12th APCTP-BLTP JINR Joint Workshop
"Modern problems in nuclear and elementary particle physics"
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Thermal Systems and Black Holes

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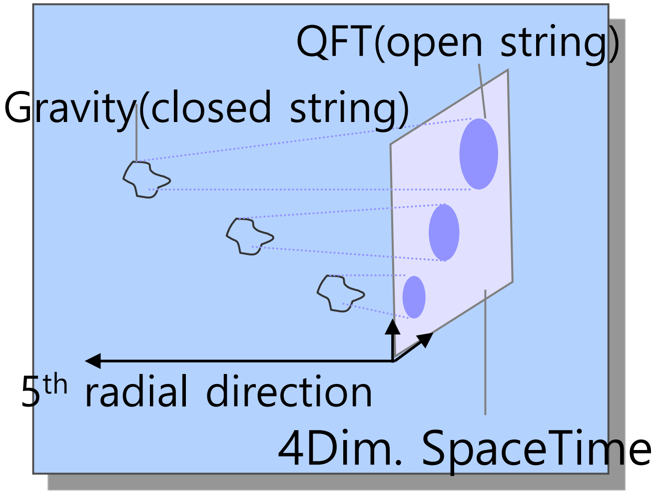
IHolography Principle (AdS/CFT Correspondence)

"2^{nd} revolution of the string theory (1994)

「(Strongly Interacting) Quantum Field Theory
in a given space time dimension (Ex):3+1=4 dim)
can be equivalently described by
the (classical) gravity theory
in one higher dimensional spacetime (Ex): 4+1=5dim).



3+1 dim. QFT (large Nc)
⇔
4+1 dim. Effective Gravity description

Energy scale in QFT corresponds to the parameter in extra "dimension"
or radial direction in AdS5 space

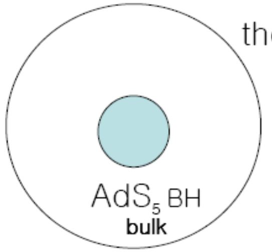
$$g_s = e^{\phi(r)} = g_{YM}^2(\mu)$$

- Temperature ↔
  Black hole geometry
  T = $T_{Hawking}$

E. Witten (1998)

$$ds_5^2 = \frac{1}{z^2}\left(f^2(z)dt^2 - (dx^i)^2 - \frac{1}{f^2(z)}dz^2\right),$$

$$f^2(z) = 1 - \left(\frac{z}{z_T}\right)^4 \qquad T = \frac{1}{\pi z_T}$$



Running coupling constants : needs deviation from the AdS5 space

$$g_s = e^{\phi(r)} = g_{YM}^2(\mu)$$

No-Hair Theorem of Black Holes

Werner Israel(1967),Brandon Carter(1971,1977), David Robinson (1975)

Stationary black holes (in 4-dim Einstein Gravity) are completely described by 3 parameters of the Kerr-Newman metric :
mass, charge, and angular momentum (M, Q, J)

Q: Can there be Black Holes with hairs?

The black hole may carry the hair in the dilaton-Gauss-Bonnnet theory.

Holographic QCD & CMT

Witten '98

<u>Goal</u> : Using the 5 dim. dual gravity, study 4dim. strongly interacting QCD & CMT

Needs the dual geometry !.

<u>Ex) Hawking-Page phase transition</u> =Transition of bulk geometry

The geometry is described by the following action

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{g}\,(-\mathcal{R}+2\Lambda)$$

$\Lambda = -\frac{6}{R^2}$ cosmological constant

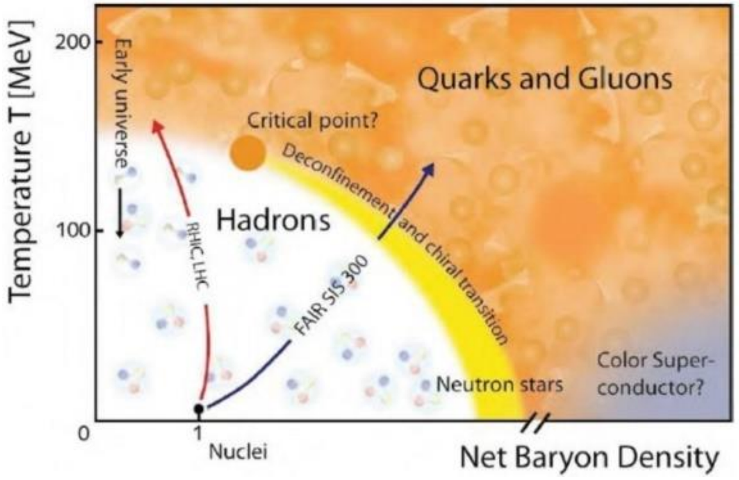
The geometry with smaller action is the stable one for given T.

$$\Delta S = \lim_{\epsilon\to 0}(S_{AdSBH} - S_{tAdS}) = \frac{\pi z_h R^3}{\kappa^2}\left(\frac{1}{z_{IR}^4} - \frac{1}{2z_h^4}\right)$$

> 0 for T < Tc

< 0 for T>Tc

[Herzog , Phys.Rev.Lett.98:091601,2007]



Low T QCD Phase transition High T

	Confinement	Deconfinement
QCD (4Dim)	Hadron	Quark-Gluon
Gravity (5Dim)	Thermal AdS	AdS Black Hole

Hawking-Page phase transition

AdS space :
no coupling running ->
Needs deviation from
the AdS geometry

Holography

(asymptotic) AdS Black Hole ↔ Quantum System
in d+1 dim. in d dim.

Instability of Black Holes ↔ instability of Quantum System

Hence,
instability of AdS BH ↔ phase transitions in Quantum System

* Running of coupling constants -> Geometry (of Black holes) is beyond simple Einstein Gravity.

Higher Curvature Gravity theory can be a candidate!

Why Higher Curvatures - Gauss-Bonnet Term?

Low energy effective theory from string theory
→ Einstein Gravity + higher curvature terms
Gauss-Bonnet term is the simplest leading term.

Q : What is the physical effects of the Higher Curvature terms?

1) Effects to the Black Holes.

No-Hair Theorem of Black Holes

Stationary black holes (in 4-dim Einstein Gravity) are completely described by 3 parameters of the Kerr-Newman metric :
mass, charge, and angular momentum (M, Q, J)

Werner Israel(1967),
Brandon Carter(1971,1977),
David Robinson (1975)

Hairy black hole solution ?
In the dilaton-Gauss-Bonnnet theory → Yes!

Exists the minimum mass of BH
Affects the stability, etc.

2) Effects in the Early Universe, and
implication to the quantum boundary theory.

Note.Cosmological Effects of the Gauss–Bonnet term – Inflation

- An action with a Gauss–Bonnet term:

$$S = \int d^4x\,\sqrt{-g}\left[\frac{1}{2\kappa^2}\,R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \,- V(\phi) - \frac{1}{2}\xi(\phi)R_{GB}^2\right]$$

Gauss–Bonnet term

$$R_{GB}^2 = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

$$G_{\mu\nu}=\kappa^2\left(T_{\mu\nu} + T_{\mu\nu}^{GB}\right)$$

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi + V(\phi) - \frac{1}{2}g_{\mu\nu}\left(g^{\rho\sigma}\partial_\rho\phi\partial_\sigma\phi + 2V\right)$$

$$\kappa^2 = 8\pi G \qquad G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\,R$$

$$T_{\mu\nu}^{GB} = 4\left(\partial^\rho\partial^\sigma\xi R_{\mu\rho\nu\sigma} - \Box\xi R_{\mu\nu} + 2\partial_\rho\partial_{(\mu}\xi R_{\nu)}^{\rho} - \frac{1}{2}\partial_\mu\partial_\nu\xi R\right) - 2\left(2\partial_\rho\partial_\sigma\xi R^{\rho\sigma} - \Box\xi R\right)g_{\mu\nu}$$

$$\Box\phi - V_{,\phi}(\phi) - \frac{1}{2}T^{GB} = 0 \qquad T^{GB} = \xi_{,\phi}(\phi)R_{GB}^2$$

- FLRW Universe metric: $ds^2 = -\,dt^2 + a^2(t)\left(\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + sin^2\theta\, d\varphi^2)\right)$

Einstein and Field equations yield:

$$H^2 = \frac{\kappa^2}{3}\left(\frac{1}{2}\dot\phi^2 + V - \frac{3K}{\kappa^2a^2} + 12\dot\xi H\left(H^2 + \frac{K}{a^2}\right)\right)$$

$$\dot H = -\frac{\kappa^2}{2}\left(\dot\phi^2 - \frac{2K}{\kappa^2a^2} - 4\ddot\xi\left(H^2 + \frac{K}{a^2}\right) - 4\dot\xi H\left(2\dot H - H^2 - \frac{3K}{a^2}\right)\right)$$

$$\ddot\phi + 3H\dot\phi + V_{,\phi} + 12\xi_{,\phi}\left(H^2 + \frac{K}{a^2}\right)\left(\dot H + H^2\right) = 0$$

Inflation with a Gauss–Bonnet

- Action $$S = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa^2} R - \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) - \frac{1}{2}\xi(\phi)R^2_{GB}\right]$$

Einstein and Field equations yield:

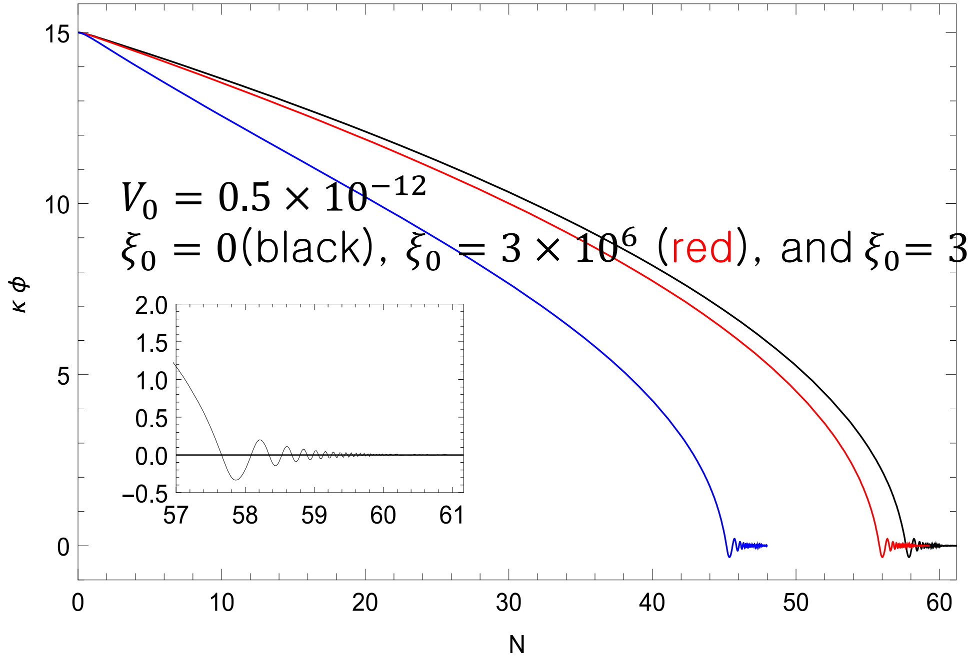
$$H^2 = \frac{\kappa^2}{3}\left(\frac{1}{2}\dot{\phi}^2 + V - \frac{3K}{\kappa^2 a^2} + 12\dot{\xi}H\left(H^2 + \frac{K}{a^2}\right)\right)$$

$$\dot{H} = -\frac{\kappa^2}{2}\left(\dot{\phi}^2 - \frac{2K}{\kappa^2 a^2} - 4\ddot{\xi}\left(H^2 + \frac{K}{a^2}\right) - 4\dot{\xi}H\left(2\dot{H} - H^2 - \frac{3K}{a^2}\right)\right)$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} + 12\xi_{,\phi}\left(H^2 + \frac{K}{a^2}\right)\left(\dot{H} + H^2\right) = 0$$

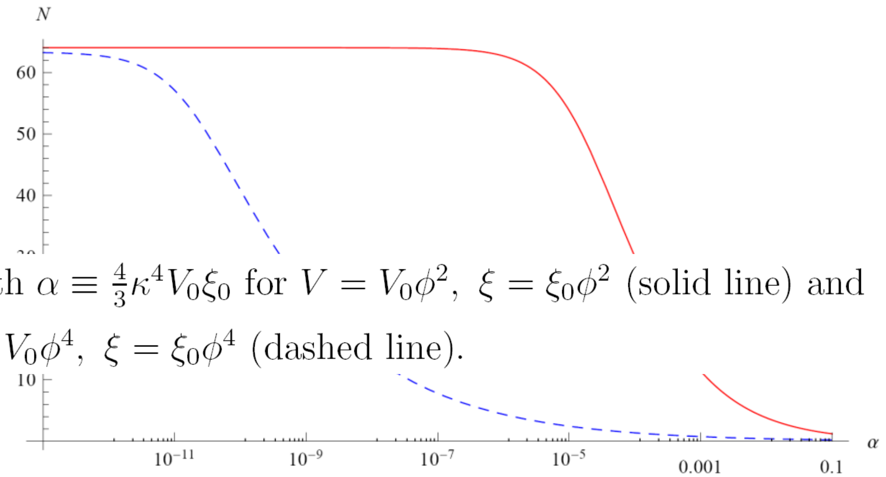
S. Koh, BHL, W. Lee, Tumurtushaa
PRD90 (2014) no.6, 063527

S. Koh, BHL, W. Lee, Tumurtushaa
Phys.Rev. D95 (2017) no.12, 123509



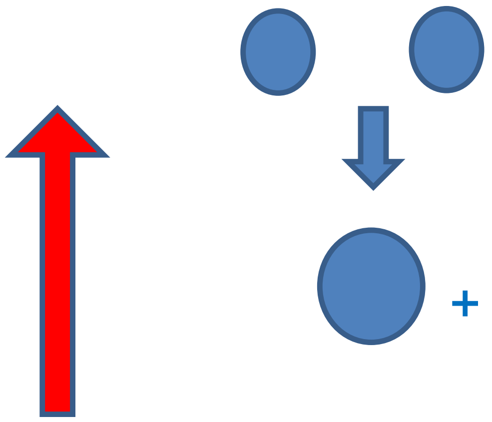
$V_0 = 0.5 \times 10^{-12}$
$\xi_0 = 0$(black), $\xi_0 = 3 \times 10^6$ (red), and $\xi_0 = 3 \times 10^7$ (blue).

The duration of inflation gets shorter as the Gauss–Bonnet coupling constant increases.
(making the effective potential steeper)



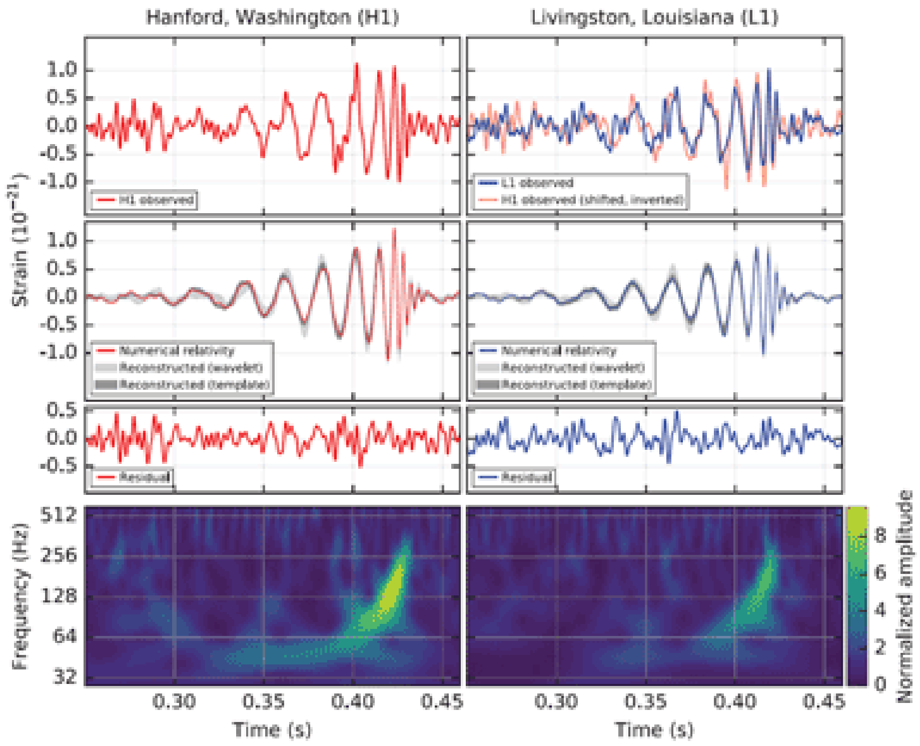
e-folding numbers with $\alpha \equiv \frac{4}{3}\kappa^4 V_0 \xi_0$ for $V = V_0\phi^2$, $\xi = \xi_0\phi^2$ (solid line) and $V = V_0\phi^4$, $\xi = \xi_0\phi^4$ (dashed line).

Colliding Black Holes :
A Black Hole Merger + Gravitational Wave



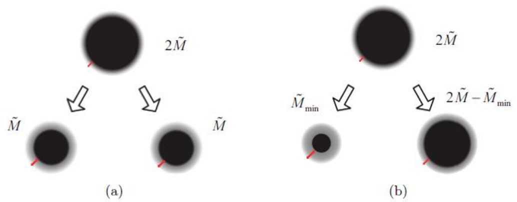
+ Gravitational Wave

GW150914



Q: Reverse Process ?

Can a Black Hole be unstable splitting into two Black Holes ?



Contents

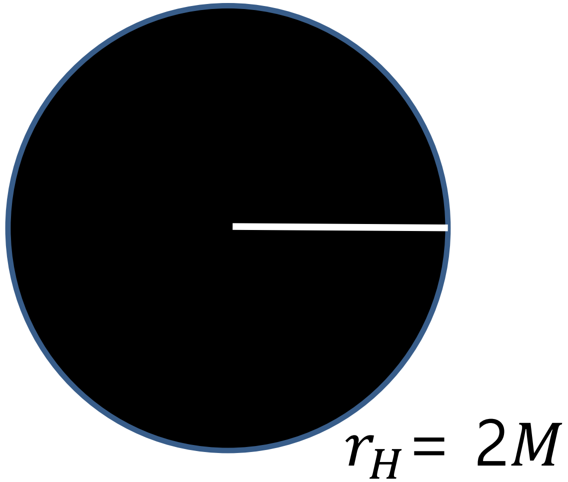
2. Black Holes in the Dilaton Einstein Gauss-Bonnet (DEGB) theory

Review : Einsten theory – Schwarzschild Black Hole

Action

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa^2} R - \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right]$$

where $g = \det g_{\mu\nu}$ and $\kappa \equiv 8\pi G$

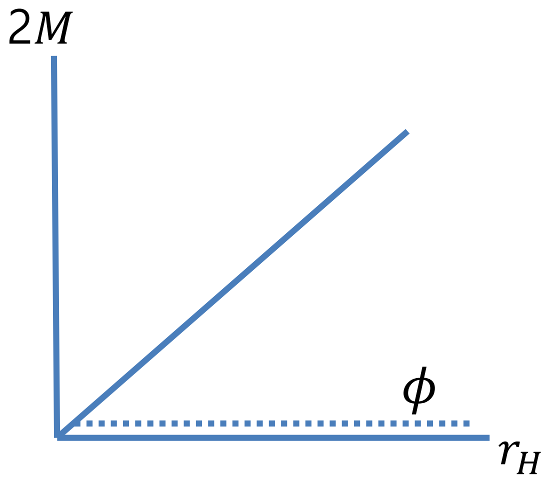


Black Hole solution

$$ds^2 = -\left(1-\frac{2M}{r}\right) dt^2 + \frac{dr^2}{\left(1-\frac{2M}{r}\right)} + r^2 d\Omega^2$$

Horizon

$r_H = 2M$



$\phi = 0$ No hair

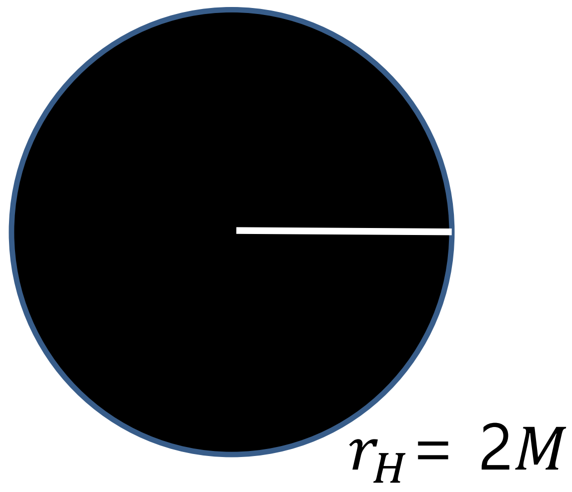
Review : AdS Black Holes

Brown, Creighton & Mann (1994)

Action

$$S = \int d^4x\,\sqrt{-g}\left[\frac{1}{2\kappa^2}\,R + \Lambda - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right]$$

where $g = \det g_{\mu\nu}$ and $\kappa \equiv 8\pi G$



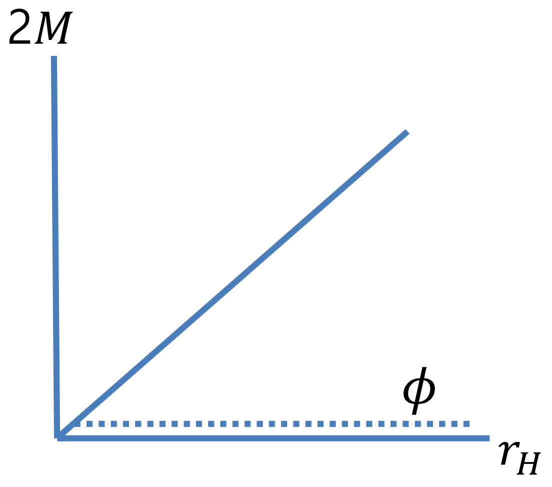
Black Hole solution

$$ds^2 = -N^2(r)\,dt^2 + f^{-2}(r)\,dr^2 + r^2 d\Omega^2$$

$$N^2(r) = f^2(r) = (1 - \frac{2M}{r} + \frac{r^2}{l^2})$$

Horizon $r_H = 2M$

$\phi = 0$ No hair



Note :

1) For $\Lambda = 0$, the theory becomes the Einstein gravity.

2-1) Einsten Gauss-Bonnet (EGB) theory

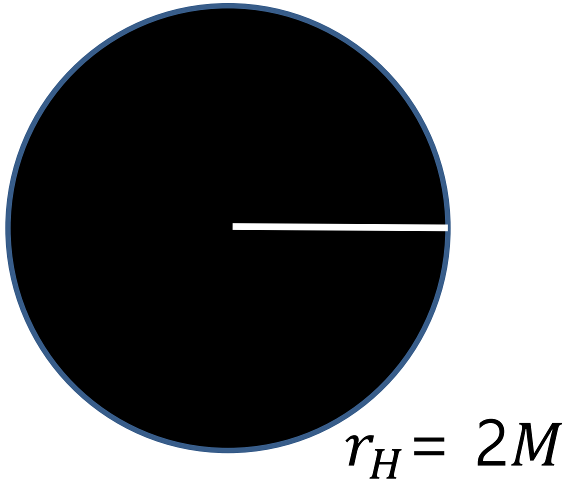
W.Ahn, B. Gwak, BHL, W.Lee, Eur.Phys.J.C (2015)

Action

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa^2} R + \alpha R^2_{GB} - \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right]$$

where $g = \det g_{\mu\nu}$ and $\kappa \equiv 8\pi G$ $\quad R^2_{\rm GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$

The Gauss-Bonnet term

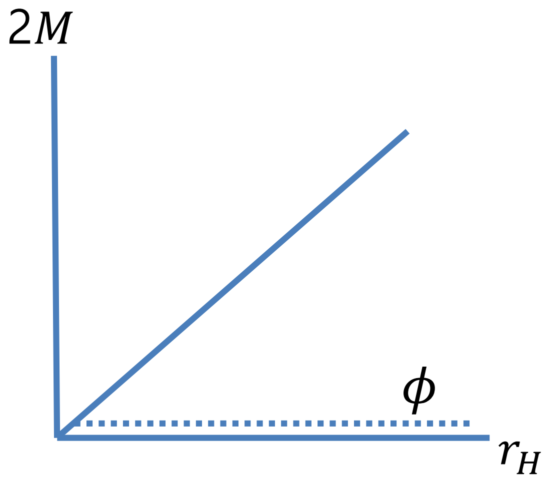


Black Hole solution

$$ds^2 = -\left(1-\frac{2M}{r}\right) dt^2 + \frac{dr^2}{\left(1-\frac{2M}{r}\right)} + r^2 d\Omega^2$$

$\phi = 0$ No hair

Horizon $r_H = 2M$



Note :

1) For the coupling $\alpha = 0$, the theory becomes the Einstein gravity.

2) GB term is a surface term, not affecting the e.o.m. Hence,
The black hole solution is the same as that of the Schwarzschild one.

3) However, the GB term contributes to the black hole entropy and influence stability.

2-2) Dilaton-Einstein-Gauss-Bonnet (DEGB) theory : Hairy black holes

Action

$$I = \int_{\mathcal{M}} \sqrt{-g} d^4x \left[\frac{R}{2\kappa} - \frac{1}{2} \nabla_\alpha \Phi \nabla^\alpha \Phi + \boxed{\alpha e^{-\gamma\Phi} R^2_{\rm GB}} \right] + \oint_{\partial\mathcal{M}} \sqrt{-h} d^3x \frac{K - K_o}{\kappa} , \quad (1)$$

where $g = \det g_{\mu\nu}$ and $\kappa \equiv 8\pi G$

Guo, N.Ohta & T.Torii, Prog.Theor.Phys. 120,581(2008);121 ,253 (2009);
N.Ohta &Torii,Prog.Theor.Phys.121,959; 122,1477(2009);124,207 (2010);
K.i.Maeda,N.Ohta Y.Sasagawa, PRD80, 104032(2009); 83,044051 (2011)
N. Ohta and T. Torii, Phys.Rev. D 88 ,064002 (2013).

The GB term :

$$R^2_{\rm GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

Note :

1) For $\gamma = 0$, DEGB theory becomes the Einstein-Gauss-Bonnet (EGB) theory,
with the GB term becoming the boundary term

2) The symmetry under $\gamma \to -\gamma, \Phi \to -\Phi$. allows choosing γ positive without loss of generality.

3) α scaling :The coupling α dependency could be absorbed by the $r \to r/\sqrt{\alpha}$ transformation.
However, the behaviors for the α = 0 case cannot be generated in this way.
Hence, we keep the parameter α, to show a continuous change to α = 0.

Q : signature of α ?

The Einstein equations and the scalar field equation are

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa\left(\partial_\mu\Phi\partial_\nu\Phi - \frac{1}{2}g_{\mu\nu}\partial_\rho\Phi\partial^\rho\Phi + T^{GB}_{\mu\nu}\right), \quad (2)$$

$$\frac{1}{\sqrt{-g}}\partial_\mu[\sqrt{-g}g^{\mu\nu}\partial_\nu\Phi] - \alpha\gamma e^{-\gamma\Phi}R^2_{GB} = 0, \quad (3)$$

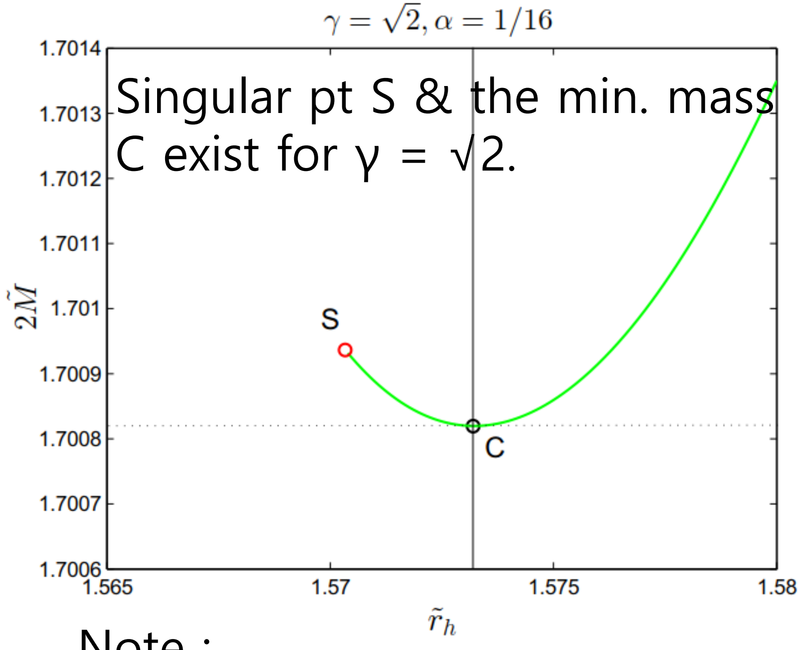
Note :

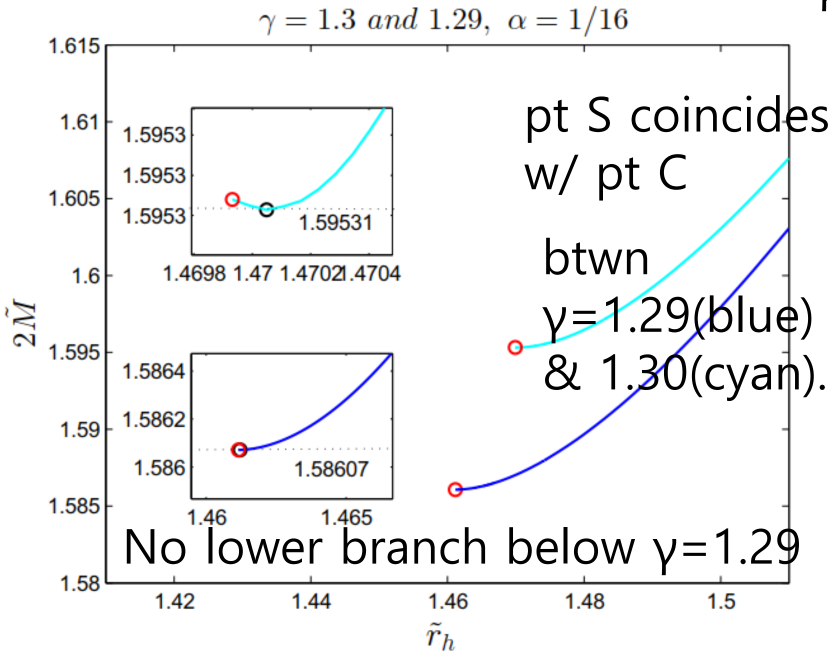
1) All the black holes in the DEGB theory with given non-zero couplings α and γ have hairs.
I.e., there does not exist black hole solutions without a hair in DEGB theory.
(If we have Φ = 0, dilaton e.o.m. reduces to R^2_{GB} = 0. so it cannot satisfy the dilaton e.o.m..)

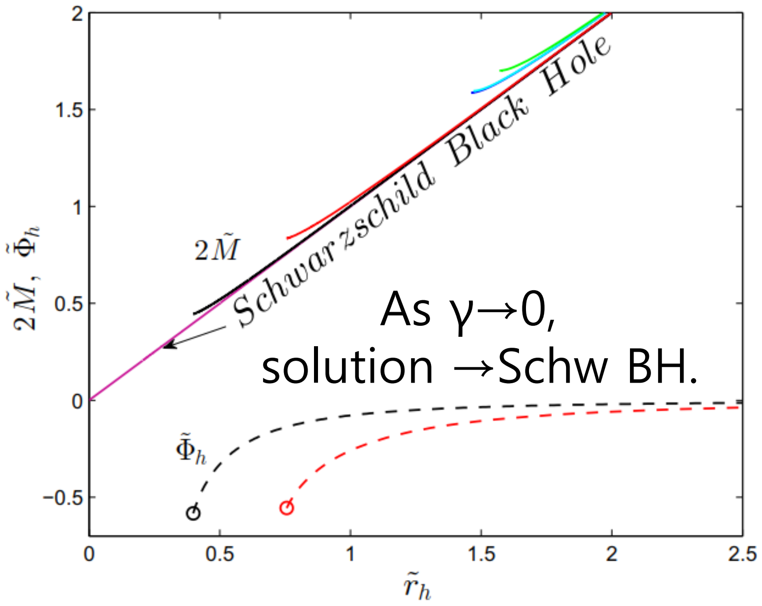
2) Hair Charge Q is not zero, and is not independent charge either.

Coupling γ dependency of the minimum mass for fixed α = 1/16 >0.

γ=√2(green),γ=1.3(cyan),γ=1.29(blue)
γ=1/2(red), γ=1/6(black), γ=0(purple)







Note :

1. For large γ, sing. pt S & extremal pt C (with minimum mass $\tilde{M}$) exist.
2. The solutions between point S and C are unstable for perturbations and end at the singular point S , I.e., there are two black holes for a given mass in which the smaller one is unstable under perturbations.
3. As γ smaller, the singular point S gets closer to the minimum mass point C.
4. Below γ=1.29, the solutions are perturbatively stable and approach the Schwarzschild black hole in the limit of γ going to zero. These solutions depend on the coupling γ.
5. If DEGB BH horizon becomes larger, the scalar field goes to 0, and the BH becomes a Schwarzschild BH.

GB term → "repulsive" gravity effects !!!

Q: How about the properties, such as Stability & implication to the cosmology, etc ?

Note : Black Hole Stability

Perturbative Gravitational (in)stability

Perturbations of a black hole space-time

by adding fields or by perturbing the metric.

The typical equations in the linear approximation :

$$-\frac{d^2 R}{dr_*^2} + V(r,\omega)R = \omega^2 R,$$

Quasinormal modes :
solutions of the wave equation, satisfying specific boundary conditions at the black hole horizon and far from the black hole.

The quasinormal spectrum of a stable black hole is an infinite set of complex frequencies which describes damped oscillations.

If there is at least one growing mode, the space−time is unstable with the instability growth rate proportional to the imaginary part of the growing QNM.

Note : Most of the 4 dim. BHs are perturbatively stable.

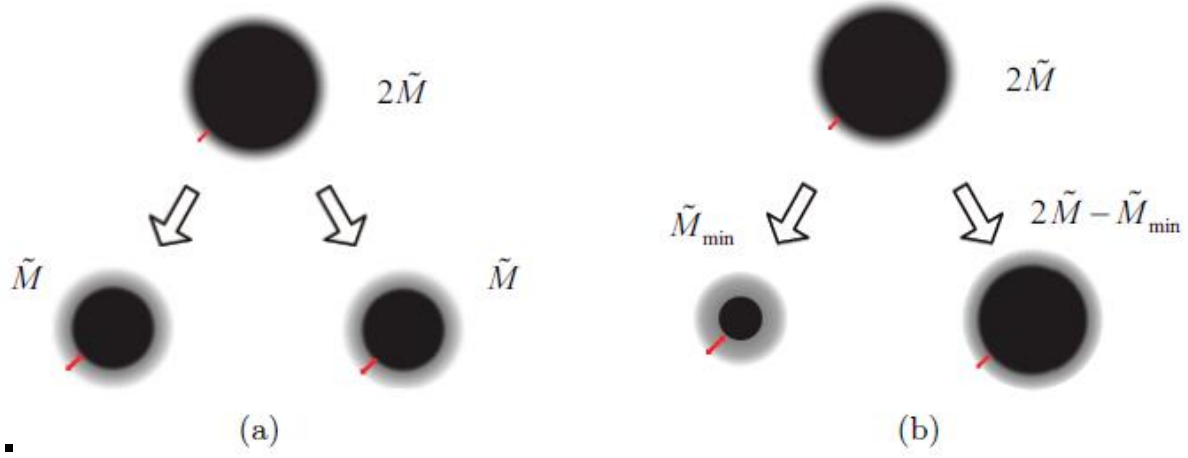
NonperturbativeBlack Hole Stability

Fragmentation instability

is based on the entropy preference between the solutions.

Emparan and Myers, JHEP 0309, 025 (2003).

Apply thermodynamic 2nd law to initial (one black hole) and final(fragmented two black holes) phase.



entropy of 1 BH < entropy of 2 fragmented BHs

$\longrightarrow$ (transition to) instability

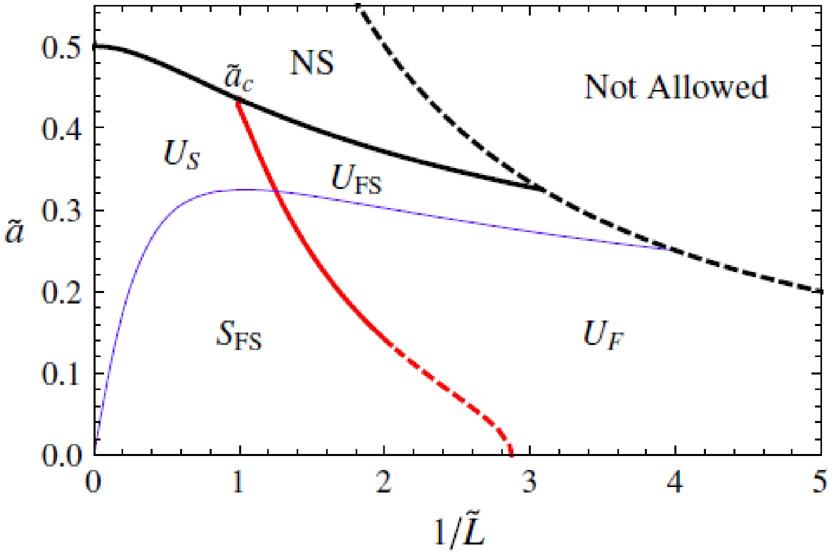
Myers-Perry blackhole : Rotating Black hole in higher dimensions
There doesn't exist any upper limit on the angular momentum

Myers-Perry blackhole becomes unstable for large angular momentum into fragmentation.

RN blackhole is also thermodynamically unstable in specific parameter region.

Rotating AdS black hole and charged AdS black hole show the fragmentation instability in some parameter range

Fragmentation allows the upper or lower bound of black hole charges.



B. Gwak and BHL, PRD91 (2015) 6, 064020.

B.Gwak, BHL, D. Rho, Phys.Lett. B761 (2016)

S_{FS} stable under both fragmentation and superradiance.
U_F unstable under fragmentation.
U_S unstable under superradiance.
U_{FS} unstable under both fragmentation and superradiance.

This final phase is specified by a mass ratio $\delta = \frac{\tilde{m}}{\tilde{M}}$. We denote final phase as $(\delta, 1-\delta)$. $\bar{\delta} \leq \boldsymbol{\delta} \leq$ ½ $\bar{\delta}$ minimum mass ratio given as $\bar{\delta} = \frac{\tilde{M}_{min}}{\tilde{M}}$. The minimum mass ratio has a finite value in DGB black hole, because the black hole has minimum mass $\tilde{M}_{min}$. The black holes can be fragmented only when it exceeds twice of minimum mass. With a black hole mass below twice of minimum mass, there are no fragmented black hole solutions, so these black holes are absolutely stable.

The mass and momenta of the black hole are related

$$\tilde{M} = \sqrt{(\delta\tilde{M})^2 + P_1^2} + \sqrt{(1-\delta)^2\tilde{M}^2 + P_2^2}\,. \qquad (21)$$

The linear momenta are arbitrary, so we set $P_1 = P_2 = 0$ to maximize the total entropy of the final phase. In this condition, the black hole slightly breaks into two black holes with negligible momenta. The initial phase decays to the final phase if the entropy is larger than that of the initial phase.

For Schwarzschild black hole

$$\frac{S_f}{S_i} = \frac{(\delta\,\tilde{r}_h)^2 + ((1-\delta)\,\tilde{r}_h)^2}{\tilde{r}_h^2} = \delta^2 + (1-\delta)^2\,, \qquad (22)$$

The entropy ratio is always smaller than 1.
Therefore, a Schwarzschild black hole is always stable under fragmentation.
The entropy ratio marginally approaches 1 in
$\delta \to 0,$

These phenomena become different in the theory with the higher order of curvature term.

Fragmentation Instability for DGB Black Holes

We investigate the fragmentation instability using a numerical analysis.

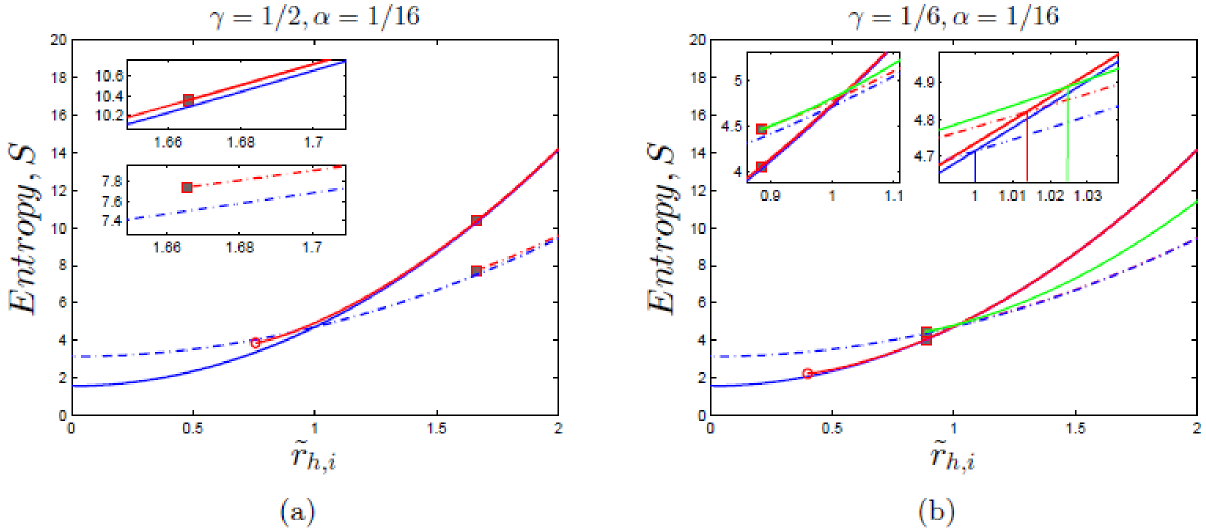
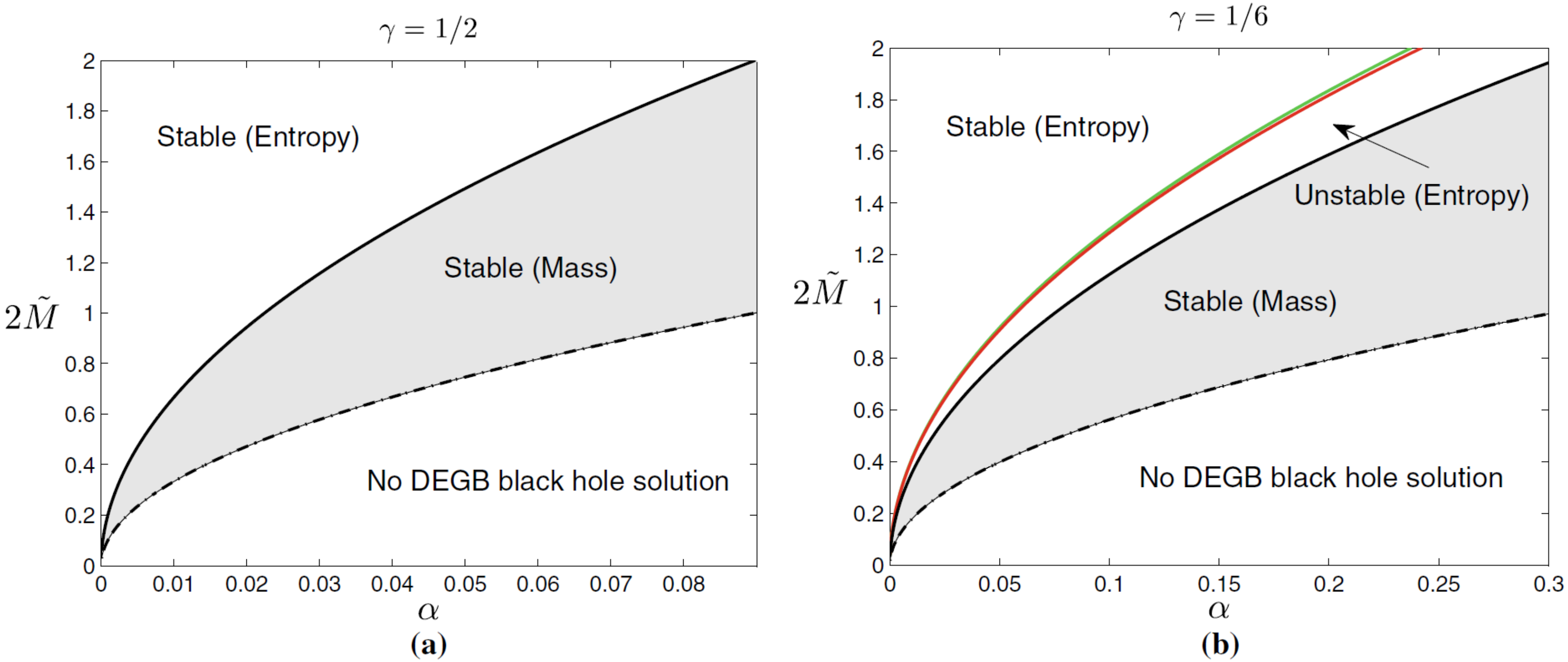
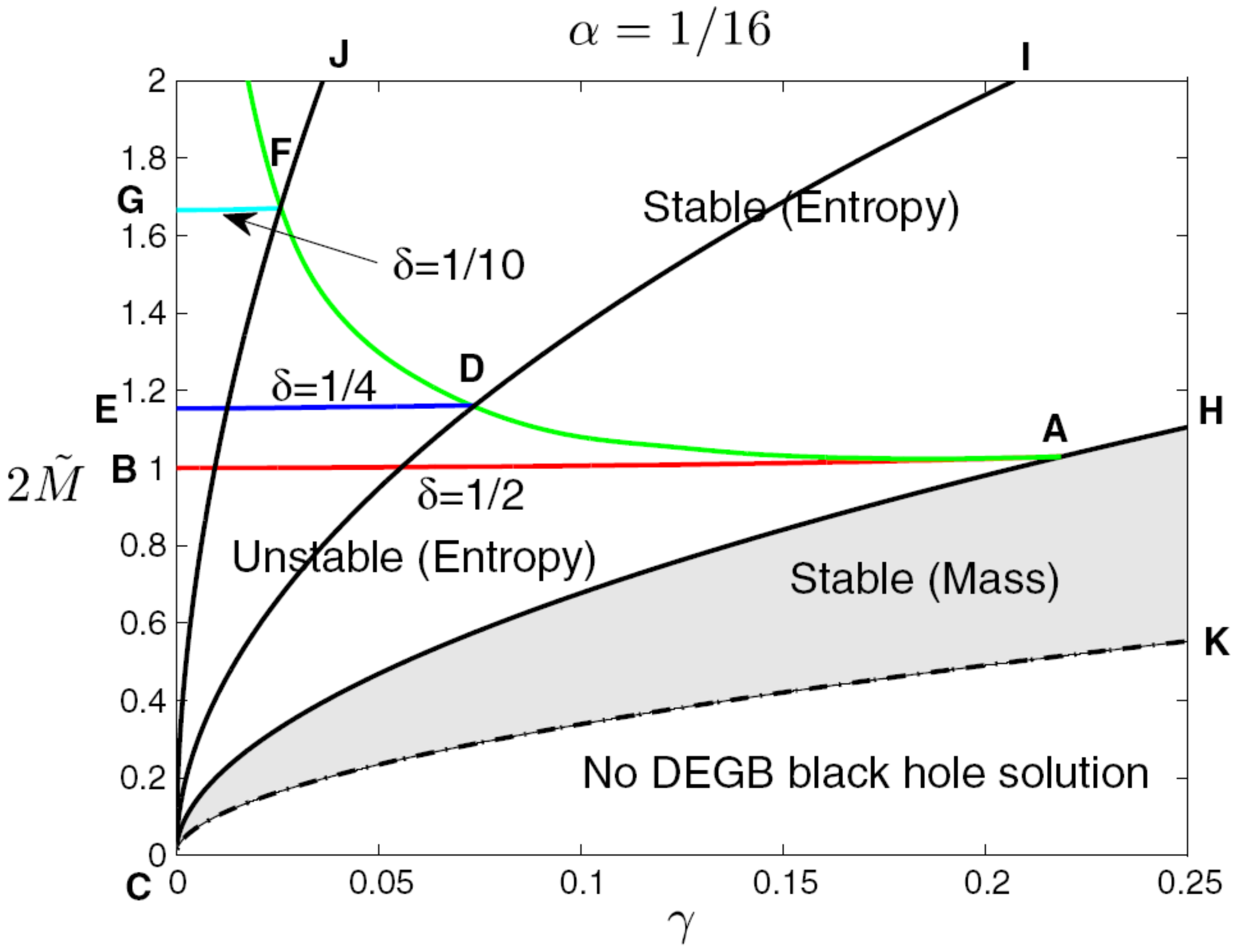


Figure 6: The initial and final phase entropies with respect to $r_{h,i}$ for given couplings γ and α. The blue solid line and blue dashed-dot line are initial and final phase entropies in EGB theory as a reference for $(\frac{1}{2}, \frac{1}{2})$. The red solid line and red dashed-dot line are initial and final phase entropies in DGB theory for $(\frac{1}{2}, \frac{1}{2})$. Initial phase exists above red circle for minimum mass. Final phase exists above red box for $(\frac{1}{2}, \frac{1}{2})$. The green solid line represents fragmentation for marginal mass ratio $\bar{\delta}$.



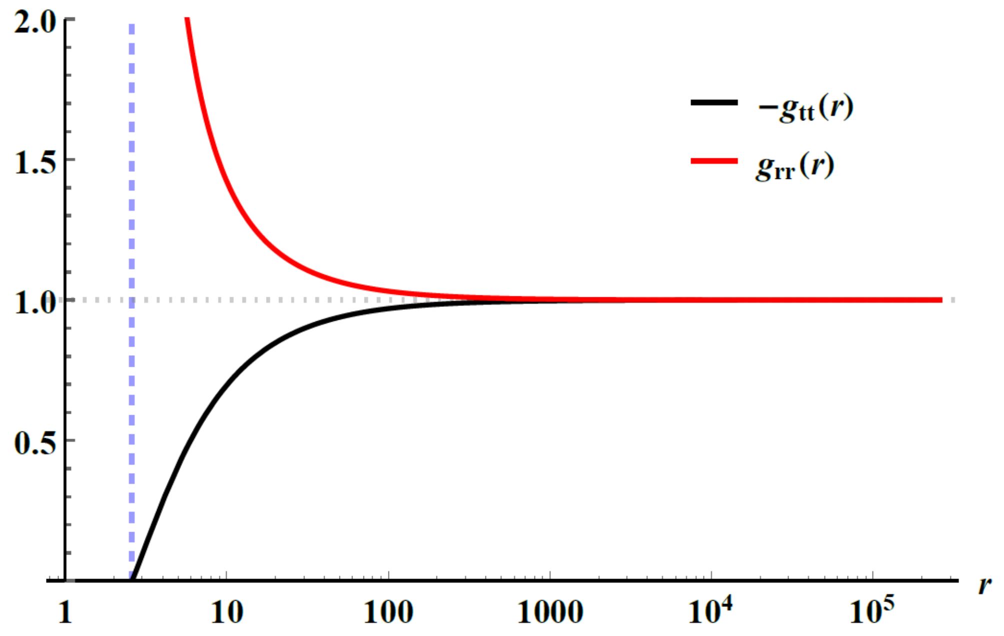
The phase diagrams with respect to α and $\widetilde{M}$ in fixed γ .
The red solid line represents (1/2 ,1/2) fragmentation.
The green solid line represent $(\bar{\delta}, 1-\bar{\delta})$ fragmentation



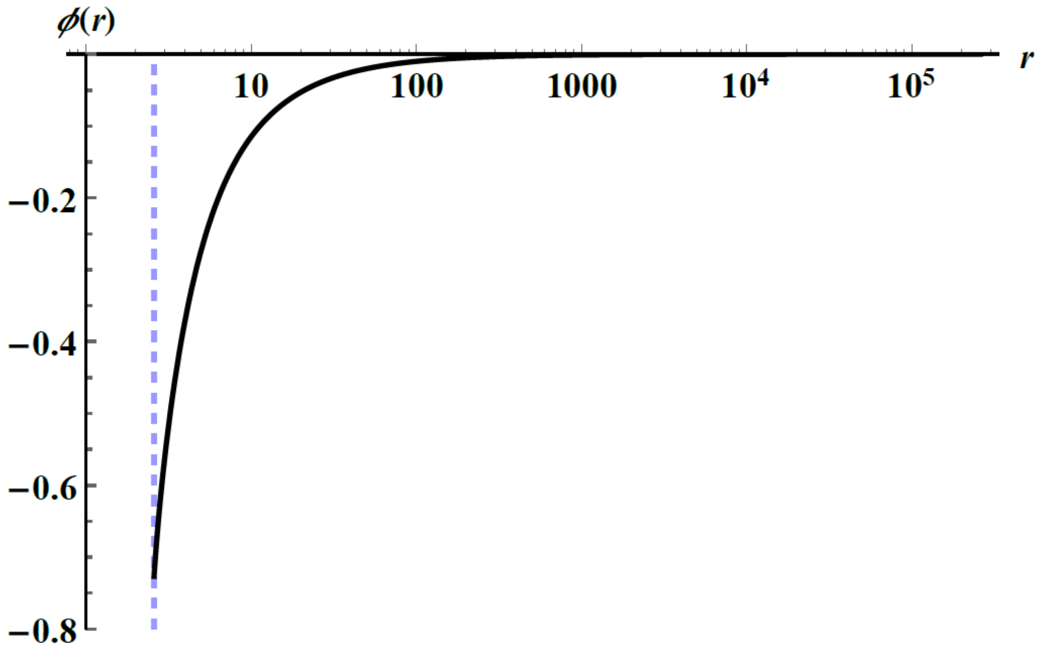
The phase diagrams with respect to γ and $\widetilde{M}$ for fixed α for (1/2 ,1/2) (red solid line), (1/4 , 3/4) (blue solid line), (1/10 , 9/10) (cyan solid line), and $(\bar{\delta}, 1-\bar{\delta})$ (green solid line) fragmentation

A Dilaton Black Hole solution (α < 0)

BHL, W. Lee, D. Rho,
in preparation



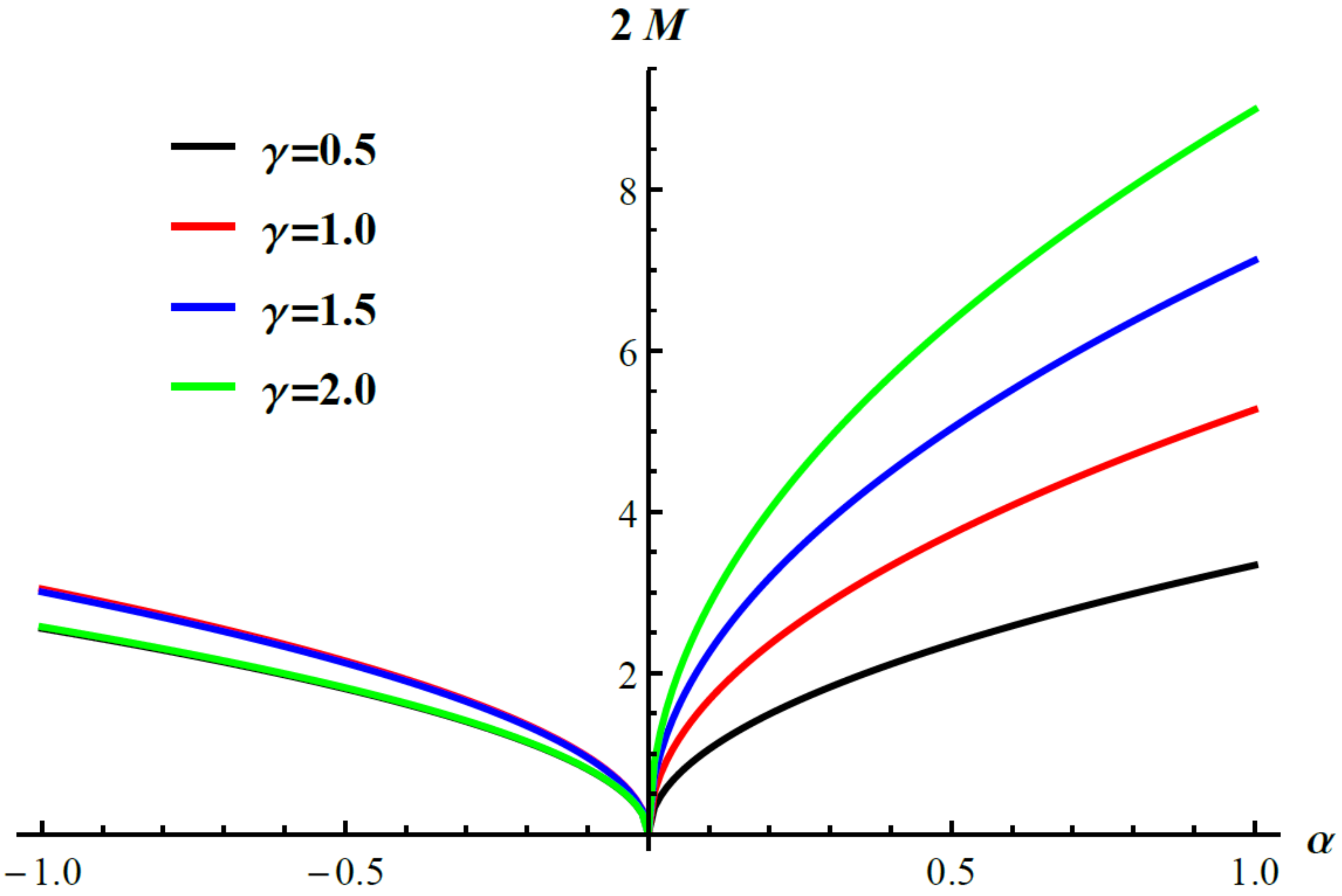
(a) $-g_{tt}(r)$ and $g_{rr}(r)$ vs. r



(b) $\phi(r)$ vs. r

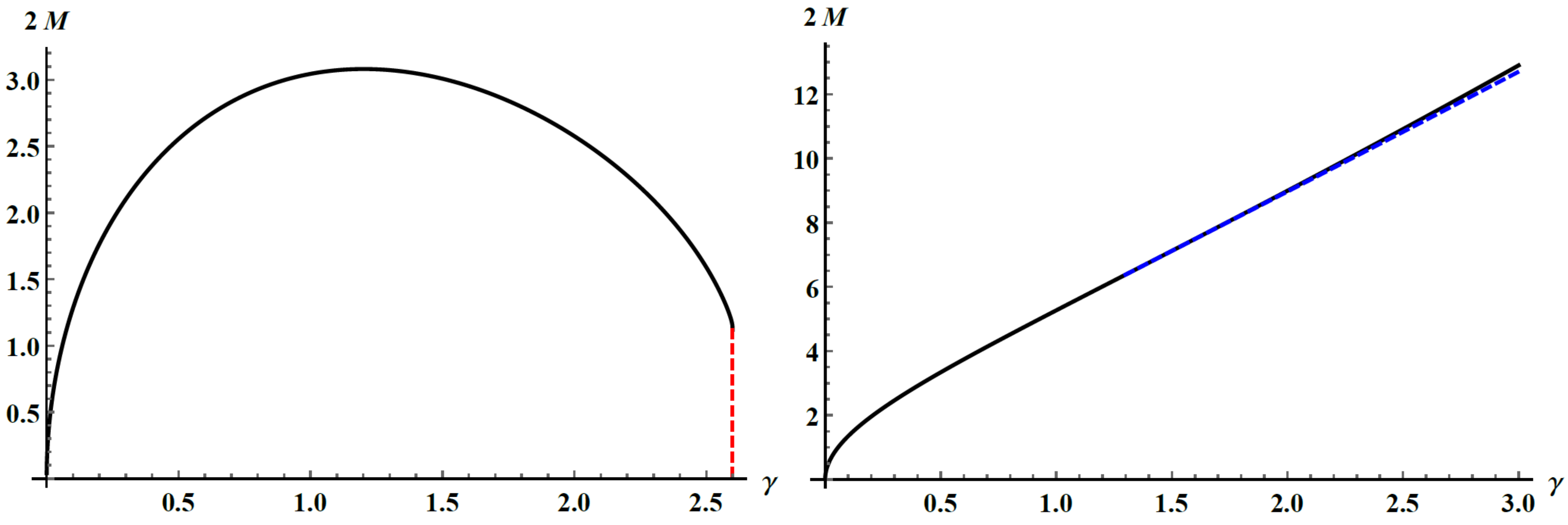
Figure 1: A dilaton black hole solution.

The mass of dilation black hole vs. α with several γ values



Remark : "Attractive" "repulsive"

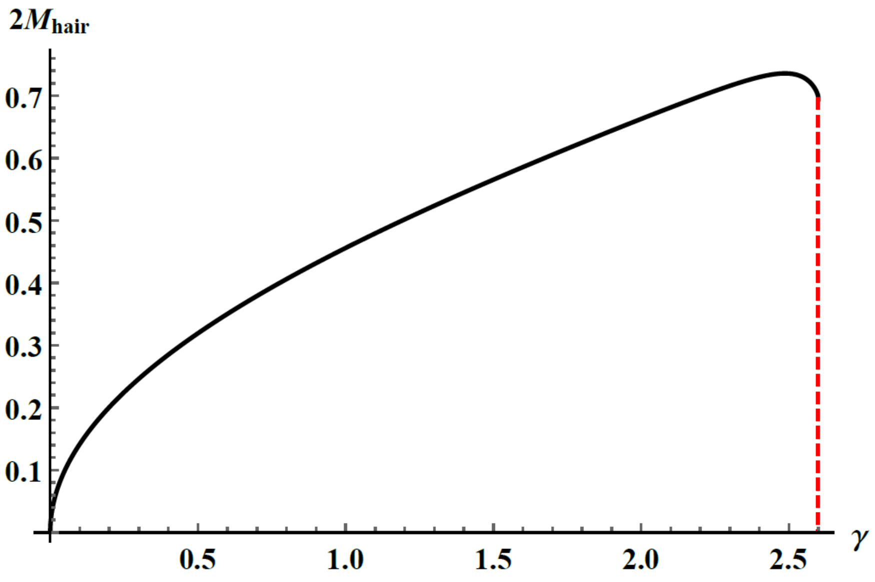
Mass versus γ



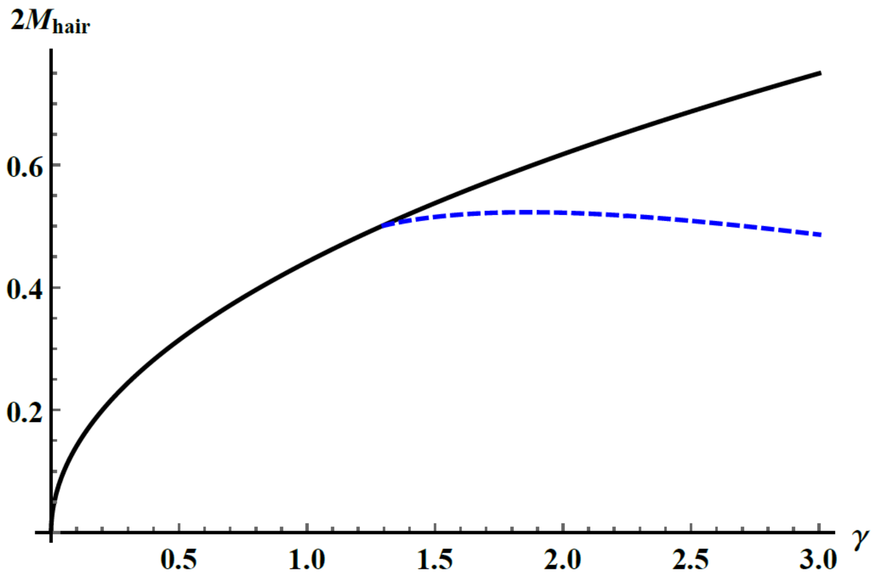
(a) The black hole mass vs. γ with $\alpha = -1$.

(b) The black hole mass vs. γ with $\alpha = 1$.

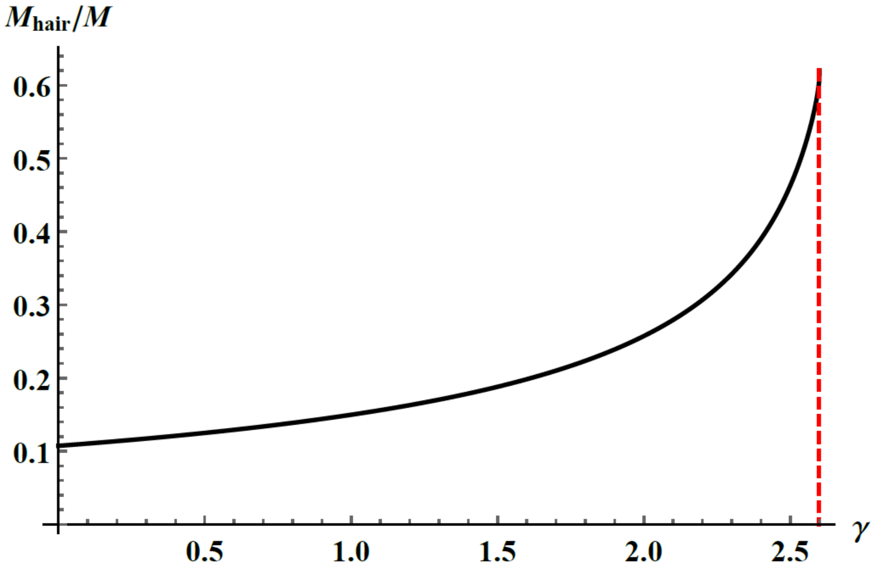
Figure 5: Several mass figures with respect to γ. The black line represent the black holes with maximum value of ϕ_h which are having minimum black hole radius r_h. The red dashed line represents the maximized γ to get the black hole solution with negative α. The blue dashed line represent the black holes having the minimum masses.



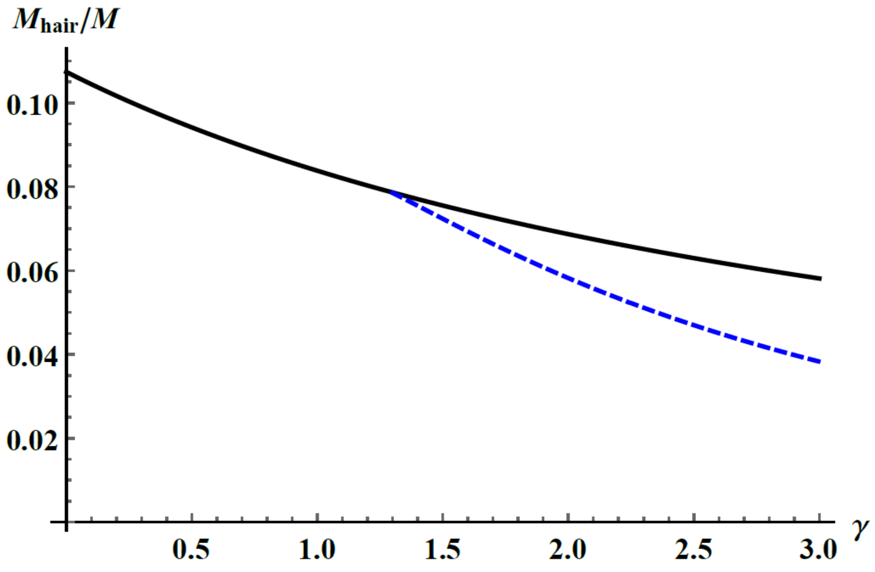
(c) The hairy mass vs. γ with $\alpha = -1$.



(d) The hairy mass vs. γ with $\alpha = 1$.



(e) The hairy mass ratio vs. γ with $\alpha = -1$.



(f) The hairy mass ratio vs. γ with $\alpha = 1$.

4.Summary

Holography Principle : Finite temperature system <-> Black Hole Geometry

Running coupling constants -> Needs beyond AdS geometry

We have studied the Black Hole with Gauss-Bonnet term

- Numerically constructed the static DGB **hairy** black hole in asymptotically flat spacetime

  There exists **minimum mass**, etc. ("repulsive" gravity effect)
- **Fragmentation instability of black holes:**

The BH solution and its properties are strongly dependent on the signature of the Gauss-Bonnet term

When the scalar field on the horizon is the maximum, the DGB black hole solution has the minimum horizon size.

The amount of black hole hair decreases as the DGB black hole mass increases. DGB black hole configurations go to EGB black hole cases for small α **and** γ.

Summary - continued

Most of the 4–dim. black holes are perturbatively stable.

The DGB black hole phase is unstable under fragmentation, even if these phases are stable under perturbation.

We have found the phase diagram of the fragmentation instability for a black hole mass and two couplings.

There exists region
unstable under fragmention while perturbatively stable.

Further details of the BH properties for $\alpha < 0$ are under investigation.

Implication to the finite temperature systems still needs further study.

Thank You!