Charge Symmetry Breaking Effects in Pion and Kaon Structure

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12^{th} APCTP-BLTP JINR Joint Workshop

Outline

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Background

- QCD is the fundamental theory of the strong interactions (hadronic system and interactions)
- Charge symmetry is u–and d– quarks are identical, $m_u = m_d$. In the charge symmetry, the charge symmetry operator P_{cs} is a isospin rotation by 180^o about the y-axis, where the z-axis related to charge
- In QCD the sources of CSB comes originally from the electromagnetic interactions and the mass difference between the down and up quarks $m_u \neq m_d$, $\delta m = m_d - m_u \sim +3\mathrm{MeV}$
- At the quark level, P_{CS} acts on the two light quark flavors

$$P_{CS}|\,d\rangle \;\; = \;\; |\,u\rangle \;\; and \;\; P_{CS}|\,u\rangle = -|\,d\rangle \tag{1}$$

- At quark level, the CSB effect is expected at a level 1 % or smaller
- The charge symmetry is exact : $e^{i\pi I_2}$

Background

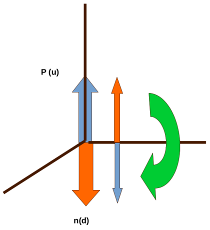

Figure: Cartoon of the charge symmetry operation

- Empirically CSB effects are clearly evident in proton-neutron mass difference, the differing masses between the charged and neutral pion and kaon states
- The QCDSF-UKQCD collaboration found *inter alia* that the QCD CSB effects between kaons are much larger than between the proton and neutron using dynamical lattice simulation of $QED + QCD$ [1]

[1] *Horsley, R, JPG**43** (2016), 10LT02*

Background

- In different area, CSB is important background in extraction of the strange electromagnetic form factor and PDFs of the nucleon
- CSB in the PDFs of the nucleon is vital to understanding the NuTeV anomaly [2]
- Beyond mass difference and effects in low energy nuclear physics, such as the Nolen-Schiffer anomaly, the experimental study of CSB effects is challenging. Definitive experiments are certainly needed
- Examples of promising experiments include parity violating deep elastic scattering (DIS) on the deutron and π^+/π^- production in SIDIS from the nucleon, which are planned at Jefferson Lab.
- Interesting possibilities exist at an electron-ion collider (EIC) such as charged current reactions and using pion induced Drell-Yan reactions

[2] *Zeller, GP, PRL**88** (2002), 091802*

Background

- CSB proposal in JLAB

Jefferson Lab PAC 37 Proposal

E12-09-002; Charge Symmetry Violating Quark Distributions via Precise Measurement of π^+/π^- Ratios in Semi–inclusive Deep Inelastic Scattering.

December 1, 2010

J. Arrington, R. Dupré, D. Geesaman, K. Hafidi (spokesperson[1]), R. J. Holt, D. H. Potterveld, P. E. Reimer, X. Zhan

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CSB and Pion and Kaon in the BSE-NJL Model

The three flavor NJL Lagrangian – containing only four fermion interactions [3]

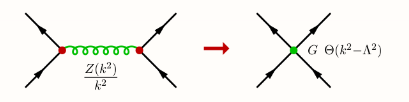
$$\mathscr{L}_{NJL} = \bar{\psi}[i\partial\!\!\!/ - \hat{m}_q]\psi + G_\pi \sum_{a=0}^{8} \left[(\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi}\lambda_a\gamma_5\psi)^2 \right] - G_\rho \sum_{a=0}^{8} \left[(\bar{\psi}\lambda_a\gamma^\mu\psi)^2 + (\bar{\psi}\lambda_a\gamma^\mu\gamma_5\psi)^2 \right] \quad (2)$$

- $\psi = (u, d, s)$ denotes the quark field with the flavor components
- G_π and G_ρ are four-fermion coupling constants
- $\lambda_1, \cdots, \lambda_8$ are Gell-Mann matrices in flavor space and $\lambda_0 \equiv \sqrt{\frac{2}{3}}\mathbb{1}$
- $\hat{m}_q = \mathrm{diag}(m_u, m_d, m_s)$ denotes the current quark matrix

[1] PH, Ian Cloet, and A. Thomas, PRC94 (2016), 035201

CSB and Pion and Kaon in the BSE-NJL Model

- In the NJL model, the gluon exchange is replaced by four-fermion contact interaction



- NJL model has a lack of confinement (it can be simply seen quark propagator has a pole)
- Therefore we regularize using the proper time regularization

$$\frac{1}{D^n} = \frac{1}{(n-1)!}\int_0^\infty d\tau\tau^{n-1}e^{-\tau D} \rightarrow \frac{1}{(n-1)!}\int_{\frac{1}{\Lambda_{UV}^2}}^{\frac{1}{\Lambda_{IR}^2}} d\tau\tau^{n-1}e^{-\tau D} \quad (3)$$

where $\Lambda_{IR} \sim \Lambda_{QCD} = 0.24\ \mathrm{GeV}$ and Λ_{UV} is determined.

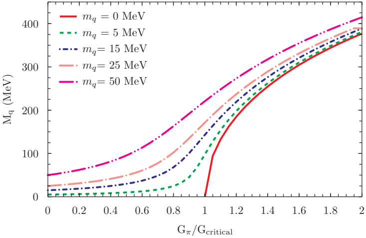
NJL Gap Equation in the proper-time regularization takes the form

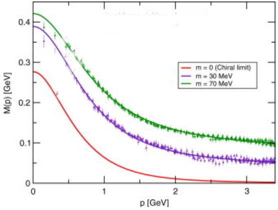
$$M_q = m_q + M_q \frac{3G_\pi}{\pi^2} \int d\tau \frac{e^{-\tau M_q^2}}{\tau^2} \tag{4}$$

- Chiral quark condensates is defined by $\langle\bar{\psi}\psi\rangle = -\frac{3M_q}{2\pi^2}\int d\tau \frac{e^{-\tau M_q^2}}{\tau^2}$
- Mass is generated through interaction vacuum $\rightarrow \langle\bar{\psi}\psi\rangle \neq 0$
- The NJL model naturally describes the appearance of a non-zero quark condensate, which is directly linked to dynamically generated dressed quark masses, and pions and kaon as Goldstone bosons

CSB and NJL Gap Equation

Results for NJL gap equation [4]and DSE model [5]



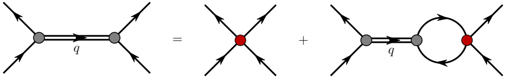


- The NJL model valid for very small *p* and work well in the low energy of QCD
- Therefore the NJL model is an ideal tool with which to study CSB effects in hadron structure because it is the dressed quark masses, not current quark masses, that determine the size of CSB at scales similar to Λ_{QCD}

[2] *PH, Ian Cloet, and A. Thomas, PRC**94** (2016), 035201*

[3] *CD Roberts,PPNP**61** (2008),50-65*

Bethe Salpeter Equation for the pion and kaon



- In the NJL model, T-matrix is given by

$$T(q) = \mathcal{K} + \int \frac{d^4k}{(2\pi)^4} \mathcal{K} S(q+k) T(q) S(k)$$

- The solution to the BSE in the pion and kaon

$$T_\alpha(q)_{ab,cd} = [\gamma_5 \lambda_\alpha]_{ab}\, t_\alpha(q) \left[\gamma_t \lambda_\alpha^\dagger\right] \tag{5}$$

- The reduced t-matrix in this channel take a form

$$\begin{aligned} t_\alpha(q) &= \frac{-2iG_\pi}{1 + 2G_\pi \Pi_\pi(q^2)} \\ t_\beta^{\mu\nu}(q) &= \frac{-2iG_\rho}{1 + 2G_\rho \Pi_\beta(q^2)} \left(g^{\mu\nu} + 2G_\rho \Pi_\beta(q^2) \frac{q^\mu q^\nu}{q^2} \right) \end{aligned} \tag{6}$$

Bethe Salpeter Equation of the pion and kaon

- The bubble diagrams appearing read

$$\begin{aligned}
\Pi_\pi(q^2) &= 6i\int \frac{d^4k}{(2\pi)^4} Tr_D\left[\gamma_5 S_l(k)\gamma_5 S_l(k+q)\right], \\
\Pi_K(q^2) &= 6i\int \frac{d^4k}{(2\pi)^4} Tr_D\left[\gamma_5 S_l(k)\gamma_5 S_s(k+q)\right], \\
\Pi^{aa}_\nu(q^2) &= 6i\int \frac{d^4k}{(2\pi)^4} Tr_D\left[\gamma^\mu S_a(k)\gamma^\nu S_a(k+q)\right]
\end{aligned} \tag{7}$$

- The kaon and pion masses is given by the pole of the t-matrix

$$\begin{aligned}
1 + 2G_\pi \Pi_\pi(k^2 = m_\pi^2) &= 0 \\
1 + 2G_\pi \Pi_K(k^2 = m_K^2) &= 0
\end{aligned} \tag{8}$$

Pion and Kaon Masses

The meson masses are defined by the pole in the corresponding t-matrix and therefore the kaon and pion masses are given by

$$
\begin{aligned}
m_{\pi^0}^2 &= \left[\frac{m_u}{M_u}\right]\frac{1}{G_\pi \mathcal{I}_{uu}(m_\pi^2)} + \left[\frac{m_d}{M_d}\right]\frac{1}{G_\pi \mathcal{I}_{dd}(m_\pi^2)} \\
m_K^2 &= \left[\frac{m_s}{M_s} + \frac{m}{M_l}\right]\frac{1}{G_\pi \mathcal{I}_{ls}(m_K^2)} + (M_s - M_l)^2
\end{aligned} \tag{9}
$$

where $\mathcal{I}_{ll}$ and $\mathcal{I}_{ls}$ in the proper time regularization scheme are defined by

$$
\mathcal{I}_{ab}(k^2) = \frac{3}{\pi^2}\int_0^1 dx \int \frac{d\tau}{\tau} e^{-\tau(x(x-1)k^2 + xM_b^2 + (1-x)M_a^2)} \tag{10}
$$

where $M_{a,b}$ are the dressed quark masses that appear in the meson. These results illustrate the Goldstone boson nature of the pion and kaon.

The meson-quark-quark coupling constants and Pion and Kaon Decay Constants

The residue at a pole in the $\bar{q}q$ $t-$matrix defines the effective meson-quark -quark coupling constants:

$$
\begin{aligned}
Z_\pi(q^2) &= -\frac{\partial \Pi_\pi(q^2)}{\partial q^2}\,|_{q^2=m_\pi^2} \\
Z_K(q^2) &= -\frac{\partial \Pi_K(q^2)}{\partial q^2}\,|_{q^2=m_K^2} \\
Z_\rho(q^2) &= -\frac{\partial \Pi_\rho(q^2)}{\partial q^2}\,|_{q^2=m_\rho^2}
\end{aligned}
\tag{11}
$$

Pion and kaon decay constant in the proper time regularization is given by

$$
\begin{aligned}
f_\pi &= \frac{N_C\sqrt{Z_\pi}M}{4\pi^2}\int_0^1 dx \int \frac{d\tau}{\tau} e^{-\tau(k^2(x^2-x)+M^2)} \\
f_K &= \frac{N_C\sqrt{Z_K}}{4\pi^2}[(1-x)M_2 + xM_1]\int_0^1 dx \int \frac{d\tau}{\tau} e^{-\tau(k^2(x^2-x)+xM_2^2-(x-1)M_1^2)}
\end{aligned}
\tag{12}
$$

Result for the CSB effect on properties of the pion and kaon

Results for the current and dressed quark masses, neutral pion, kaon and ρ^+ masses, neutral pion leptonic decay constant, meson-quark-quark coupling constant and the model parameters that vary with m_u/m_d.

m_u/m_d	m_u	m_d	M_u	M_d	m_{π^0}	$m_{K^\pm}$	m_{K^0}	m_{ρ^+}	f_{π^0}	Z_{π^0}	$Z_{\pi^\pm}$	$Z_{K^\pm}$	Z_{K^0}	G_π	G_ρ	UV
0	0	32.9	387	412	137.84	483	507	775.67	92.83	17.830	17.842	20.73	21.04	19.06	10.731	644.52
0.1	2.99	29.9	390	410	138.56	486	504	775.44	92.89	17.837	17.846	20.76	21.01	19.05	10.746	644.64
0.3	7.58	25.3	393	406	139.38	489	501	775.19	92.95	17.846	17.850	20.80	20.97	19.05	10.764	644.77
0.5	11.0	21.9	396	404	139.76	491	499	775.07	92.98	17.850	17.852	20.83	20.94	19.05	10.773	644.83
0.7	13.5	19.3	398	402	139.93	493	497	775.02	92.99	17.852	17.853	20.86	20.91	19.04	10.776	644.86
0.9	15.6	17.3	399	401	139.99	494	496	775.00	93.00	17.853	17.853	20.88	20.89	19.04	10.778	644.87
1	16.4	16.4	400	400	140	495	495	775	93	17.853	17.853	20.89	20.89	19.04	10.778	644.87

- The current quark mass ratio $r_{ud} = m_u/m_d$ is a free parameter, which adjust to study CSB effects
- For give r_{ud} we have $m_{u,d} = m_0 \pm \delta m$, where $\delta m = m_0(1 - r_{ud})/(1 + r_{ud})$.
- We find that $m_s = 356\ \mathrm{MeV}$ ($M_s = 611\ \mathrm{MeV}$) and $m_0 = 16.4\ \mathrm{MeV}$ and therefore $m_s/m_0 = 21.7$ (empirical 27.5 ± 1.0)

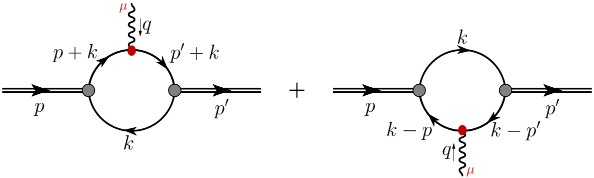
Result for the CSB effect on properties of the pion and kaon

- For realistic current quark mass ratio of $m_u/m_d = 0.5$, we have CSB effect in the current quarks of $(m_d - m_u)/(m_u + m_d) = 33$ %
- For the dressed quark masses, we have $(M_d - M_u)/(M_u + M_d) = 1$ %. It is clear that in the infrared dynamical chiral symmetry breaking (DCSB) dramatically reduces the size of CSB effects that may be expected from the current quark masses
- For the same m_u/m_d ratio, we find $m_{\pi^\pm} - m_{\pi^0} = 0.24$ MeV, which is smaller than the empirical value of 4.59 MeV and is therefore in agreement with expectation that this mass-splitting is dominated by QED effects
- For the kaon mass we have $(m_{K^0} - m_{K^\pm})/(m_{K^0} + m_{K^\pm}) = 0.8$ % with $m_{K^0} - m_{K^\pm} = 7.8$ MeV, which is twice the empirical splitting of 3.93 MeV and therefore QED effects must reduce in this mass splitting, which is the finding of the lattice calculation
- CSB effects in f_α,and $g_{\alpha qq}$ are found to be negligible and the same is found for the chiral condensate

FORM FACTOR IN THE CONFINING NJL MODEL

Form Factor in NJL model

Diagrammatic representation of the electromagnetic current for the pion and kaon



Feynman diagram for quark [left] and for the anti quark [right]

Form Factor in NJL model

The matrix element of the electromagnetic current for a pseudoscalar mesons reads

$$J^{\mu}_{\alpha}(p', p) \;=\; \left(p'^{\mu} + p^{\mu}\right) F_{\alpha}(Q^2), \quad \alpha = \pi, K \tag{13}$$

where p and $p^{'}$ denote the initial and final four momentum of the state, $q^2 = (p^{'} - P)^2 = -Q^2$ and $F_{\alpha}(Q^2)$ is the pion or kaon form factor. The pseudoscalar meson form factor in the NJL model are given by the sum of the two Feynman diagrams, which are respectively given by

$$\begin{aligned} j^{\mu}_{1,\alpha}(p^{'}, p) &= iZ_{\alpha} \int \frac{d^4k}{(2\pi)^4} Tr\left[\gamma_5 \lambda^{\dagger}_{\alpha} S(p^{'} + k) \hat{Q}\gamma^{\mu} S(p+k)\gamma_5 \lambda_{\alpha} S(k)\right] \\ j^{\mu}_{2,\alpha}(p^{'}, p) &= iZ_{\alpha} \int \frac{d^4k}{(2\pi)^4} Tr\left[\gamma_5 \lambda_{\alpha} S(k-p) \hat{Q}\gamma^{\mu} S(k-p^{'})\gamma_5 \lambda^{\dagger}_{\alpha} S(k)\right] \end{aligned} \tag{14}$$

where the Tr is over Dirac, color and flavor indices. The index α labels the state and the λ_{α} are the corresponding flavor matrices

Form Factor in NJL model

We will focus on the quark sector and total form factors for π^+, K^+ and K^0, we find

$$
\begin{aligned}
F_{\pi^+}^{(bare)}(Q^2) &= (e_u - e_d) f_\pi^{ll}(Q^2) \\
F_{K^+}^{(bare)}(Q^2) &= e_u f_K^{ls}(Q^2) - e_s f_K^{sl}(Q^2) \\
F_{K^0}^{(bare)}(Q^2) &= e_d f_K^{ls}(Q^2) - e_s f_K^{sl}(Q^2)
\end{aligned} \tag{15}
$$

The results are denoted as "*bare*" because the quark-photon vertex is elementary result, that is, $\Lambda_{\gamma q}^{\mu(bare)} = \hat{Q}\gamma^\mu$. The quark-sector form factors for a hadron α are defined by

$$
F_\alpha(Q^2) = e_u F_\alpha^u(Q^2) + ed F_\alpha^d(Q^2) + e_s F_\alpha^s(Q^2) + \cdots \tag{16}
$$

therefore the "*bare*" pseudoscalar meson quark-sector form factors are easily read from the total form factor equation above

Form Factor in NJL model

The first superscript on the body form factors, $f_\alpha^{ab}(Q^2)$, indicates the struck quark and the second the spectator, where

$$\begin{aligned} f_\alpha^{ab}(Q^2) &= \frac{3Z_\alpha}{4\pi^2}\int_0^1 dx \int \frac{d\tau}{\tau} e^{-\tau(M_a^2+x(1-x)Q^2)} \\ &+ \frac{3Z_\alpha}{4\pi^2}\int_0^1 dx \int_0^{1-x} dz \int d\tau \\ &\times e^{-\tau((x+z)(x+z-1)m_\alpha^2+(x+z)M_a^2+(1-x-z)M_b^2+xzQ^2)} \\ &\times \left[(x+z)m_\alpha^2 + (M_a - M_b)^2(X+Z) + 2M_b(M_a - M_b)\right] \end{aligned} \tag{17}$$

Importantly, these expression satisfy charge conservation.

Form Factor in NJL model

In general the quark-photon vertex is not elementary ($\hat{Q}\gamma^\mu$) but instead dressed, with this dressing given by the inhomogeneous BSE. The general solution for the dressed quark-photon vertex for a quark of flavor q has the form

$$\Lambda^{\mu}_{\gamma Q}(p', p) = e_q\gamma^\mu + \left(\gamma^\mu - \frac{q^\mu \not{q}}{q^2}\right) F_Q(Q^2) \to \gamma^\mu F_{1Q}(Q^2) \tag{18}$$

where the final result is used because the $\frac{q^\mu \not{q}}{q^2}$ term cannot contribute to a hadron electromagnetic current because of current conservation

Form Factor in NJL model

For the dressed u, d and s quarks we find

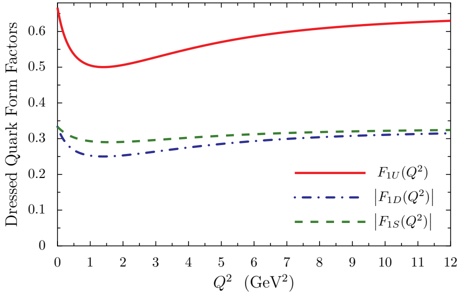
$$\begin{aligned} F_{1U/D}(Q^2) &= e_{u/d}\frac{1}{1+2G_\rho \Pi_v^{ll}(Q^2)} \\ F_{1S}(Q^2) &= e_s\frac{1}{1+2G_\rho \Pi_v^{ss}(Q^2)} \end{aligned} \tag{19}$$

where the explicit form of the bubble diagram is

$$\Pi_v^{qq}(Q^2) = \frac{3Q^2}{\pi^2}\int_0^1 dx \int \frac{d\tau}{\tau} x(1-x) e^{-\tau\left[M_q^2 + x(1-x)Q^2\right]} \tag{20}$$

Form Factor in NJL model

The dressed quark form factors obtained as solutions to the inhomogeneous BSE:



In the limit $Q^2 \to \infty$ these form factors reduce to the elementary quark charges, as expected because of asymptotic freedom in QCD. For small Q^2 these results are similar to expectations from vector meson dominance, where the dressed u and d quarks are dressed by ρ and ω mesons and the dressed s quark by ϕ meson.

Form Factor in NJL model

The complete results for the pseudoscalar meson form factor – with a dressed quark-photon vertex – read

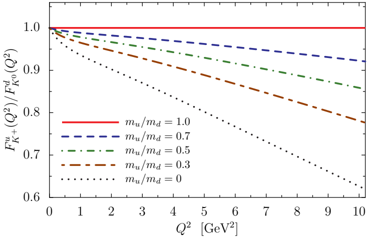
$$F_{\pi^+}(Q^2) = F_{1U}(Q^2) f^{ud}_{\pi^+}(Q^2) - F_{1D}(Q^2) f^{du}_{\pi^+}(Q^2), \tag{21}$$

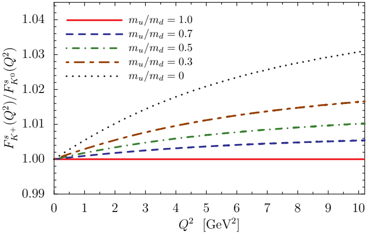
$$F_{K^+}(Q^2) = F_{1U}(Q^2)\, f^{us}_{K^+}(Q^2) - F_{1S}(Q^2)\, f^{su}_{K^+}(Q^2), \tag{22}$$

$$F_{K^0}(Q^2) = F_{1D}(Q^2)\, f^{ds}_{K^0}(Q^2) - F_{1S}(Q^2)\, f^{sd}_{K^0}(Q^2), \tag{23}$$

Elastic Form Factor Results

Results for CSB effects between the u quark sector form factors in the K^+ and the d quark sector form factors in the K^0



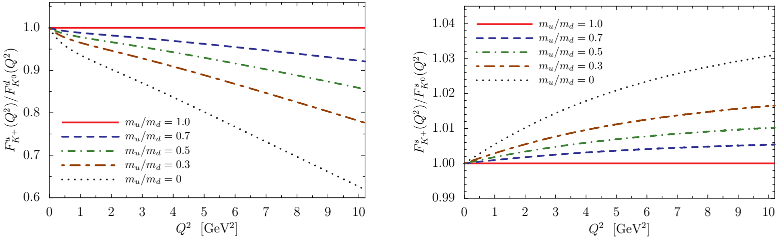


$\rightarrow$ We find the ratio $F^u_{K^+}(Q^2)/F^D_{K^0}(Q^2) < 1$ and the CSB effect grow with increasing Q^2

$\rightarrow$ For the kaon, these effects are about twice that of the pion for large Q^2. We therefore find that $r^u_{K^+} > r^d_{K^0}$, which is in agreement with the expectation from the fact that $M_u < M_d$

Elastic Form Factor Results

Results for CSB effects between the u quark sector form factors in the K^+ and the d quark sector form factors in the K^0

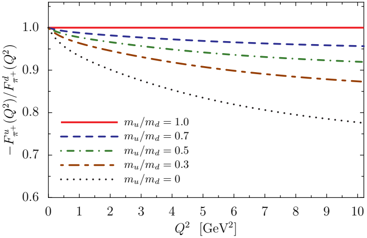


$\rightarrow$ For $m_u/m_d = 0.5$, we find the CSB effects in the quark radii is about 0.6 %, which is almost similar to that found in the pion

$\rightarrow$ Comparison between the s quark sectors in K^+ and the K^0, which is a measure of the environment sensitivity for the s quark in both mesons. These effects are an order of magnitude smaller that the CSB effects

Elastic Pion Form Factor Results

Results for ratio of the u and d quark sector form factors in the π^+ for various values of m_u/m_d



$\rightarrow$ We find the ratio decreases from unity as m_u/m_d gets smaller. This reflects $r^u_\pi > r^d_\pi$ ($m_u/m_d < 1$, we have $M_u < M_d$ and the lighter dressed u–quark has larger probability to be further from the charge center of the π^+)

$\rightarrow$ For realistic value of $m_u/m_d = 0.5$, we have find the CSB effects of the size about 7 % and increasing the Q^2 we find the CSb effects increase substantially, reaching about 8 % at $Q^2 = 10\ \mathrm{GeV}^2$

Result for the quark sector radii

Results for the quark sector radii in the π^+, K^+ and K^0 given in units of fm.

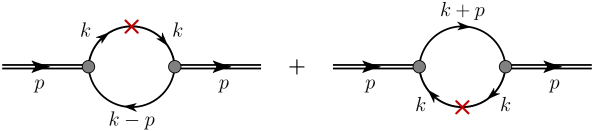
m_u/m_d	$r^u_{\pi^+}$	$r^d_{\pi^+}$	$r^u_{K^+}$	$r^d_{K^0}$	$r^s_{K^+}$	$r^s_{K^0}$
0	0.634	−0.608	0.650	0.625	−0.436	−0.438
0.1	0.632	−0.610	0.647	0.627	−0.436	−0.438
0.3	0.628	−0.614	0.644	0.631	−0.437	−0.438
0.5	0.625	−0.616	0.641	0.633	−0.437	−0.437
0.7	0.623	−0.618	0.639	0.635	−0.437	−0.437
0.9	0.621	−0.620	0.638	0.636	−0.437	−0.437
1	0.621	−0.621	0.637	0.637	−0.437	−0.437

- We predict that the CSB effect from the u and d quark mass difference should initially increase with Q^2, then at scales $Q >> \Lambda_{QCD}$ when the effect perturbative QCD start dominate, they should begin to decrease and then vanish in the asymptotic limit

VALENCE QUARK DISTRIBUTION IN THE CONFINING NJL MODEL

Valence quark distribution

The valence quark distribution functions of the pion or kaon are given by the two Feynman diagrams



The operator insertion $\gamma^+\delta\left(k^+ - xp^+\right)\hat{P}_q$, where $\hat{P}_q$ is the projection operator for quarks of flavor q:

$$
\begin{aligned}
\hat{P}_{u/d} &= \frac{1}{2}\left(\frac{2}{3}\mathbb{1} \pm \lambda_3 + \frac{1}{\sqrt{3}}\lambda_8\right) \\
\hat{P}_s &= \frac{1}{3}\mathbb{1} - \frac{1}{\sqrt{3}}\lambda_8
\end{aligned}
\tag{24}
$$

Valence quark distribution

The valence quark and anti-quark distributions in the pion or kaon are given by

$$
\begin{aligned}
q_\alpha(x) &= iZ_\alpha \int \frac{d^4k}{(2\pi)^4} \delta\left(k^+ - xp^+\right) \\
&\times \; Tr\left[\gamma_5 \lambda_\alpha^\dagger S(k) \gamma^+ \hat{P}_q S(k) \gamma_5 \lambda_\alpha S(k-p)\right] \\
\bar{q}_\alpha(x) &= -iZ_\alpha \int \frac{d^4k}{(2\pi)^4} \delta\left(k^+ + xp^+\right) \\
&\times \; Tr\left[\gamma_5 \lambda_\alpha S(k) \gamma^+ \hat{P}_q S(k) \gamma_5 \lambda_\alpha^\dagger S(k+p)\right] \qquad (25)
\end{aligned}
$$

To evaluate these expression we first take the moments

$$
\mathcal{A}_n = \int_0^1 dx x^{n-1} q(x) \qquad (26)
$$

where $n = 1, 2, 3, \cdots$ is an integer.

Valence quark distribution

Using the Ward-like identity $S(k)\gamma^+ S(k) = \frac{-\partial S(k)}{\partial k_+}$ and introducing the Feynman parameterization, the quark and anti-quark distributions can then be straightforwardly determined. For the valence quark and anti-quark distributions of the K^+ we find:

$$\begin{aligned} q_K(x) &= \frac{3Z_K}{4\pi^2}\int d\tau e^{-\tau\left[x(x-1)m_K^2 + xM_s^2 + (1-x)M_l^2\right]} \\ &\times \left[\frac{1}{\tau}x(1-x)\left[m_K^2 - (m_l - M_s)^2\right]\right] \\ \bar{q}_K(x) &= \frac{3Z_K}{4\pi^2}\int d\tau e^{-\tau\left[x(x-1)m_K^2 + xM_l^2 + (1-x)M_s^2\right]} \\ &\times \left[\frac{1}{\tau}x(1-x)\left[m_K^2 - (m_l - M_s)^2\right]\right] \end{aligned} \tag{27}$$

Results for the π^+ are obtained by $M_s \to M_l$ and $Z_K \to Z_\pi$, giving the result $u_\pi(x) = \bar{d}_\pi(x)$

Valence quark distribution

The quark distributions satisfy the baryon number and momentum sum rules, which for the K^+ read:

$$\int_0^1 dx\,[u_{K^+}(x) - \bar{u}_{K^+}(x)] \;=\; \int_0^1 [\bar{s}_{K^+}(x) - s_{k^+}(x)] = 1 \qquad (28)$$

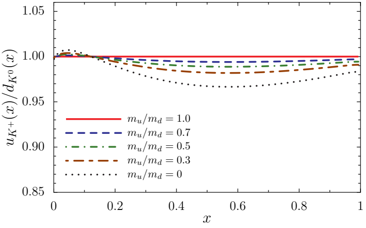
for the number sum rules and at the model scale the momentum sum rules is given by

$$\int_0^1 dxx\,[u_{K^+}(x) + \bar{u}_{K^+}(x) + \bar{s}_{K^+}(x) + s_{k^+}(x)] \;=\; 1 \qquad (29)$$

Analogous results holds for the remaining kaons and the pions.

RESULTS FOR CSB IN THE VQDFs

Ratio of the u quark distribution in the K^+ to the d quark distribution in the K^0, after QCD evolution to a scale of $Q^2 = 5\,\mathrm{GeV}^2$, for various values of current quark mass ratio m_u/m_d. *Right side:* Ratio of the $\bar{s}$ quark distribution in the K^+ to the same PDF in the K^0 at a scale of $Q^2 = 5\,\mathrm{GeV}^2$. This ratio is a measure of environment sensitivity effects

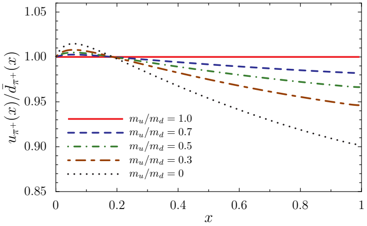


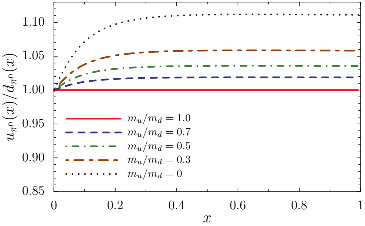
$\rightarrow$ We find that these CSB effects are at the few percent level, making CSB effects in the kaon PDFs much smaller than these effects in the pion, which is opposite with what we found for the pion and kaon EMFFs

$\rightarrow$ For the realistic m_u/m_d, we find the CSB effects at the few percent level, that are greater than unity and maximal when $x \rightarrow 1$.

Results for CSB in the VQDFs

Ratio of the u quark distribution to the $\bar{d}$ quark distribution in the π^+, after QCD evolution to a scale of $Q^2 = 5\,\text{GeV}^2$, for various values of current quark mass ratio m_u/m_d. *Right side:* Ratio of the u quark distribution to the d quark distribution in the neutral pion, at a scale of $Q^2 = 5\,\text{GeV}^2$





→ We find that the ratio $u_{\pi^o}(x)/d_{\pi^0}(x)$ is always greater than unity when CSB effects are included. In contrast to the π^+, this implies that the lighter u–quark carries more lightcone momentum in the π^0 than the heavier d–quark

→ In general, we find that the CSB effects in the PDFs are much smaller than in the EMFFs at Q^2

Result for the quark sector radii

Results for moments of the quantities: $\delta q_{\pi^+}(x) = \bar{d}_{\pi^+}(x) - u_{\pi^+}(x)$, $\delta q_{\pi^0}(x) = d_{\pi^0}(x) - u_{\pi^0}(x)$ and $\delta q_K(x) = d_{K^0}(x) - u_{K^+}(x)$.

m_u/m_d	$< x\,\delta q_{\pi^+} >$	$< \delta q_{\pi^0} >$	$< x\,\delta q_{\pi^0} >$	$< x\,\delta q_K >$
0	0.0174	−0.0532	−0.0266	0.0086
0.1	0.0143	−0.0435	−0.0218	0.0070
0.3	0.0094	−0.0286	−0.0143	0.0046
0.5	0.0058	−0.0177	−0.0089	0.0029
0.7	0.0031	−0.0094	−0.0047	0.0015
0.9	0.0009	−0.0028	−0.0014	0.0005
1	0	0	0	0

- These results are at the model scale of $Q^2 = 0.16\,\mathrm{GeV}^2$, where there are no sea quarks, so the first moments of $\delta q_{\pi^+}(x)$ and $\delta q_K(x)$ must vanish, and are therefore not tabulated

Summary and Outlook

- We have calculated the form factor and parton distribution of the kaon and pion in the confining NJL model
  $\Rightarrow$ Our results on FF and PDF of the kaon and pion are excellent agreement with the available experimental data
- We have observed the CSB effects in the pion and kaon
  $\Rightarrow$ we found that the effect of CSb arising from the light quark mass differences is surprisingly large in the quark sector elastic form factors at large Q^2
  $\Rightarrow$ For the realistic value of $m_u/m_d = 0.5$, one finds CSB at the 15 % level in ratio $F^u_{K^+}(Q^2)/F^d_{K^0}(Q^2$ at $Q^2 = 10\mathrm{GeV}^2$. The analogous changes in the PDFs are considerably smaller in magnitude, reaching 3 % as $x \to 1$ in the pion ratio $u_{\pi^+}(x)/\bar{d}_{\pi^+}(x)$, compared with just 1 % in the ratio $u_{K^+}(x)\ d_{K^0}(x)$ for the kaon.
- Testing these predictions presents considerable experimental challenges.

THANK YOU VERY MUCH FOR ATTENTION

Any questions or comments ??