The S_{E1} factor of radiative α capture on ^{12}C in cluster EFT

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Outline

- Introduction: ^{12}C$(\alpha,\gamma)^{16}$O process in the stars
- EFT for the ^{12}C$(\alpha,\gamma)^{16}$O process at low energies
- Numerical results
- Results and discussion

1. Introduction

- 90% of human body consists of ^{12}C and ^{16}O.
- ^{12}C and ^{16}O are synthesized during helium burning process in the stars.
- ^{12}C/^{16}O ratio in the universe is mostly determined by the ^{12}C(α,γ)^{16}O process.
- Meanwhile about 20% uncertainty of S-factors of the ^{12}C(α,γ)^{16}O process (NACRE-II) exists after more than a half century long intensive studies for the process.

The main goal is to determine S_{E1}-factor for the process with 5-10% theoretical uncertainty in the future studies.

^{12}C(α,γ)^{16}O process

- Level diagram of ^{16}O

260 L.R. Buchmann, C.A. Barnes / Nuclear Physics A 777 (2006) 254–290

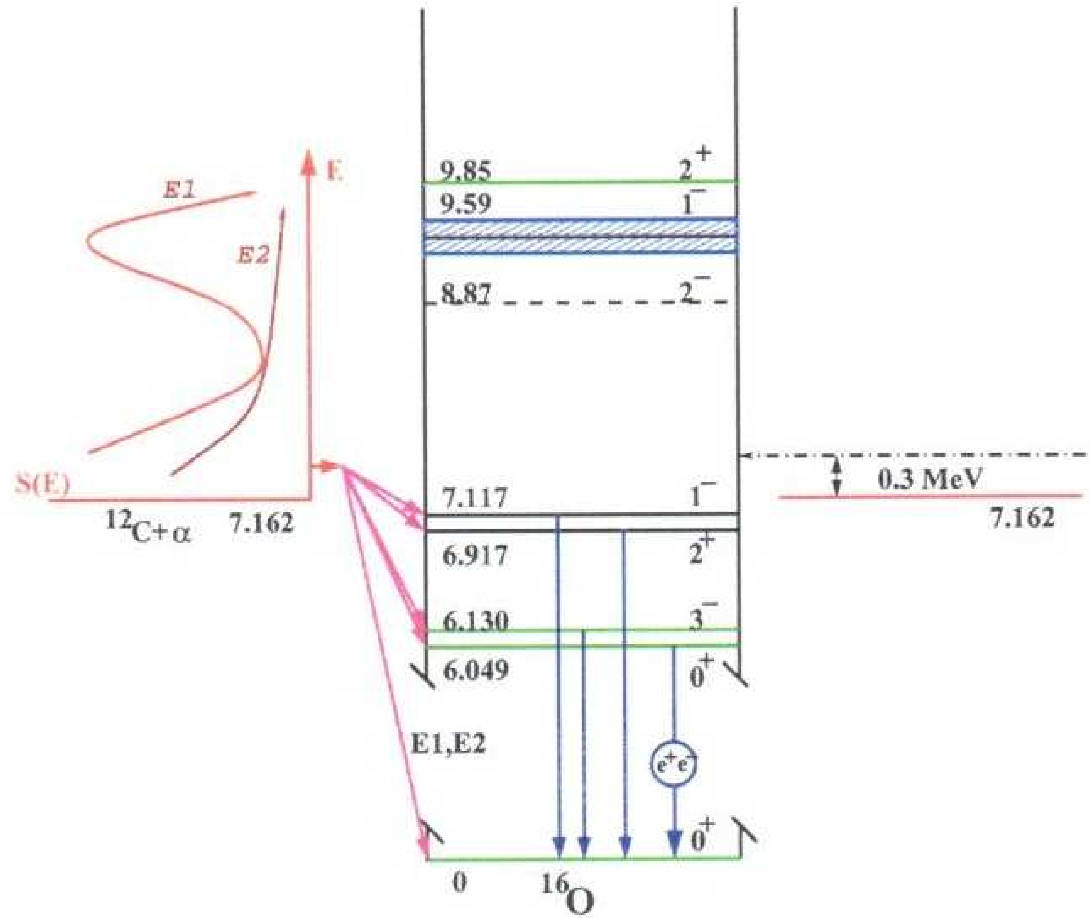


Fig. 1. ^{16}O states relevant to the ^{12}C(α,γ)^{16}O reaction.

- Effective Field Theories (EFTs)
  - Model independent approach
  - Separation scale
  - Counting rules
  - Parameters should be fixed by experiments

2. ^{12}C(α, γ)^{16}O in EFT

- Typical momentum of the process
  at $T_G \simeq 0.3$ MeV; $Q \sim \sqrt{2\mu T_G} \sim 40$ MeV
  The α and ^{12}C states; elementary-like states
- Separation (large) scale
  Excited energies of α and ^{12}C; large scale
  The large momentum scale, $\Lambda_H \sim 150$ MeV
- Expansion parameter, $Q/\Lambda_H \sim 1/3$

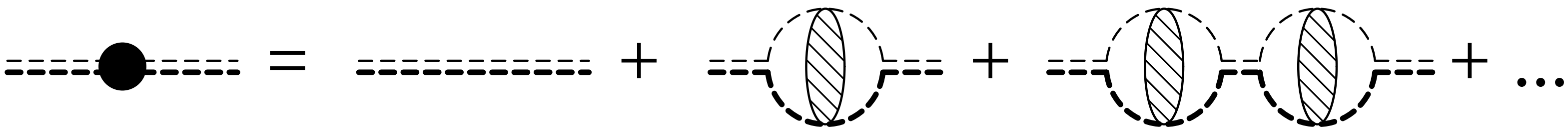
- Effective Lagrangian

$$\mathcal{L} = \phi_\alpha^\dagger \left(iD_0 + \frac{\vec{D}^2}{2m_\alpha} + \cdots \right) \phi_\alpha + \phi_C^\dagger \left(iD_0 + \frac{\vec{D}^2}{2m_C} + \cdots \right) \phi_C$$
$$+ \sum_{n=0}^{3} C_n^{(0)} d^\dagger \left[iD_0 + \frac{\vec{D}^2}{2(m_\alpha + m_C)} \right]^n d - y^{(0)} \left[d^\dagger (\phi_\alpha \phi_C) + (\phi_\alpha \phi_C)^\dagger d \right]$$
$$+ \sum_{n=0}^{3} C_n^{(1)} d_i^\dagger \left[iD_0 + \frac{\vec{D}^2}{2(m_\alpha + m_C)} \right]^n d_i - y^{(1)} \left[d_i^\dagger (\phi_\alpha O_i^{(1)} \phi_C) + (\phi_\alpha O_i^{(1)} \phi_C)^\dagger d_i \right]$$
$$-h^{(1)} \frac{y^{(0)} y^{(1)}}{\mu} \left[(\mathcal{O}_i^{(1)} d)^\dagger d_i + \text{H.c.} \right] + \cdots ,$$

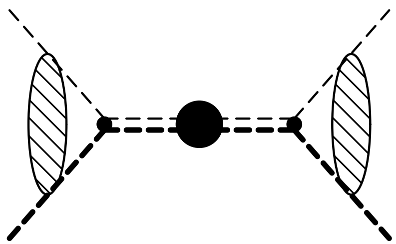
with

$$O_l^{(1)} = i \left(\frac{\overrightarrow{D}_C}{m_C} - \frac{\overleftarrow{D}_\alpha}{m_\alpha} \right)_i , \quad \mathcal{O}_i^{(1)} = \frac{iD_i}{m_O} ,$$
$$h^{(1)} = g^{(1)} + p g^{(1)'} ,$$

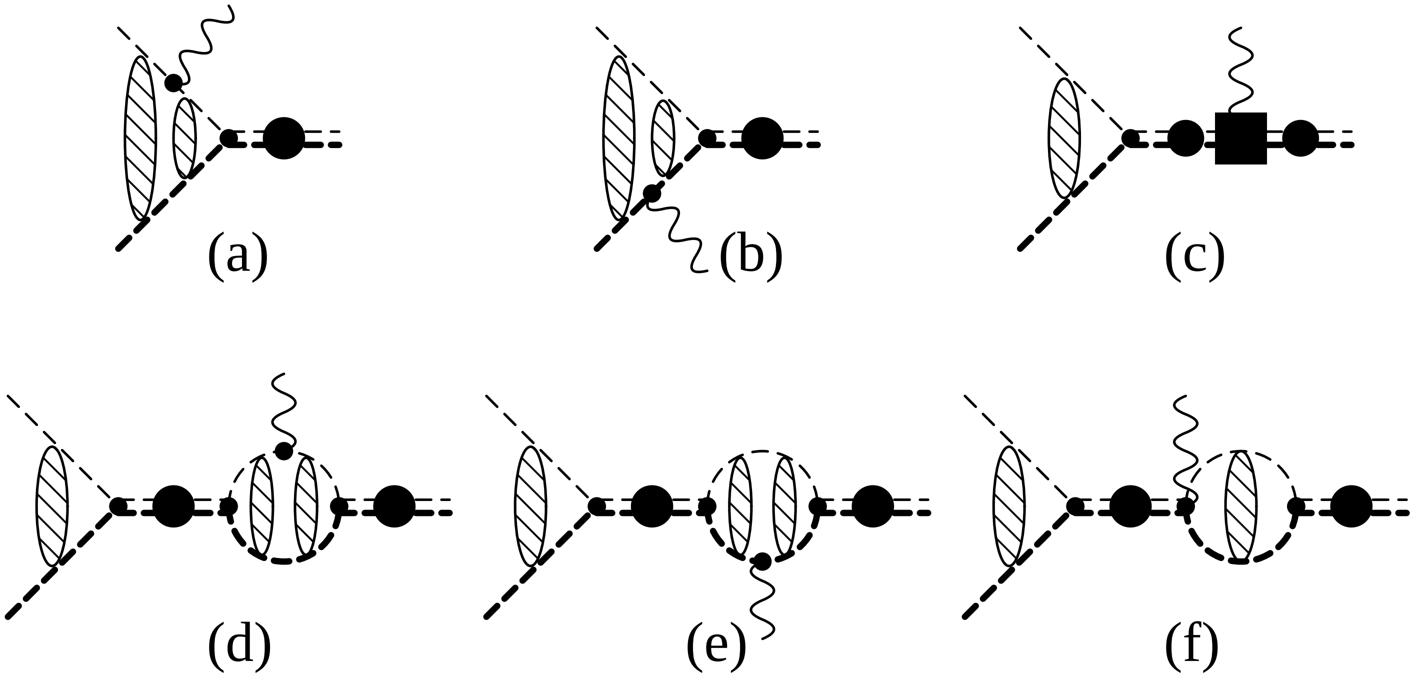
- Diagrams for dressed composite ^{16}O propagator



- Diagrams for elastic α-^{12}C scattering



- Diagrams for ^{12}C$(\alpha,\gamma)^{16}$O process



- Radiative capture amplitude

$$A^{(l=1)} = \vec{\epsilon}^{\,*}_{(\gamma)} \cdot \hat{p} X^{(l=1)} ,$$

where

$$X^{(l=1)} = X^{(l=1)}_{(a+b)} + X^{(l=1)}_{(c)} + X^{(l=1)}_{(d+e)} + X^{(l=1)}_{(f)} ,$$

with

$$X^{(l=1)}_{(a+b)} = 2y^{(0)} \sqrt{Z_{gs}} e^{i\sigma_1} \Gamma(1+\kappa/\gamma_0) \times \int_0^\infty dr r W_{-\kappa/\gamma_0, \frac{1}{2}}(2\gamma_0 r) \left[\frac{Z_\alpha \mu}{m_\alpha} j_0\left(\frac{\mu}{m_\alpha} k' r \right) - \frac{Z_C \mu}{m_C} j_0 \left(\frac{\mu}{m_C} k' r \right) \right] \times \left\{ \frac{\partial}{\partial r} \left[\frac{F_1(\eta, pr)}{pr} \right] + 2 \frac{F_1(\eta, pr)}{pr^2} \right\} ,$$

$$X^{(l=1)}_{(c)} = +y^{(0)} (g^{(1)R} + p g^{(1)\prime}) \sqrt{Z_{gs}} \frac{6\pi Z_O}{\mu m_O} \frac{e^{i\sigma_1} p \sqrt{1+\eta^2} C_\eta}{K_1(p) - 2\kappa H_1(p)} ,$$

$$
\begin{aligned}
X^{(l=1)}_{(d+e)} &= +i\frac{2}{3}y^{(0)}\sqrt{Z_{gs}}\frac{e^{i\sigma_1}p^2\sqrt{1+\eta^2}C_\eta}{K_1(p)-2\kappa H_1(p)}\Gamma(1+\kappa/\gamma_0)\Gamma(2+i\eta) \\
&\times \int_{r_C}^{\infty} dr r W_{-\kappa/\gamma_0,\frac{1}{2}}(2\gamma_0 r)\left[\frac{Z_\alpha\mu}{m_\alpha}j_0\left(\frac{\mu}{m_\alpha}k'r\right)-\frac{Z_C\mu}{m_C}j_0\left(\frac{\mu}{m_C}k'r\right)\right] \\
&\times \left\{\frac{\partial}{\partial r}\left[\frac{W_{-i\eta,\frac{3}{2}}(-2ipr)}{r}\right]+2\frac{W_{-i\eta,\frac{3}{2}}(-2ipr)}{r^2}\right\}, \\
X^{(l=1)}_{(f)} &= -3y^{(0)}\sqrt{Z_{gs}}\,\mu\left[-2\kappa H(\eta_{b0})\right]\left(\frac{Z_\alpha}{m_\alpha}-\frac{Z_C}{m_C}\right)\frac{e^{i\sigma_1}p\sqrt{1+\eta^2}C_\eta}{K_1(p)-2\kappa H_1(p)},
\end{aligned}
$$

and

$$
K_1(p) = -\frac{1}{a_1}+\frac{1}{2}r_1p^2-\frac{1}{4}P_1p^4+Q_1p^6\,,
$$

- S_{E1} factor

$$S_{E1}(E) = \sigma_{E1}(E) E e^{2\pi\eta} \,,$$

where

$$\sigma_{E1}(E) = \frac{4}{3}\frac{\alpha_E \mu E'_\gamma}{p(1+E'_\gamma/m_O)}|X^{(l=1)}|^2 C \,,$$

with $E'_\gamma \simeq B_0 + E - \frac{1}{2m_O}(B_0+E)^2$.

- Renormalization of divergence from the loops
  - The loops of the diagrams (a) and (b) are finite.
  - The loops of the diagrams (d) and (e) lead to a log divergence in the r integral in $X_{(d+e)}^{(l=1)}$. We introduce a short range cutoff r_C, and the divergence is renormalized by $g^{(1)'}$ term of $X_{(c)}^{(l=1)}$.
  - The loop of the diagram (f) is diverge, and the divergence is renormalized by $g^{(1)R}$ term of $X_{(c)}^{(l=1)}$.

- Modification of the counting rules
  - The p-wave dressed ^{16}O propagator is enhanced, and non-pole amplitude, $X_{(a+b)}^{(l=1)}$ turns out to be negligible
  - Approximately two structure (momentum dependence) are remained in the transition amplitude, while there are three unknown constants, $g^{(1)R}$, $g^{(1)'}$, C.
  - We choose $g^{(1)R} = 0$ and fix $g^{(1)'}$ and C to the data

Numerical results

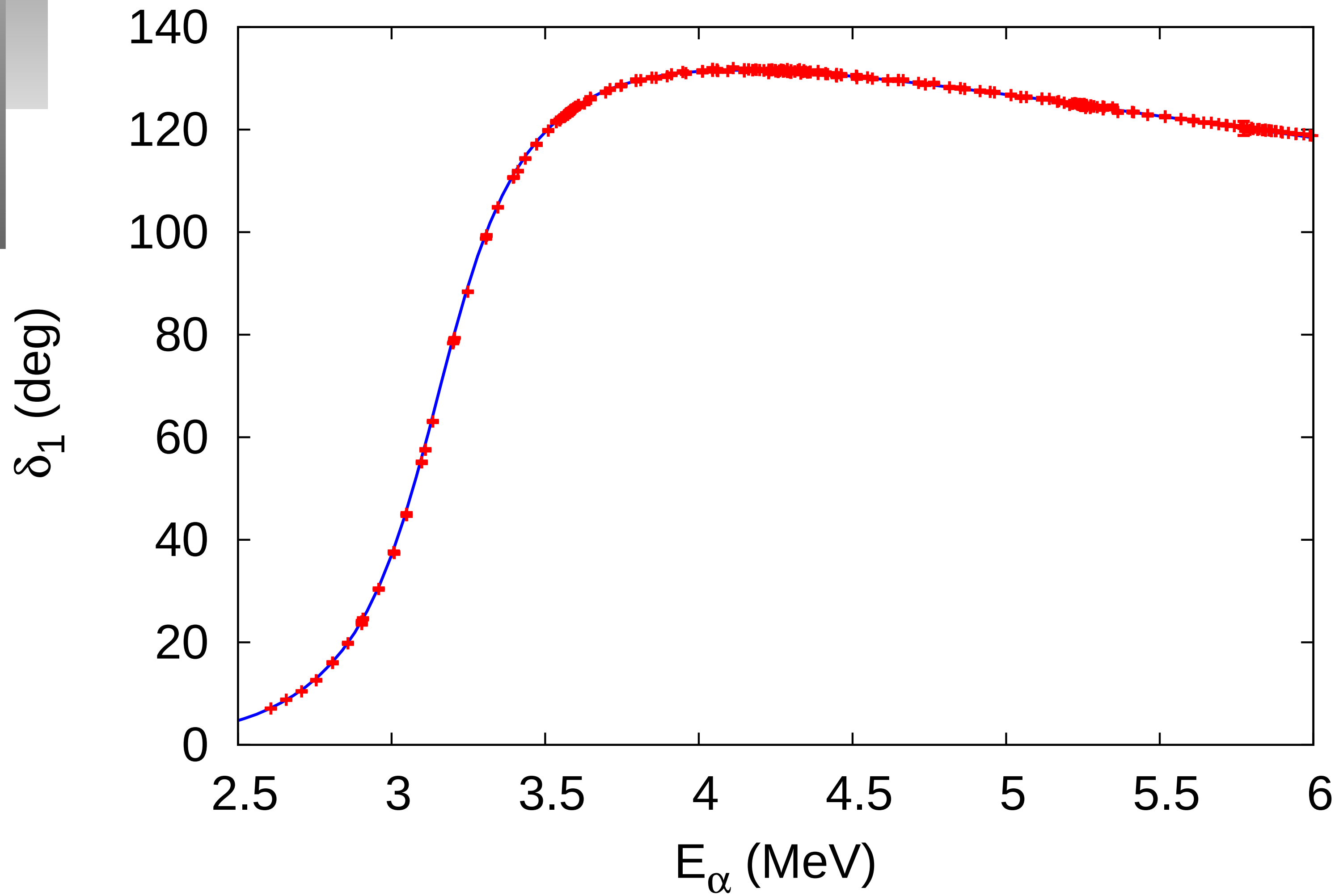
- Phase shift of the elastic α-^{12}C scattering for $l = 1$

The effective range parameters are fitted to the phase shift data from the Tischhauser *et al.*'s paper, and we have

$$r_1 = 0.415255(9)\ \text{fm}^{-1}\,, \quad P_1 = -0.57484(9)\ \text{fm}\,,$$
$$Q_1 = 0.02016(2)\ \text{fm}^3\,,$$

where the number of the data is $N = 273$ and $\chi^2 = 504$, and thus $\chi^2/N = 1.85$, and a_1 is obtained by using the binding energy of the 1_1^- state as

$$a_1 = -1.67 \times 10^5\ \text{fm}^3\,.$$



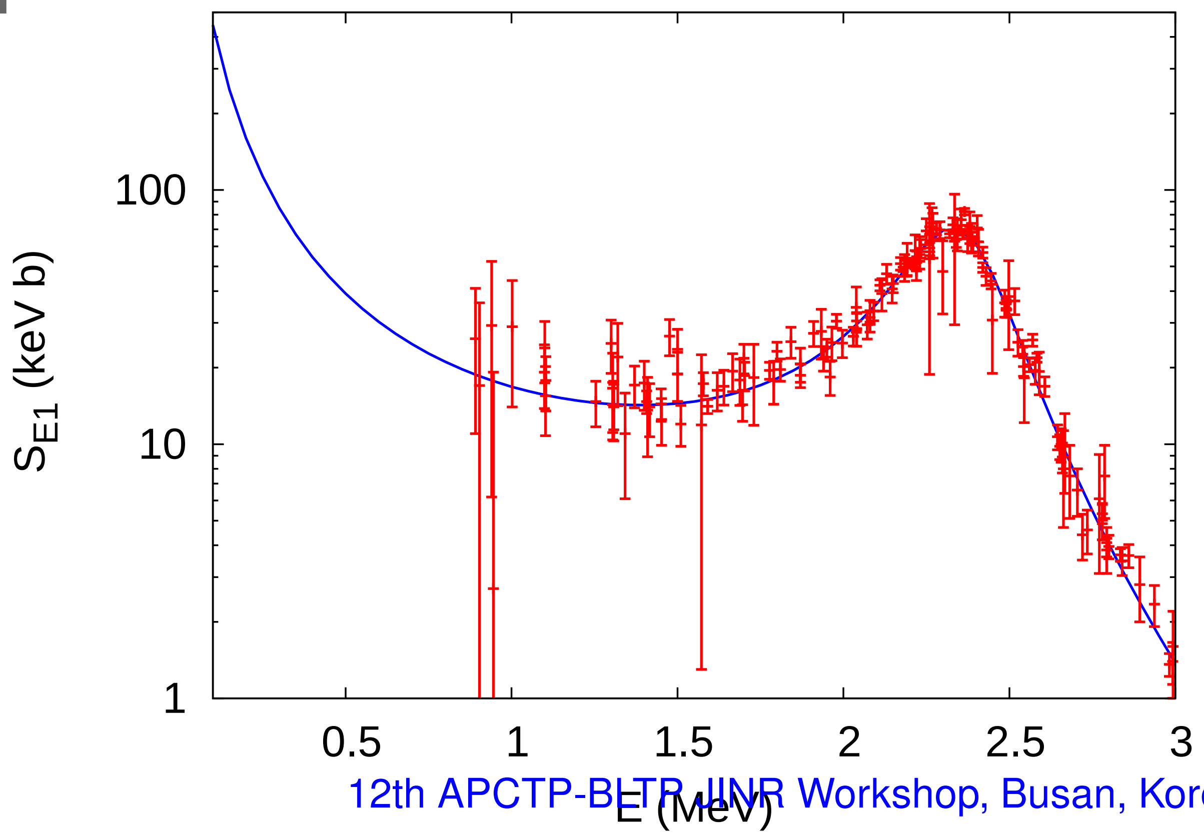
140
120
100
80
60
40
20
0
2.5
3
3.5
4
4.5
5
5.5
6
δ1 (deg)
Eα (MeV)

Numerical results: the S_{E1} factor

The two parameters are fitted to the S_{E1} data, and we have

$$g^{(1)'} = -94.2(13)\ \text{MeV}^2\,, \quad C = 2.04(9) \times 10^{-2}\,,$$

where $N = 151$ and $\chi^2 = 248$, and thus $\chi^2/N = 1.64$.



- S_{E1}-factor at E_G

$$S_{E1} = 86 \pm 4 \text{ keV·b}\,.$$

This result is in good agreement with the previous estimate reported recently: 86 ± 22 by Tang *et al.* (2010), 83.4 by Schurmann *et al.* (2012), 100 ± 22 by Oulebsir *et al.* (2012), 80 ± 18 by Xu *et al.* (2013), 98.0 ± 7.0 by An *et al.* (2015), 86.3 by dwBoer *et al.* (2017).

Results and discussion

- The EFT approach has been applied to the study of the S_{E1} factor of the ^{12}C(α,γ)^{16}O process.
- Our result of S_{E1} at E_G is in good agreement with the previous estimates reported recently.
- We find a small, about 5%, error in the S_{E1}, but we have not estimated a theoretical uncertainty yet.
- Necessary to study higher order terms of the process, however it may not easy to fix additional parameters due to the present quality of the data set of S_{E1}. It may be better studying the other quantities at low energies, the β delayed α emission spectrum of ^{16}N or the γ angular distribution of the radiative capture process.