Phase diagram of two-color QCD matter at finite baryon and axial isospin densities

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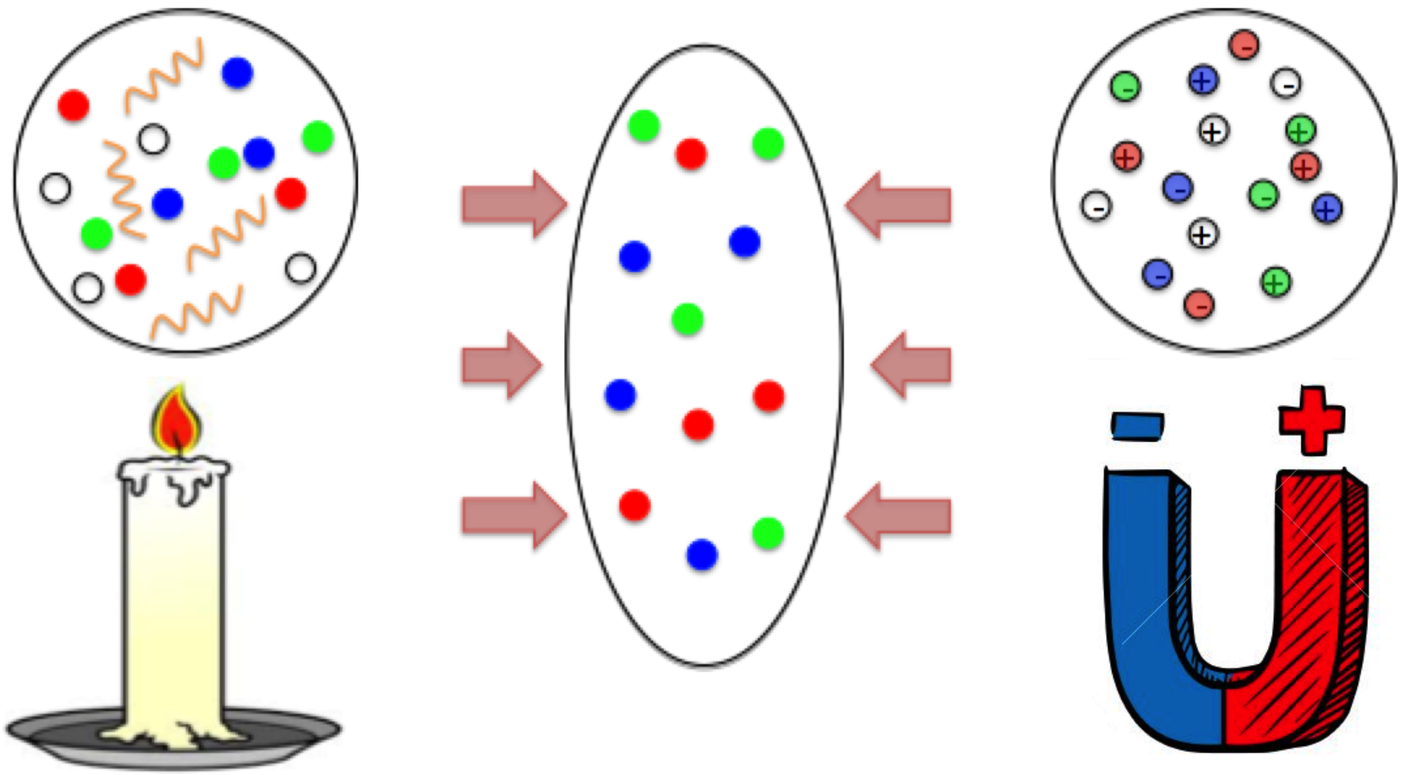
Outline

- Motivations
  - ✓ Different phases of QCD occur in the universe
  - ✓ QCD simplifies in extreme environments
  - ✓ The behaviors of different matter can be similar at the regime of transition
- QCD like theories
- Phase diagram in the plane of $\mu - \nu_5$

Why Electromagnetic Fields

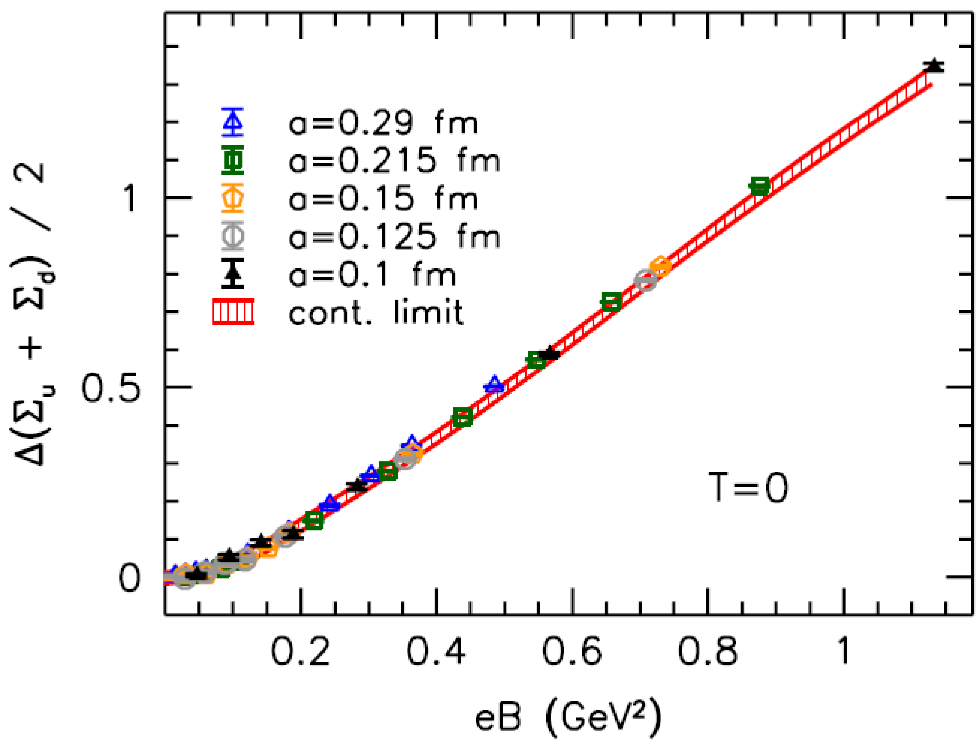
Heavy ion collisions create the strongest magnetic fields in the Laboratory.



Different excited freedoms at different environments

Magnetic Catalysis at Zero Temperature

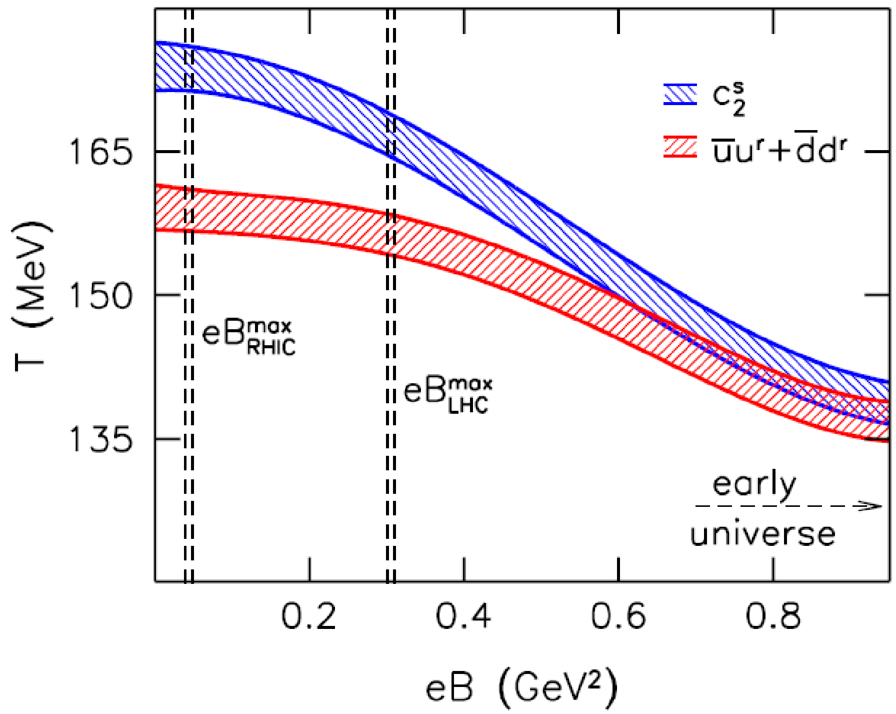
Linear dependence of chiral condensate for large B: c.f. (V. P. Gusynin, V. A. Miransky, and I. A. Shovkovy, Phys. Rev. D 52, 4747)



Lattice data. c.f. (Bali et al., JHEP 1202, 044)

The QCD phase diagram in the $T - B$ plane

Low energy effective models of QCD predict(ed) an increasing critical temperature $T_\chi(B)$, but



T_χ investigated as a function of magnetic field. c.f. (Bali et al., JHEP 1202, 044)

Propagators at Constant Magnetic fields

- spin-0 c.f. (A. Erdas and G. Feldman, Nucl. Phys. B 343, 597)

$$D(k) = \int_0^\infty \frac{ds}{\cos(eBs)} \exp\left[is\left(k_{\parallel}^2 + k_{\perp}^2 \frac{\tan(eBs)}{eBs} - m^2\right)\right]$$

where $k_{\parallel} = (k_0, 0, 0, k_3)$ and $k_{\perp} = (0, k_1, k_2, 0)$.

- spin-$\frac{1}{2}$ c.f. (J. Schwinger, Phys. Rev. 82, 664)

$$\not{S}(k) = \int_0^\infty \frac{ds}{\cos(eBs)} e^{is\left(k_{\parallel}^2 + k_{\perp}^2 \frac{\tan(eBs)}{eBs} - m^2\right)} \left[(m + \not{k}_{\parallel})\, e^{-ieBs\gamma_3} + \frac{\not{k}_{\perp}}{\cos(eBs)}\right]$$

- spin-1 c.f. (A. Erdas and G. Feldman, Nucl. Phys. B 343, 597)

$$G^{\mu\nu}(k) = \int_0^\infty \frac{ds}{\cos(eBs)} e^{is\left(k_{\parallel}^2 + k_{\perp}^2 \frac{\tan(eBs)}{eBs} - m^2\right)} \left[F^{\mu\nu}\frac{\sin(2eBs)}{B} - g_{\parallel}^{\mu\nu} - g_{\perp}^{\mu\nu}\cos(2eBs)\right]$$

where $g_{\parallel}^{\mu\nu} = \mathbf{diag}(+1, 0, 0, -1)$; $g_{\perp}^{\mu\nu} = \mathbf{diag}(0, -1, -1, 0)$ and $F^{\mu\nu}$ is the electromagnetic field tensor. 3 different tensor structures !

Landau Levels

- Another expression of fermion propagator in the B field:

$$S(k) = i \exp\left[-\frac{k_\perp^2}{eB}\right] \sum_{n=0}^{\infty} \frac{(-1)^n D_n(eB, k)}{k_0^2 - k_3^2 - M^2 - 2neB}$$

- Free energy spectrum

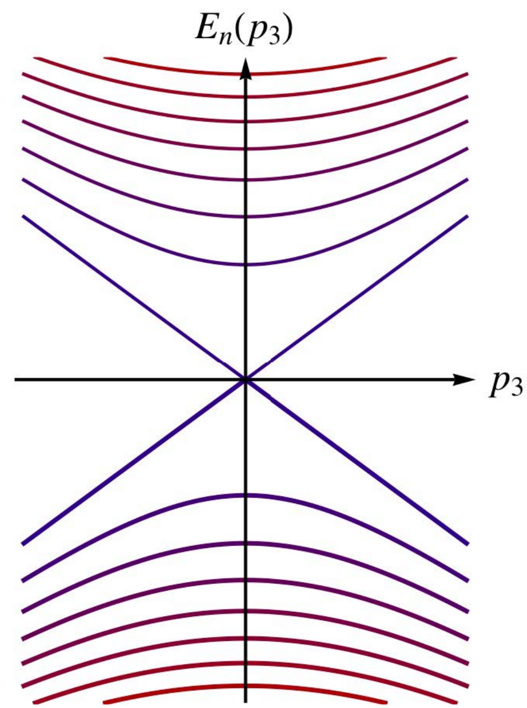
$$E_n^{3+1}(k_3) = \pm\sqrt{(2n + 2s_3 + 1)|eB| + k_3^2 + m^2}$$

where s_3 is the projection of the spin on the B field and $n = 0, 1, 2, \ldots$ is the orbital quantum number.

- Lowest Landau Level assumption of fermions in 3-direction while $B \to \infty$

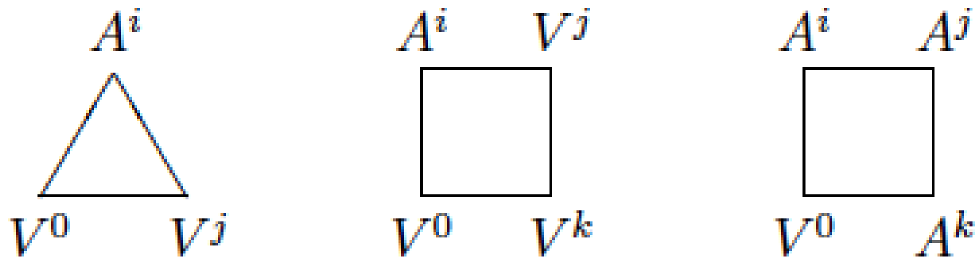
$$E_0^{3+1}(k_3) = \pm|k_3|$$

- Is the effective mass of fermions able to treat as $m_{\text{eff}}^2 = m^2 + 2neB$ at n-th level ?



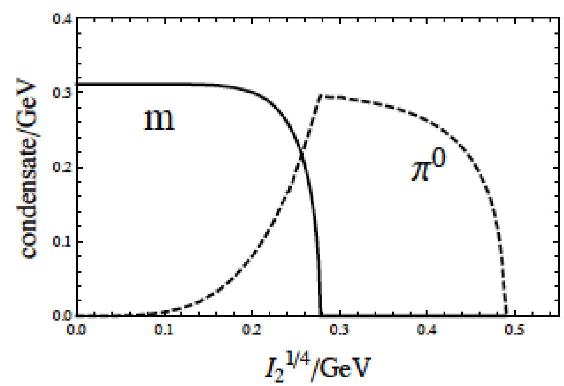
CP Odd Effects in QCD

$$\pi^0 \to \gamma\gamma,\ \pi^- \to e^-\bar{\nu}_e\gamma,\ \gamma\pi^0 \to \pi^+\pi^-$$



Anomalous loop diagrams in two flavor QCD

When $\mathrm{Tr}[\gamma_5 ...] \neq 0$, the pion condensates are allowed due to the odd parity domain being created. c.f. (G. Cao and X. G. Huang, Phys. Lett. B, 757, 1 (2016).)



Equivalent Replacement

How to get a quick physics answer in phenomena?

☞ As one of candidate to explain the inverse magnetic catalysis, QCD phase transition has been intensively studied at finite chiral chemical potential μ_5. c.f. (JC, P. Chu and M. Huang, Phys. Rev. D, 88, 054009 (2013).)

Indeed, nonzero n_5 inducing in electromagnetic field is expressed by: c.f. (K. Fukushima, D. E. Kharzeev and H. J. Warringa, Phys. Rev. D 78, 074033 (2008))

$$\partial_\mu \left(j_5^3\right)^\mu = -\frac{e^2}{16\pi^2}\varepsilon^{\alpha\beta\mu\nu}F_{\alpha\beta}F_{\mu\nu}\cdot \mathrm{tr}\left[\tau^3 Q^2\right]$$

where F is the electromagnetic field strength.

It turns out $\nu_5\bar{\psi}\gamma_0\gamma_5\tau_3\psi$ is added into the Lagrangian because of the coupling of quarks to electromagnetism, i.e., electromagnetic anomaly.

Fermion-Sign Problem

$$D(\mu) = \not{D} + m - \gamma_0 \mu$$

$$\gamma_5 D^\dagger(\mu) \gamma_5 = D(-\mu^*) \Longrightarrow \det D(\mu) = (\det D(-\mu^*))^*$$

Some QCD-like theories (two-color QCD or finite isospin QCD) free of the sign problem, where the analytic continuation can be compared with Monte Carlo determinations obtained directly at real chemical potentials

- Dirac operator has anti-unitary symmetry

  $$[T, D(\mu)] = 0 \Longrightarrow U^{-1} D(\mu) U = D(\mu)^\dagger \quad T^2 = (UK)^2 = \pm 1$$

  two-color QCD $\quad D(\mu)\, t_2 C \gamma_5 = t_2 C \gamma_5 D(\mu)^*$

- QCD with μ_I

  finite isospin QCD $\quad \det D(\mu_I) \cdot \det D(-\mu_I) = |\det D(\mu_I)|^2$

QCD-like Theory in $N_c = 2$

The global symmetries of the $\mathrm{SU}(N_f)$ flavor space can be extended to $\mathrm{SU}(2N_f)$

$$\Psi = \begin{pmatrix} \psi_L \\ \tilde{\psi}_R \end{pmatrix}, \quad \Psi^\dagger = \left(\psi_L^\dagger, \tilde{\psi}_R^\dagger\right)$$

at $N_c = 2$ since it allows to rotate ψ_R^C into ψ_L where $\tilde{\psi}_R = -it_2 C\psi_R^*$ and C is the complex conjugate operator in spinor space.

- Symmetry is established by the connection between quarks and antiquarks
- Color-neutral bound states of two quarks, bosonic baryons
  mesons and scalar diquarks become degenerate!
- Similar phase diagram as for N_c=3
  pion and diquark condensation at finite μ_I and μ_B, respectively

SU(4) Group

It has 10 symmetric and 5 anti-symmetric generators:

$$S_a = \frac{1}{2\sqrt{2}} \begin{pmatrix} \tau_a & 0 \\ 0 & -\tau_a^T \end{pmatrix}, \quad \text{for } a = 0, 1, 2, 3;$$

$$S_a = \frac{1}{2\sqrt{2}} \begin{pmatrix} 0 & B_a \\ B_a^\dagger & 0 \end{pmatrix}, \quad \text{for } a = 4, \ldots, 9$$

with $B_{(4,5)} = i^{(0,1)}\tau_0$, $B_{(6,7)} = i^{(0,1)}\tau_3$ and $B_{(8,9)} = i^{(0,1)}\tau_1$.

$$X_i = \frac{1}{2\sqrt{2}} \begin{pmatrix} \tau_i & 0 \\ 0 & \tau_i^T \end{pmatrix}, \quad \text{for } i = 1, 2, 3;$$

$$X_i = \frac{1}{2\sqrt{2}} \begin{pmatrix} 0 & D_i \\ D_i^\dagger & 0 \end{pmatrix}, \quad \text{for } i = 4, 5$$

with $D_{(4,5)} = i^{(0,1)}\tau_2$.

Patterns of symmetry breaking

$$\mathcal{L}_{\text{mass}} = \frac{m_0}{2}\left(\Psi^T i t_2 C E_4 \Psi - \Psi^{*T} i t_2 C E_4 \Psi^*\right)$$

The mass operator

$$E_4 = \begin{pmatrix} 0 & \tau_0 \\ -\tau_0 & 0 \end{pmatrix}$$

obey the relation $S_a^T E_4 + E_4 S_a = 0$ for $S^a \in \text{Sp}(2)$

$$\text{SU}(4) \to \text{Sp}(2) \quad \text{at } m \neq 0$$

The chemical potential μ is introduce by $\mu \Psi^\dagger B_0 \Psi$ via the $\mathcal{L}_{\text{kin}} = \Psi^\dagger i \sigma^\mu D_\mu \Psi$

$$B_0 = -\gamma_0 E_4 = \begin{pmatrix} \tau_0 & 0 \\ 0 & -\tau_0 \end{pmatrix}$$

breaks

$$\text{SU}(4) \to \text{SU}_L(2) \times \text{SU}_R(2) \quad \text{at } \mu \neq 0$$

Incorporating External Fields in ChPT

- Including external gauge fields in QCD

$$\mathcal{L}_\psi = \bar{\psi}_L \not{D}_L \psi_L + \bar{\psi}_R \not{D}_R \psi_R$$

$$(D_L)_\mu = \partial_\mu + ig\mathcal{A}_\mu + iL_\mu \quad (D_R)_\mu = \partial_\mu + ig\mathcal{A}_\mu + iR_\mu$$

Covariant derivative is required to compensate the gauge transformation of $\psi_L \to L(x)\psi_L$ and $\psi_R \to R(x)\psi_R$

- Incorporate external gauge fields in ChPT

$$\Sigma \to L(x)\Sigma R^\dagger(x)$$
$$D_\mu\Sigma = \partial_\mu\Sigma + iL_\mu\Sigma - i\Sigma R_\mu^\dagger$$

Bilinear fields Σ are modified to keep local invariance after transformation

Chiral Lagrangian and Its Predictions

The isospin density, $\nu\,\bar{\psi}\gamma_0\tau_3\psi$, is written as $\nu\,\Psi^\dagger I_0 \Psi$ with $I_0 = \mathrm{Diag}(\tau_3, -\tau_3)$. Applying the gauge transformation

$$\Psi \to V\,\Psi, \quad V = \exp\left(i\theta^i X_i\right)$$

The Lagrangian term of leading order in ChPT is shown as

$$\mathcal{L}_{\chi\mathrm{PT}} = \frac{f_\pi^2}{4}\mathrm{Tr}\left(D_\mu\Sigma\right)^\dagger\left(D_\mu\Sigma\right) - c\,\mathrm{Tr}\left(\Sigma^\dagger + \Sigma\right),$$

The isospin baryon current is embedded in $\pi_\pm$ mesons

$$\mathcal{L}_{\chi\mathrm{PT}} = \mathcal{L}_{\pi_i} \sim \mathrm{Tr}\left[\tau_3\nu, \Sigma\right]\left[\Sigma^\dagger, -\tau_3\nu\right] \sim +\nu^2\mathrm{Tr}\left(\tau_3\Sigma\tau_3\Sigma^\dagger\right),$$

one has $[X_i, I_0] \neq 0$ for $i = 1, 2$.

Chiral Lagrangian and Its Predictions, cont

Similarly, one has $[X_i, B_0]$ becoming nonzero for $i = 4, 5$ and

$$\mathcal{L}_{\chi\text{PT}} = \mathcal{L}_{\Delta,\Delta^*} \sim \text{Tr}\left\{\tau_0\mu, \Sigma\right\}\left\{\Sigma^\dagger, -\tau_0\mu\right\} \sim -\mu^2\text{Tr}\left(\Sigma\Sigma^\dagger\right)$$

The different commutation bracket is corresponding to different composition

It predicts that $\langle\bar{q}q\rangle \neq 0$ and $\langle qq\rangle \neq 0$ for $\mu \geq \frac{m_\pi}{2}$.

$\nu_5\bar{\psi}\gamma_0\gamma_5\tau_3\psi$ is expressed as $\nu_5\Psi^\dagger I_5\Psi$ with $I_5 = \gamma_5 I_0 = \text{Diag}(\tau_3, \tau_3)$ where $[X_i, I_5] \neq 0$ for $i = 1, 2, 4, 5$ and $[S_a, I_5] \neq 0$ for $a = 1, 2, 8, 9$. Therefore,

$$\mathcal{L}_{\chi\text{PT}} = \mathcal{L}_{\pi_i} \sim \text{Tr}\left[\tau_3\nu_5, \Sigma\right]\left[\Sigma^\dagger, \tau_3\nu_5\right] \sim -\nu_5^2\text{Tr}\left(\tau_3\Sigma\tau_3\Sigma^\dagger\right)$$

$$\mathcal{L}_{\chi\text{PT}} = \mathcal{L}_{\Delta,d} \sim \text{Tr}\left\{\tau_3\nu_5, \Sigma\right\}\left\{\Sigma^\dagger, \tau_3\nu_5\right\} \sim +\nu_5^2\text{Tr}\left(\tau_3\Sigma\tau_3\Sigma^\dagger\right)$$

Diquarks are energy favored and $\text{Sp}(2)$ reduce to $\text{O}(4)$ at high baryon density

NJL Model at $N_c = 2$ w.r.t. Diquarks

In terms of Nambu-Gorkov bispinors

$$\psi = \frac{1}{\sqrt{2}} \begin{pmatrix} q \\ C\bar{q}^T \end{pmatrix}, \quad \bar{\psi} = \frac{1}{\sqrt{2}} \left(\bar{q}, q^T C\right),$$

$$\nu_5 = \mu_L^u - \mu_R^u = \mu_R^d - \mu_L^d \Rightarrow \left(\psi_L^u \psi_R^d + \psi_R^u \psi_L^d\right)$$

τ_1 sector locates in the flavor symmetric channel and spin direction of $s = 1, s_z = 0$ is selected due to the Pauli principle.

$$\mathcal{L}_{\rm NJL} = \bar{\psi}\left(i\gamma^\mu \partial_\mu + \mu\gamma_0 + \nu_5\gamma_0\gamma_5\tau_3 - m_0\right)\psi + \mathcal{L}_{\bar{q}q} + \mathcal{L}_{qq}$$

$$\mathcal{L}_{\bar{q}q} = \frac{G}{2}\left[(\bar{\psi}\psi)^2 + (\bar{\psi}\, i\gamma_5 \vec{\tau}\psi)^2\right]$$

$$\mathcal{L}_{qq} = \frac{H}{2}(\bar{\psi}\, i\gamma_5\tau_2 t_2 C\bar{\psi}^T)(\psi^T C\, i\gamma_5\tau_2 t_2\psi) - \frac{H}{4}(\bar{\psi}\gamma_3\tau_1 t_2 C\bar{\psi}^T)(\psi^T C\gamma_3\tau_1 t_2\psi)$$

Energy Dispersion

Applying $\pi_3 = -\frac{G}{2}\langle\bar{\psi}\, i\gamma_5\tau_3\psi\rangle$, $\Delta = -\frac{H}{2}\langle\psi^T\, i\gamma_5\tau_2 t_2 C\psi\rangle$, $\Delta^* = -\frac{H}{2}\langle\bar{\psi}\, i\gamma_5\tau_2 t_2 C\bar{\psi}^T\rangle$, $d = -\frac{H}{4}\langle\psi^T\gamma_3\tau_1 t_2 C\psi\rangle$ and $d^* = -\frac{H}{4}\langle\bar{\psi}\gamma_3\tau_1 t_2 C\bar{\psi}^T\rangle$,

$$\Omega_{\mathrm{eff}} = \frac{1}{2}\bar{\psi}\mathcal{S}^{-1}\left(p;\pi_3,\Delta,d\right)\psi - \frac{\pi_3^2 + |\Delta|^2 + |d|^2}{4G}.$$

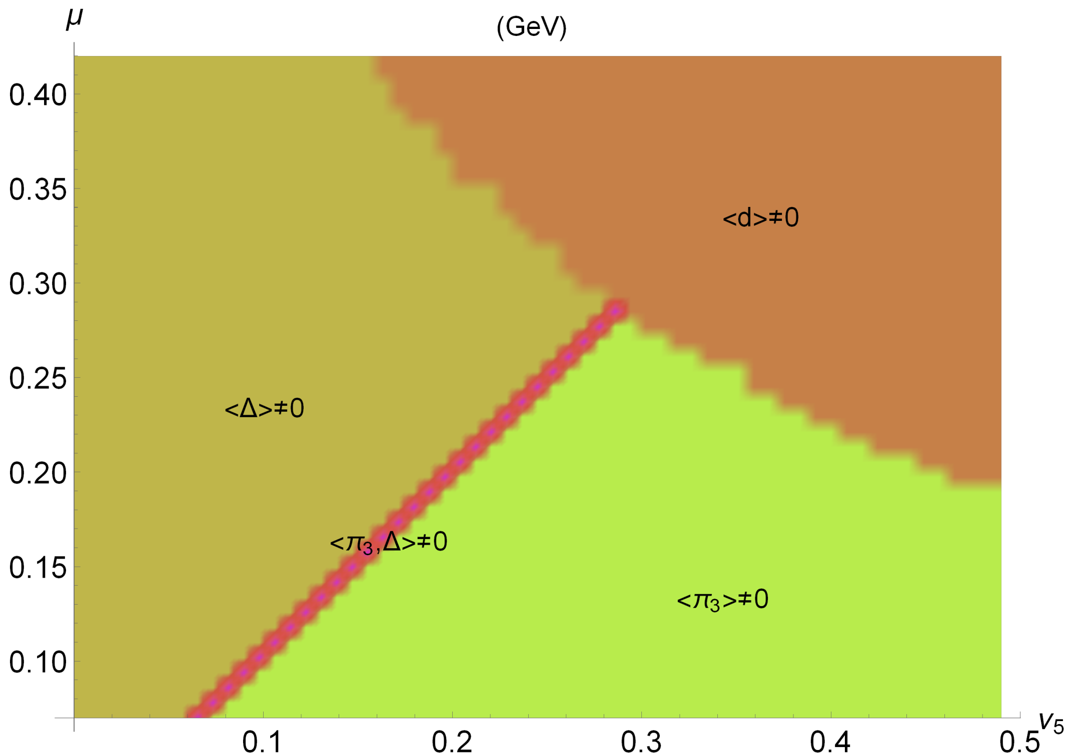
Here, the inverse propagator of fermion is

$$\mathcal{S}^{-1}(p) = \begin{pmatrix} \not{p} - M + \gamma_0\mu + \gamma_0\gamma_5\tau_3\nu_5 & \gamma_5\tau_2\Delta + \gamma_3\tau_1 d \\ -\gamma_5\tau_2\Delta^* - \gamma_3\tau_1 d^* & \not{p} - M^T - \gamma_0\mu + \gamma_0\gamma_5\tau_3\nu_5 \end{pmatrix}$$

The energy dispersion in the chiral limit with $\pi_3 = \Delta = \Delta^* = 0$ for real d

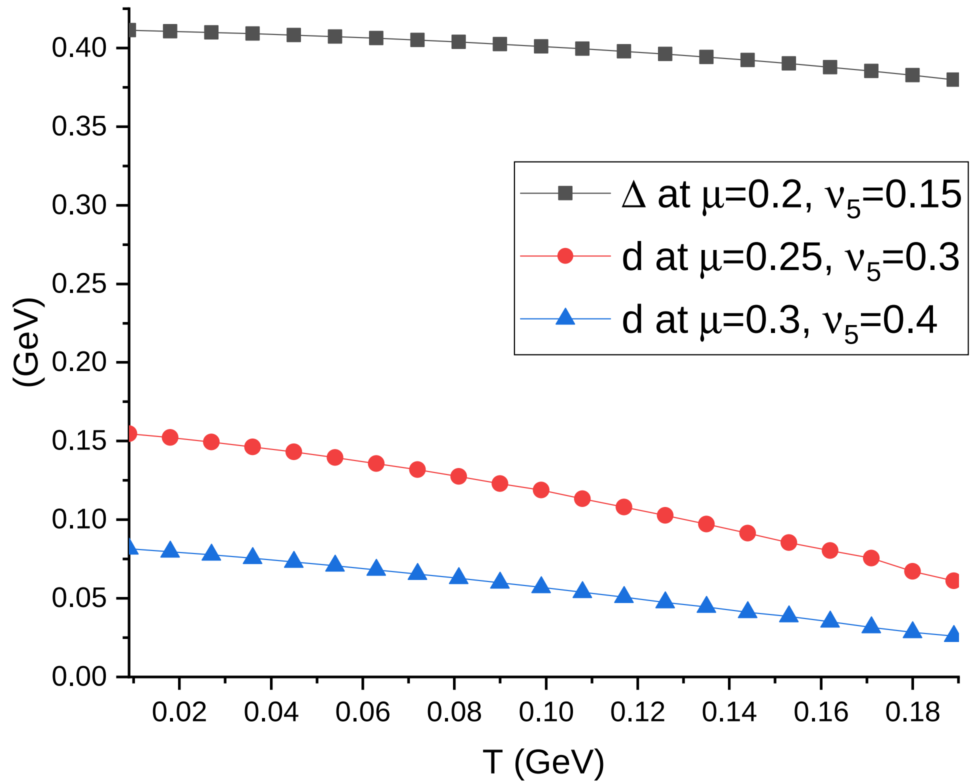
$$E_i(p) = \pm\sqrt{d^2 + \mathbf{p}^2 + (\mu \pm \nu_5)^2 \pm 2\sqrt{d^2 p_z^2 + \mathbf{p}^2\left(\mu \pm \nu_5\right)^2}}.$$

Phase Diagram in QC_2D



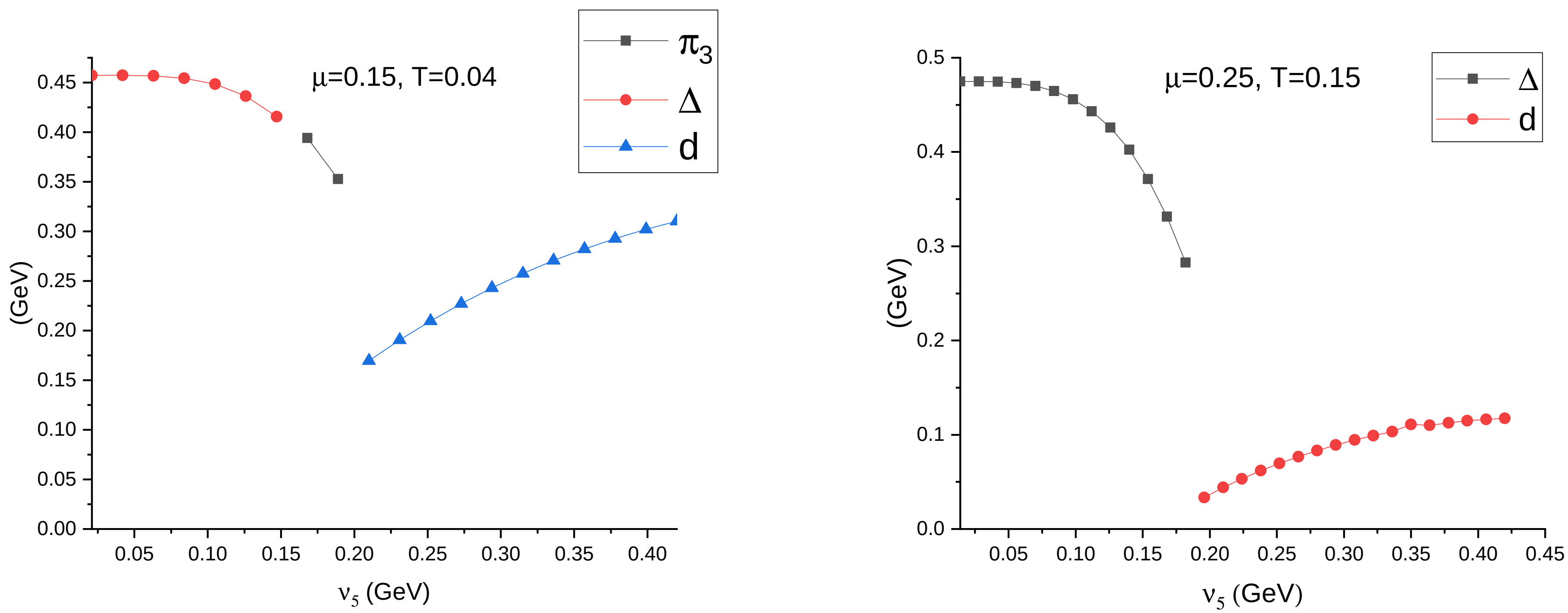
Phase diagram of two-color QCD within two fundamental quarks in the $\mu - \nu_5$ plane. It contains four different states: superfluid pion π_3, condenses of scalar diquark Δ and axial vector diquark d and a mixed state composed by π_3 and Δ. c.f. (JC, arXiv:1808.01928)

Results at Finite Temperature (I)



Scalar diquark condensate Δ and axial vector diquark d as a function of T at given μ and ν_5. The units of μ and T are GeV. c.f. (JC, arXiv:1808.01928)

Results at Finite Temperature (II)



Neutral pion condensate π_3, scalar diquark condensate Δ and axial vector diquark d as a function of ν_5 at given μ and T. The units of μ and T are GeV. c.f. (JC, arXiv:1808.01928)

Constraints in Hybrid Star from Modern Cooling Data

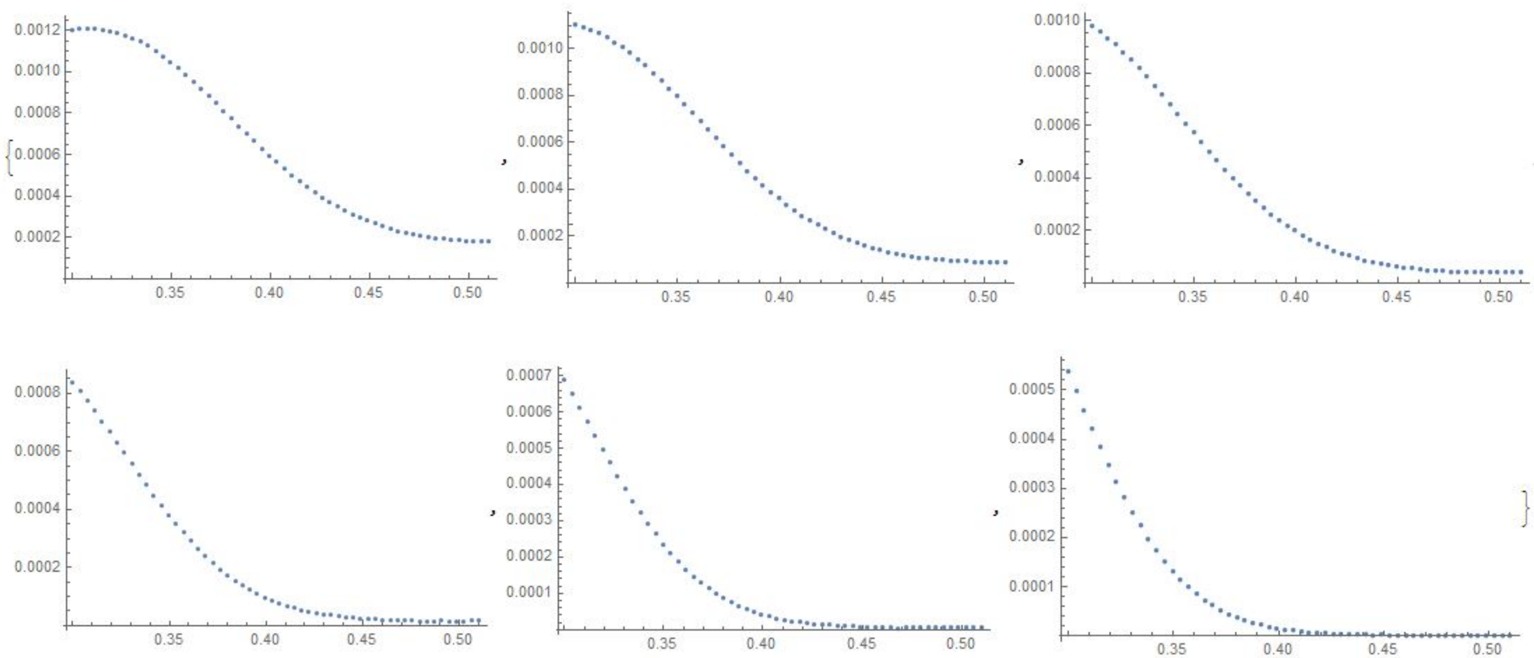
Emission of neutrino processed is applied in quark matter for cooling the star

- All quarks need to be paired
- The smallest gaps should be in the range $10 \sim 100\,\mathrm{keV}$
- The smallest gaps should have a decreasing density dependence in the relevant domain of $\mu_{\mathrm{crit}} \leq \mu \leq 0.5\mathrm{GeV}$

c.f. (H. Grigorian, D. Blaschke and D. Voskresensky, Phys. Rev. C 71, 045801)

Fitted Cooling Behaviors at $N_c = 3$

A linear combination of colors, $\lambda_{A=2,5,7}$ locking with spin, $(C\gamma_{i=1,2,3})$, it remains an unbroken global SO(3) of mixture and the gap is isotropic



The decreasing gap of spin one diquark d in the range of $\mu \in (0.35, 0.5)$ GeV for $\nu_5 = 0.25 \pm 0.025$ GeV. c.f. (JC and C. Zhou, Preliminary)

Summary

- Explore the new phases for QCD matter after turning on $\mathbf{E} \cdot \mathbf{B}$.
- Take home message: the possible cooling mechanism in neutron stars.
- Include Polyakov loop dynamics.

Thank You for Your Attention!