# Particle production by oscillating curvature in $R + R^2$ cosmology


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*based on common work with A. Dolgov and R. Singh*


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# General Relativity (GR):

$$S_{EH} = -\frac{m_{Pl}^2}{16\pi} \int d^4x \sqrt{-g}\, R$$

describes basic properties of the universe in very good agreement with observations.

- $m_{Pl} = 1.22 \cdot 10^{19}$ GeV is the Planck mass

Beyond the frameworks of GR:

$$S_F = -\frac{m_{Pl}^2}{16\pi} \int d^4x \sqrt{-g}\, [R + F(R)]$$

$F(R) = -R^2/(6m^2)$:

- was suggested by V.Ts. Gurovich and A.A. Starobinsky for elimination of cosmological singularity (JETP **50** (1979) 844).
- It was found that the the addition of the $R^2$-term leads to inflationary cosmology. (A. A. Starobinsky, Phys. Lett. **B91**, 99 (1980))

In any cosmological scenario the problem of graceful exit from inflation and the problem of the universe heating are of primary importance.

- Review: A. De Felice and S. Tsujikawa, Living Rev. Rel. 13, 3 (2010)

# I. Cosmological Equations in $R^2$-theory

- The term describing particle production is included as a source into equation for the energy density evolution.

# II. Solution *ab ovo* to $\Gamma t \lesssim 1$, but $mt \gg 1$

1. Solutions at inflationary and post-inflationary epoch
2. Asymptotic solutions at $\tau \gg 1$: RD and MD stages
3. Energy influx to cosmological plasma from the scalaron decay

During this time the universe evolution was quite different from the General Relativity one.

# III. Solution at $\Gamma t \gtrsim 1$: approaching to GR cosmology

- GR is recovered when the energy density of matter becomes larger than that of the exponentially decaying scalaron.
- This approach is delayed: not at $\Gamma t \sim 1$, but at $\Gamma t \sim \ln(m/\Gamma) \Rightarrow$ modification of high temperature baryogenesis, variation of the frozen abundances of heavy DM particles, necessity of reconsideration of the formation of PBHs, etc.

# IV. Conclusions

# I. Cosmological Equations in $R^2$-theory

Let us consider the theory described by the action:

$$S_{tot} = -\frac{m_{Pl}^2}{16\pi}\int d^4x\sqrt{-g}\left(R - \frac{R^2}{6m^2}\right) + S_m$$

- $m$ is a constant parameter with dimension of mass

The modified Einstein equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \frac{1}{3m^2}\left(R_{\mu\nu} - \frac{1}{4}Rg_{\mu\nu} + g_{\mu\nu}D^2 - D_\mu D_\nu\right)R = \frac{8\pi}{m_{Pl}^2}T_{\mu\nu}$$

- $D^2 \equiv g^{\mu\nu}D_\mu D_\nu$ is the covariant D'Alembert operator.

The energy-momentum tensor of matter $T_{\mu\nu}$

$$T^\mu_\nu = diag(\varrho, -P, -P, -P)$$

where $\varrho$ is the energy density, $P$ is the pressure of matter.

The matter distribution is homogeneous and isotropic

$$P = w\varrho$$

- non-relativistic: $w = 0$, relativistic: $w = 1/3$, vacuum-like: $w = -1$

## FRW: $ds^2 = dt^2 - a^2(t)\left[dr^2 + r^2 d\vartheta^2 + r^2 \sin^2\vartheta\, d\varphi^2\right]$, $H = \dot{a}/a$

The curvature scalar:

$$R = -6\dot{H} - 12H^2$$

The covariant conservation condition $D_\mu T^\mu_\nu = 0$:

$$\dot{\varrho} = -3H(\varrho + P) = -3H(1+w)\varrho$$

Trace equation:

$$D^2 R + m^2 R = -\frac{8\pi m^2}{m_{Pl}^2}\, T^\mu_\mu$$

For homogeneous field, $R = R(t)$, and with $P = w\varrho$:

$$\ddot{R} + 3H\dot{R} + m^2 R = -\frac{8\pi m^2}{m_{Pl}^2}(1-3w)\varrho$$

This is the Klein-Gordon (KG) type equation for massive scalar field $R$, which is sometimes called "scalaron". It differs from KG by the liquid friction term.

## $\ddot{R} + 3H\dot{R} + m^2 R = -\frac{8\pi m^2}{m_{Pl}^2}(1-3w)\varrho$

This equation:

- does not include the effects of particle production by the curvature scalar;
- is a good approximation at inflationary epoch, when particle production by $R(t)$ is absent, because $R$ is large and friction is large, so $R \to 0$ slowly.

At some stage, when $\boldsymbol{H}$ becomes smaller than $\boldsymbol{m}$, $\boldsymbol{R}$ starts to oscillate efficiently producing particles.

- It commemorates the end of inflation, the heating of the universe, and the transition from the accelerated expansion (inflation) to a de-accelerated one.
- The latter resembles the usual Friedmann matter dominated expansion regime but differs in many essential features.

Particle production: for the harmonic potential can be approximately described by an additional liquid friction term $\Gamma\dot{R}$, where

$$\Gamma = \frac{m^3}{48 m_{Pl}^2}$$

- Ya.B. Zeldovich, A. Starobinsky, JETP Lett. 26, 252 (1977); A. Vilenkin, Phys. Rev. **D32**, 2511 (1985); EA, A. Dolgov, L. Reverberi, JCAP **1202** (2012) 049.

# Particle Production: friction term approximation

Equation for $R$ acquires an additional friction term:

$$\ddot{R} + (3H + \Gamma)\dot{R} + m^2 R = -\frac{8\pi m^2}{m_{Pl}^2}(1 - 3w)\varrho$$

Particle production leads to an emergence of the source term in Eq. for $\varrho$:

$$\dot{\varrho} = -3H(1 + w)\varrho + \frac{mR_{ampl}^2}{1152\pi}$$

where $R_{ampl}$ is the amplitude of $R(t)$-oscillations.

- The state of the cosmological matter depends not only upon the spectrum of the decay products but also on the thermal history of the produced particles.
- Depending on that, the parameter $w$ may be not exactly equal to 0 or 1/3. The equation of state can be not simple $P = w\varrho$ with constant $w$.
- Two limiting values $w = 0$ and $1/3$ are possible simple examples.
- Different values of $w$ would not change the presented results significantly.

# Dimensionless Equations

Dimensionless time variable and dimensionless functions

$$\tau = tm, \quad H = mh, \quad R = m^2 r, \quad \varrho = m^4 y, \quad \Gamma = m\gamma.$$

The system of dimensionless equations

$$h' + 2h^2 = -r/6$$
$$r'' + (3h + \gamma)r' + r = -8\pi\mu^2(1 - 3w)y$$
$$y' + 3(1 + w)h\, y = S[r]$$

- prime denotes derivative over $\tau$, $\mu = m/m_{Pl}$, $\gamma = \mu^2/48$

The source term is taken as:

$$S[r] = \frac{\langle r^2 \rangle}{1152\pi}.$$

- $\langle r^2 \rangle$ means amplitude squared of harmonic oscillations, $r^2_{ampl}$, of the dimensionless curvature $r(\tau)$.
- For nonharmonic oscillations we approximate $\langle r^2 \rangle$ as $2(r')^2$ or $(-2r''r)$.

# II. Solution *ab ovo* to $\Gamma t \lesssim 1$

- Solution at inflationary epoch

# Duration of Inflation

The initial conditions should be chosen in such a way that at least 70 e-foldings during inflation are ensured:

$$N_e = \int_0^{\tau_{inf}} h \, d\tau \geq 70$$

We can roughly estimate the duration of inflation neglecting higher derivatives in equations for $h$ and $r$.

- A. A. Starobinsky, Phys. Lett. **B91**, 99 (1980); A. De Felice and S. Tsujikawa, Living Rev. Rel. 13, 3 (2010); arXiv:1002.4928

Simplified system to estimate the duration of inflation ($\boldsymbol{y = 0}$, $\gamma \ll \mathbf{1}$):

$$\boldsymbol{h^2 = -r/12, \quad 3hr' = -r}$$

Solutions:

$$\boldsymbol{\sqrt{-r(\tau)} = \sqrt{-r_0} - \tau/\sqrt{3}, \;\; h(\tau) = (\sqrt{-3r_0} - \tau)/6, \;\; r_0 = r(\tau = 0)}$$

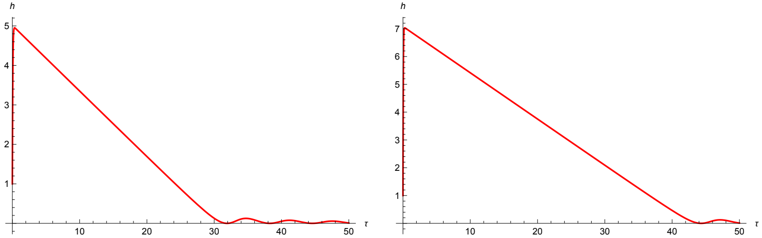
The duration of inflation is roughly determined by the condition $\boldsymbol{h = 0}$, i.e.

$$\boldsymbol{\tau_{inf} = \sqrt{-3r_0} \;\Rightarrow\; N_e \approx r_0/4}$$

Exact System: $h' + 2h^2 = -r/6, \quad y' + 3(1+w)h\,y = S[r]$

$$r'' + (3h+\gamma)r' + r = -8\pi\mu^2(1-3w)y$$

## Numerical solutions: Evolution of $\boldsymbol{h(\tau)}$ at the inflationary stage



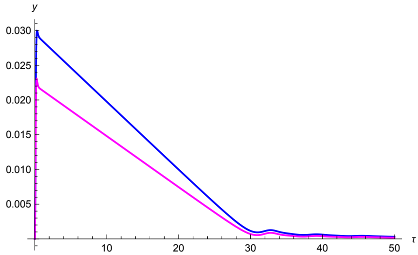

- Initial values of dimensionless curvature $r_0 = 300$ (left) and $r_0 = 600$ (right).
- Initially $h_{in} = 0$, but it quickly reaches the value $h(0) = \sqrt{-r_0/12}$.
- The numbers of e-foldings: $\boldsymbol{N_e \approx r_0/4 = 75}$ (left) and **150** (right).

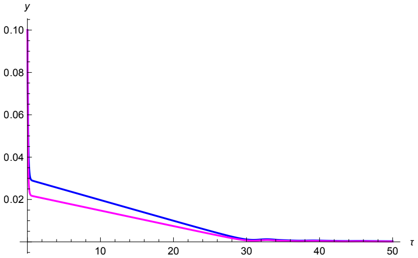
An excellent agreement with numerical solutions demonstrates high precision of the slow roll approximation and weak impact of particle production at (quasi)inflationary stage.

Exact System: $h' + 2h^2 = -r/6, \quad y' + 3(1+w)h\,y = S[r]$

$$r'' + (3h + \gamma)r' + r = -8\pi\mu^2(1-3w)y$$

Evolution of the dimensionless energy density of matter $\boldsymbol{y(\tau)}$ during inflation for $w = 0$ (blue) and $w = 1/3$ (magenta).



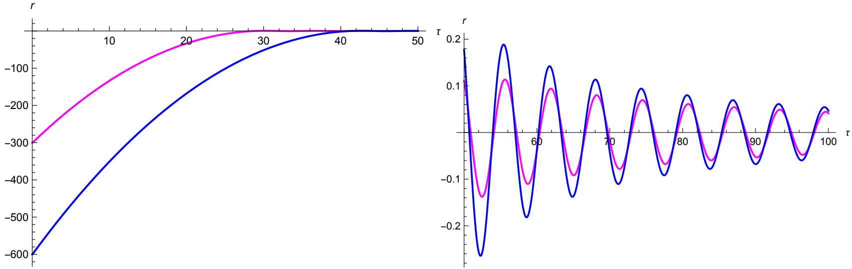

- *Left panel*: initially $y_{in} = 0$. *Right panel*: $y_{in} = 0.1$.

The initial fast rise of $\varrho$ from zero during short time is generated by the $\boldsymbol{S[r]}$-term. The results are not sensitive to the form of $\boldsymbol{S[r]}$.

Exact System: $h' + 2h^2 = -r/6, \quad y' + 3(1+w)h\,y = S[r]$

$$r'' + (3h + \gamma)r' + r = -8\pi\mu^2(1-3w)y$$

Evolution of the dimensionless curvature scalar $\boldsymbol{r(\tau)}$ for $\boldsymbol{r_{in} = -300}$ (magenta) and $\boldsymbol{r_{in} = -600}$ (blue)




- *Left panel*: evolution during inflation.
- *Right panel*: evolution after the end of inflation, the curvature scalar starts to oscillate.

# II. Solution *ab ovo* to $\Gamma t \lesssim 1$

- Solution at inflationary epoch
- Solution at post-inflationary epoch

The behavior of $\boldsymbol{R}$, $\boldsymbol{H}$ and $\varrho$, or dimensionless quantities $\boldsymbol{r}$ , $\boldsymbol{h}$, and $\boldsymbol{y}$ is drastically different at the vacuum-like dominated stage (inflation) and during scalaron dominated stage, which followed the inflationary epoch.

# Numerical solutions at post-inflationary epoch

We will find the laws of evolution of $r(\tau)$, $h(\tau)$, and $y(\tau)$ after inflation till $\gamma\tau \sim 1$, solving the system

$$h' + 2h^2 = -r/6$$
$$r'' + (3h + \gamma)r' + r = -8\pi\mu^2(1 - 3w)y$$
$$y' + 3(1 + w)h\, y = S[r]$$

The numerical solutions:

- from the end of inflation $\to$ large $\tau \gg 1$, but not too large
- the numerical procedure for huge $\tau \sim 1/\gamma$ becomes unstable

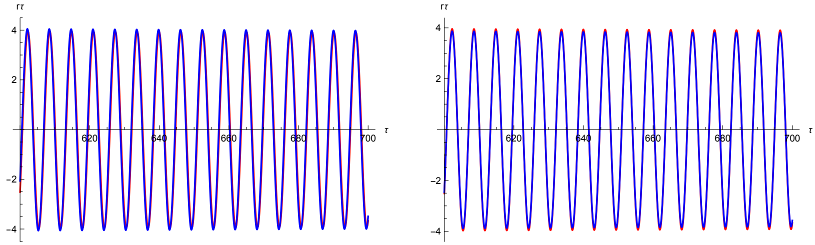
Analytical solutions: asymptotically valid at any large $\tau$ up to $\tau \sim 1/\gamma$.

Very good agreement between numerical and analytical solutions at large but not huge $\tau$ allows to trust asymptotic analytical solution at huge $\tau$.

Exact System: $h' + 2h^2 = -r/6, \quad y' + 3(1+w)h\,y = S[r]$

$$r'' + (3h+\gamma)r' + r = -8\pi\mu^2(1-3w)y, \;\; \mu = m/m_{Pl} = 0.1, \;\; \gamma = \mu^2/48$$

Evolution of the curvature scalar, $\boldsymbol{\tau r(\tau)}$, in post-infationary epoch.

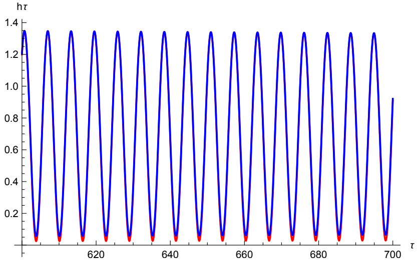



*Left panel* ($\boldsymbol{w = 1/3}$) : initially $\boldsymbol{r_{in} = -300}$ (red), $\boldsymbol{r_{in} = -600}$ (blue). There is absolutely no difference between the curves.
*Right panel* ($\boldsymbol{r_{in} = -300}$): $\boldsymbol{w = 1/3}$ (red) and $\boldsymbol{w = 0}$ (blue). The difference is minuscule.
The source term here is taken as $\boldsymbol{S[r] = (r')^2/1152\pi}$. The results are not sensitive to its form.

Exact System: $h' + 2h^2 = -r/6, \quad y' + 3(1+w)h\,y = S[r]$

$$r'' + (3h+\gamma)r' + r = -8\pi\mu^2(1-3w)y, \;\; \mu = m/m_{Pl} = 0.1$$

Evolution of the Hubble parameter, $h\tau$, in post-inflationary epoch for $w = 1/3$ (red) and $w = 0$ (blue)



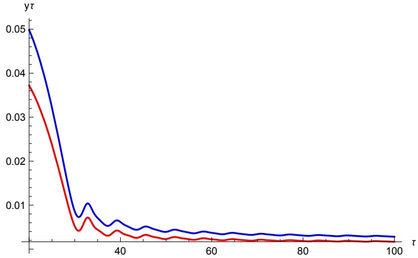

- The dependence on $w$ is very weak, except for small values of $h$.

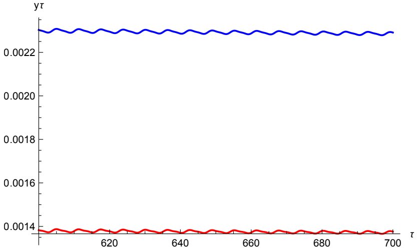
If $h$ is very close to zero, it may become negative because of numerical error due to insufficient precision.

Exact System: $h' + 2h^2 = -r/6, \quad y' + 3(1+w)h\, y = S[r]$

$$r'' + (3h + \gamma)r' + r = -8\pi\mu^2(1-3w)y$$

Energy density of matter as a function of time for
$\boldsymbol{w = 1/3}$ (red) and $\boldsymbol{w = 0}$ (blue)




- Evolution of $\boldsymbol{y\tau}$ at small $\boldsymbol{\tau}$ (left) and at large $\boldsymbol{\tau}$ (right).

The magnitude of $\boldsymbol{\varrho}$ for these two values of $\boldsymbol{w}$ are noticeably different in contrast to other relevant quantities, $\boldsymbol{r}$ and $\boldsymbol{h}$, which very weakly depend upon $\boldsymbol{w}$.
The product $\boldsymbol{y\tau}$ tends to a constant value with rising $\boldsymbol{\tau}$ till $\boldsymbol{\gamma\tau}$ remains small.
This behavior much differs from the standard matter density evolution $\boldsymbol{\varrho \sim 1/t^2}$.

# Asymptotic solution at $\tau \gg 1$, $\gamma\tau \lesssim 1$ and $w = 1/3$

Simple form of numerical solutions at large $\tau$:

- $r$ oscillates with the amplitude decreasing as $1/\tau$ around zero
- $h$ also oscillates almost touching zero with the amplitude also decreasing as $1/\tau$ around some constant value close to 2/3.

In the case $w = 1/3$ we have the system of equations

$$h' + 2h^2 = -r/6, \tag{1}$$

$$r'' + 3hr' + r = 0, \tag{2}$$

$$y' + 4h\,y = \frac{< r^2 >}{1152\pi},$$

We search for the asymptotic expansion of $h$ and $r$ at $\tau \gg 1$ in the form:

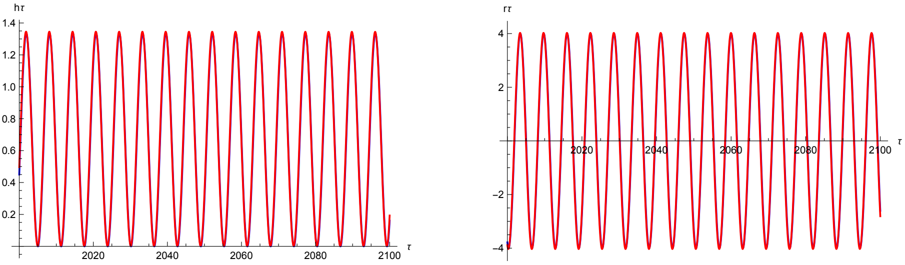
$$r = \frac{r_1 \cos(\tau + \theta_r)}{\tau} + \frac{r_2}{\tau^2}, \quad h = \frac{h_0 + h_1 \sin(\tau + \theta_h)}{\tau}$$

- $r_j$ and $h_j$ are some constant coefficients to be calculated from Eqs.(1)-(2)
- the constant phases $\theta_j$ are determined through the initial conditions and will be adjusted by the best fit of the asymptotic solution to the numerical one

Finally we find:

$$h = \frac{2}{3\tau}\left[1 + \sin\left(\tau + \theta\right)\right], \quad r = -\frac{4\cos(\tau + \theta)}{\tau} - \frac{4}{\tau^2}$$

Comparison of numerical calculations with analytical estimates for the adjusted "by hand" phase $\theta = -2.9\pi/4$




- *Left panel*: comparison of numerical solution for $h\tau$ (red) with analytic estimate (blue).
- *Right panel*: the same for numerically calculated $r\tau$.

The difference between the red and blue curves is not observable.

## Equation for energy density: $y' + 4h\,y = < r^2 > /(1152\pi)$

$\langle r^2 \rangle = 16/\tau^2$ is the square of the amplitude of the harmonic oscillations.

Analytical integration gives:

$$y(\tau) = \frac{1}{72\pi} \int_{\tau_0}^{\tau} \frac{d\tau_2}{\tau_2^2} \exp\left[-4 \int_{\tau_2}^{\tau} d\tau_1 h(\tau_1)\right]$$

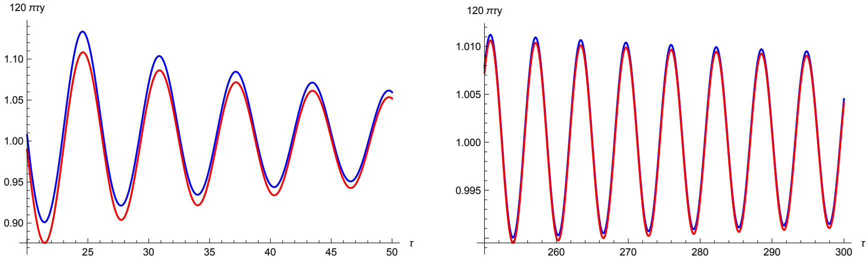
- $\tau_0 \ll \tau$ is some initial value of the dimensionless time.

Taking asymptotical $h(\tau)$ we can find an asymptotic behavior of $y(\tau)$:

$$y_{1/3} = \frac{1}{120\pi\tau} + \frac{1}{45\pi}\frac{\cos(\tau+\theta)}{\tau^2} - \frac{1}{27\pi\tau^2}\int_{\epsilon}^{1} \frac{d\eta_2}{\eta_2^{1/3}} \cos(\tau\eta_2 + \theta)$$

- the subindex (1/3) indicates that $w = 1/3$
- $\epsilon = \tau_0/\tau \ll 1$. The last integral is proportional to $1/\tau^{2/3}$ and is subdominant.

# Asymptotic behavior of the energy density for $w = 1/3$




Comparison of the integral solution

$$y(\tau) = \frac{1}{72\pi} \int_{\tau_0}^{\tau} \frac{d\tau_2}{\tau_2^2} \exp\left[-4 \int_{\tau_2}^{\tau} d\tau_1 h(\tau_1)\right]$$

for the dimensionless energy density $120\pi\tau\, y(\tau)$ with the asymptotic expression $120\pi\tau\, y_{1/3}(\tau)$ for moderately large $\tau$ (*left panel*) and very large $\tau$ (*right panel*).

# Asymptotic solution at $\tau \gg 1$, $\gamma\tau \lesssim 1$ and $w = 0$

For $w = 0$ equations take the form

$$h' + 2h^2 = -r/6$$
$$r'' + 3hr' + r = -8\pi\mu^2 y$$
$$y' + 3h\,y = S[r]$$

$\mu^2 \ll 1 \Rightarrow$ the impact of the r.h.s. in Eq. for $r$ is not essential $\Rightarrow$ we can use:

$$h = \frac{2}{3\tau}\left[1 + \sin(\tau + \theta)\right], \quad r = -\frac{4\cos(\tau + \theta)}{\tau} - \frac{4}{\tau^2}$$

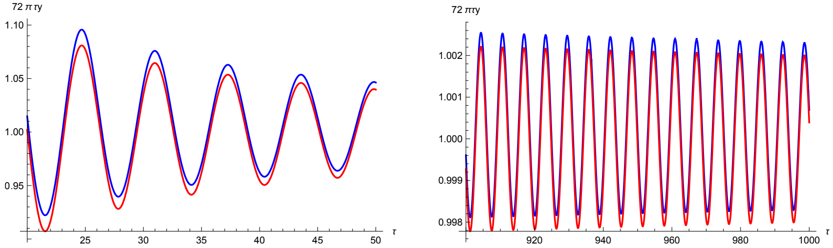
The only essential difference with the $w = 1/3$ case arises in the equation governing the evolution of the energy density, $y(\tau)$.

There appears coefficient (-3) in the exponent, instead of (-4):

$$y(\tau) = \frac{1}{72\pi}\int_{\tau_0}^{\tau}\frac{d\tau_2}{\tau_2^2}\exp\left[-3\int_{\tau_2}^{\tau} d\tau_1 h(\tau_1)\right]$$

# Asymptotic behavior of the solution for $w = 0$

$$y_0 = \frac{1}{72\pi\tau} + \frac{\cos(\tau+\theta)}{36\pi\tau^2}$$




Comparison of the integral solution

$$y(\tau) = \frac{1}{72\pi}\int_{\tau_0}^{\tau}\frac{d\tau_2}{\tau_2^2}\,\exp\left[-3\int_{\tau_2}^{\tau} d\tau_1 h(\tau_1)\right]$$

for the dimensionless energy density $72\pi\tau\, y(\tau)$ with the asymptotic expression $72\pi\tau\, y_0(\tau)$ for moderately large $\tau$ (*left panel*) and very large $\tau$ (*right panel*).

## II. Solution *ab ovo* to $\Gamma t \lesssim 1$

- Energy influx to cosmological plasma from the scalaron decay

# Energy influx to cosmological plasma from the scalaron decay

Energy conservation demands equality of the energy influx induced by

$$S[r] = \frac{\langle r^2 \rangle}{1152\pi}$$

to the loss of scalaron energy density due to its decay with the width $\Gamma = m^3/(48 m_{Pl}^2)$.

To check that let us consider a simplified model:

$$A_R = \frac{m_{Pl}^2}{48\pi m^4} \int d^4x \sqrt{-g} \left[ \frac{(DR)^2}{2} - \frac{m^2 R^2}{2} - \frac{8\pi m^2}{m_{Pl}^2} T_\mu^\mu R \right]$$

which leads to the proper equation of motion

$$D^2 R + m^2 R = -\frac{8\pi m^2}{m_{Pl}^2} T_\mu^\mu$$

To determine the energy density of the scalaron field we have to redefine this field in such a way that the new field is canonically normalized, that is its kinetic term enters the action with the coefficient 1/2.

## Energy influx to cosmological plasma from the scalaron decay

Canonically normalized scalar field:

$$\Phi = \frac{m_{Pl}}{\sqrt{48\pi}\, m^2} R$$

Correspondingly, the energy density of the scalaron field:

$$\varrho_R = \frac{\dot{\Phi}^2 + m^2 \Phi^2}{2} = \frac{m_{Pl}^2 (\dot{R}^2 + m^2 R^2)}{96\pi m^4}$$

The energy production rate is given by:

$$\dot{\varrho}_R = 2\Gamma \varrho_R = \frac{\dot{R}^2 + m^2 R^2}{2304\pi m} = \frac{m^3}{72\pi t^2}$$

- The coefficient 2 in front of $\Gamma$ appears because a pair of particles is produced in the scalaron decay.
- We take $\Gamma = m^3/(48 m_{Pl}^2)$, $r = -4\cos(\tau + \theta)/\tau - 4/\tau^2$ and differentiate only the quickly oscillating factor.

# Energy influx to cosmological plasma from the scalaron decay

Let us compare result

$$\dot{\varrho}_R = 2\Gamma\varrho_R = \frac{\dot{R}^2 + m^2 R^2}{2304\pi m} = \frac{m^3}{72\pi t^2}$$

with

$$\dot{\varrho} = -3H(1+w)\varrho + \frac{m R_{ampl}^2}{1152\pi} \quad or \quad S[r] = \frac{\langle r^2 \rangle}{1152\pi}$$

- we take the amplitude of harmonic oscillations of $R$ equal to $R_{ampl} = 4m/t$.

The contribution of the particle production is exactly the same as above:

$$\dot{\varrho}_{source} = \frac{m R_{ampl}^2}{1152\pi} = \frac{m^3}{72\pi t^2}.$$

According to the results obtained above the cosmological evolution in $\boldsymbol{R^2}$-gravity is strongly different from the usual FRW-cosmology.

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- Firstly, the energy density of matter in $R^2$ modified gravity at RD stage drops down as

$$\varrho_{R^2} = \frac{m^3}{120\pi t} \quad \textit{instead of} \quad \varrho_{GR} = \frac{3H^2 m_{Pl}^2}{8\pi} = \frac{3m_{Pl}^2}{32\pi t^2}$$

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- Secondly, the Hubble parameter quickly oscillates with time

$$h = \frac{2}{3\tau}\left[1 + \sin(\tau + \theta)\right]$$

almost touching zero, and it is the same for $w = 1/3$ and $w = 0$.

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- The curvature scalar drops down as $m/t$ and oscillates changing sign

$$r = -\frac{4\cos(\tau + \theta)}{\tau} - \frac{4}{\tau^2}$$

instead of being proportional to the trace of the energy-momentum tensor of matter, which is identically zero at RD stage and monotonically decreases with time, as $1/t^2$ at MD stage.

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$$\varrho_{R^2} = \frac{m^3}{120\pi t} \quad \textit{instead of} \quad \varrho_{GR} = \frac{3H^2 m_{Pl}^2}{8\pi} = \frac{3m_{Pl}^2}{32\pi t^2}$$

- Secondly, the Hubble parameter quickly oscillates with time

$$h = \frac{2}{3\tau}\left[1 + \sin\left(\tau + \theta\right)\right]$$

almost touching zero, and it is the same for $w = 1/3$ and $w = 0$.

- The curvature scalar drops down as $m/t$ and oscillates changing sign

$$r = -\frac{4\cos(\tau + \theta)}{\tau} - \frac{4}{\tau^2}$$

instead of being proportional to the trace of the energy-momentum tensor of matter, which is identically zero at RD stage and monotonically decreases with time, as $1/t^2$ at MD stage.

- It is noteworthy that $R$ is not related to the energy density of matter as is true in GR.

# III. Solution at $\Gamma t \gtrsim 1$

# Solution at $\gamma\tau \gtrsim 1$, $\gamma = \mu^2/48$ and $\mu = m/m_{Pl}$

$$h' + 2h^2 = -r/6$$
$$r'' + (3h+\gamma)r' + r = -8\pi\mu^2(1-3w)y$$
$$y' + 3(1+w)h\,y = S[r]$$

A straightforward numerical solution of this system quickly becomes unreliable due to very small exponential suppression factor $\exp\left(-\gamma\tau/2\right)$, when $\gamma\tau \gg 1$.

The case of relativistic matter, $w = 1/3$:

$$r'' + (3h+\gamma)r' + r = 0$$

Eliminating the first derivative $r'$ by introducing the new function $v$, we find:

$$r = \exp\left[-\gamma(\tau-\tau_0)/2 - (3/2)\int_{\tau_0}^{\tau} d\tau_1 h(\tau_1)\right] v(\tau)$$

where

$$v(\tau) = -4\gamma\cos\left(\tau+\theta\right)$$

NB. Curvature $r$ exponentially vanishes at large $\gamma\tau/2 \Longrightarrow$ r.h.s. of Eqs. for $h$ and $y$ tends to $0$ with the same speed, restoring the normal cosmology at RD stage.

## Nonrelativistic dominance: $w = 0$ or some deviations from $w = 1/3$

We study

$$r'' + (3h + \gamma)r' + r = -8\pi\mu^2(1 - 3w)y$$

- non-zero r.h.s. might change the asymptotical exponential decrease of $r$.

Making the same transformation as previously, we find for the curvature scalar:

$$r = r_{hom} + r_{inh}$$

where $r_{hom}$ is a solution of the homogeneous equation:

$$r_{hom} = r_0 \cos(\tau + \theta_r)\, \mathbf{exp}\left[-\frac{\gamma}{2}(\tau - \tau_0) - \frac{3}{2}\int_{\tau_0}^{\tau} d\tau_2 h(\tau_2)\right]$$

and the inhomogeneous part of the solution is:

$$r_{inh} = -8\pi\mu^2(1 - 3w)\int_{\tau_0}^{\tau} d\tau_1 y(\tau_1)\sin(\tau - \tau_1)\exp\left[-\frac{\gamma}{2}(\tau - \tau_1) - \frac{3}{2}\int_{\tau_1}^{\tau} d\tau_2 h(\tau_2)\right]$$

- the solution of the homogeneous equation, $r_h$, drops down exponentially as $e^{-\gamma\tau/2}$
- the inhomogeneous part does not; integral for $r$ is dominated by $\tau_1$ close to $\tau$.

The value of $(1 - 3w)$ is not yet specified here, we only assume that it is nonzero.

# The transition from the modified $R^2$-regime to GR

Assuming $$h(\tau) = \frac{h_1 + h_2 \sin(\tau + \theta_h)}{\tau}, \quad y(\tau) = \frac{y_1}{\tau^\beta}$$

we find the asymptotic solution:

$$r_{inh} \approx \frac{16\pi\mu^2(1-3w)\, y_1}{\tau^\beta} \left[ \left( \frac{\tau}{\tau_0} \right)^{\beta - 3h_1/2} e^{-\gamma(\tau-\tau_0)/2} \cos(\tau - \tau_0) - 1 \right]$$

- $\gamma\tau > 1$: the first term dies down, but the last non-oscillating term survives.
- In this limit the particle production by $R$ vanishes, or strongly drops down.

We have:

$$r = r_{hom} + r_{inh}$$

- $r_{hom}$ decreases as $e^{-\gamma\tau/2}$
- $r_{inh} \sim \mu^2 \ll 1$, but does not drop down exponentially due to the last term in the square brackets

The GR regime is restored when the second term becomes comparable with the exponentially decreasing one.

# The transition from the modified $R^2$-regime to GR

It is natural to expect that the GR regime starts roughly at $\tau \gtrsim 1/\gamma$.

Simple estimate for $w = 0$:

We have to compare the value of the curvature scalar

$$r = 2\mu^2/(9\tau)$$

with homogeneous solution for the curvature:

$$r \sim 4 \exp\left(-\gamma\tau/2\right)/\tau$$

These expressions become comparable at

$$\gamma\tau \approx 2\ln\left(1/\mu^2\right) \sim \ln\left(m_{Pl}/m\right)$$

Similar arguments cannot be applied to $w = 1/3$, because in this case $R_{GR} \equiv 0$.

In realistic case $w$ differs from zero either due to presence of massive particles in the primeval plasma or because of the conformal anomaly.

# Cosmological history in $R^2$-gravity: **4** distinct epoch

# Cosmological history in $R^2$-gravity: 4 distinct epoch

1. An exponential (inflationary) expansion: the universe was void and dark with slowly decreasing curvature scalar $R(t)$. The initial value of $R$ should be quite large, $R > 300m^2$, to ensure sufficiently long inflation ($N_e \geq 70$).

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2. Scalaron dominated epoch: $R$ dropped down and started to oscillate as

$$R \sim m \cos(mt)/t$$

The curvature oscillations resulted in the onset of creation of usual matter, which remains subdominant.

The universe expansion is described by unusual law with the Hubble parameter

$$H = (2/3t)[1 + \sin(mt)]$$

Such a regime was realised asymptotically for large time, $mt \gg 1$, but $\Gamma t \lesssim 1$.

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4. **After this time we arrive to the usual GR cosmology.**

# To be continued...

Unusual cosmological evolution during the time $t < 1/\Gamma$ would lead to:

- noticeable modification of the cosmological baryogenesis scenarios
- variation of the probability of formation of primordial black holes
- change of the frozen density of dark matter particles
- etc..

In particular, it opens window for heavy lightest supersymmetric particles to be the cosmological dark matter (work in progress..)

The END

Thank You for Your Attention