

Comparison with recent works



[A] F.J. Fattoyev, J. Piekarewicz, and C.J. Horowitz, arXiv:1711.06615v2
[B] P.G. Krastev, and B.-A. Li, arXiv: 1801.04620v1
[C] E. Annala, T. Gorda, A. Kurkela, and A. Vuorinen, arXiv:1711.02644v2
[D] Y.Lim and J. Holt, arXiv:1803.02803

Kim et al., arxiv:1805.00219

Red line: $\Lambda(1.4M_\odot) = 2.88 * 10^{-6}\ (R/km)^{7.5}$ (fitting function in [C])

Prospects

- Both **Masses & Tidal Deformability of NS**
  can be measured simultaneously
  by GW generated from NS mergers
- Expecting more GWs from NS binary mergers

Binary interactions are always interesting

Thanks