

# Radiative corrections to quasi-elastic neutrino-nucleon scattering

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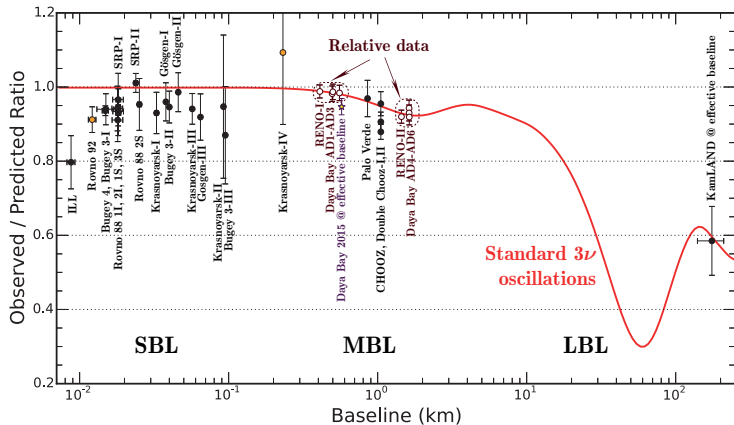
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- Motivation
- Types of radiative corrections
- Magnitude of QED corrections
- Example: RC to  $\nu$  DIS
- Preliminary results for LLA corrections
- RC to muon decay spectrum
- Remarks on higher-order corrections
- Estimates of theoretical uncertainties
- Outlook

# Motivation

- The continuously increasing experimental precision due to higher statistics, modern detectors, and analysis techniques
- The general importance of the neutrino physics for understanding of the SM and going beyond it
- The progress in the methods of RC calculations
- Development of high-power computer tools (Monte-Carlo generators etc.)

# Reactor antineutrino anomaly



The average ratio  $\mu = 0.943 \pm 0.023$  is significantly below 1  
 Figure from [D.V.Naumov, V.A.Naumov, D.S.Shkirmanov, Phys. Part. Nucl. '2017]

# CERN Neutrino Platform (CENF)

The **CERN Neutrino Platform** is CERN's undertaking to foster and contribute to fundamental research in neutrino physics at particle accelerators worldwide, as recommended by the 2013 European Strategy for Particle Physics.



- Assist the various groups in their R&D phase (detectors and components) in the short and medium term and give coherence to a fragmented European Neutrino Community
- Provide to the community a test beam infrastructure (charged particles)
- Bring R&D at the level of technology demonstrators in view of major technical decisions
- Continue R&D on beam, as a possible base for further collaborations
- Support the short baseline activities (infrastructure & detectors)
- Support the long baseline activities (infrastructure & detectors)

# CERN Neutrino Platform (CENF)



Involvement of CERN NP in current and future LBN experiments  
Figure from [S. Bordini, PoS(EPS-HEP2017)483]

# Types of Radiative Corrections (RC)

## Different classifications:

- 1) QED, QCD, (electro)weak, mixed, ...
- 2) improved Born approximation, one-loop, leading logs, higher orders etc.
- 3) leading order, next-to-leading order, NNLO, ...
- 4) perturbative, re-summed, non-perturbative, ...
- 5) virtual (loop) RC, soft photon emission, hard Bremsstrahlung, light pair creation, vacuum polarization, ...

For quasi-elastic  $\nu$  scattering on nuclei at  $E_\nu \sim \text{GeV}$  we need **at first**

- nonperturbative effects of nucleon form factors
- one-loop QED corrections

# Magnitude of QED RC

We have several small and large parameters in expansions:

- $\alpha/(2\pi) \approx 0.001$
- $(\alpha/(2\pi))^2 \approx 10^{-6}$
- $L \equiv \ln(E_e^2/m_e^2) \approx 16$  the **large log** for  $E_e = 1$  GeV
- $(m_e^2/E_\nu^2) \ll 1$ , but for  $\mu$  and  $\tau$  ...
- there can be other enhancement and suppression factors due to concrete experimental conditions

Knowing the **experimental precision tag** is crucial. E.g. for **1%** precision tag we need to control all effects of the order of **a few permille**



# Example: RC to $\nu$ DIS (I)

$$\nu_\mu + N \longrightarrow \nu_\mu(\mu) + X$$

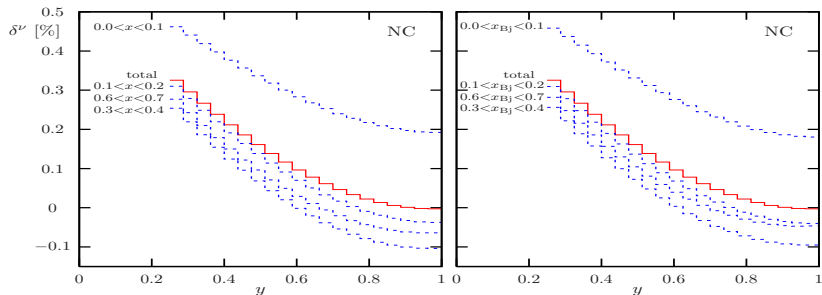
$$R^\nu = \frac{\sigma_{NC}^\nu(\nu_\mu N \rightarrow \nu_\mu X)}{\sigma_{CC}^\nu(\nu_\mu N \rightarrow \mu^- X)}$$

A.B. Arbuzov, D.Yu. Bardin, L.V. Kalinovskaya, [Radiative Corrections to Neutrino Deep Inelastic Scattering Revisited](#), JHEP 06 (2005) 078

Calculation was done for the **NOMAD** experiment motivated by the **NuTeV anomaly** in the NC/CC ratio

# Example: RC to $\nu$ DIS (II)

Size of RC to **neutral current** DIS

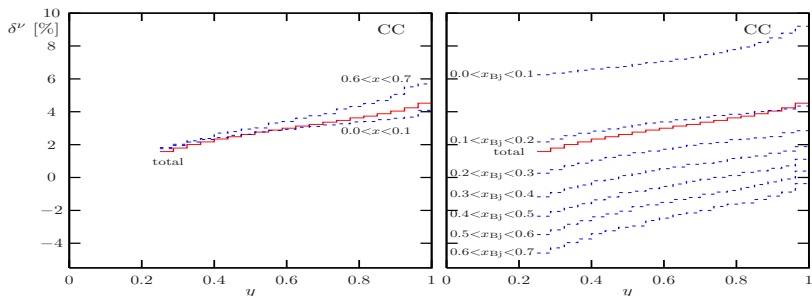


$$p_q^\mu = x P_N^\mu, \quad x_{Bj} = \frac{-(p_l - k_l)^2}{2M_N E_{\text{had+phot}}^{\text{LAB}}}$$

**N.B.** No large logs here

# Example: RC to $\nu$ DIS (III)

Size of RC to **charged current** DIS



**N.B.** Large logs dominate here:  $(\alpha/\pi) \ln(Q^2/m_\mu^2)$

# Preliminary results for LLA RC (I)

**N.B.** At the very beginning DGLAP evolution equations were first written for (scalar) QED. Later on they were extensively exploited in QCD. But **QED DGLAP** are very useful also to estimate the leading part of one-loop and higher-order QED radiative corrections.

LLA (LO) QED DGLAP:

[1] E.A.Kuraev and V.S.Fadin, **On Radiative Corrections to  $e^+ e^-$  Single Photon Annihilation at High-Energy**, Sov. J. Nucl. Phys. **41** (1985) 466 (758 citations)

[2] A.De Rujula, R.Petronzio, A.Savoy-Navarro, **Radiative Corrections to High-Energy Neutrino Scattering**, Nucl. Phys. B **154** (1979) 394 (177 citations)

NLO QED DGLAP:

[3] F.A.Berends, W.L. van Neerven, G.J.H.Burgers, **Higher Order Radiative Corrections at LEP Energies**, Nucl. Phys. B **297** (1988) 429

[4] A.Arbuzov, K.Melnikov,  **$O(\alpha^2 \ln(m_\mu/m_e))$  corrections to electron energy spectrum in muon decay**, Phys. Rev. D **66** (2002) 093003.

# Preliminary results for LLA RC (II)

The **master formula** for  $\nu n \rightarrow ep$  in the NLO approximation:

$$d\sigma(Q^2) = \int_{\bar{z}}^1 \frac{dz}{z} \left( d\sigma^{(0)} \left( \frac{Q^2}{z} \right) + d\bar{\sigma}^{(1)} \left( \frac{Q^2}{z} \right) + \mathcal{O}(\alpha^2 L^0) \right) \mathcal{D}_{ee}^{\text{frg}}(z)$$

where  $d\bar{\sigma}^{(1)}$  is  $\mathcal{O}(\alpha)$  correction with “massless electron” in  $\overline{\text{MS}}$  scheme

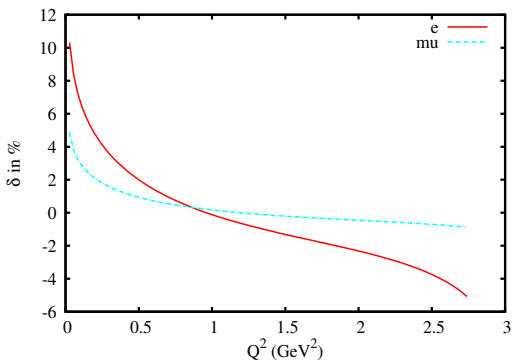
$\mathcal{D}_{ee}^{\text{frg}}(z)$  is the QED fragmentation function  $e \rightarrow e$

$$\begin{aligned} \mathcal{D}_{ee}^{\text{frg}}(z) &= \delta(1-z) + \frac{\alpha}{2\pi} \left( \ln \frac{\Lambda^2}{m_e^2} - 1 \right) \left[ \frac{1+z^2}{1-z} \right]_+ \\ &+ \frac{1}{2} \left( \frac{\alpha}{2\pi} \right)^2 \left( \ln \frac{\Lambda^2}{m_e^2} - 1 \right)^2 P^{\otimes 2}(z) + \dots \end{aligned}$$

where  $\Lambda$  is the **factorization scale**,  $\Lambda \sim E_e$

# Preliminary results for LLA RC (III)

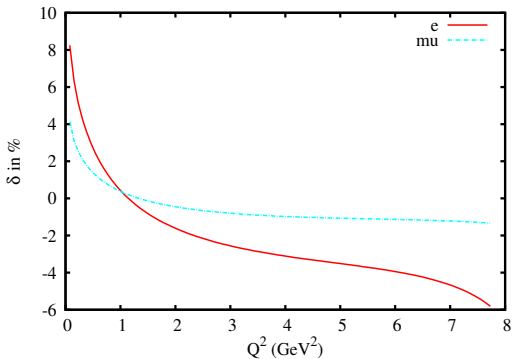
$$E_\nu = 2 \text{ GeV} \quad \delta = \left[ \frac{d\sigma^{\text{corrected}}}{d\sigma^{\text{Born}}} - 1 \right] \cdot 100\%$$



A simplified set of nucleon form factors was used for numerical calculations. Which one should be taken?

# Preliminary results for LLA RC (IV)

$$E_\nu = 5 \text{ GeV} \quad \delta = \left[ \frac{d\sigma^{\text{corrected}}}{d\sigma^{\text{Born}}} - 1 \right] \cdot 100\%$$



# RC to muon decay spectrum

Nuclear beta decays yield  $E_\nu \sim$  a few MeV. So effects of the order  $m_e/E_\nu$  might be important even in  $\mathcal{O}(\alpha)$

For the muon neutrino case  $m_\mu/E_\nu$  is typically not small at all

N.B. In most cases we get **mass effects**  $\sim m_e^2/E_\nu^2$

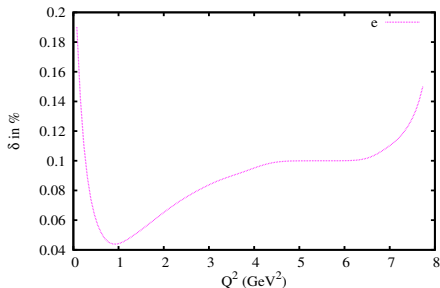
Complete one-loop QED corrections with exact electron mass dependence to **muon decay spectrum** were computed first in [A.A. PLB' 2002]. Those results can be transformed into corrections to neutrino quasi-elastic scattering

$$\mu \rightarrow e + \bar{\nu}_e + \nu_\mu (\gamma) \longleftrightarrow \nu_e + n \rightarrow e + p (\gamma)$$

The work is in progress



# Remarks on higher-order corrections



In higher orders we should estimate first the leading log effects of the order  $\alpha^2 \ln(E^2/m_e^2)$ . That is easy both for photonic and  $e^+e^-$  pair corrections

Typically,  $\mathcal{O}(\alpha^2)$  pair RC are a few times less than  $\mathcal{O}(\alpha^2)$  photonic ones, see e.g. [A.A. JHEP'2001](#)

# Kinoshita–Lee–Nauenberg theorem

The large logs are singular if  $m_e \rightarrow 0$ . They are so-called **mass singularities**. But for **sufficiently inclusive** observables such mass singularities should cancel out in accord with the **Kinoshita–Lee–Nauenberg theorem**.

In practice (roughly speaking), the condition for the cancellation is the **calorimetric** registration of charged particles together with collinear photons which accompany them.

Another important principle for radiative corrections:

**the more you cut — the more you get**

i.e. corrections tend to increase for tight experimental cuts or for “very exclusive” observables.

# Estimates of theoretical uncertainties

For leading logs like in QCD we can estimate the uncertainty by variation of the energy scale under the large log:  $L \rightarrow L \pm \ln(2)$ .

But further reduction of the uncertainty will be (soon) reached by having the **complete one-loop** result plus leading-log higher order RC

**Problem:** form factors extracted from experiments might (in fact do) include a part of QED radiative corrections, if the RC have not been properly treated in the data analysis.

# Outlook

1. Nowadays radiative corrections became **relevant** for many neutrino experiments
2. The size of RC crucially depend on the experimental set-up. There can't be a ready-to-use result for all cases
3. **Leading log** results for quasi-elastic  $\nu_e$  and  $\nu_\mu$  scattering are presented
4. **Double counting** between QED RC and nucleon form factors should be excluded
5. **Complete one-loop** QED RC will be presented soon
6. Mass effects should be treated with care
7. Regions of **max** and **min**  $Q^2$  deserve special treatment
8. Implementation of RC into a **Monte-Carlo** code have to be discussed