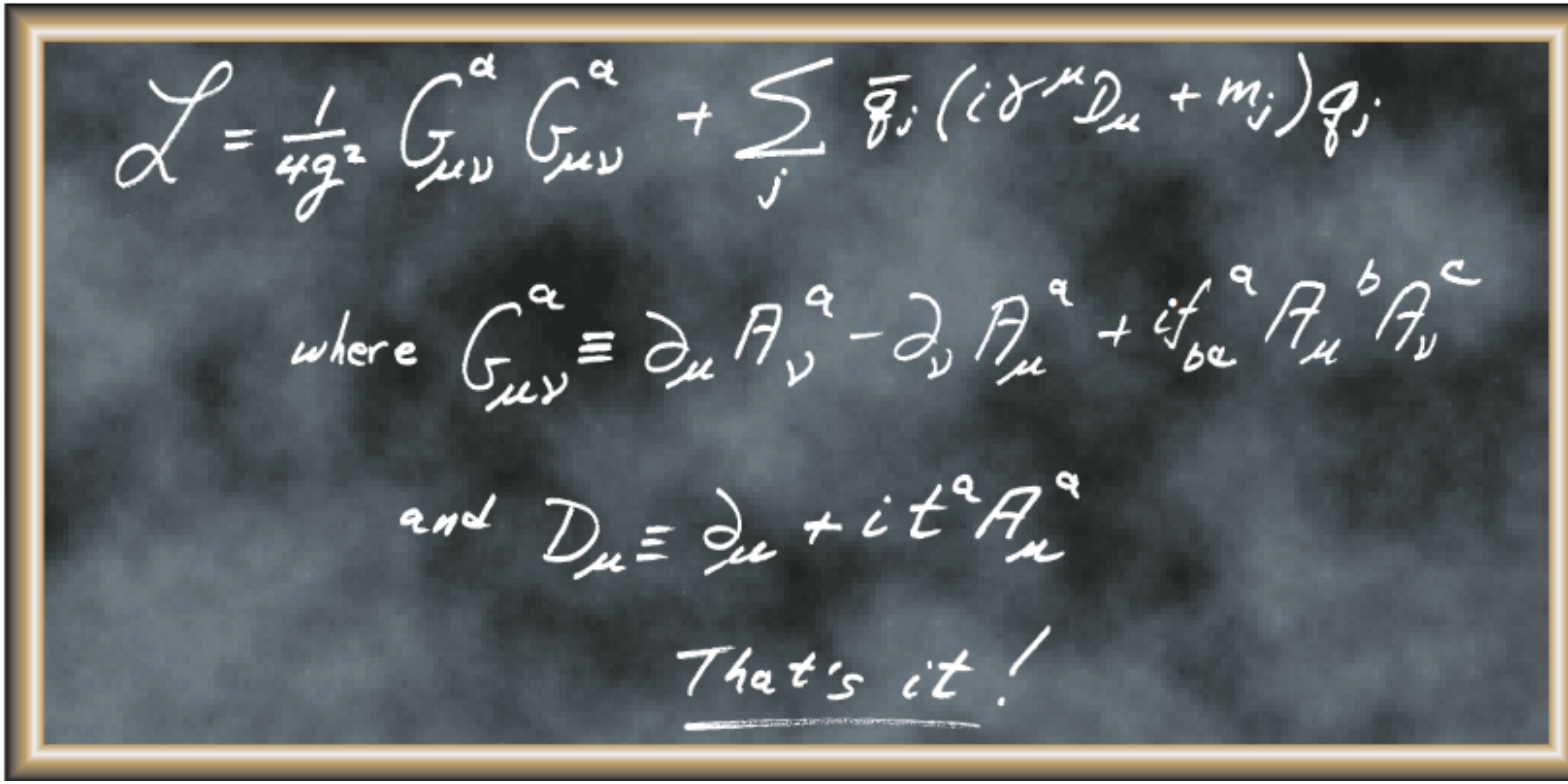
12th APCTP-BLTP JINR JOINT WORKSHOP
Modern Problems in Nuclear and Elementary Particle Physics
Busan (Korea)

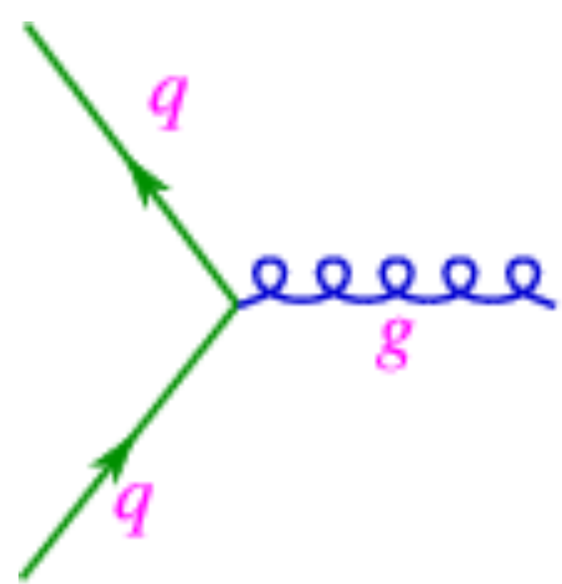
The elusive Glueballs

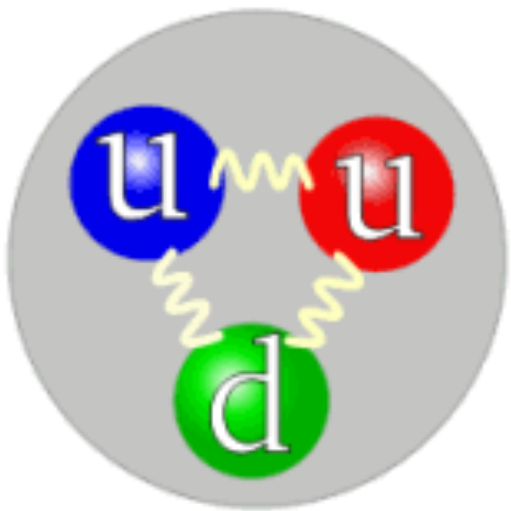
Vicente Vento
IFIC
Universidad de Valencia
Spain

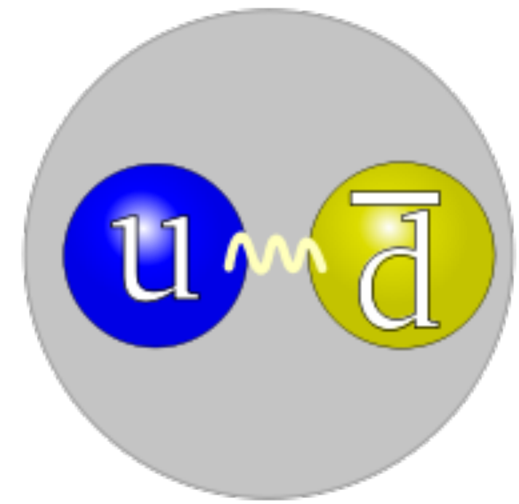
Nikolai Kochelev
Simon Kiesewetter
Vincent Mathieu
Matteo Rinaldi

Introduction





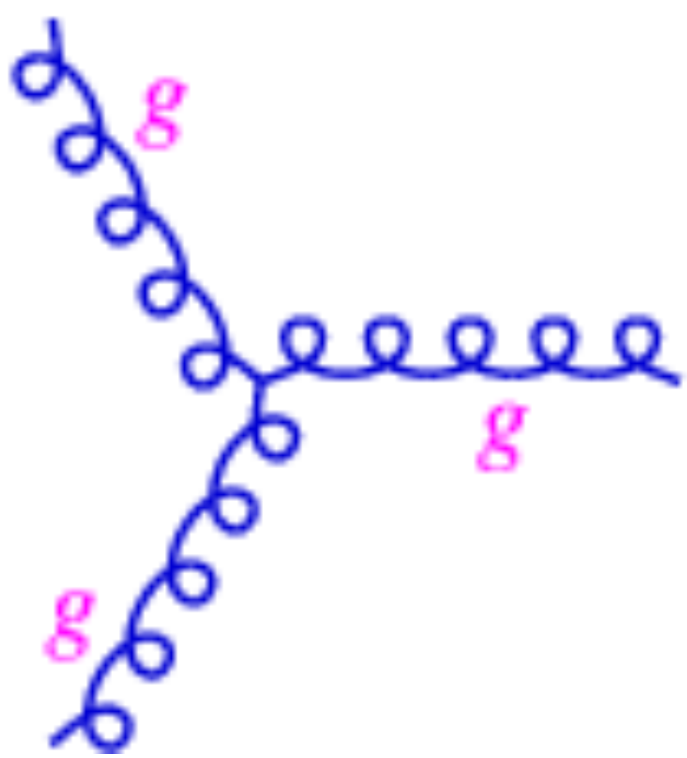


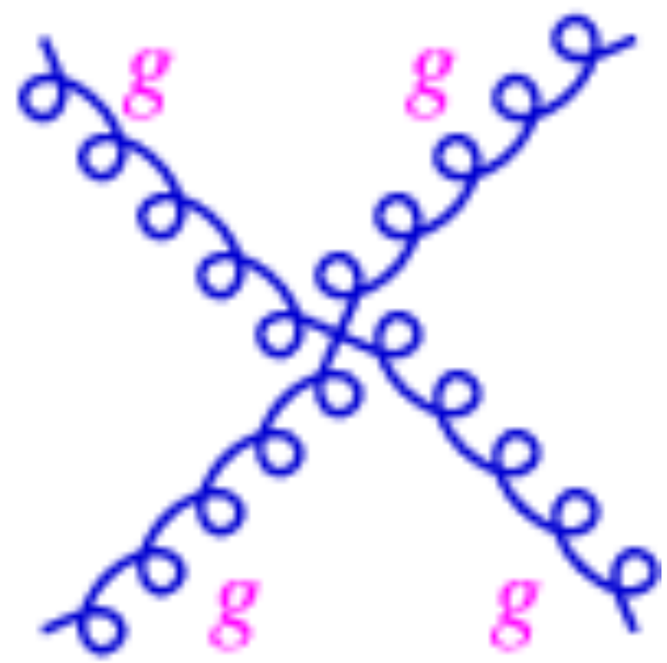


The problem is always to understand the vacuum !

$$\mathcal{L} = \frac{1}{4g^2} G^a_{\mu\nu} G^a_{\mu\nu}$$

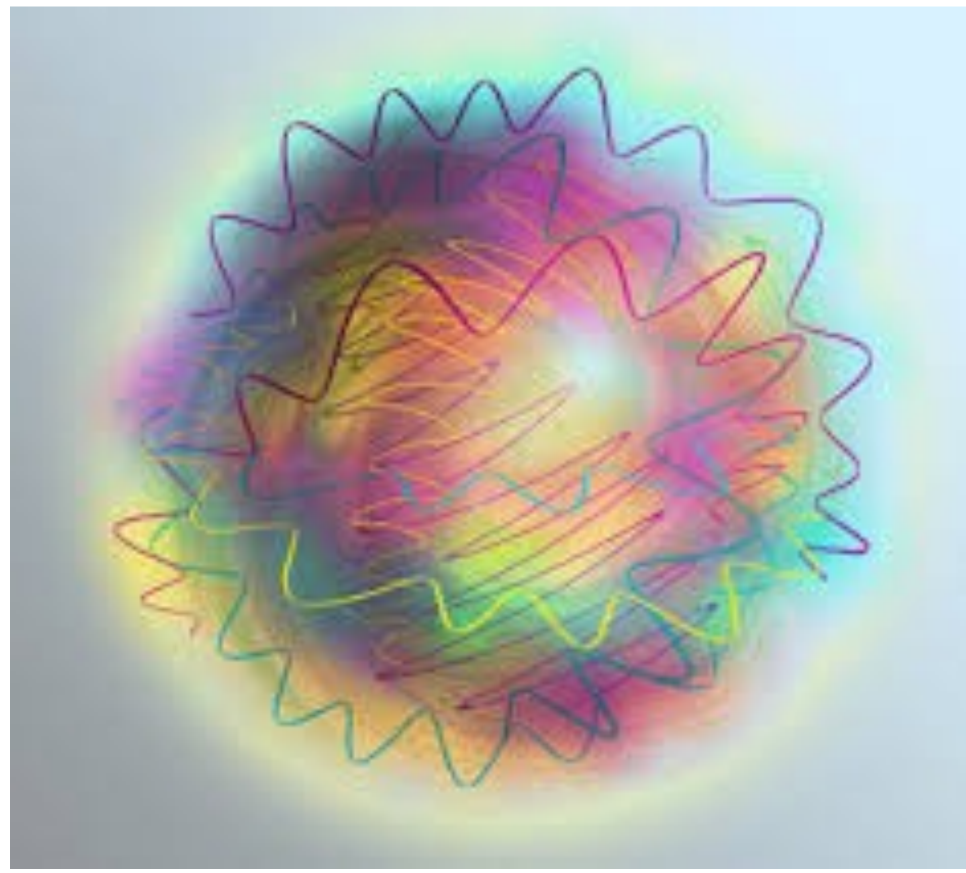
$$G^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + i f^a_{bc} A^b_\mu A^c_\nu$$





1975-Fritzsch-Minkowski
Glueballs

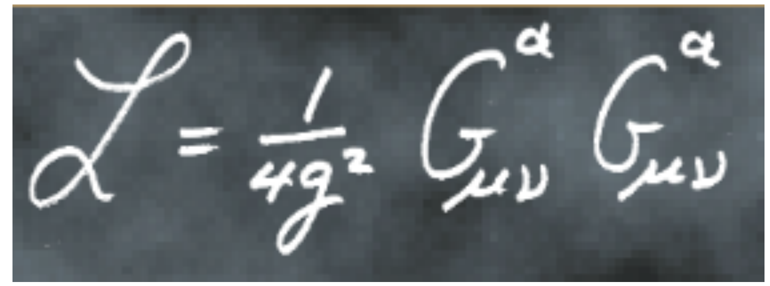
8 x 8 = 1+…   8 x 8 x 8 = 1 + …

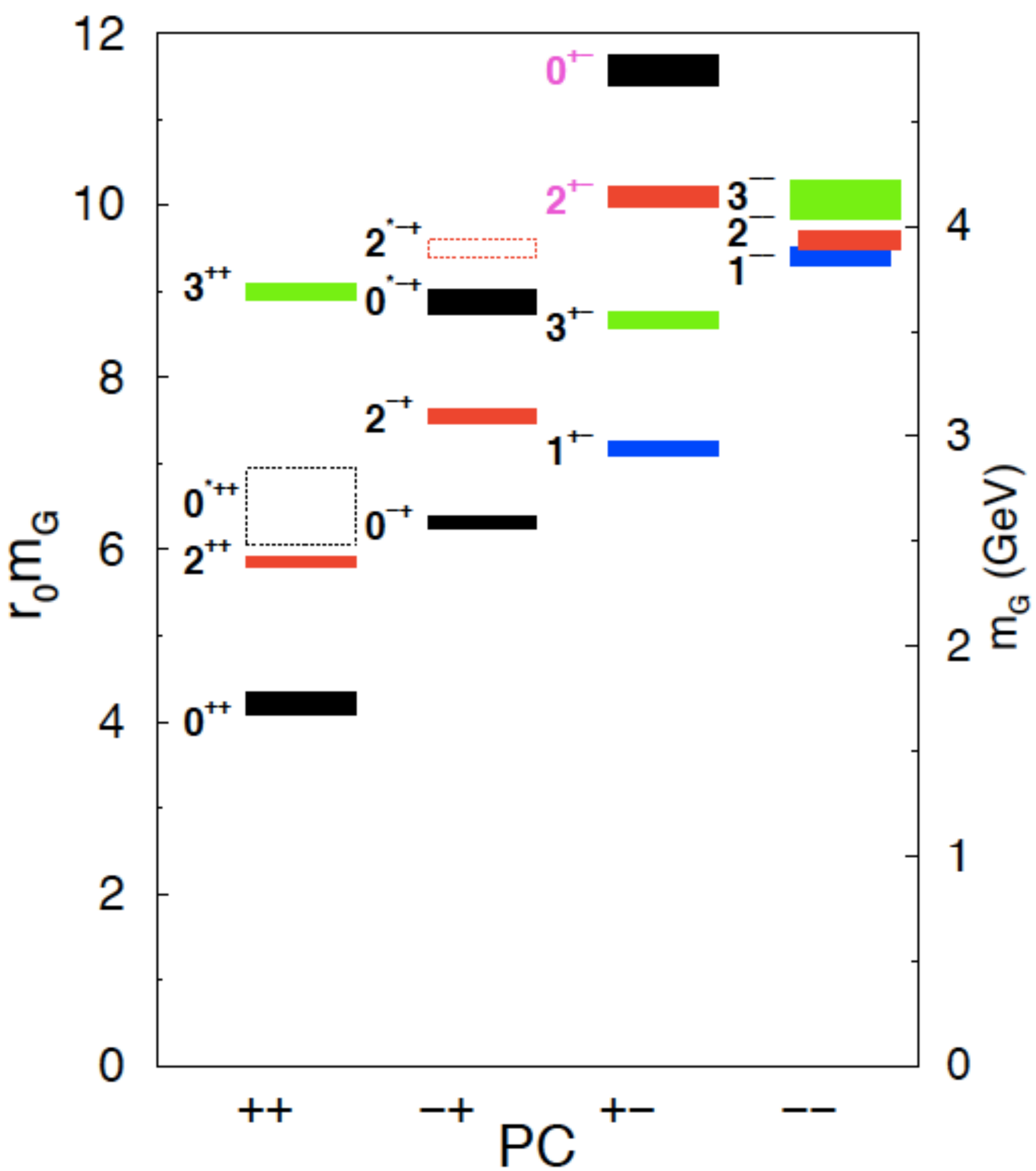


The existence of glueballs would provide an experimental tool for understanding the behaviour of soft gluonic fields and the vacuum in QCD which would be instrumental in understanding the confinement mechanism.

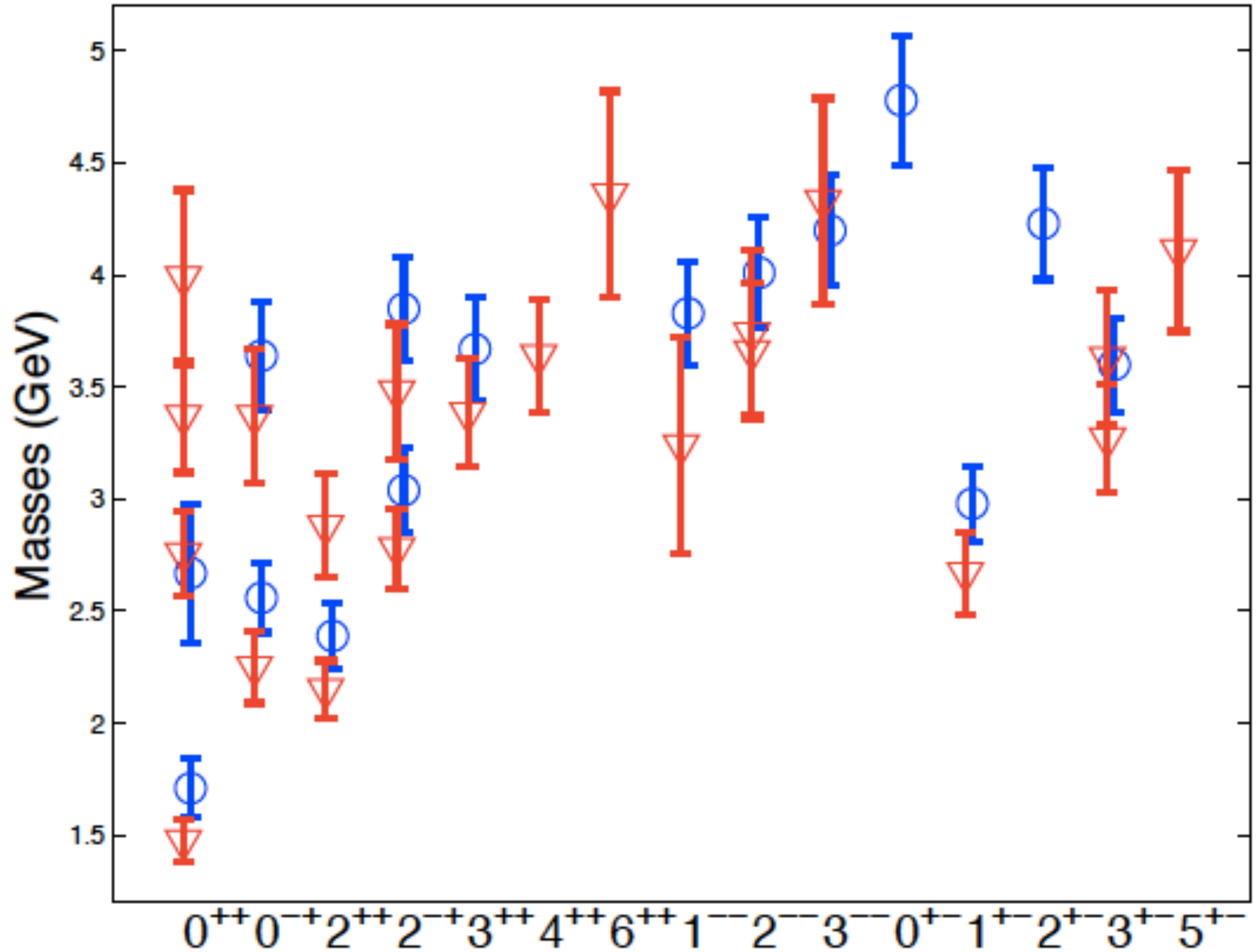
Glueballs Theory

Lattice QCD



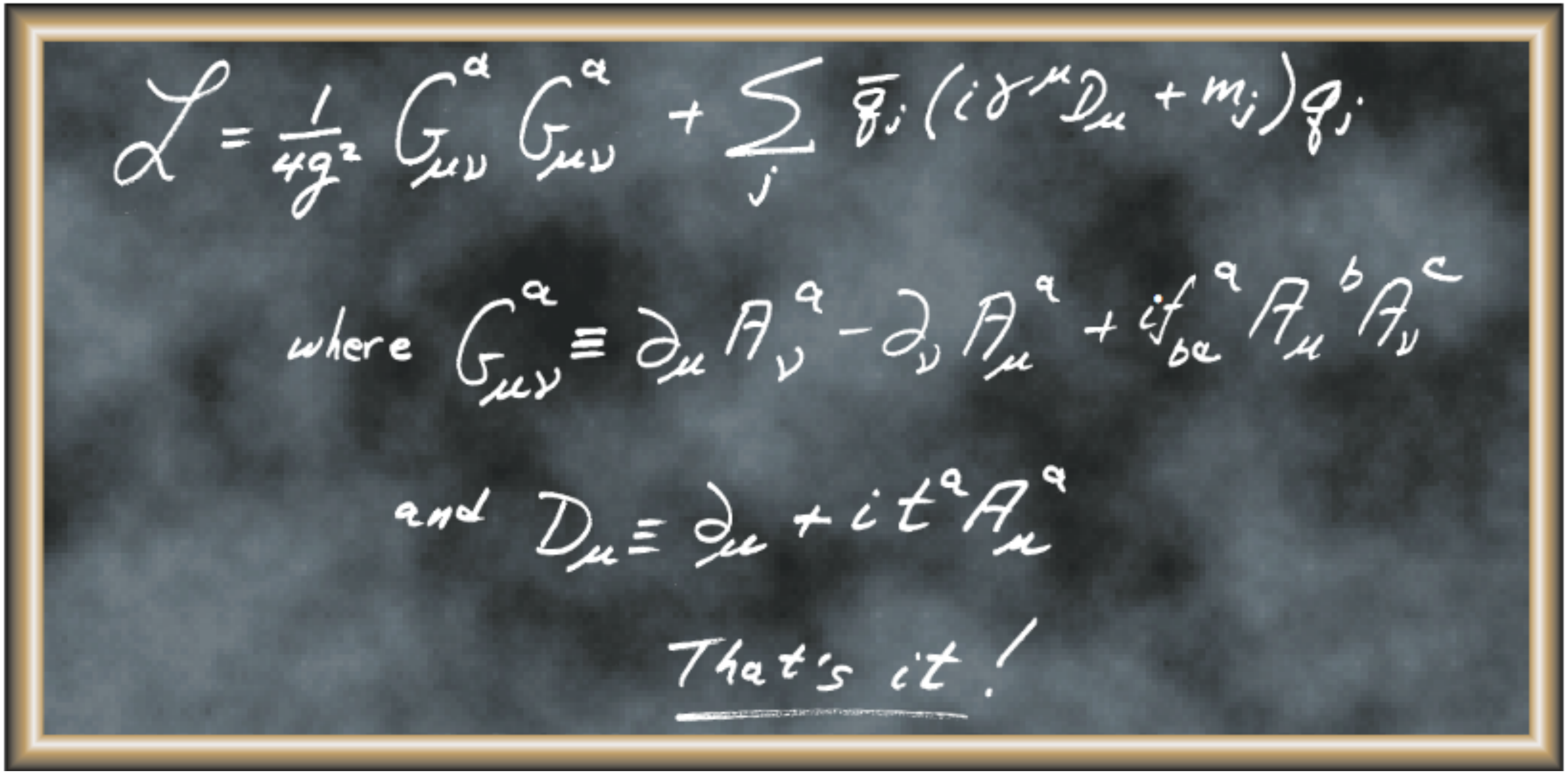


J^{PC}	Other J	$r_0 m_G$	m_G (MeV)
0^{++}		4.21 (11)(4)	1730 (50)(80)
2^{++}		5.85 (2)(6)	2400 (25)(120)
0^{-+}		6.33 (7)(6)	2590 (40)(130)
0^{*++}		6.50 (44)(7)†	2670 (180)(130)
1^{+-}		7.18 (4)(7)	2940 (30)(140)
2^{-+}		7.55 (3)(8)	3100 (30)(150)
3^{+-}		8.66 (4)(9)	3550 (40)(170)
0^{*-+}		8.88 (11)(9)	3640 (60)(180)
3^{++}	6, 7, 9, ...	8.99 (4)(9)	3690 (40)(180)
1^{--}	3, 5, 7, ...	9.40 (6)(9)	3850 (50)(190)
2^{*-+}	4, 5, 8, ...	9.50 (4)(9)†	3890 (40)(190)
2^{--}	3, 5, 7, ...	9.59 (4)(10)	3930 (40)(190)
3^{--}	6, 7, 9, ...	10.06 (21)(10)	4130 (90)(200)
2^{+-}	5, 7, 11, ...	10.10 (7)(10)	4140 (50)(200)
0^{+-}	4, 6, 8, ...	11.57 (12)(12)	4740 (70)(230)



J^{PC}	0^{++}	2^{++}	0^{++}	2^{++}	0^{++}	0^{++}
MP	1730 ± 94	2400 ± 122	2670 ± 222			
YC	1719 ± 94	2390 ± 124				
LTW	1475 ± 72	2150 ± 104	2755 ± 124	2880 ± 164	3370 ± 180	3990 ± 277
Lattice	1631 ± 50	2313 ± 68	2713 ± 127	2880 ± 164	3370 ± 180	3990 ± 277

[5] E. Gregory, A. Irving, B. Lucini, C. McNeile, A. Rago, C. Richards and E. Rinaldi, JHEP **1210** (2012) 170 [arXiv:1208.1858 [hep-lat]].
[6] C. J. Morningstar and M. J. Peardon, Phys. Rev. D **60** (1999) 034509 [hep-lat/9901004].
[7] Y. Chen, A. Alexandru, S. J. Dong, T. Draper, I. Horvath, F. X. Lee, K. F. Liu and N. Mathur *et al.*, Phys. Rev. D **73** (2006) 014516 [hep-lat/0510074].
[8] B. Lucini, M. Teper and U. Wenger, JHEP **0406** (2004) 012 doi:10.1088/1126-6708/2004/06/012 [hep-lat/0404008].



J^{PC}	Mass MeV		
	Unquenched	Quenched	
	Gl	Mp	Ky
0^{++}	1795(60)	1730(50)(80)	1710(50)(80)
0^{++}	3760(240)	2670(180)(130)	

Is unquenching all of QCD?
NO!
Meson-glueball mixing!!

A full unquenched QCD lattice calculation which incorporates meson-glueball mixing should reproduce the physical spectrum. This calculation is not yet possible, however several approximate studies have been carried out and they all tend to decrease the mass of the lowest scalar and pseudscalar glueballs, but not so the mass of the tensor glueball.

Another interesting result would be to determine the decay channels of the glueballs and calculate their widths to provide with clear experimental signals. Since lattice QCD is worked out in Euclidean space-time these calculations are difficult and their results have not achieved consensus in the community

$$\mathcal{L} = \frac{1}{4g^2} G^a_{\mu\nu} G^a_{\mu\nu} + \sum_j \bar{q}_j \left(i\gamma^\mu D_\mu + m_j \right) q_j$$

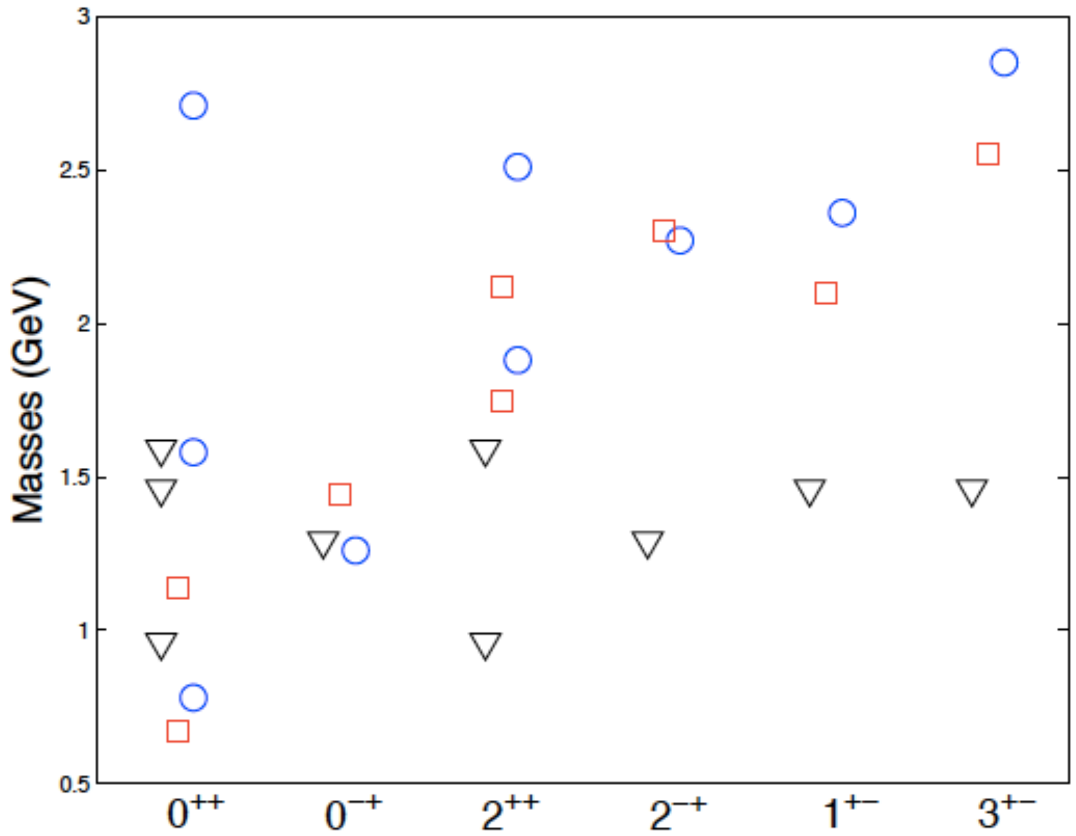
where $G^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + i f^a_{bc} A^b_\mu A^c_\nu$

and $D_\mu \equiv \partial_\mu + i t^a A^a_\mu$

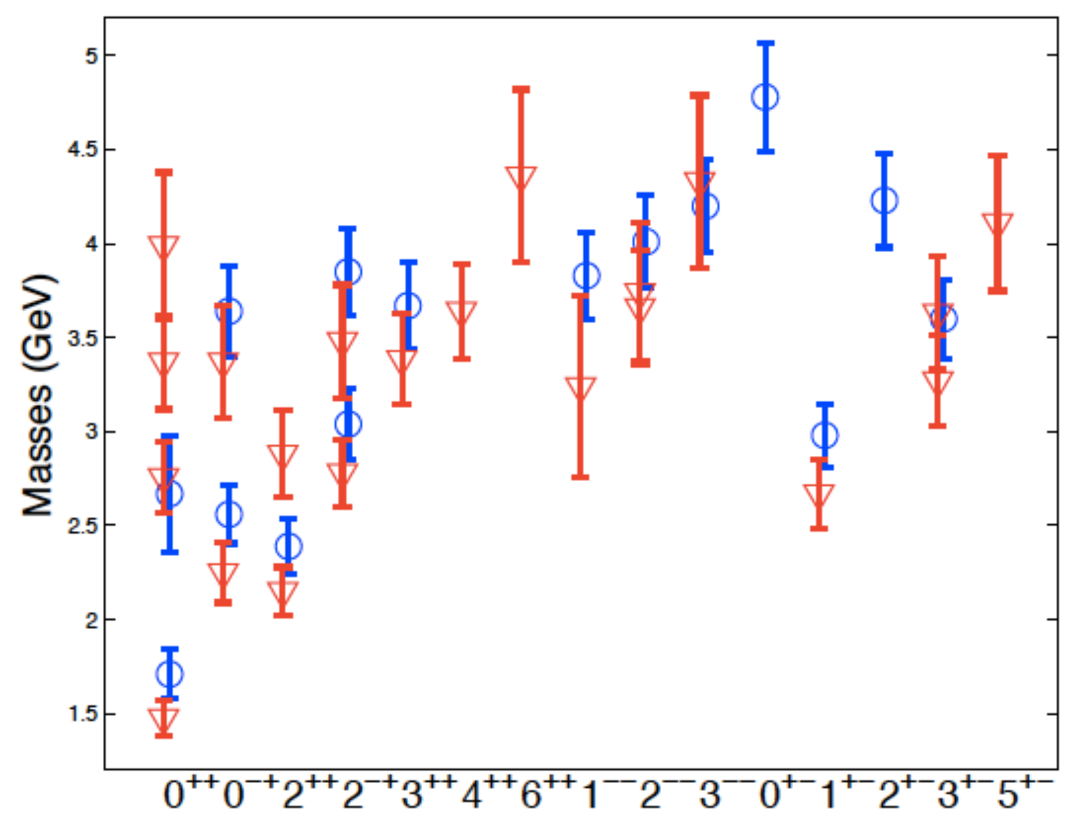
<u>That's it!</u>

Non-perturbative calculations

Bag model (Jaffe -Johnson)

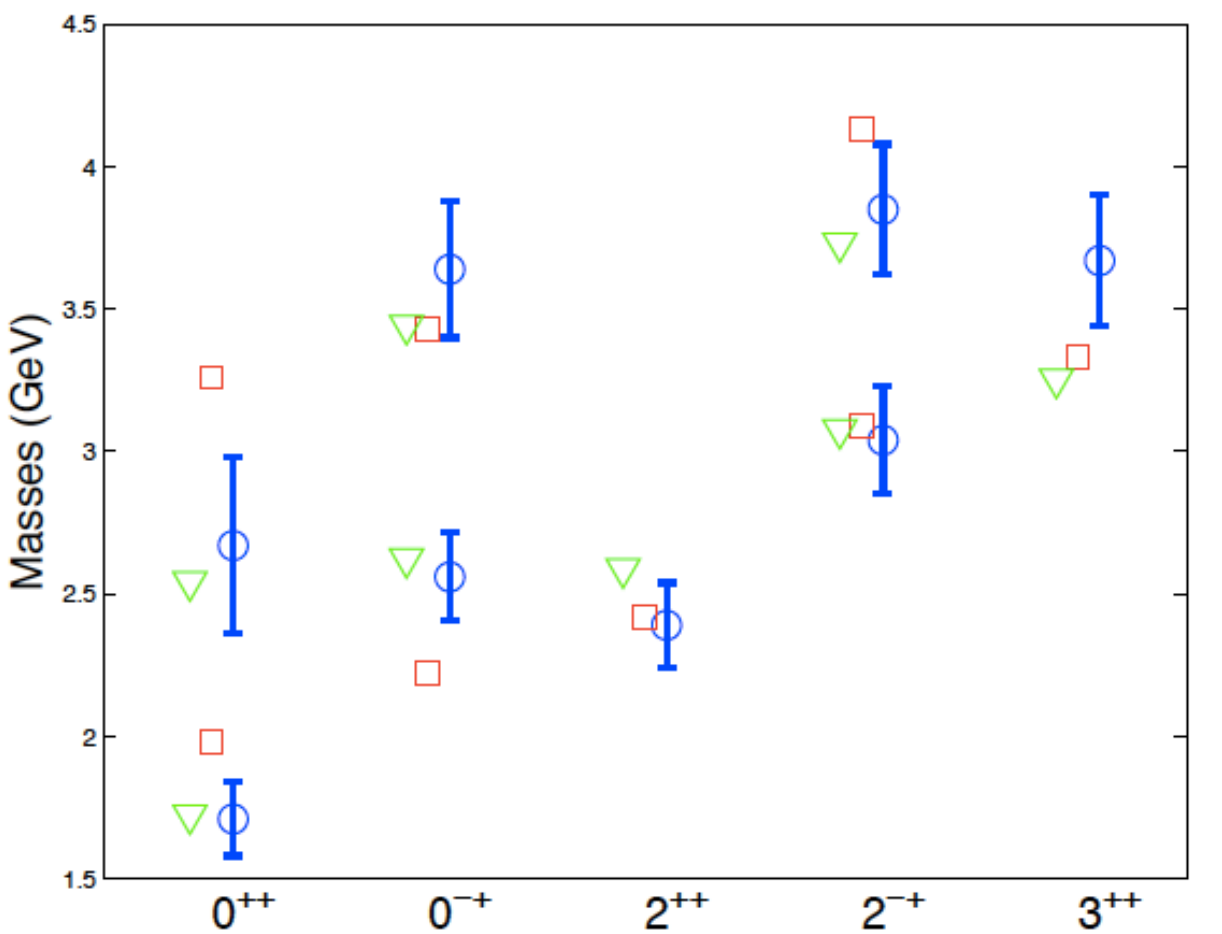


Lattice QCD



Caveat: coupling constant fitted to conventional hadrons

Relativistic Potential Models (Mathieu-Buisseret-Semay)



J^{PC}	ref. [44]	ref. [47]	ref. [45]	ref. [51]	ref. [17, 19]	ref. [18]
0^{++}	(1.00)	1.98(1.00)	1.72(1.00)	1.41(1.00)	1.71(1.00)	1.48(1.00)
	(1.40)	3.26(1.65)	2.54(1.48)	2.41(1.71)	2.67(1.56)	2.76(1.89)
0^{-+}	(1.00)	2.22(1.12)	2.62(1.52)	2.28(1.62)	2.56(1.50)	2.25(1.54)
	(1.40)	3.43(1.73)	3.44(2.00)	3.35(2.38)	3.64(2.13)	3.37(2.31)
2^{++}	(1.16)	2.42(1.22)	2.59(1.51)	2.30(1.63)	2.39(1.40)	2.15(1.47)
	(1.35)	3.11(1.57)	3.08(1.79)	3.32(2.35)		2.88(1.97)
2^{-+}	(1.35)	3.09(1.56)	3.08(1.79)	2.70(1.91)	3.04(1.78)	2.78(1.90)
	(1.68)	4.13(2.09)	3.73(2.17)	3.73(2.65)	3.89(2.27)	3.48(2.38)
3^{++}	(1.42)	3.33(1.68)	3.25(1.89)		3.67(2.15)	3.39(2.32)
4^{++}	(1.63)	3.99(2.02)	3.77(2.19)			3.64(2.49)
	(1.71)	4.28(2.16)	3.96(2.30)			
4^{-+}	(1.71)	4.27(2.16)	3.96(2.30)			
5^{++}			4.21(2.45)			
6^{++}			4.60(2.67)			4.36(2.98)

Caveat: they do fit the lowest lying glueball

One step beyond!

Sum rules, 1/N and the scalar glueball

-Dominguez and Paver, Bordes, Penarrocha and Gimenez , and Kisslinger and Johnson obtain by means of low energy theorems and/or sum rule calculations with (or without) instanton contributions a low lying (mass < 700 MeV), narrow (< 100 MeV) scalar glueball.

-Vento using broken OZI dynamics, 1/N and mixing obtains also a scalar glueball (mass< 700 MeV) and also small width (< 100 MeV)

-Narison and collaborators using a two (substracted and unsubstracted) sum rules prefer a broader (200-800 MeV), heavier (700-1000 MeV) gluonium whose properties imply a strong violation of the Okubo-Zweig-Ishimura's (OZI) rule.

-Forkel in a sophisticated sum rule calculation using the instanton vacuum obtains the lowest gluonium at 1250 ± 200MeV with a large width (300 MeV). However, he has some strength at lower masses which he is not able to ascribe to a resonance in the fits.

- The eta(1405) was proposed by the Mark II experiment as a possible pseudo scalar glueball. But lattice QCD gives masses above 2 GeV.

Question: is lattice QCD missing the low lying glueballs? How does mixing affect the glueball masses?

Glueballs Phenomenology

Scalar

J^{PC}	Name	Mass MeV	Width MeV	Comment
0^{++}	$f_0(500)$	400-550	400-700	
0^{++}	$f_0(980)$	990±20	40-100	
0^{++}	$f_0(1370)$	1200-1500	200-500	
0^{++}	$f_0(1500)$	1505 ± 6	109 ± 7	
0^{++}	$f_0(1710)$	1720 ± 6	135 ± 8	
0^{++}	$f_0(2020)$	1992 ± 16	442 ± 60	needs confirmation
0^{++}	$f_0(2100)$	2013 ± 8	209 ± 19	needs confirmation
0^{++}	$f_0(2200)$	2189 ± 13	238 ± 50	needs confirmation

Pseudoscalar

J^{PC}	Name	Mass MeV	Width MeV	Comment
0^{-+}	$\eta(545)$	$548 \pm 0.$	~ 1 KeV	
0^{-+}	$\eta'(958)$	$958 \pm 0.$	$0.2 \pm 0.$	
0^{-+}	$\eta(1295)$	1294 ± 4	55 ± 5	
0^{-+}	$\eta(1405)$	1409 ± 3	51 ± 3	
0^{-+}	$\eta(1475)$	1476 ± 4	85 ± 9	
0^{-+}	$\eta(1760)$	1751 ± 15	240 ± 30	needs confirmation
0^{-+}	$\eta(2225)$	2221 ± 13	185 ± 40	needs confirmation

Tensor

J^{PC}	Name	Mass MeV	Width MeV	Comment
2^{++}	$f_2(1270)$	1275 ± 1	185 ± 3	
2^{++}	$f_2'(1525)$	1525 ± 5	73 ± 6	
2^{++}	$f_2(1950)$	1944 ± 12	472 ± 18	
2^{++}	$f_2(2010)$	2011 ± 80	202 ± 60	
2^{++}	$f_2(2300)$	2297 ± 28	149 ± 40	
2^{++}	$f_2(2340)$	2339 ± 60	319 ± 80	

-Non perturbative técniques point to a low lying scalar glueball mass< 1000 MeV . The f0(500) could hide a glueball strongly mixed with a meson state. They also point to a higher lying glueball m >1250, which could be hiding in the big bump at the 1310.

-Other treatments Amsler-Close have made a point in favor of the f0(1500) or Chanowitz and his chiral suppresion scheme in favor of the f0(1710) . The latter do not exclude the low mass glueballs.

-In the pseudoscalar sector the eta and eta prime might be mixed meson glueball states.

- Relations between AdS/CFT and the PDG

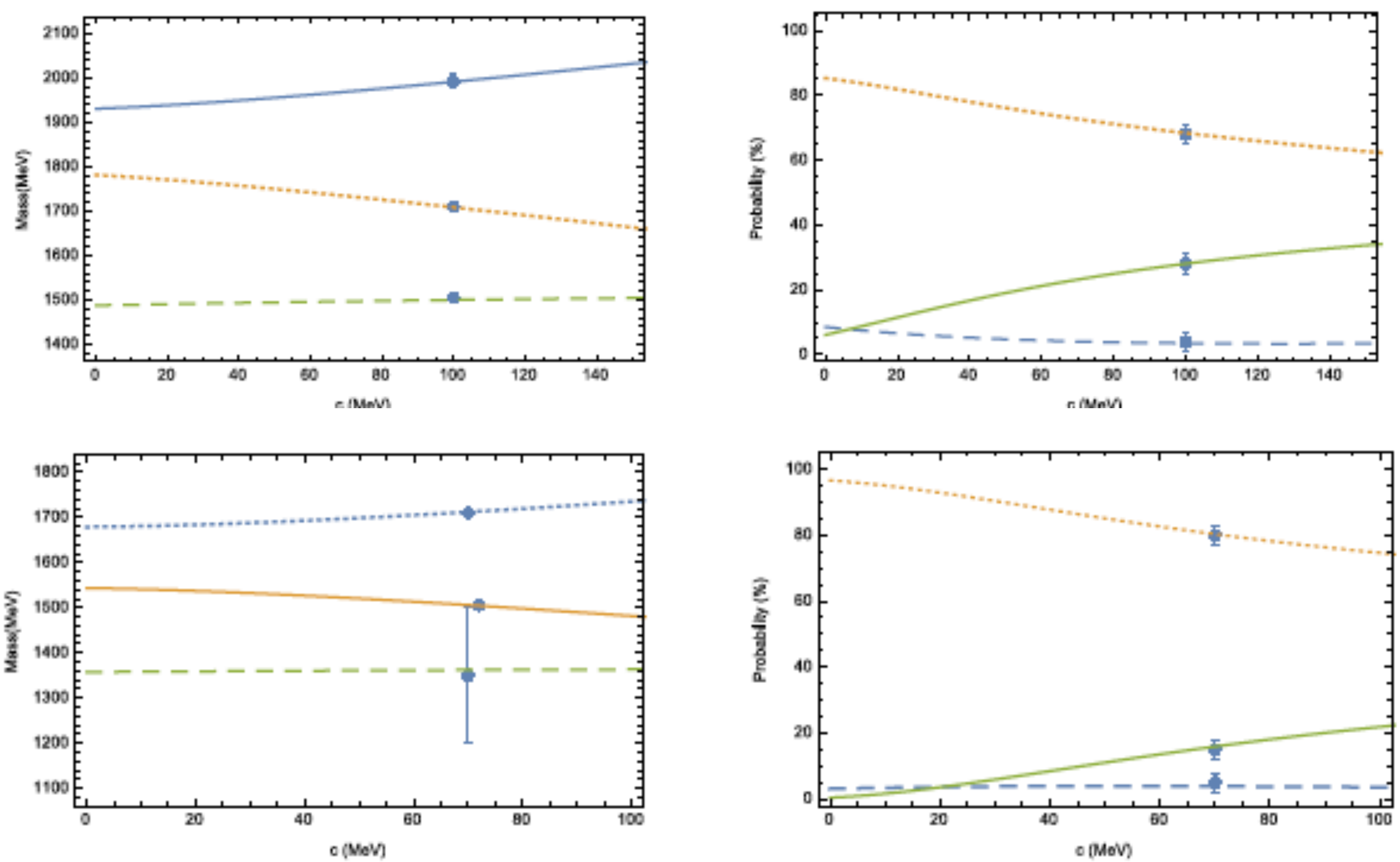
Looking for glueballs

Mixing f0(1710) and f0(1500)

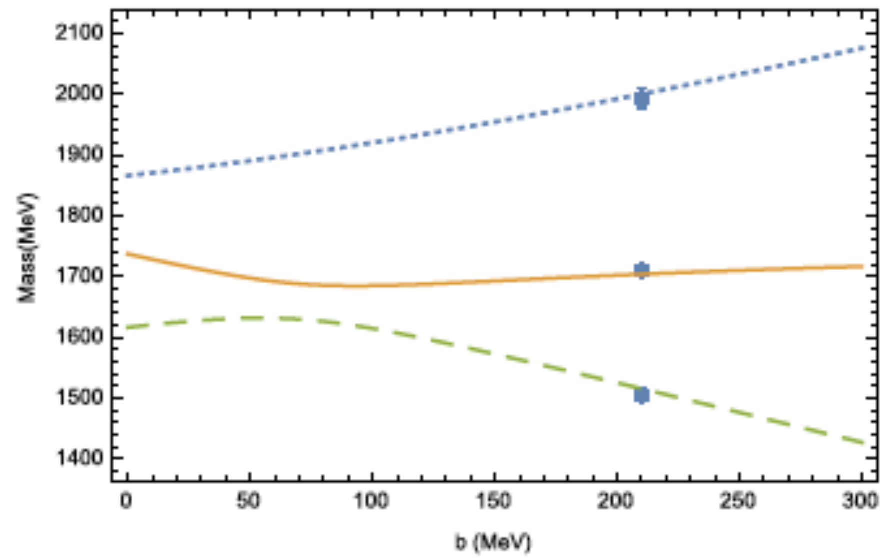
$$\begin{pmatrix} m_{s1} & a & b \\ a & m_{s2} & c \\ b & c & m_g \end{pmatrix}$$

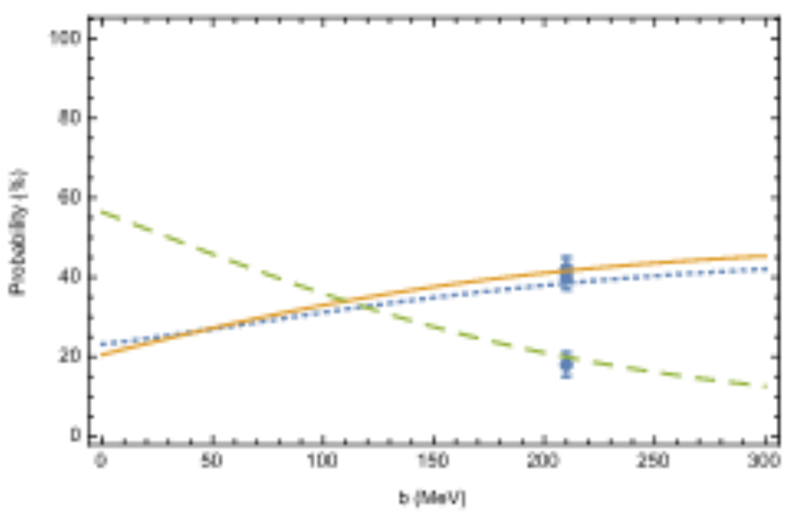
ms ~1 + O(Nc^-1)~1500 MeV
mg~1 + O(Nc^-2)~1500 MeV
a~b~c~ O(Nc^-2) < 200 MeV

Weak mixing f0(1710)

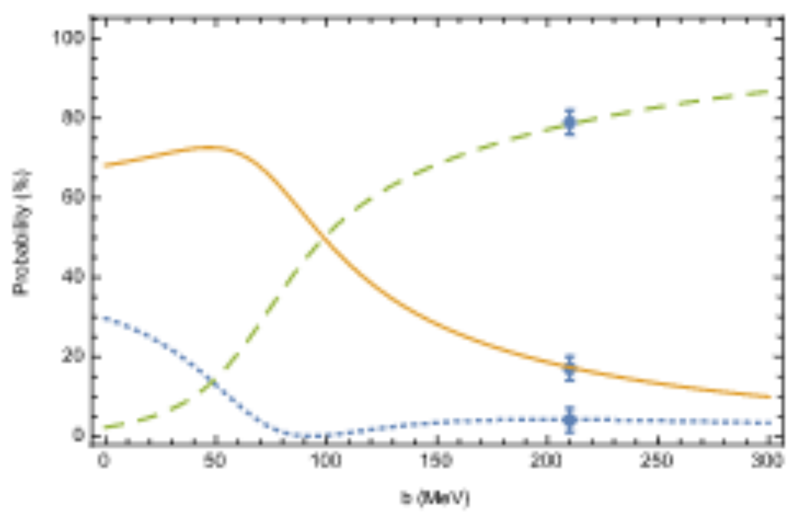


Strong mixing f0 (1500)

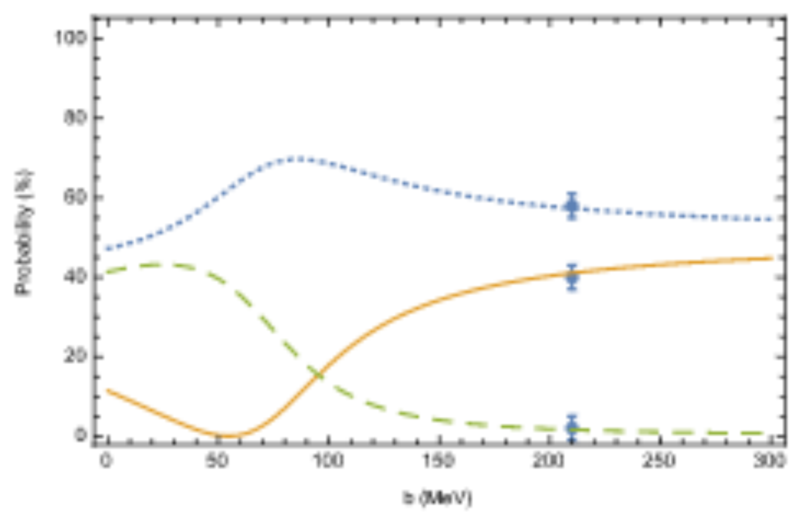




f0(1500)



f0(1710)



f0(2020)

Pseudoscalar glueballs

Mixing eta(545), eta'(958) and glueball: Chiral Lagrangian with pseudoscalar glueball

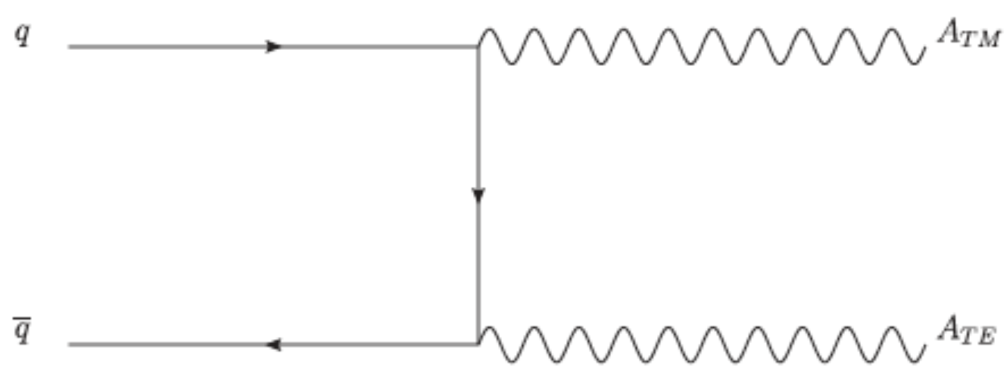
The traditional chiral scheme leads for the masses of eta and eta' to a relation in terms of one mixing angle.

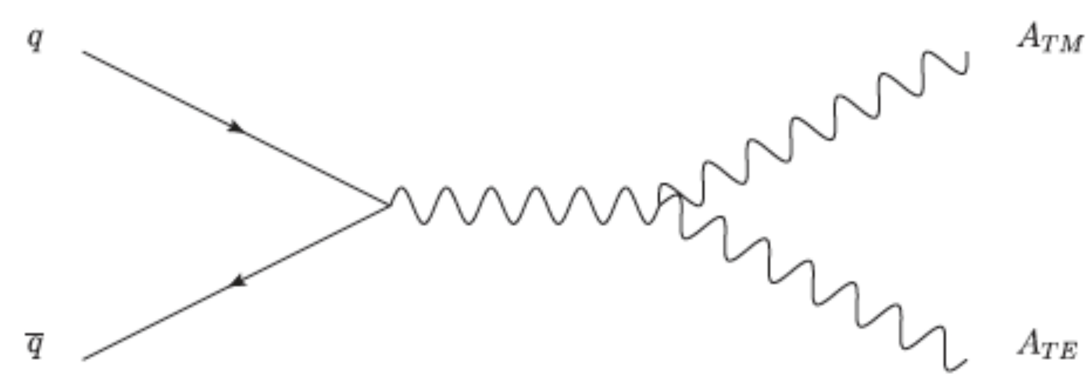
$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix}$$

and to a mass relation

$$M_\eta^2 = \frac{1}{3}[4M_K^2 - M_\pi^2 + 2\sqrt{2}(M_K^2 - M_\pi^2)\tan\theta],$$
$$M_{\eta'}^2 = \frac{1}{3}[4M_K^2 - M_\pi^2 - 2\sqrt{2}(M_K^2 - M_\pi^2)\cot\theta].$$

Which does not provide the physical mass ratio!





We incorporate a pseudoscalar glueball to the chiral scheme to implement the axial anomaly together with the eta_0. In order to fit the data we need to include a coupling between the octet meson and the glueball, whose ultimate origin is the difference in mass between the up and down quarks and the strange quarks. The effective theory leads to a 3x3 mass matrix whose physical states are the eta, eta’ and glueball.

We are able to parametrize the mass matrix in terms of two mixing angles which depend on the glueball mass in a complicated manner.

$$\begin{pmatrix} \eta \\ \eta' \\ \Theta \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \\ G \end{pmatrix}$$

In order to determine fix parameters we look at J/Psi decays to eta and eta', and eta, eta' decays to photons fixing the mass of eta and eta' to the physical masses.

$$\frac{\Gamma(J/\psi \to \eta'\rho)}{\Gamma(J/\psi \to \eta\rho)} \quad \frac{\Gamma(J/\psi \to \eta'\omega)}{\Gamma(J/\psi \to \eta\omega)} \quad \frac{\Gamma(J/\psi \to \eta'\phi)}{\Gamma(J/\psi \to \eta\phi)} \quad \frac{\Gamma(J/\psi \to \eta'\gamma)}{\Gamma(J/\psi \to \eta\gamma)}$$

$$\frac{\Gamma(\eta' \to \omega\gamma)}{\Gamma(\omega \to \eta\gamma)} \quad \frac{\Gamma(\eta' \to \rho\gamma)}{\Gamma(\rho \to \eta\gamma)} \quad \frac{\Gamma(\phi \to \eta'\gamma)}{\Gamma(\phi \to \eta\gamma)}$$

$$\frac{\Gamma(\eta \to \gamma\gamma)}{\Gamma(\pi^0 \to \gamma\gamma)} \quad \frac{\Gamma(\eta' \to \gamma\gamma)}{\Gamma(\pi^0 \to \gamma\gamma)}$$

$$\begin{pmatrix} \eta \\ \eta' \\ \Theta \end{pmatrix} = \begin{pmatrix} 0.9938 & 0.1114 & 0 \\ -0.0904 & 0.8065 & 0.5842 \\ 0.0651 & -0.5806 & 0.8116 \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \\ gg \end{pmatrix}$$

Note that the eta' has a large glueball component.

Our fit leads to a mass compatible with Lattice QCD

$$2100 \text{ MeV} < M_\Theta < 2300 \text{ MeV}$$

We are able to predict J/Psi decays into the pseudoscalar glueball

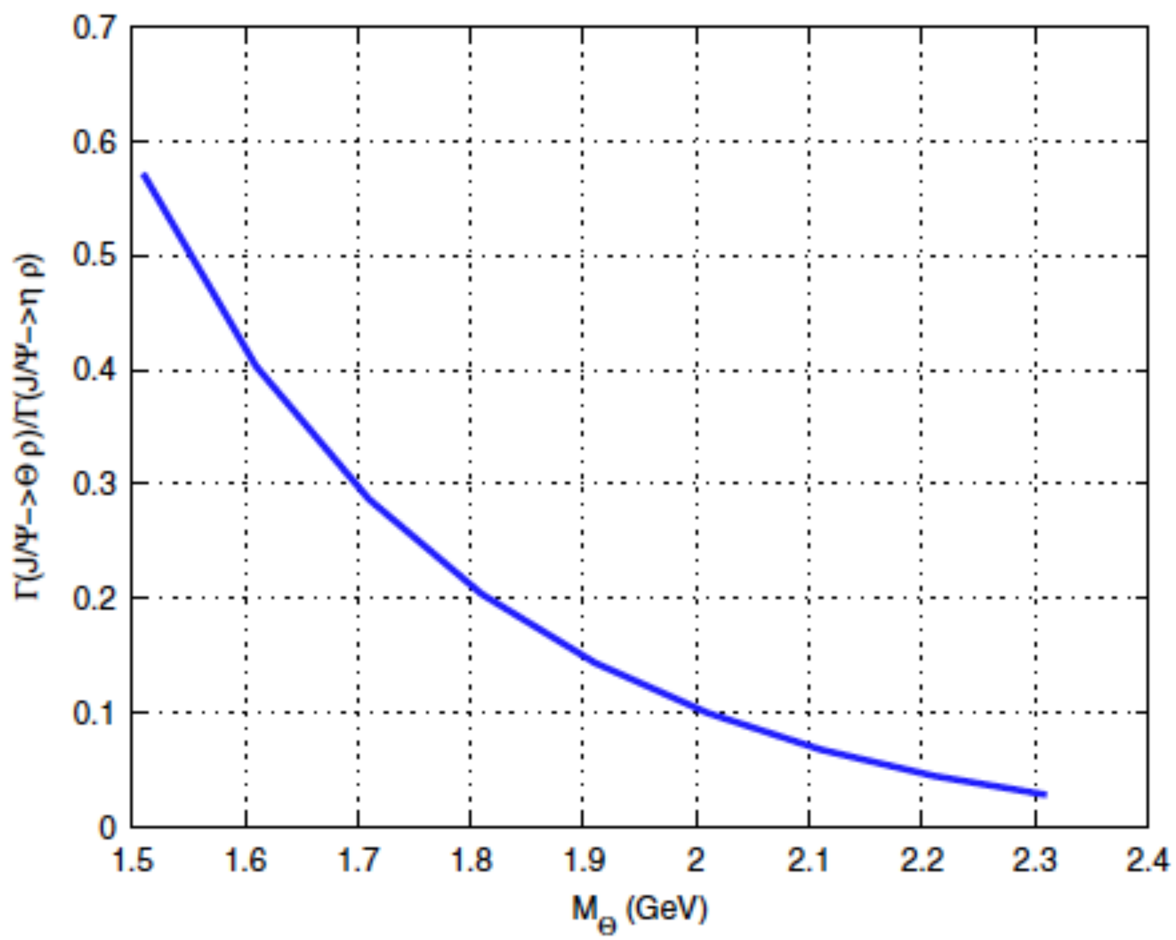


FIG. 16 (color online). $\Gamma(J/\psi \to \Theta\rho)/\Gamma(J/\psi \to \eta\rho)$ as a function of M_Θ.

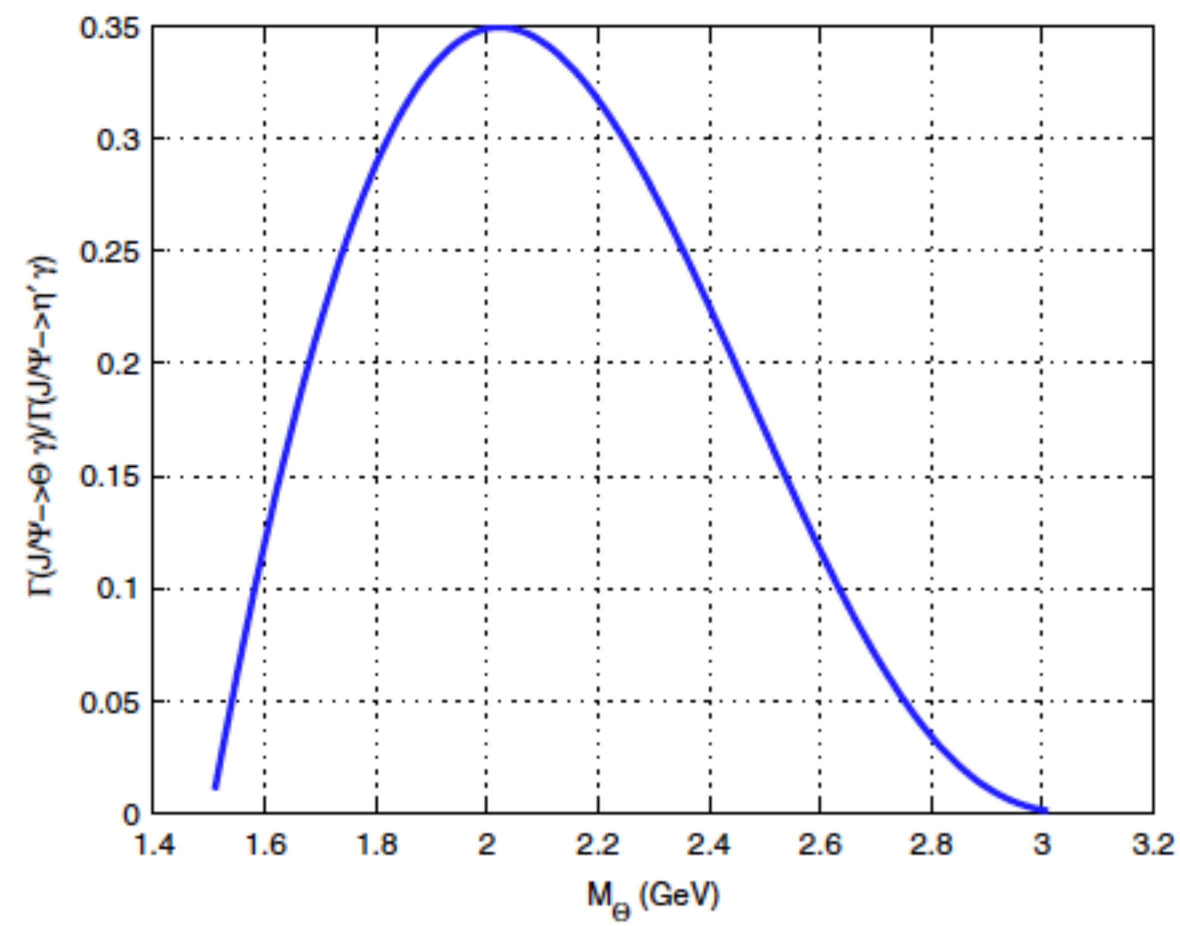
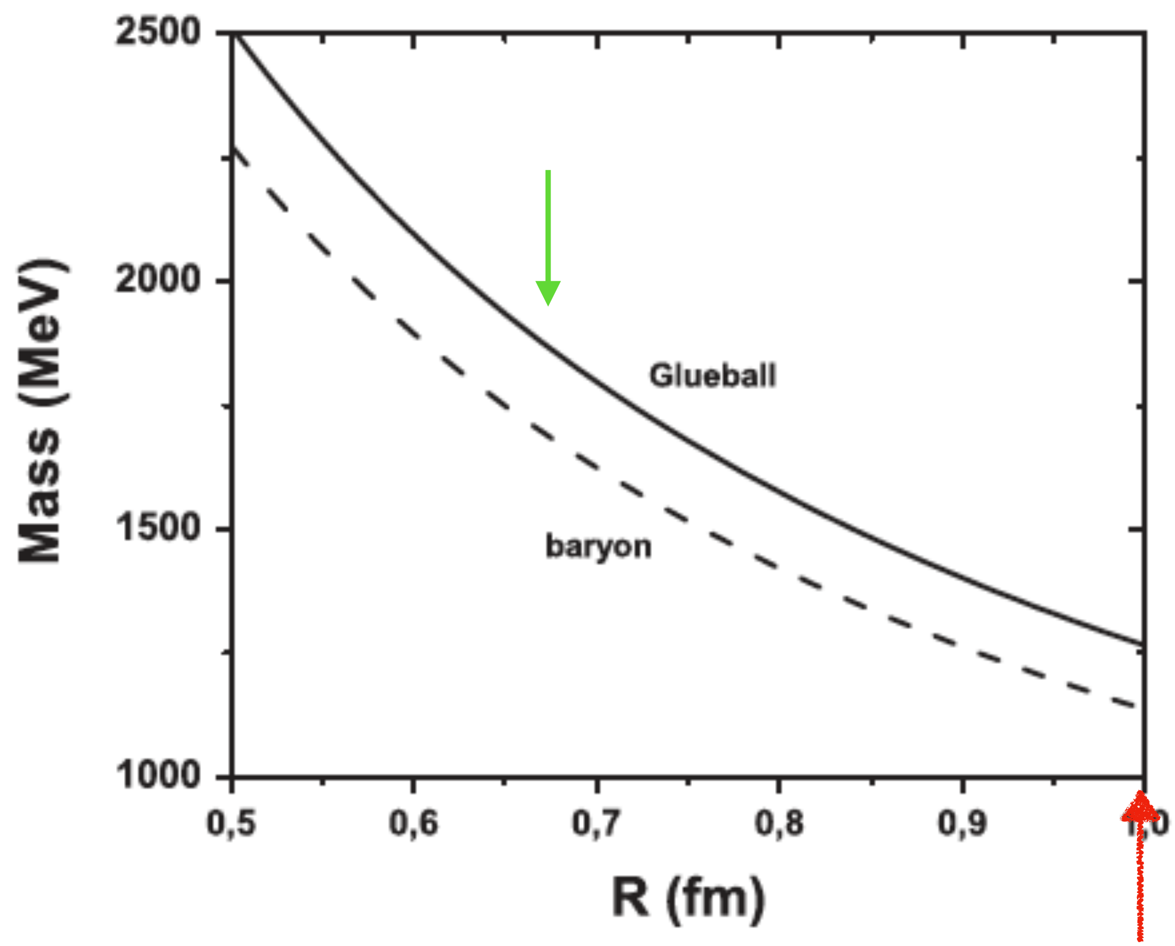


FIG. 17 (color online). $\Gamma(J/\psi \to \Theta\gamma)/\Gamma(J/\psi \to \eta'\gamma)$ as a function of M_Θ.

Bag Model

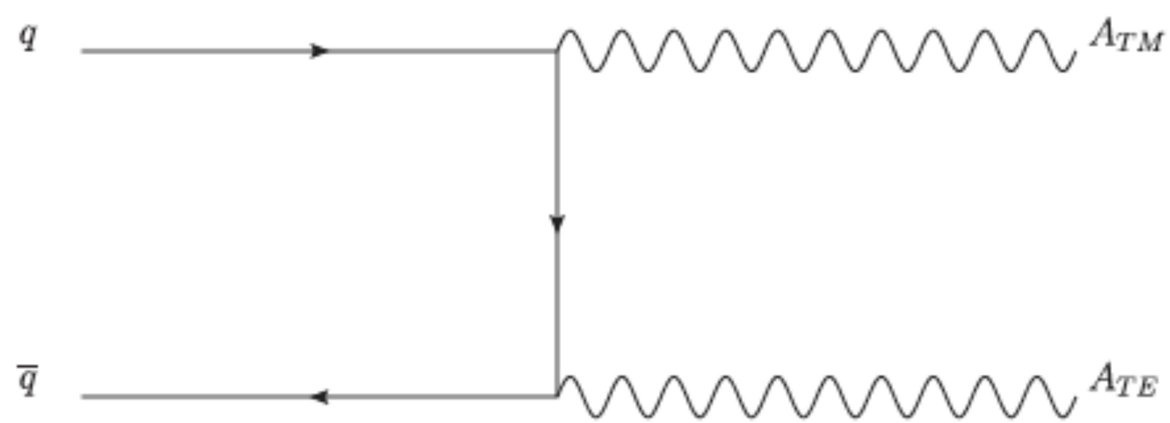
We studied eta,eta’glueball mixing in a sophisticated bag model with center of mass motion corrections and recoil corrections.

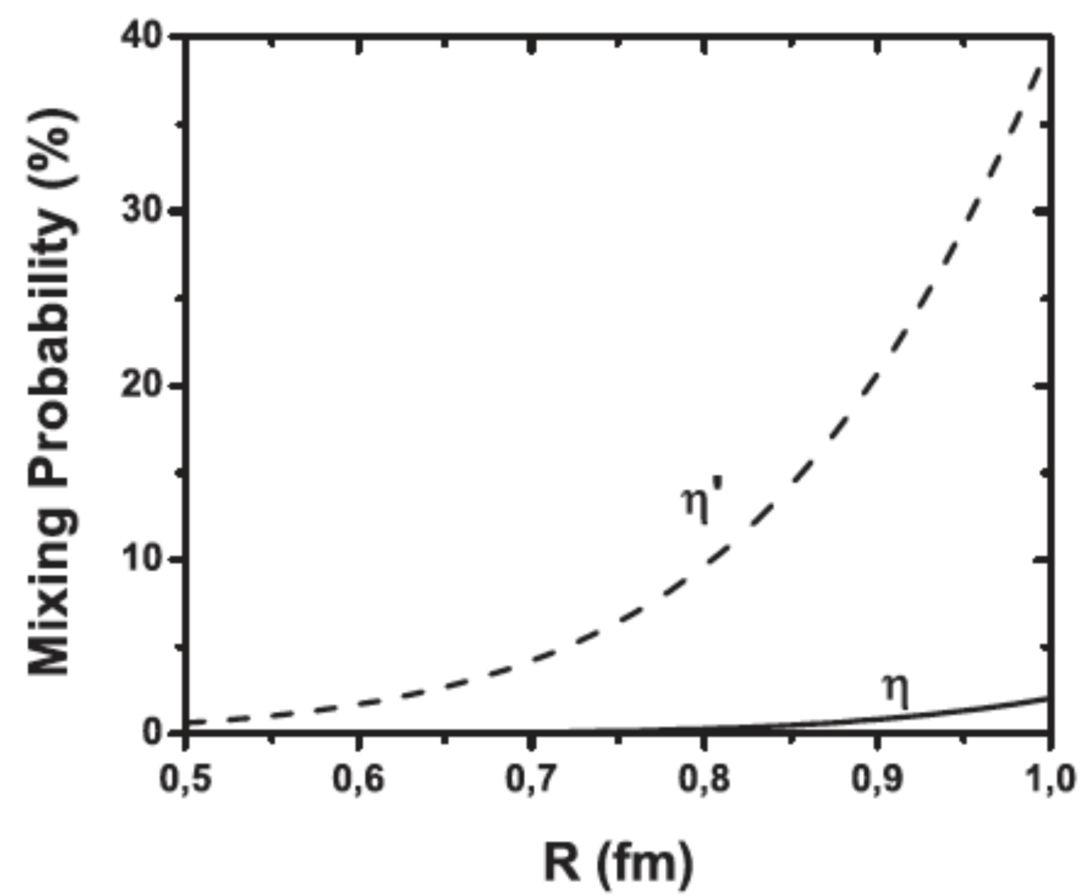
It was argued by Kuti that the problem with the calculation of Jaffe and Johnson was that they took the same parameters as for the baryons. If the scale is fixed to the lowests mass lattice glueball, the spectrum is quite compatible with lattice QCD.



The off-diagonal term in the energy expectation value is given by

$$E = \langle G|N\Big\{2g^2 \int d^3x\,\bar{\psi}(x)\gamma^\mu A(x)^a_\mu t^a \int d^4x' S_F(x,x')\gamma^\nu \times A(x')^b_\nu t^b\,\psi(x')\Big\}|\eta(\eta')\rangle \quad (32)$$





AdS/CFT Back to the scalar spectrum

Top down approach of Brower, Mathur and Tan

To arrive at QCD_4 as the target theory, one begins with the eleven dimensional M theory on $AdS^7 \times S^4$ or 10-d type IIA string theory after compactifing the "eleventh" dimension (on a very small circle of radius R_0). Again following a suggestion by Witten [2], the "temperature" is raised with a second compact radius R_1 in a direction τ parallel to the type IIA D4-branes. On the "thermal" circle, the fermionic modes have anti-periodic boundary conditions breaking conformal and all SUSY symmetries. Therefore, if all goes as conjectured, in the scaling limit $g^2 N = g_s N\beta/R_1 \to 0$ there should be a fixed point mapping type IIA string theory in an background AdS^7 black hole metric,

$$ds^2 = (r^2 - \frac{1}{r^4})d\tau^2 + r^2 \sum_{i=0,1,2,3,4} dx_i^2 + (r^2 - \frac{1}{r^4})^{-1} dr^2 + d\Omega_4^2 \,, \tag{4}$$

into pure $SU(N)$ Yang Mills or quarkless QCD_4.

To compute the glueball excitation for QCD_4 and QCD_3, in the extreme strong coupling limit, one simply needs to find the spectrum of harmonic fluctuations for the bosonic supergravity fields around these AdS black hole backgrounds. The "warp factor" in the radial "fifth" co-ordinate forms a "cavity" so that all modes are discrete and massive.

AdS^7 black hole metric $$ds^2 = (r^2 - \frac{1}{r^4})d\tau^2 + r^2 \sum_{i=1,2,3,4,11} dx_i^2 + (r^2 - \frac{1}{r^4})^{-1} dr^2 + \frac{1}{4} d\Omega_4^2 \,,$$

States from 11-d G_{MN}				States from 11-d A_{MNL}		
$G_{\mu\nu}$	$G_{\mu,11}$	$G_{11,11}$	m_0 (Eq.)	$A_{\mu\nu,11}$	$A_{\mu\nu\rho}$	m_0 (Eq.)
G_{ij} 2^{++}	C_i $1^{++}_{(-)}$	ϕ 0^{++}	4.7007 (T_4)	B_{ij} 1^{+-}	C_{123} $0^{+-}_{(-)}$	7.3059 (N_4)
$G_{i\tau}$ $1^{-+}_{(-)}$	C_τ 0^{-+}		5.6555 (V_4)	$B_{i\tau}$ $1^{--}_{(-)}$	$C_{ij\tau}$ 1^{--}	9.1129 (M_4)
$G_{\tau\tau}$ 0^{++}			2.7034 (S_4)	G^α_α State 0^{++}		10.7239 (L_4)

Table 1: IIA Classification for QCD_4. Subscripts to J^{PC} designate $P_\tau = -1$.

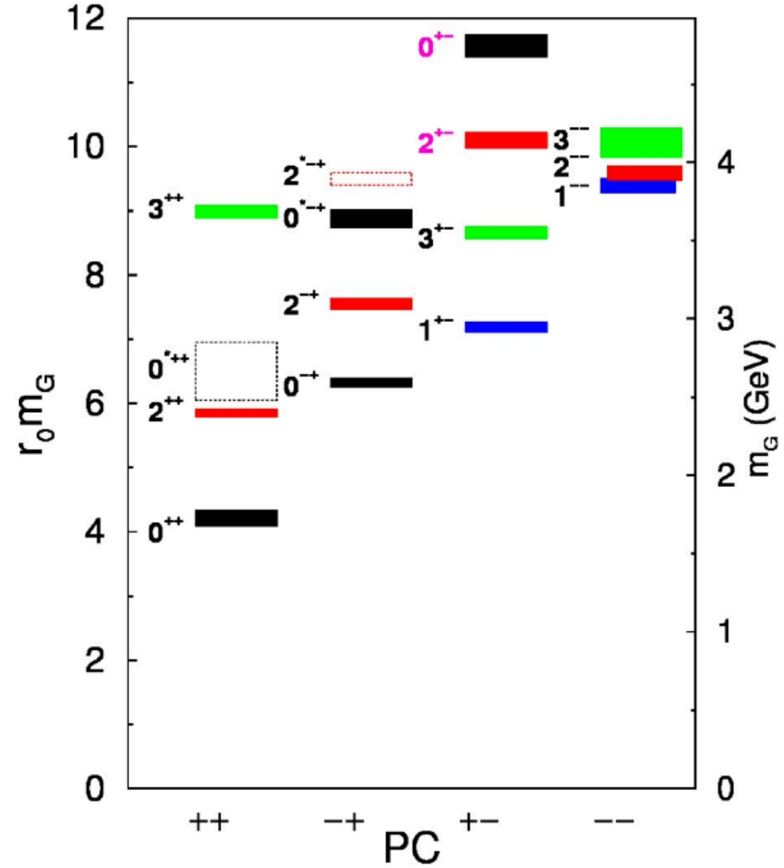
$$
\begin{aligned}
- \quad & \frac{d}{dr}(r^7 - r)\frac{d}{dr}T_4(r) - (m^2 r^3)T_4(r) = 0 \,, \\
- \quad & \frac{d}{dr}(r^7 - r)\frac{d}{dr}V_4(r) - (m^2 r^3 - \frac{9}{r(r^6-1)})V_4(r) = 0 \,, \\
- \quad & \frac{d}{dr}(r^7 - r)\frac{d}{dr}S_4(r) - (m^2 r^3 + \frac{432 r^5}{(5r^6-2)^2})S_4(r) = 0 \,, \\
- \quad & \frac{d}{dr}(r^7 - r)\frac{d}{dr}N_4(r) - (m^2 r^3 - 27r^5 + \frac{9}{r})N_4(r) = 0 \,, \\
- \quad & \frac{d}{dr}(r^7 - r)\frac{d}{dr}M_4(r) - (m^2 r^3 - 27r^5 - \frac{9r^5}{r^6-1})M_4(r) = 0 \,, \\
- \quad & \frac{d}{dr}(r^7 - r)\frac{d}{dr}L_4(r) - (m^2 r^3 - 72r^5)L_4(r) = 0 \,.
\end{aligned}
$$

Equation: J^{PC}:	T_4 $2^{++}/1^{++}/0^{++}$	V_4 $1^{-+}/0^{-+}$	S_4 0^{++}	N_4 $1^{+-}/0^{+-}$	M_4 $1^{--}/1^{--}$	L_4 0^{++}
n = 0	22.097	31.985	7.308	53.376	83.046	115.002
n = 1	55.584	72.489	46.986	109.446	143.582	189.631
n = 2	102.456	126.174	94.485	177.231	217.399	277.282
n = 3	162.722	193.287	154.981	257.958	304.536	378.099
n = 4	236.400	273.575	228.777	351.895	405.018	492.169
n = 5	323.541	368.087	315.976	459.131	518.059	619.547
n = 6	424.195	474.268	416.666	579.706	646.088	760.252
n = 7	538.487	594.231	530.950	713.638	786.559	914.307
n = 8	666.479	729.102	658.996	860.939	939.557	1081.732
n = 9	808.398	875.315	800.860	1021.613	1108.010	1262.518

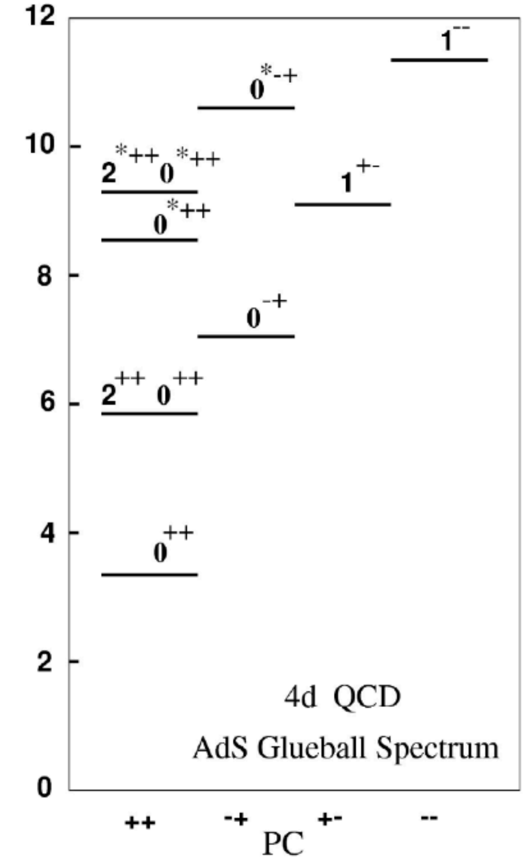
Table 2: The mass spectrum, m_n^2, for QCD_4 Glueballs

Lattice glueballs vs. supergravity glueballs

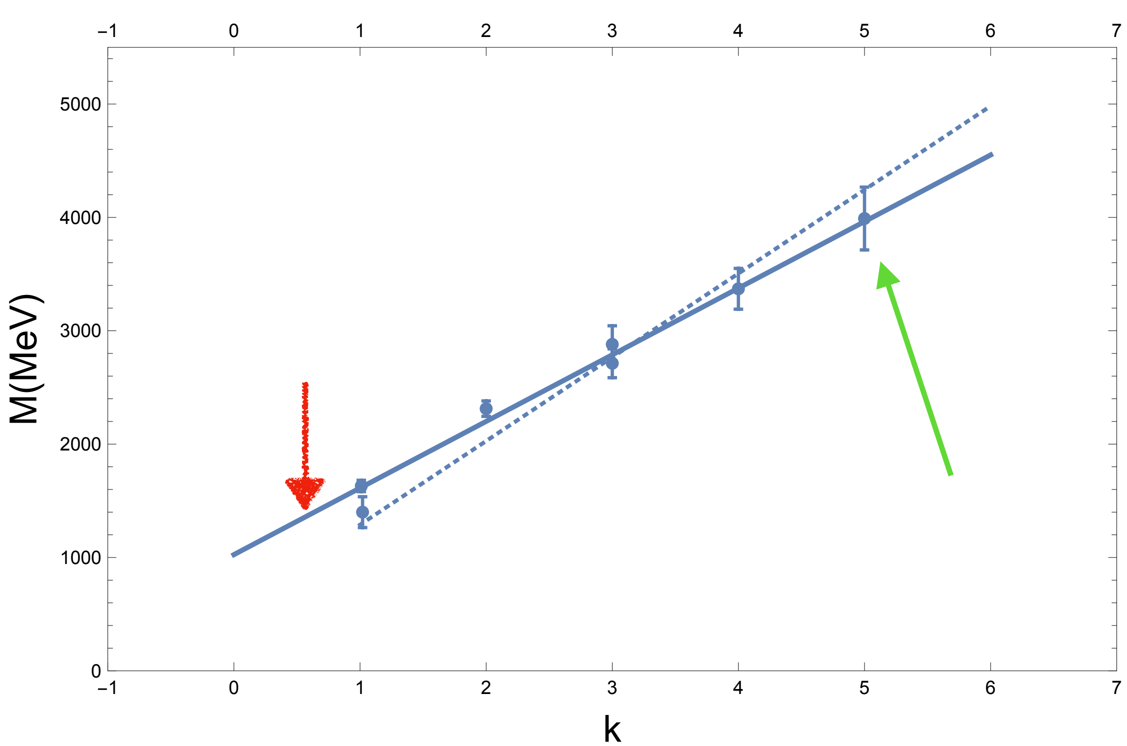
Morningstar & Peardon hep-lat/9901004:



Brower, Mathur & Tan 2000:



Using the graviton



Bottom-up approach

Holographic models of the scalar sector of QCD

- chiral dynamics of QCD (a few operators)
- Scalar mesons: a_0(980, 1450), f_0(980, 1370, 1505)…
- Scalar (& vector) glueballs : bound-states of gluons (well defined in the large N limit)

M^4 QCD operators

- left- and right-handed currents :

  $j^a_{L\,\mu}$ & $j^a_{R\,\mu}$ (Δ=3,p=1)
- chiral order parameter :

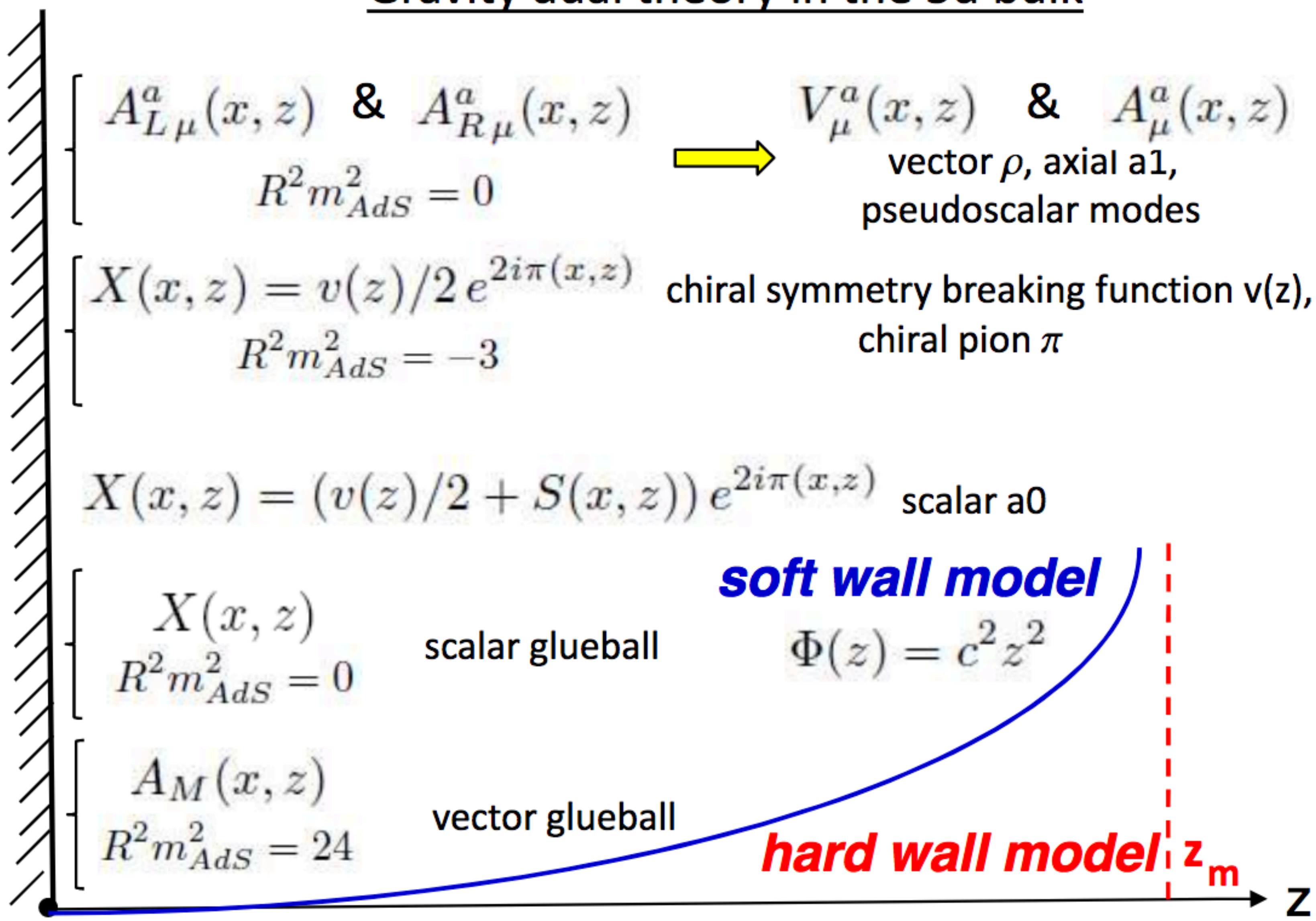
  $\bar{q}_R q_L$ (Δ=3,p=0)
- scalar meson operator :

  $\mathcal{O}^A_S = \bar{q}\,T^A\,q$ (Δ=3,p=0)
- scalar glueball operator :

  $\mathcal{O}_S = Tr\,(G^2)$ (Δ=4,p=0)
- vector glueball operator :

  $\mathcal{O}_V = Tr\,(G(DG)G)$ (Δ=7,p=1)

Gravity dual theory in the 5d bulk



$A^a_{L\,\mu}(x,z)$ & $A^a_{R\,\mu}(x,z)$, $R^2 m^2_{AdS} = 0$ ⇒ $V^a_\mu(x,z)$ & $A^a_\mu(x,z)$: vector ρ, axial a1, pseudoscalar modes

$X(x,z) = v(z)/2\,e^{2i\pi(x,z)}$, $R^2 m^2_{AdS} = -3$: chiral symmetry breaking function v(z), chiral pion π

$X(x,z) = (v(z)/2 + S(x,z))\,e^{2i\pi(x,z)}$ scalar a0

$X(x,z)$, $R^2 m^2_{AdS} = 0$: scalar glueball

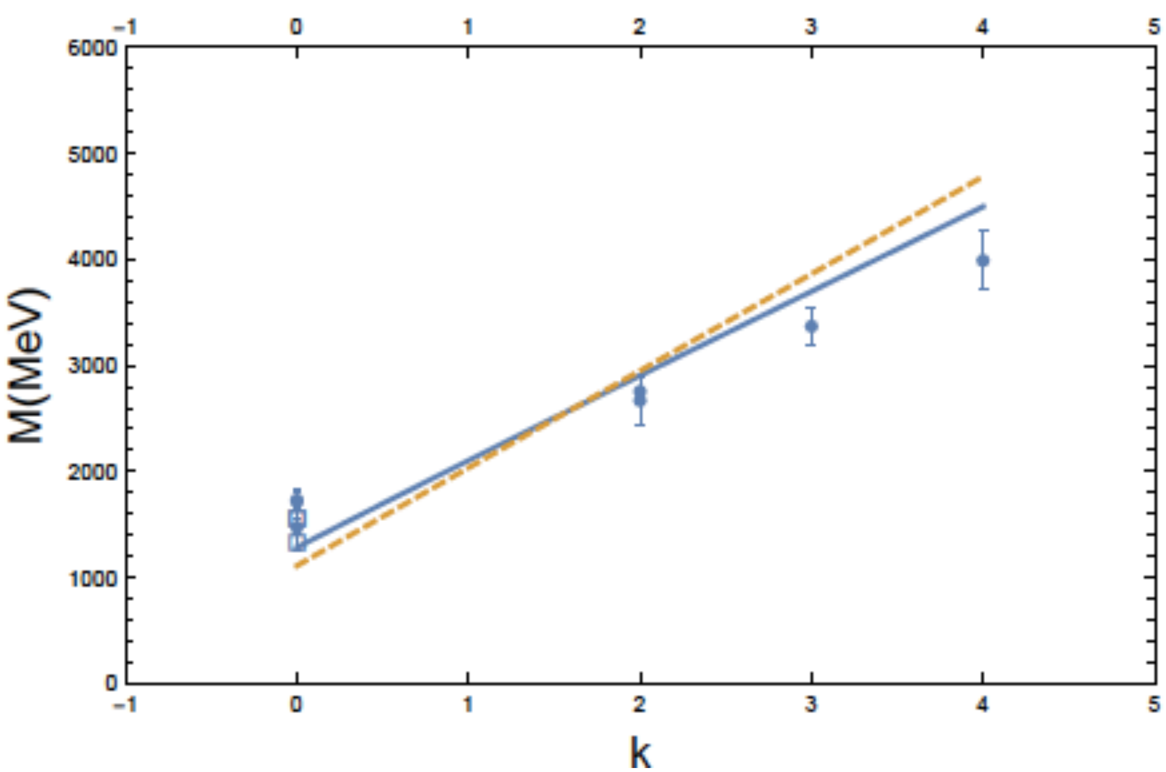
$\Phi(z) = c^2 z^2$ (soft wall model)

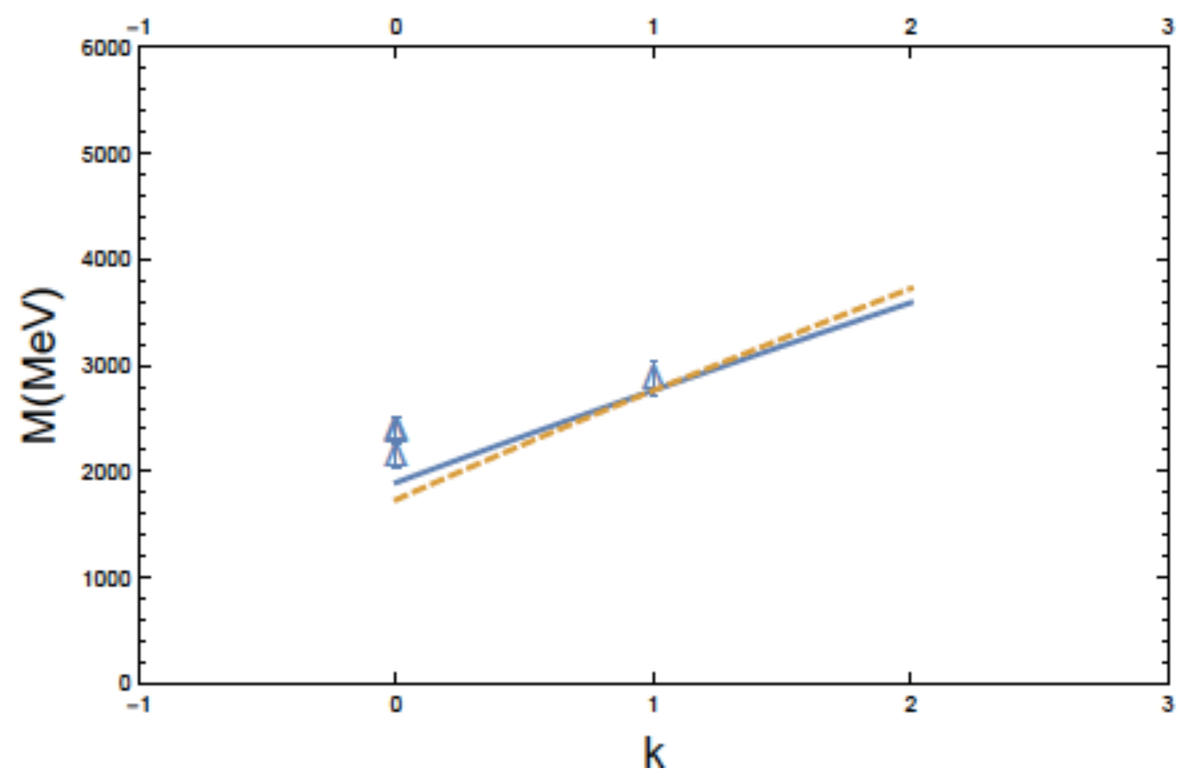
$A_M(x,z)$, $R^2 m^2_{AdS} = 24$: vector glueball

Hardwall model a slice of AdS5

$$ds^2 = \frac{R^2}{z^2}(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu) + R^2 d\Omega_5$$

$$\partial_z^2\Phi - \frac{3}{z}\partial_z\Phi + (\eta_{\mu\nu}\partial_\mu\partial_\nu - \frac{(\mu R)^2}{z^2})\Phi = 0. \qquad (\mu R)^2 = J(J+4)$$



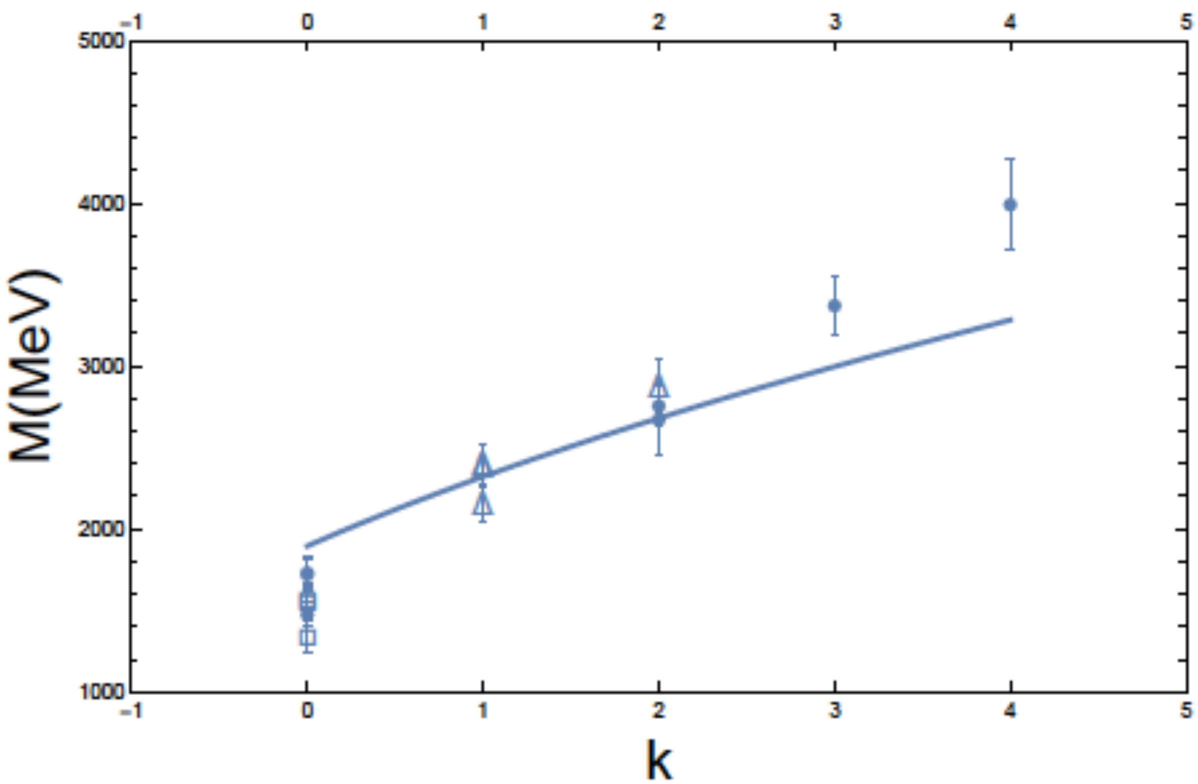


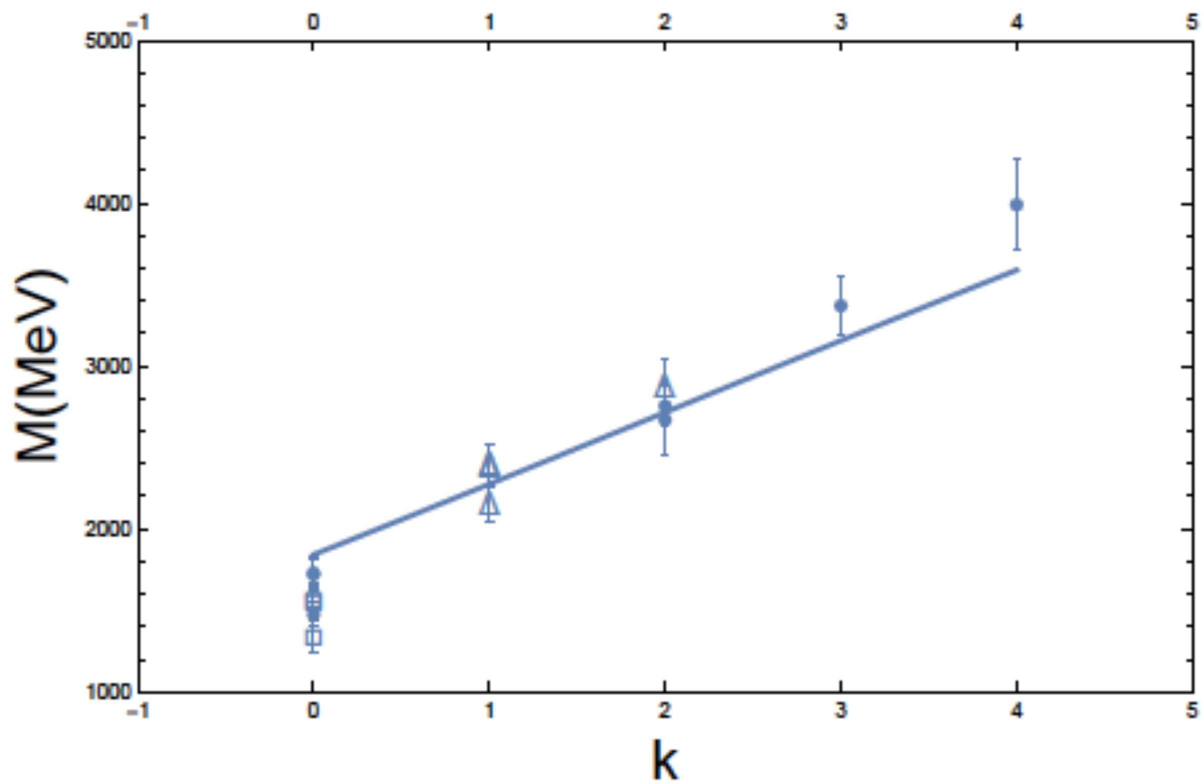
H. Boschi-Filho and N. R. F. Braga,

Softwall models modified AdS5 with dilaton

$$\mathcal{I} = \int d^5x \sqrt{-g} e^{-\Phi} \mathcal{L}$$

$$g_{MN}(z) = e^{-\alpha^2(z^2/R^2)} \frac{R^2}{z^2}(-1,1,1,1,1) \qquad \Phi(z) = \beta^2 z^2$$



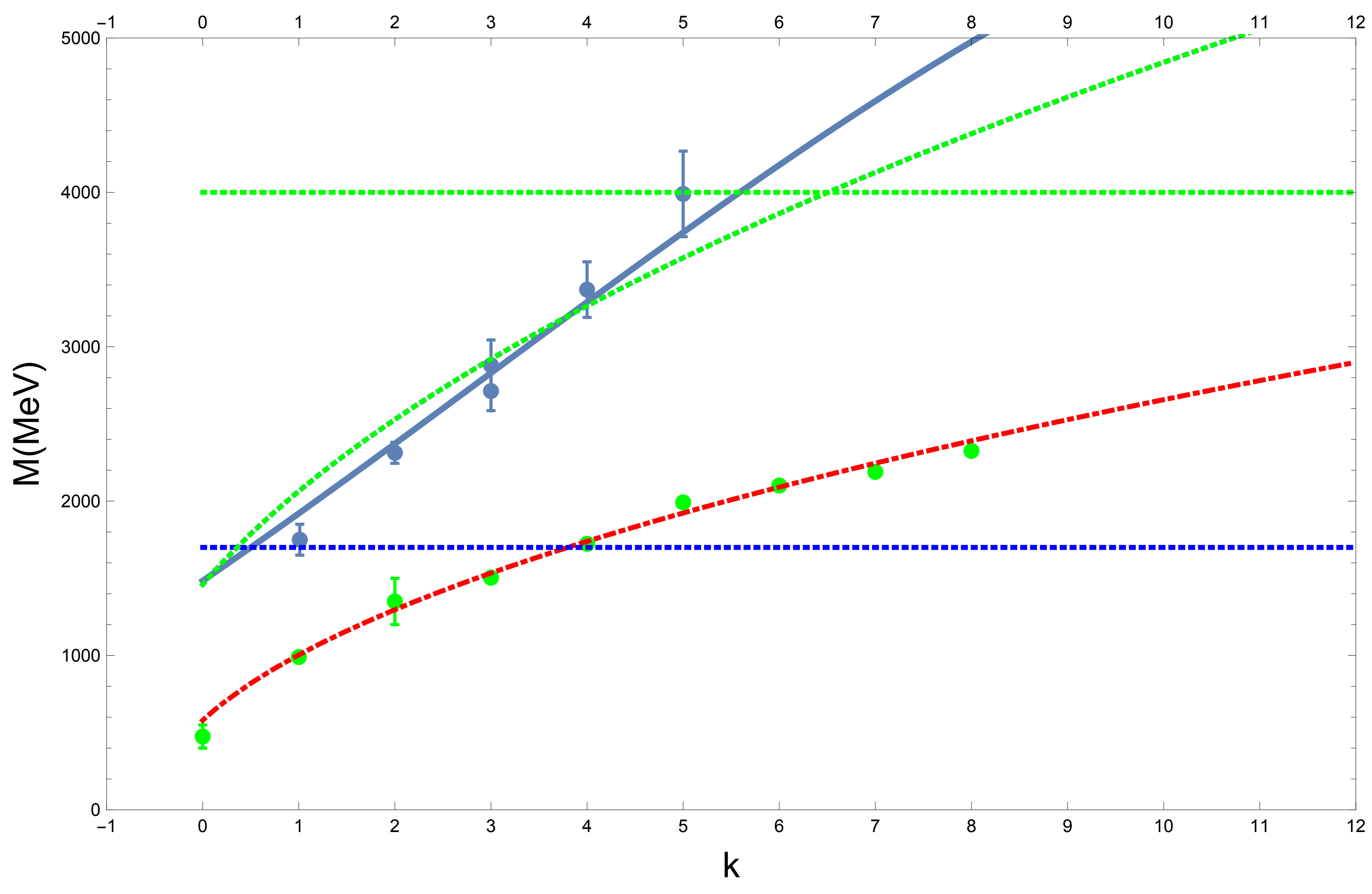


P. Colangelo, F. De Fazio, F. Jugeau and S. Nicotri,

Rinaldi-Vento

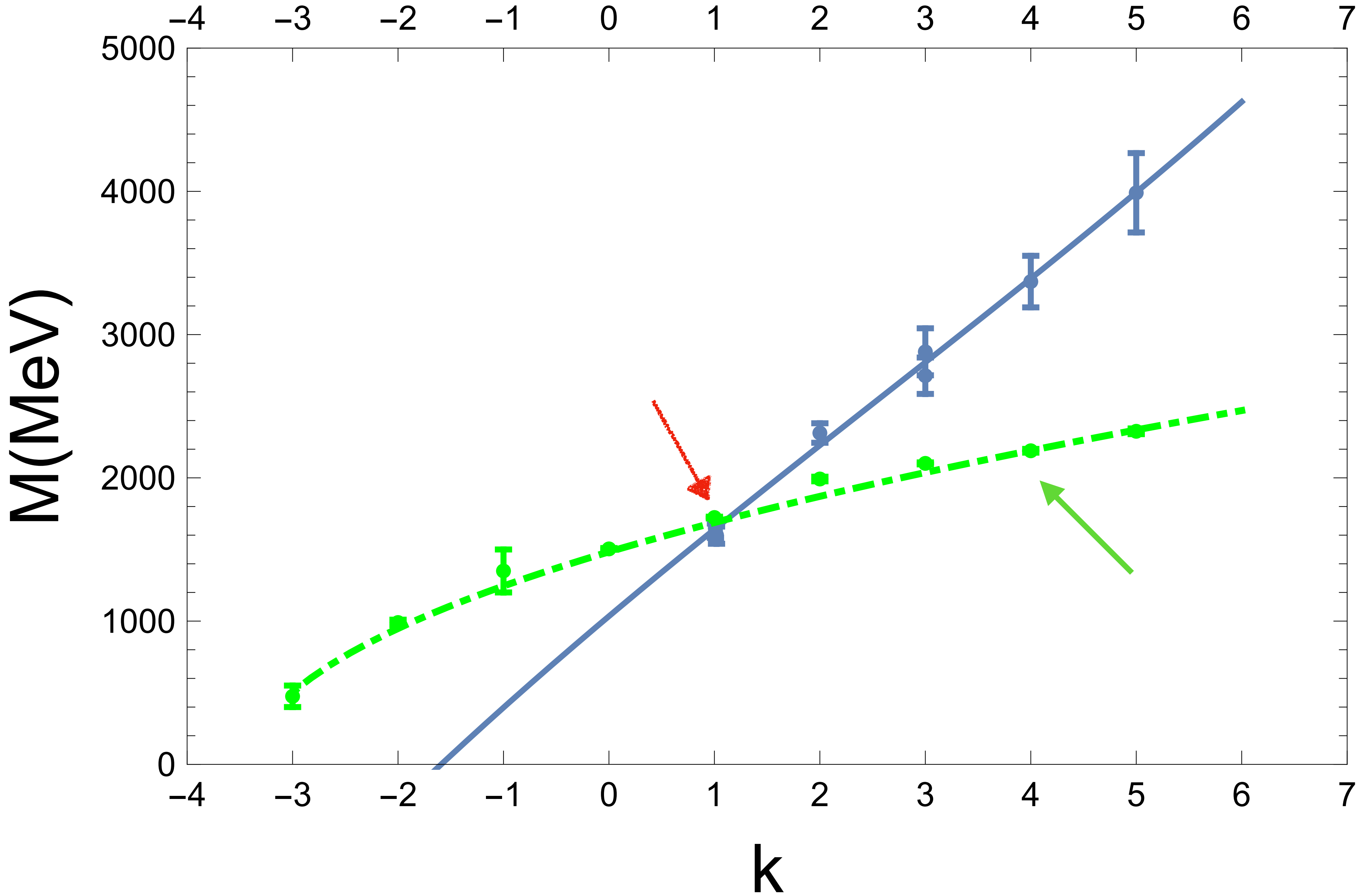
Comparing physical mesons and lattice QCD glueballs

Bottom - up approach



J. Erlich, E. Katz, D. T. Son
and M. A. Stephanov,

Top-down approach



Concluding remarks

- Scalar spectrum: the wishful glueballs f0(1710) and f0(1500)
  i) f0(1710) arises in a scheme with small mixing and is mostly glueball
  ii) f0(1500) arises in a scheme with large mixing and is ~50% glueball.
  The f0(2020) also ~50% glueball.

- The pseudoscalar glueball arises in a strong mixing scenario with the eta' almost 40% glueball and an additional heavy mixed glueball with mass > 2000 MeV.

- AdS/CFT might support a glueball with mass below 1000 MeV.

- AdS/CFT supports strong mixing at the f0(1710)

- AdS/CFT predicts that high mass glueballs will appear unmixed.

Thank you for your attention!!!

Additional information

Chiral Lagrangian

$$\mathcal{L}_0 = \frac{F^2}{4}\langle \partial_\mu U^\dagger \partial^\mu U \rangle + \frac{F^2 B}{2}\langle \mathcal{M} U^\dagger + U \mathcal{M}^\dagger \rangle.$$

$$U = \exp\left(i \frac{\sqrt{2}\mathcal{P}}{F} \right),$$

$$\mathcal{P} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_8}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta_8 \end{pmatrix}$$

$$\mathcal{P} \to \mathcal{P} + \eta_0 \mathbf{1}_3/\sqrt{3}.$$

$$\mathcal{L}_A = \frac{F^2}{16}\frac{\alpha}{N}\left\langle \ln\left(\frac{\det U}{\det U^\dagger}\right)\right\rangle^2 = -\frac{3}{2}\frac{\alpha}{N}\eta_0^2$$

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix}$$

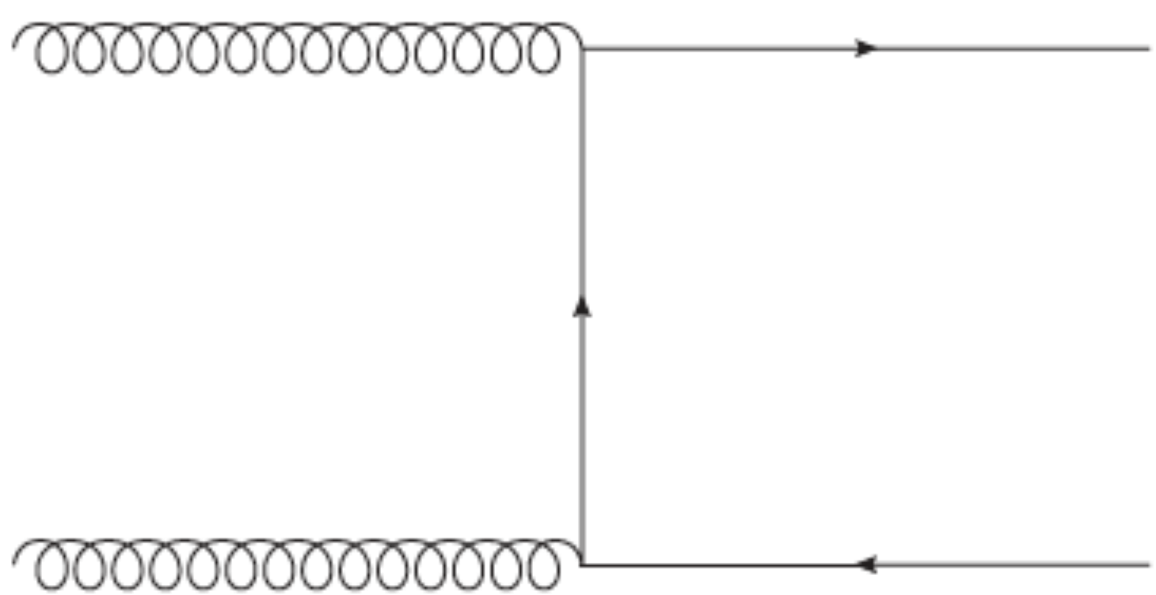
$$\mathcal{M}^2_{80} = \frac{1}{3}\begin{pmatrix} 4M_K^2 - M_\pi^2 & -2\sqrt{2}(M_K^2 - M_\pi^2) \\ -2\sqrt{2}(M_K^2 - M_\pi^2) & 2M_K^2 + M_\pi^2 + 3\alpha \end{pmatrix}$$

$$\mathcal{L}_A = i\frac{F}{4}\sqrt{\frac{\alpha}{N}}Y\left\langle \ln\left(\frac{\det U}{\det U^\dagger}\right)\right\rangle + \frac{1}{2}Y^2$$

$$Y = (\eta_{\text{aux}} + \tilde{g})$$

$$\mathcal{M}^2_{80g} = \frac{1}{3}\begin{pmatrix} 4M_K^2 - M_\pi^2 & -2\sqrt{2}(M_K^2 - M_\pi^2) & 0 \\ -2\sqrt{2}(M_K^2 - M_\pi^2) & 2M_K^2 + M_\pi^2 + 3\alpha & 3\beta \\ 0 & 3\beta & 3\gamma \end{pmatrix}$$

$$\mathcal{M}^2_{80g} = \begin{pmatrix} W & Z & \delta \\ Z & Y + \alpha & \beta \\ \delta & \beta & \gamma \end{pmatrix}.$$



Bag model

The off-diagonal term in the energy expectation value is given by

$$E = \langle G|N\Big\{2g^2 \int d^3x\,\bar{\psi}(x)\gamma^\mu A(x)^a_\mu t^a \int d^4x' S_F(x,x')\gamma^\nu \times A(x')^b_\nu t^b\,\psi(x')\Big\}|\eta(\eta')\rangle \quad (32)$$