Ground hadron states in composite superconformal string model

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Motivation

- Aim: description of hadron interaction (mesons and baryons) - amplitudes and spectrum.
- Description of mesons and baryons at low and intermediate energies (0,1 - 7 GeV).
  - QCD in this energy region has coupling constant $g \gg 1$.
  - We have to find approach which contains small parameter.
- String theories in tree approximation gives linear Regge trajectories.
- Unitary (string loop) corrections lead to nonlinear Regge trajectories.
- Consistent construction can lead to necessary smallness of these corrections.

Classical string models

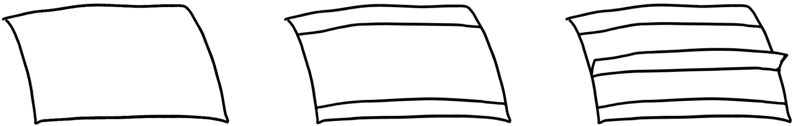
- The absence of ghosts (negative norm states) in physical states spectrum for $\alpha_0 = 1$ in the case of open strings.
- Open string sector contains massless vector particle (gluon).
- Closed string sector contains massless tensor particle (graviton), $\alpha'_{closed} \sim \varkappa$.
- Relation for scale parameters of open and close string:

$$\alpha'_{open} = 2\alpha'_{closed}$$

- Classical string models describe interaction on Planck scale

$$\alpha' \sim \frac{1}{m_{Pl}^2}$$

Composite superconformal string model



- The absence of ghosts (negative norm states) in physical states spectrum for the intercept $\alpha_0 = 1/2$ in the case of open string.
- Possibility to make independent scale parameters α'_{open} and α'_{closed} due to new topology.
- meson momentum:

$$p = \sqrt{\alpha'_H}(k_1 - k_2), \quad \alpha'_H \sim 1 GeV^{-2}$$

- baryon momentum:

$$p = \sqrt{\alpha'_H}(k_1 + k_2) + \sqrt{\alpha'_{Pl}} k_f, \quad \alpha'_{Pl} \sim 10^{-38} GeV^{-2}$$

- Superconformal symmetry on two-dimensional surface.

Formulation of string models

- Functional integration:

$$A_N = \int \prod dz_i \int DX(z) \int Dg(z) e^{S(z)} V_1(z_1) ... V_N(z_N),$$

  where z-coordinate on two-dimensional surface, $X(z)$-fields on two-dimensional surface, $g(z)$-metric on two-dimensional surface.

- For tree approximation it is convenient to use amplitude in VV-formalism:

$$A_N = \int \prod dz_i \langle 0 | V_1(z_1) ... V_N(z_N) | 0 \rangle,$$

  where

$$V_i(z_i) = z_i^{-L_0} V(1) z_i^{L_0}$$

Vertex operator

- Ground state emission vertex operator

$$\hat{V}(z_i) = z_i^{-L_0} \left\{ G, \hat{W} \right\} z_i^{L_0}, \quad \hat{W} \sim : e^{ikX} :$$

where G is the super Virasoro algebra generator for Neveu-Schwarz case.

- super Virasoro algebra:

$$\begin{aligned} [L_n, L_m] &= (n-m)L_{n+m} + \delta_{n,-m} c_1 n(n^2-1), \\ \{G_r, G_s\} &= 2L_{r+s} + c_2 \left(r^2 - \tfrac{1}{4}\right) \delta_{r,-s}, \\ [L_n, G_r] &= \left(\tfrac{n}{2} - r\right) G_{n+r}. \end{aligned}$$

- Additional supercurrent symmetry nilpotent operator Ξ

$$[\hat{V}, \Xi] = 0$$

- Additional supercurrent symmetry leads to $m_\pi = 0$

Supergenerator G, two-dimensional fields

$$
\begin{aligned}
G &= \sum_{\mu=0}^{3} \partial X_\mu H_\mu + \sum_{a=1}^{6} I^a \theta^a + \qquad \text{(on the basic surface)} \\
&+ \sum_{i} (Y_\mu^{(i)} f_\mu^{(i)} + \phi_{(1)}^{(i)} \phi_{(2)}^{(i)} \phi_{(3)}^{(i)}) + \qquad \text{(on the edge surfaces)} \\
&+ \sum_{f} (Y_\mu^{(f)} f_\mu^{(f)} + \phi_{(1)}^{(f)} \phi_{(2)}^{(f)} \phi_{(3)}^{(f)}) \qquad \text{(on the ridge surfaces)}
\end{aligned}
$$

- Lorentz index $\mu = 0...3$.
- Index $a = 1...6$. The I^6 field has hadron scale, $I^1 ... I^5$ fields gives neglectible contribution into tree approximation.
- Linear realization of supersymmetry for $\partial X_\mu H_\mu$, $I^a \theta^a$, $Y_\mu^{(i)} f_\mu^{(i)}$.
- Nonlinear realization of supersymmetry for ϕ fields $[G_r, \phi_{(1)}^{(i)} \phi_{(2)}^{(i)}] = \phi_{(3)}^{(i)}$ and $\{G_r, \phi_{(1)}^{(i)}\} = \phi_{(2)}^{(i)} \phi_{(3)}^{(i)}$

Nucleon vertex operator

- To make possible transition $N\bar{N} \to \pi$ it is necessary to have two types of nucleon vertex operator $V_N = V^{NS} + V^{BH}$:
  $\hat{V}^{NS}_{i,i+1}(z_j) = z_j^{-L_0} \left\{ G_r, \hat{W}_{i,i+1} \right\} z_j^{L_0}$ with odd number of anticommuting field components
  $\hat{V}^{BH}_{i,i+1}(z_j) = z_j^{-L_0} \left\{ G_r, \hat{F}\hat{W}_{i,i+1} \right\} z_j^{L_0}$ with even number of anticommuting field components
- $\left[\left\{ G_r \hat{F} \right\} \hat{W} \right] = 0$
- Conformal spin of vertex operator should be equal to 1.
- Requirement of π-meson to be a state with minimal mass.

Spin-parity strucrure

- Eigenvectors λ_i of zero components I_0^a of fields carry quark quantum numbers - spin and isospin.
- Nucleon isospin-spin structure $TJ^P = \frac{1}{2}\frac{1}{2}^+$:
  $(\tilde{\lambda}_i \lambda_j)\lambda_f$ for $\hat{V}^{BH}$,
  $(\tilde{\lambda}_i \gamma_5 \tau^a \lambda_j)\lambda_f \gamma_5 \tau^a$ for $\hat{V}^{NS}$.
- π-meson structure:
  $$(\bar{\lambda}_k \tau^{(i)} \gamma_5 \lambda_j)$$
- η-meson structure:
  $$(\bar{\lambda}_i \gamma_5 \lambda_j)$$

The zeroth components

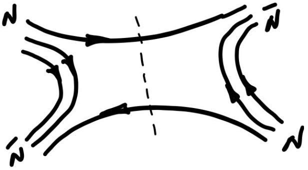
- Two-dimensional fields $Y^{(i)}_{\mu}(z) = \sum_n Y^{(i)}_{n\mu} z^n$
- The eigenvalue of $Y^{(i)}_{0\mu}$ is $\sqrt{\alpha'_H} k^{(i)}_{\mu}$
- The eigenvalue of $Y^{(f)}_{0\mu}$ is $\sqrt{\alpha'_{Pl}} k^{(f)}_{\mu}$
- The eigenvalue of $\partial X_{0\mu}$ is $\sum \sqrt{\alpha'_{Pl}} k^{(f)}_{\mu} + \sum \sqrt{\alpha'_H} k^{(i)}_{\mu}$
- The eigenvalue of I^6_0 is

$$I^6_0 = g_6 \left[\left(\sum_a \left(\sum_i T^{(a)(i)} \right) T^{(a)(f) ridge} \right) + \delta \sum_{a, i \neq j} T^{(a)(i)} T^{(a)(j)} \right],$$

where a is the number of isotopic component,
$T^{(a)(l)} = \frac{1}{2} \bar{\lambda}^{(l)} \tau^a \lambda^{(l)}$ are isotopic generators.

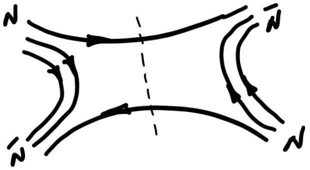
π-meson

- π-meson has spin-parity $J^P = 0^-$, isospin $T = 1$.
- π-meson quantum numbers appear in $\bar{V}_{BH} V_{NS}$ channel.
- Different isospin structure of V_{BH} and V_{NS} leads to different eigenvalues of zero component of field l^6 for $T = 1$ and $T = 0$: $l_0^{isovec} \neq l_0^{isoscal}$.
- On mass shell conditions $L_0 = 1$ lead to following expressions:
  V_{NS}: $\frac{1}{2} - \frac{m_N^2}{2} + \xi^2 + \frac{l_0^{(isovec)2}}{2} = 1$
  V_{BH}: $\frac{1}{2} - \frac{m_N^2}{2} + \xi^2 + \frac{l_0^{(isoscal)2}}{2} = \frac{1}{2}$
  π: $\frac{1}{2} - \frac{m_\pi^2}{2} + \xi^2 + \frac{l_0^{(\pi)2}}{2} = 1$

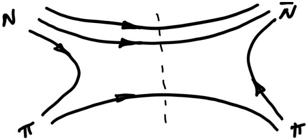


η-meson

- η-meson has spin-parity $J^P = 0^-$, isospin $T = 0$.
- η-meson quantum numbers appear in $\bar{V}_{BH} V_{BH}$ channel.
- The mass conditions $L_0 = 1$ for η-meson and π-meson give us relatoin for their masses:
  $m_\pi^2 + \frac{1}{2} m_\rho^2 = m_\eta^2$.
- We choose $m_N^2 = \frac{3}{2} m_\rho^2$, take into account $m_\pi^2 = 0$ and then get $m_\eta = 544$ MeV.
- For ground states of the first and the second daugter Regge trajectories we predict: 1140 MeV for $\eta(1295)$, 1520 MeV for $\eta(1405)$.



Interaction amplitude of π meson and nucleon

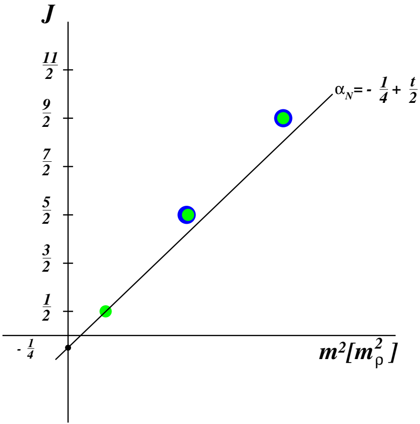


$$A_{\pi N} \sim \frac{\Gamma(\frac{1}{2} - \alpha_{N^+}(t))\Gamma(1-\alpha^{\rho}(s))}{\Gamma(\frac{3}{2} - \alpha_{N^+}(t) - \alpha^{\rho}(s))} + \frac{\Gamma(\frac{3}{2} - \alpha_{\Delta^-}(t))\Gamma(1-\alpha^{\rho}(s))}{\Gamma(\frac{3}{2} - \alpha_{\Delta^-}(t) - \alpha^{\rho}(s))} + \\ + \frac{\Gamma(\frac{3}{2} - \alpha_{\Delta^+}(t))\Gamma(1-\alpha^{\rho}(s))}{\Gamma(\frac{3}{2} - \alpha_{\Delta^+}(t) - \alpha^{\rho}(s))} + \frac{\Gamma(\frac{3}{2} - \alpha_{N^-}(t))\Gamma(1-\alpha^{\rho}(s))}{\Gamma(\frac{3}{2} - \alpha_{N^-}(t) - \alpha^{\rho}(s))}.$$

Where

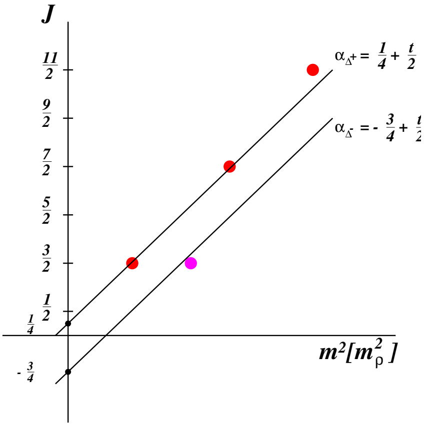
$\alpha_{N^+}(t) = -\frac{1}{4} + \frac{t}{2}$, $\alpha_{N^-}(t) = -\frac{1}{4} + \frac{t}{2}$,
$\alpha_{\Delta^+}(t) = \frac{1}{4} + \frac{t}{2}$, $\alpha_{\Delta^-}(t) = -\frac{3}{4} + \frac{t}{2}$.

N Regge trajectories



- Green dots: $m(\frac{1}{2}^+) = 940\,MeV$, $m(\frac{5}{2}^+) = 1680\,MeV$, $m(\frac{9}{2}^+) = 2250\,MeV$.
- Blue dots: $m(\frac{5}{2}^-) = 1675\,MeV$, $m(\frac{9}{2}^-) = 2220\,MeV$.

Δ Regge trajectories



- Red dots: $m(\frac{3}{2}^+) = 1232\,MeV$, $m(\frac{7}{2}^+) = 1950\,MeV$, $m(\frac{11}{2}^+) = 2420\,MeV$.
- Violet dot: $m(\frac{3}{2}^-) = 1700\,MeV$.

Summary

- We formulate vertex operator for nucleon.
- We formulate wavefunctions of initial ground states.
- We get π and η mesons in $N\bar{N}$ channel.
- Using parameter $m_N^2 = \frac{3}{2} m_\rho^2$ we predict masses of η mesons.
- π N interaction amplitude reproduce nucleon and Δ spectrum properties.

Thank you for attention.