



APCTP
Asia Pacific Center for Theoretical Physics



BLTP *Bogoliubov Laboratory of
Theoretical Physics*

On the Effective Strong Coupling and Properties of Mesons and Glueball

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Motivation

- ♣ Many novel behaviors are expected in the region $\sim 1 \div 9$ GeV. Quark confinement, QCD running coupling, glueball states, etc. require a correct description of hadron dynamics in this region.
- ♣ Construct simple, reliable models and apply in different sections (e.g.):
 - **Strong interaction:** mass-dependent strong effective coupling, Fermi coupling, meson spectrum;
 - **Exotic states:** lowest-state glueball mass, radius, etc.;
 - **Weak:** leptonic decay constants;
 - **Radiative transitions:** charmonium excitations' decay widths;

“Conventional” (*quark-antiquark*) states:

$$\text{Spin: } \frac{1}{2} \otimes \frac{1}{2}$$

Ground state (*P & V mesons*) :

$$J^{PC} = 0^{-+}, 1^{-}$$

Charmonium Excitations (*S, A & T*) :

$$J^{PC} = 0^{++}, 1^{++}, 2^{++}$$

“Exotic” (*di-gluon*) states:

$$\text{Spin: } 1 \otimes 1$$

Ground state (*Scalar Glueball*):

$$J^{PC} = 0^{++}$$

Outline

- ***Two-particle bound states within ICM***
- ***Infrared regularizations***
- ***Effective strong coupling***
- ***Scalar Glueball: mass, radius***
- ***Mesons: mass spectrum,
leptonic decay constants***
- ***Fermi coupling within CCQM***
- ***Radiative transitions of charmonium excitations***
- ***Summary and Outlook***

Model: Two-particle Bound States

- Consider a QCD-inspired relativistic quantum-field model.

[G.G., PRD79 (2009); PRD81 (2010); Phys.Part.Nucl.(2014)]

$$L = -\frac{1}{4} \left(F_{\mu\nu}^A - g f^{ABC} A_\mu^B A_\nu^C \right)^2 + \sum_f \left(\bar{q}_f^a \left[\gamma_\alpha \partial_\alpha - m_f + g \Gamma_C^\alpha A_\alpha^C \right]^{ab} q_f^b \right)$$

$$F_{\mu\nu}^B \equiv \partial_\mu A_\nu^B - \partial_\nu A_\mu^B \quad \Gamma_C^\alpha \doteq i\gamma_\alpha t^C$$

- Partition Functional – Path Integral in terms of quark and gluon variables:

$$Z = \iint \delta\bar{q} \delta q \int \delta A \exp \left\{ -\int dx L[\bar{q}, q, A] \right\}$$

- LO** contributions to **quark-antiquark** and **two-gluon** bound states:

$$Z_{(\bar{q}q)} = \iint \delta\bar{q} \delta q \exp \left\{ -(\bar{q} S^{-1} q) + \frac{g^2}{2} \langle (\bar{q} \Gamma A q)(\bar{q} \Gamma A q) \rangle_D \right\}$$

$$Z_{(AA)} = \left\langle \exp \left\{ -\frac{g}{2} (f A A F) \right\} \right\rangle_D$$

$$\langle (\bullet) \rangle_D \doteq \int \delta A e^{-\frac{1}{2}(AD^{-1}A)} (\bullet)$$

- Allocate **one-gluon exchange** between **colored** currents

$$L_{qq} = \frac{g^2}{2} \sum_{f_1 f_2} \iint dx_1 dx_2 J_{\mu f_1 f_2}^B(x_1, x_2) D_{\mu\nu}^{BC}(x_1, x_2) J_{\nu f_1 f_2}^C(x_2, x_1),$$

$$J_{\mu f_1 f_2}^B(x_1, x_2) \equiv i \bar{q}_{f_1}(x_1) \gamma_{\mu} t^B q_{f_2}(x_2).$$

- Isolate **color-singlet** combination
- Perform Fierz transformation for spins ($J = S, P, A, V, T$)
- Introduce **orthonormalized system**: $\{U_Q\}$ with quantum numbers $Q = \{n, l, \dots\}$:
- **Diagonalization** on colorless quark currents $J_N(x)$ with $N = \{Q, J, f_1, f_2\}$
- **Gaussian representation**: a new path integration over auxiliary fields B_N :

$$e^{\frac{1}{2} g^2 (J_N^+ J_N)} = \iint \delta B_N^+ \delta B_N \exp \left\{ - \sum_N (B_N^+ B_N) + g \sum_N [(B_N^+ J_N) + (J_N^+ B_N)] \right\}$$

- **Explicit path-integration** over quark variables and write the effective action
- **Hadronization Ansatz**: B_N fields are identified as meson fields with N

- Z_N rewritten in terms of B_N and all quadratic field configurations are isolated:

$$Z_{(\bar{q}q)} \rightarrow Z_N = \int \prod_N \delta B_N \exp \left\{ -\frac{1}{2} (B_N [1 + g^2 \text{Tr}(V_N S_{m1} V_N S_{m2})] B_N) + W_{resid}[B_N] \right\}$$

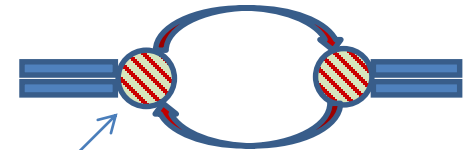
- Diagonalization of the quadratic part is equivalent to the solution of the ladder Bethe-Salpeter equation on the orthonormalized system $\{U_N\}$

$$g^2 \text{Tr}(V_N S_{m1} V_N S_{m2}) = (U_N \lambda U_{N'}) = \lambda_N(-p^2) \delta^{JJ'} \delta^{QQ'}$$

- Symmetric Bethe-Salpeter kernel is defined:

$$\alpha \cdot \lambda_J(-p^2) = \frac{4g^2 C_J}{9} \int \frac{d^4 k}{(2\pi)^4} \{V(k)\}^2 \cdot \Pi_J(k, p)$$

$$\Pi_J(p, k) = -\frac{1}{4!} \text{Tr} \left\{ \Gamma_J \tilde{S}_{m1} (\hat{k} + \xi_1 \hat{p}) \Gamma_J \tilde{S}_{m2} (\hat{k} - \xi_2 \hat{p}) \right\}$$



$$V_J(k) = \int dx \sqrt{D(x)} U_J(x) e^{ikx}$$

UV Regularization

- Renormalization:
$$U_{REN}(x) \equiv \sqrt{-\alpha \dot{\lambda}_N(M_N^2)} \cdot U_N(x)$$

$$\begin{aligned} \langle U_N | 1 + \alpha \lambda_N(-p^2) | U_N \rangle &= \langle U_N | 1 + \alpha \lambda_N(M_N^2) - \alpha \dot{\lambda}_N(M_N^2)(p^2 + M_N^2) | U_N \rangle \\ &= \langle U_{REN} | (p^2 + M_N^2) | U_{REN} \rangle \end{aligned}$$

Meson Mass Equation

$$1 + \alpha \lambda_N(M_N^2) = 0, \quad p^2 = -M_N^2$$

- Infrared (IR) divergences appear in the loop and vertice functions:

$$\Pi_J(p, k) = -\frac{1}{4!} \text{Tr} \left\{ \Gamma_J \tilde{S}_{m_1}(\hat{k} + \xi_1 \hat{p}) \Gamma_J \tilde{S}_{m_2}(\hat{k} - \xi_2 \hat{p}) \right\}$$

$$V_J(k) = \int dx \sqrt{D(x)} U_J(x) e^{ikx}$$

IR Regularization

- “**Infrared**” regularizations of **propagators** remove these divergencies:

$$\tilde{S}_m(\hat{p}) = \frac{1}{-i\hat{p} + m} = (i\hat{p} + m) \cdot \int_0^{\infty} dt \exp\{-t \cdot (p^2 + m^2)\}$$

$$\Rightarrow \tilde{S}_{IR}(\hat{p}) = (i\hat{p} + m) \cdot \int_0^{1/\Lambda^2} dt \exp\{-t \cdot (p^2 + m^2)\}$$

$$D(x) = \frac{1}{4\pi^2 x^2} = \int_0^{\infty} ds e^{-sx^2} \Rightarrow D_{IR}(x) = \int_0^{\Lambda^2/2} ds e^{-sx^2}$$

Λ – mass scale of IR
confinement region

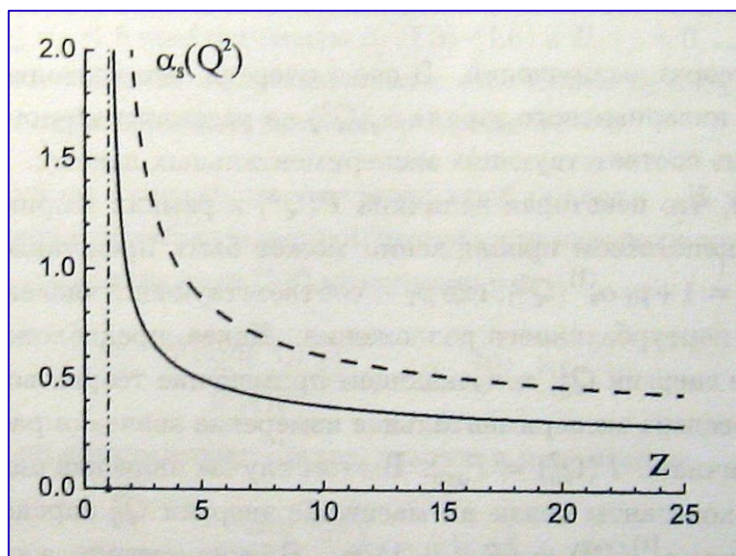
$\Lambda \rightarrow 0$ deconfinement

- Another “**Infrared**” regularization of **entire q-qbar loop** is used in the *Covariant Confined Quark Model* (see the 2nd part)

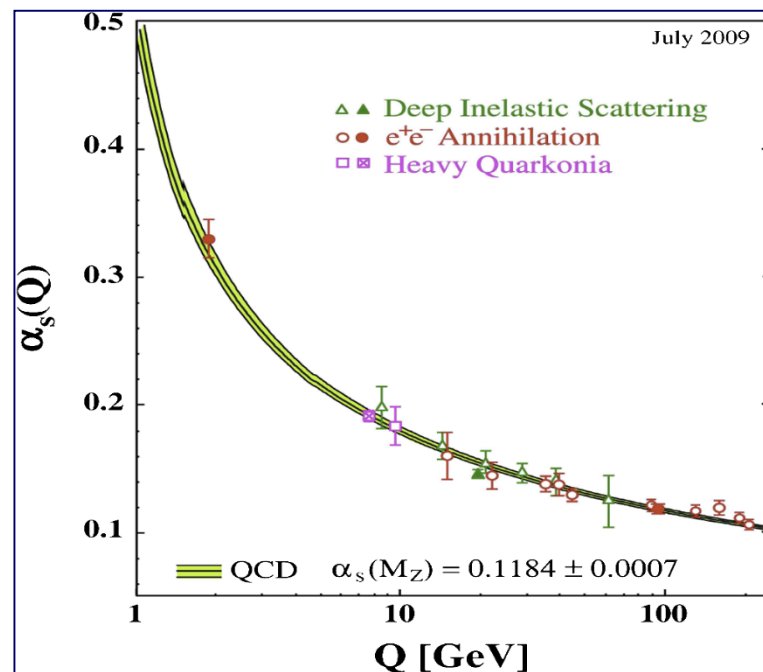
QCD Effective Coupling

THEORY: QCD predicts a dependence of $\alpha_s(Q)$ on energy scale Q .
This *dependence* is described theoretically by the Renorm Group equations.

EXPERIMENT: but its *actual value* must be obtained from experiment.
It is well determined experimentally at relatively high energies $Q > 2$ GeV.



Perturbation theory: one-loop (dashed) and two-loop (solid) approximations to $\alpha_s(Q^2)$, where $Z=Q^2/\Lambda^2$



$\alpha_s(Q < 1) \rightarrow ?$

Effective (Mass-dependent) Strong Coupling

$$1 + \tilde{\alpha}_s \lambda_N(M_N^2) = 0, \quad p^2 = -M_N^2$$

$$U_{nl\mu}(x, a) \approx T_{\ell\mu}(ax) \cdot L_n^{l+1/2}(a^2 x^2) \cdot \sqrt{D(x)} \cdot e^{-ax^2}, \quad \sum_{\mu} \int dx [U_{nl\mu}(x, a)]^2 = 1$$

[GG, J.Phys. CS 938 (2017) 012047]

- Particularly, for
 $n=l=0,$
 $m_1 = m_2 = M/2:$

$$\tilde{\alpha}_s(M) = 1 / \tilde{\lambda}(M^2)$$

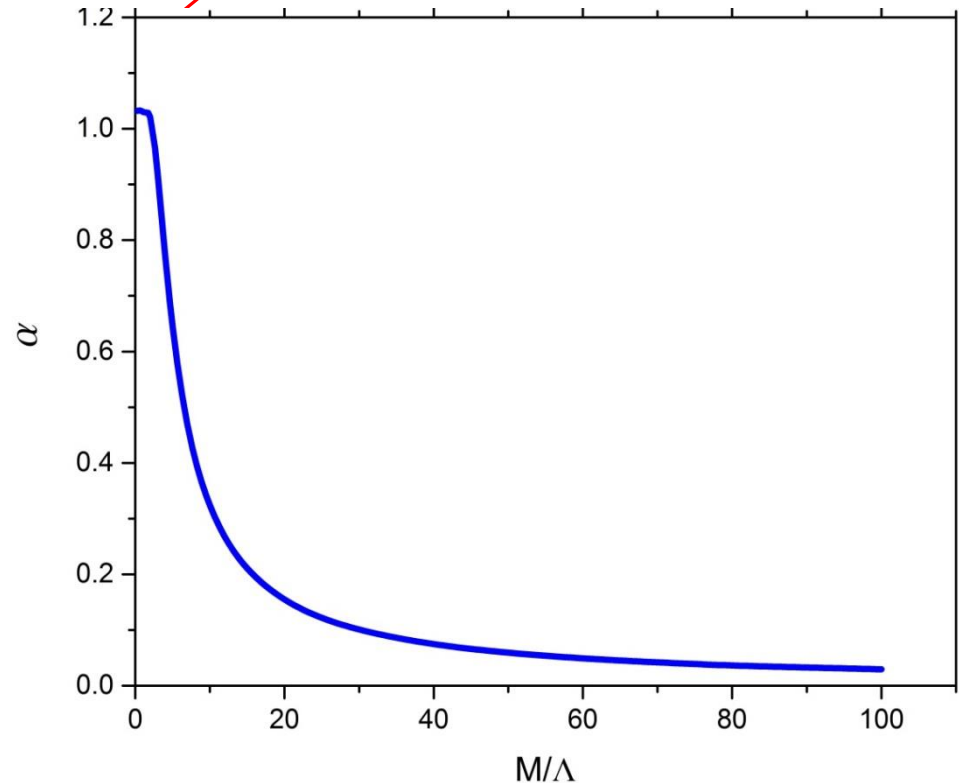
Infra-red fixed point

$$\tilde{\alpha}_s(0) \approx 1.032$$

for any $\Lambda > 0$

There exist dispersion relations between
 time-like and space-like couplings:

$$\alpha(M) \leftrightarrow \alpha(Q)$$



Glueballs (gg , ggg , ...)

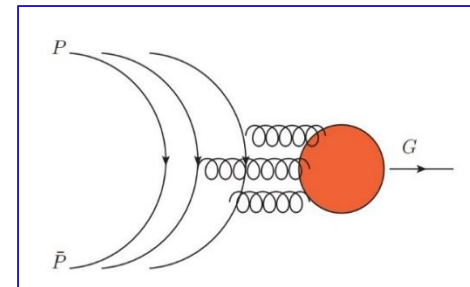
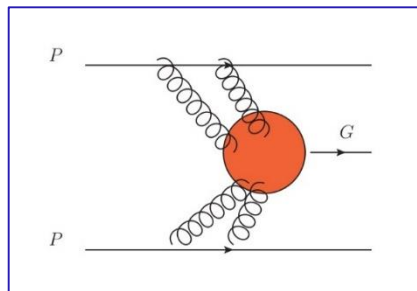
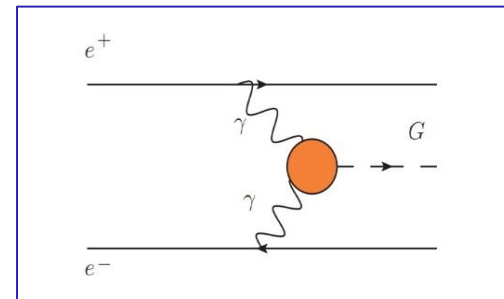
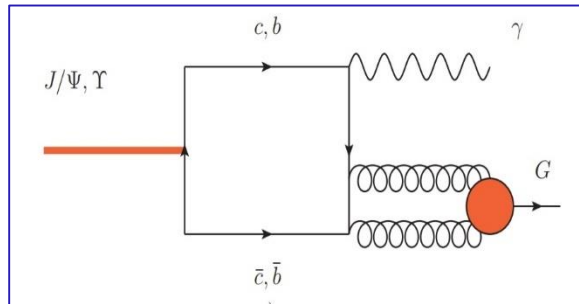
Experimental status:

Main signatures expected for glueballs:

- enhanced production in gluon-rich channels of radiative decays,
- decay branching fractions incompatible with (q-qbar) states.

Production expected in:

- heavy quarkonium radiative decays (**BESIII, Belle, BaBar, LHCb**)
- photon-photon fusion (**BESIII, Belle, BaBar**)
- central meson production (**RHIC, LHC**)
- proton-antiproton annihilation (**PANDA**)
- heavy quark (b) weak decay (**BESII, Belle, LHCb**)

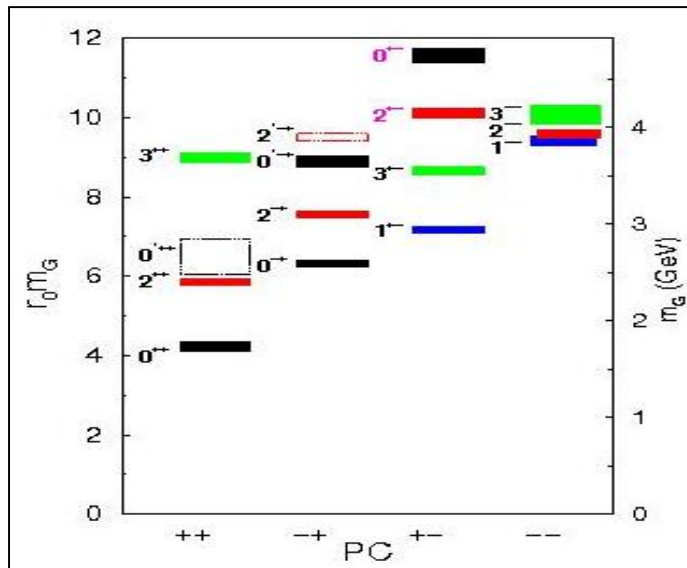


Theoretical status: QCD predicts glueballs due to self-interaction of gluons.

- MIT bag (gg) *Jaffe, Johnson*
- QCD sum rules *Narison, Forkel, SVZ, Vento, P. Zhang, Pimikov, ...*
- Constituent gluon models *Simonov, Anisovich, Lubovitskij, ...*
- Instanton-inspired models *Kochelev, Mathieu, Dong-Pil Min, ...*
- Lattice calculations *Morningstar, Peardon, Weingarten (Q), Gregory, McNeile, Chen (unQ), ...*
- Holographic Ads/QCD models *Vento, Colangelo, Rinaldi, ...*

Identification problems:

- Possible mixing with quarkonia ($q\text{-}q\text{bar}$, $q^2\text{-}q\text{bar}^2$) *[e.g., V.Vento, 2016]*
- Existence of exotic glueballs - oddballs (ggg, ...) *[e.g., P.Colangelo, 2015]*
- Many states with the same quantum numbers for $M > 1 \text{ GeV}$ *[e.g., PDG-2016]*



$$J = 0, 1, 2, 3$$

$$PC = ++, -+, +-, --$$

$$M_G \approx 1.6 - 4.9 \text{ GeV}$$

Glueballs (gg)

Scalar Glueball: (field strength square) $\mathbf{J}^{PC} = \mathbf{0}^{++}$

$$O_S = \delta^{\mu\alpha} \delta^{\nu\beta} (gG_{\mu\nu}^A)(gG_{\alpha\beta}^A) = (gG_{\mu\nu}^A)^2$$

Pseudoscalar Glueball: (topological charge density) $\mathbf{J}^{PC} = \mathbf{0}^{-+}$

$$O_P = \frac{1}{2} \varepsilon^{\mu\nu\alpha\beta} (gG_{\mu\nu}^A)(gG_{\alpha\beta}^A)$$

Tensor Glueball: (energy density) $\mathbf{J}^{PC} = \mathbf{2}^{++}$

$$O_T = \frac{1}{4} (gG_{\mu\nu}^A)^2 - (gG_{0\alpha}^A)(gG_{0\beta}^A)$$

Scalar Glueball (gg)

$$J^{PC} = 0^{++}$$

Candidates: $f_0(600)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$, $f_0(1710)$, $f_0(1790)$

Signatures:

- Large branching to decay to strange mesons $0^{++} \rightarrow K\bar{K}$
- Large branching to decay to η $0^{++} \rightarrow \eta\eta, \eta' \sigma$
- Weak coupling to photon pairs $0^{++} \rightarrow \gamma\gamma$
- Perturbative coupling to heavy quarks (c & b)

Some theoretical estimates:

$1750 \pm 50 \pm 80$ MeV

C.J.Morningstar, M.Pearidon (2004).

$1710 \pm 50 \pm 58$ MeV

Anisotropic lattice, infinite volume, continuum limit (Y.Chen 2006)

1710 ± 50 MeV

Analytic confinement model (G.Ganbold PRD79, 2009)

1795 ± 60 MeV

Unquenched Lattice 2+1-flavor $m_\pi = 360$ MeV (Gregory et al. 2012)

$1790 \pm 50 \pm 20$ MeV

UK-QCD Collaboration (2014)

1624 ± 140 MeV

Lattice 2-flavor $m_\pi = 580$ MeV (Y.Chen et al. 2016)

Scalar glueball
mass:

$$\Lambda = 236.0 \text{ MeV},$$
$$\alpha(M_G) = 0.451$$



$$M_G \approx 1738.6 \text{ MeV}$$

[GG, J.Phys. CS 938 (2017) 012047]

Scalar glueball
"radius":

$$r_G \cdot M_G = \frac{M_G}{2\Lambda} \sqrt{\frac{\int d^4x \cdot x^2 \cdot W(x) \cdot U^2(x)}{\int d^4x \cdot W(x) \cdot U^2(x)}} = 4.41$$

$$\approx 4.35 \text{ UK-QCD Collaboration (2014)}$$

$$1/r_G = M_G / 4.41 = 394 \text{ MeV}$$

$$\approx 410 \pm 20 \text{ M pure SU}_c(3) \text{ gauge theory (CJM+MP)}$$

Gluon
condensate:

$$\left\langle \frac{\alpha}{\pi} F^2 \right\rangle = \frac{16 N_c}{\pi} \alpha \Lambda^4 = 0.0214 \text{ GeV}^4$$

$$\approx 0.0223 \pm 0.0041 \text{ GeV} \text{ S.Narison, PLB706 (2012)}$$

Meson Spectrum

a) Analytic results

- Asymptotical Regge-type behaviour:

$$M_{nl}^2 \approx M_{00}^2 + (n+l) \cdot \text{const} \quad \text{for } \{n,l\} \geq 3$$

- Due to spin effect vector mesons are heavier than pseudoscalars at the same quark contents:

$$1 \approx C_J \cdot \exp(M_J^2) \cdot (M_J^2 + \text{const})$$
$$1 = C_P > C_V = 1/2$$



$$M_P^2 < M_V^2$$

- The coupling is bounded from above:

$$\alpha_s(M) = 1 / \lambda_J(M^2) \leq \alpha_s^{\max}$$

Conventional Meson Mass Estimates

Model parameters fixed by fitting the meson spectrum:

$$\Lambda = 236.0 \text{ MeV},$$

$$m_{ud} = 227.6 \text{ MeV}, \quad m_s = 420.1 \text{ MeV},$$

$$m_c = 1521.6 \text{ MeV}, \quad m_b = 4757.2 \text{ MeV}.$$

[GG, J.Phys. CS 938 (2017) 012047]

|relative errors| < 1.8%

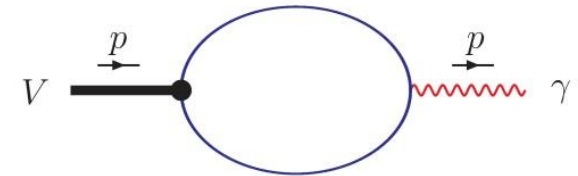
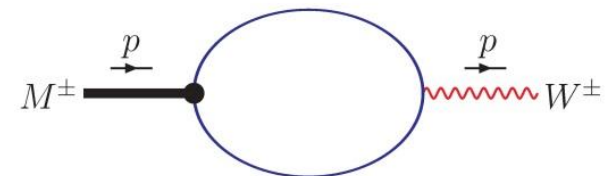
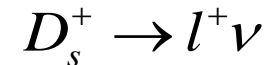
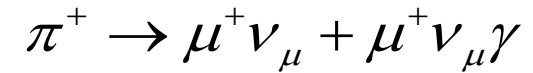
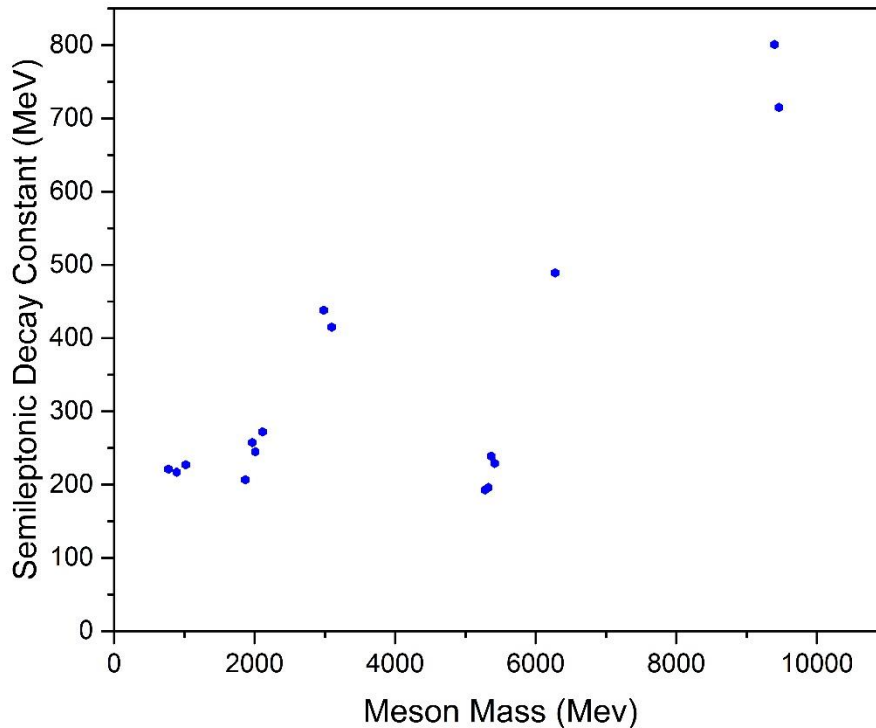
$J^{PC} = 0^{-+}$	PDG-2016	Our estim.	$J^{PC} = 1^{--}$	PDG-2016	Our estim.
D (~uc)	1869.62	1893.6	ρ (uu)	775.26	774.3
D_s (~sc)	1968.50	2003.7	K^* (us)	891.66	892.9
η_c (~cc)	2983.7	3032.5	D^* (uc)	2010.29	2003.8
B (~ub)	5279.26	5215.2	D_s^* (~sc)	2112.3	2084.1
B_s (~sb)	5366.77	5323.6	J/ ψ (~cc)	3096.92	3077.6
B_c (~cb)	6274.5	6297.0	B^* (~ub)	5325.2	5261.5
η_b (~bb)	9398.0	9512.5	Y (~bb)	9460.30	9526.4

Weak (leptonic) Decay Constants

Important value in particle physics:

$$i f_P p_\mu = \langle 0 | J_\mu(0) | U_{renorm}(p) \rangle$$

Mass dependence of weak (leptonic) decay constants (PDG 2017):



$$if_p p_\mu = \frac{g}{6} \int \frac{d^4 k}{(2\pi)^4} \int dx e^{ikx} U_R(x) \sqrt{D(x)} \text{Tr} \left\{ i\gamma_5 \tilde{S}_{m_1} (\hat{k} + \xi_1 \hat{p}) i\gamma_5 \gamma_\mu \tilde{S}_{m_2} (\hat{k} - \xi_2 \hat{p}) \right\}$$

R – “size” of meson in mass scale
in the Fourier transformation

$$\tilde{U}_R(k) \sim \int_0^1 ds f(s) \cdot \exp \left\{ -\frac{s \cdot k^2}{R^2} \right\}$$

Model parameters:

$$\Lambda = 236.0 \text{ MeV},$$

$$m_{ud} = 227.65 \text{ MeV}, \quad m_s = 420.07 \text{ MeV},$$

$$m_c = 1521.61 \text{ MeV}, \quad m_b = 4757.20 \text{ MeV}.$$

[GG, J.Phys. CS 938 (2017) 012047]

R – estimated to fit experimental
data on meson masses and
leptonic decay constants
(in **GeV**)

D	D _s	η _c	B	B _s	B _c	η _b
0.93	1.08	1.83	1.73	2.18	3.34	3.80
ρ	K*	D*	D _s *	J/ψ	B*	Υ
0.33	0.38	0.78	0.90	2.40	3.34	2.80

• *Estimated values of the leptonic decay constants:*

$J^{PC} = 0^{-+}$	Exp.data	Our estim.	$J^{PC} = 1^{--}$	Exp.data	Our estim.
f_D	206.7 ± 8.9	207	f_ρ	221 ± 1	221
f_{D_s}	257.5 ± 6.1	257	f_{K^*}	217 ± 7	217
f_{η_c}	238 ± 8	238	f_{D^*}	245 ± 20	245
f_B	192.8 ± 9.9	193	$f_{D_s^*}$	272 ± 26	271
f_{B_s}	238.8 ± 9.5	239	$f_{J/\psi}$	415 ± 7	416
f_{B_c}	489 ± 5	488	f_{B^*}	196 ± 44	196
f_{η_b}	801 ± 9	800	f_Y	715 ± 5	715

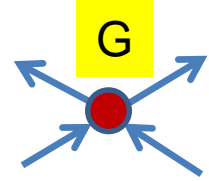
[GG, J.Phys. CS 938 (2017) 012047]

- + Agreement between estimated and experimental data is accurate.
- + The meson “size” shrinks $\sim 1/R$ as the mass grows.

Fermi Coupling

• Fermi-type model

$$L_F = \bar{q} (i\hat{\partial} - m) q + \frac{G}{2} (\bar{q} \Gamma q)^2$$



Generating functional

$$\Pi(x-y) \equiv -i \cdot \text{tr} \{ \Gamma \cdot \Phi \cdot S(x-y) \cdot \Gamma \cdot \Phi \cdot S(y-x) \}$$

CCQM used:

Vertex function:

$$\tilde{\Phi}_H(-p^2) = \exp\left(\frac{p^2}{\Lambda_H^2}\right)$$

$1/\Lambda_H \sim$ hadron “size”

Quark propagator:

$$\tilde{S}_{m_1}(\hat{p}) = \frac{m_1 + \hat{p}}{m_1^2 - p^2} = (m_1 + \hat{p}) \cdot \int_0^\infty ds_1 \exp\left[-s_1 (m_1^2 - p^2)\right]$$

Meson mass equation:

$$1 = G \cdot \tilde{\Pi}(M^2)$$

G.Ganbold, T.Gutsche, M.Ivanov, V.Lubovitsky
J.Phys. G 42, 075002 (2015).

$$\tilde{\Pi}_H(p^2) = \frac{3}{4\pi^2} \int_0^\infty \frac{dt \cdot t}{a_H^2} \int_0^1 ds \exp(-t \cdot z_0 + z_H) \cdot \left[\frac{n_H}{a_H} + m_1 m_2 + p^2 \left(\omega_1 - \frac{b}{a_H} \right) \left(\omega_2 + \frac{b}{a_H} \right) \right]$$

Integral $\int_0^\infty \frac{dt \cdot t}{a_H^2} \dots = \text{diverges}$ and a threshold singularity appears!

- “Infra-red” regularization with parameter λ has been used for the **whole loop**.

for $\lambda > 0$: $\int_0^{1/\lambda^2} \frac{dt \cdot t}{a_H^2} \dots = \text{converges!}$

- Model parameters:
fixed earlier by fitting semileptonic
decay constants and electromagnetic
decay rates of mesons.

G.Ganbold, T.Gutsche, M.Ivanov, V.Lubovitsky
J.Phys. G 42, 075002 (2015).

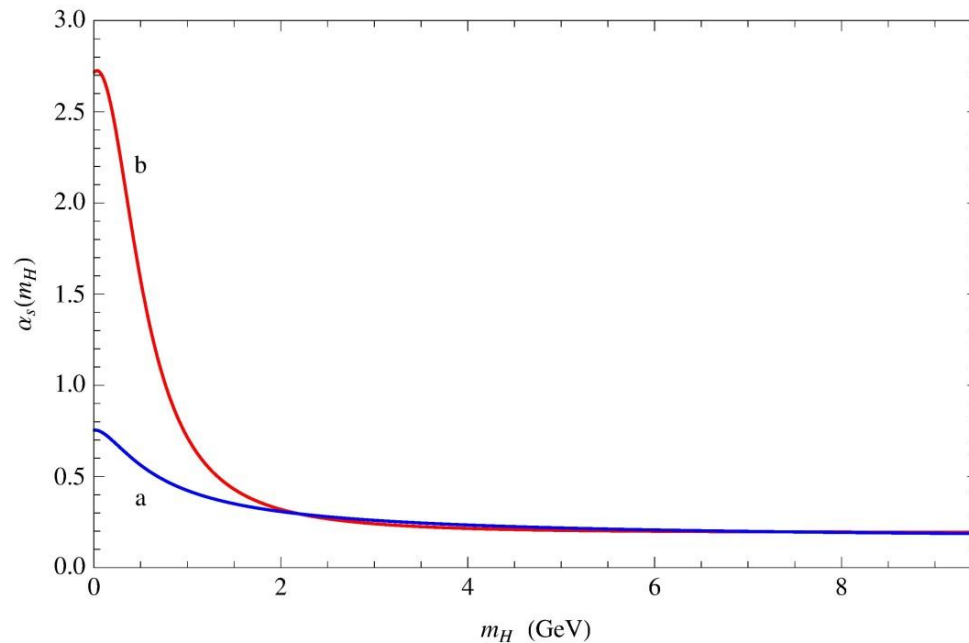
$$\begin{aligned} \lambda &= 0.181 \text{ GeV}, \\ m_{ud} &= 0.235 \text{ GeV}, \\ m_s &= 0.442 \text{ GeV}, \\ m_c &= 1.67 \text{ GeV}, \\ m_b &= 5.07 \text{ GeV} \end{aligned}$$

Fermi G coupling: Comparison with QCD Running Coupling

We compare dimensionless Fermi coupling $1.74 \lambda^2 G$ (red curve) to QCD effective charge α_s (blue curve).

G.Ganbold, T.Gutsche,
M.Ivanov, V.Lubovitsky
J.Phys. G 42, 075002 (2015).

G.Ganbold,
Phys. Rev. D 81, 094008 (2010)
Phys. Part. Nucl. 43, 79, (2012)
Phys. Part. Nucl. 45, 10, (2014).



[GG, Eur.Phys. J WC 138 (2017) 04004]

- Despite the different model origins, the behaviors of two curves are very similar each other in the region above ~ 2 GeV.
- The values at origin are mostly determined by the different confinement mechanisms. This explains the different behaviors below 2 GeV.

Charmonium Radiative Decays

Motivation:

PRL **119**, 062001 (2017)

PHYSICAL REVIEW LETTERS

week ending
11 AUGUST 2017

Observation of the Decays $\Lambda_b^0 \rightarrow \chi_{c1} p K^-$ and $\Lambda_b^0 \rightarrow \chi_{c2} p K^-$

R. Aaij *et al.**

(LHCb Collaboration)

(Received 27 April 2017; published 8 August 2017)

- ♣ First observation of these decays @LHCb in ***p-p*** collision at energy (c.m.) **8 TeV**
- ♣ Measured ratios of branching fractions:

$$\frac{\mathcal{B}(\Lambda_b^0 \rightarrow \chi_{c1} p K^-)}{\mathcal{B}(\Lambda_b^0 \rightarrow J/\psi p K^-)} = 0.242 \pm 0.014 \pm 0.013 \pm 0.009,$$
$$\frac{\mathcal{B}(\Lambda_b^0 \rightarrow \chi_{c2} p K^-)}{\mathcal{B}(\Lambda_b^0 \rightarrow J/\psi p K^-)} = 0.248 \pm 0.020 \pm 0.014 \pm 0.009,$$
$$\frac{\mathcal{B}(\Lambda_b^0 \rightarrow \chi_{c2} p K^-)}{\mathcal{B}(\Lambda_b^0 \rightarrow \chi_{c1} p K^-)} = 1.02 \pm 0.10 \pm 0.02 \pm 0.05,$$

$$\frac{\mathcal{B}(\Lambda_b^0 \rightarrow \chi_{c1} p K^-)}{\mathcal{B}(\Lambda_b^0 \rightarrow J/\psi p K^-)} = 0.242 \pm 0.014 \pm 0.013 \pm 0.009,$$

$$\frac{\mathcal{B}(\Lambda_b^0 \rightarrow \chi_{c2} p K^-)}{\mathcal{B}(\Lambda_b^0 \rightarrow J/\psi p K^-)} = 0.248 \pm 0.020 \pm 0.014 \pm 0.009,$$

$$\frac{\mathcal{B}(\Lambda_b^0 \rightarrow \chi_{c2} p K^-)}{\mathcal{B}(\Lambda_b^0 \rightarrow \chi_{c1} p K^-)} = 1.02 \pm 0.10 \pm 0.02 \pm 0.05,$$

$$\frac{B(B \rightarrow \chi_{c2} K)}{B(B \rightarrow \chi_{c1} K)}$$

Belle (2008), BaBar (2009), LHCb (2013): ***SUPPRESSED***
M.Beneke NPB811 (2009) Factorization approach: ***SUPPRESSED***

Belle Collaboration (2016): ***SUPPRESSION LESSENERD***
if additional particles are present in the final state

Charmonium orbital (triplet) excitations: χ_{cJ} ($J = 0, 1, 2$)

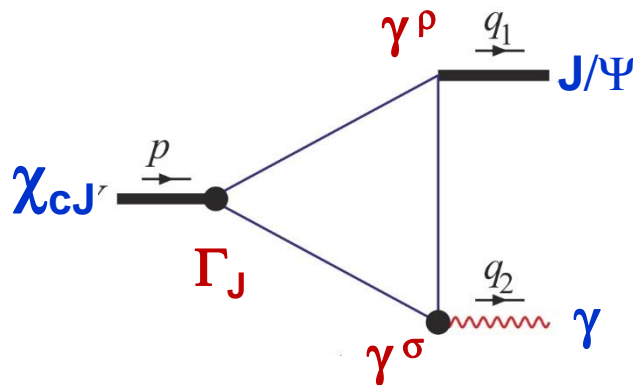
	J^{PC}	Spin	L	Mass (MeV)	Current	$\Gamma_{\text{full PDG-2018}}$
η	0^{-+}	0	0	2983.4	<i>Pseudoscalar</i>	31.8 ± 0.8 MeV
J/Ψ	1^{--}	1	0	3096.90	<i>Vector</i>	92.9 ± 2.8 keV
χ_{c0}	0^{++}	1	1	3414.75	<i>Scalar</i>	10.8 ± 0.6 MeV
χ_{c1}	1^{++}	1	1	3525.38	<i>Axial-vector</i>	0.84 ± 0.04 MeV
χ_{c2}	2^{++}	1	1	3556.20	<i>Tensor</i>	1.97 ± 0.09 MeV

Dominant (one-photon) transitions:

$$\Gamma_0 = I,$$

$$\Gamma_1 = \gamma_\mu \gamma_5,$$

$$\Gamma_2 = \frac{i}{2} (\gamma_\mu \overleftrightarrow{\partial}_\nu + \gamma_\nu \overleftrightarrow{\partial}_\mu).$$



+ 'bubble' diagrams



$$g_{\chi_{cJ} \rightarrow J/\Psi + \gamma}$$

Matrix elements and Decay widths of transitions:

in collaboration with T.Gutsche, M.A.Ivanov, V.Lubovitsky

Scalar Excitation: $J^{PC} = 0^{++}$

$X = \chi_{c0}$

Matrix element:

$$M(\chi_{c0} \rightarrow J/\Psi + \gamma) \approx D \cdot [p^\sigma q_2^\rho - g^{\sigma\rho}(pq_2)]$$

Decay width:

$$\Gamma(\chi_{c0} \rightarrow J/\Psi + \gamma) = \frac{\alpha}{24} M_{\chi_{c0}}^3 \left(1 - \frac{M_{J/\Psi}^2}{M_{\chi_{c0}}^2}\right)^3 \cdot g_{\chi_{c0} \rightarrow J/\Psi + \gamma}^2(D)$$

Axial-vector Excitation: $J^{PC} = 1^{++}$

$$X = \chi_{c1}$$

Matrix element:

$$M(\chi_{c1} \rightarrow J/\Psi + \gamma) \approx \varepsilon^{q_2\mu\rho\sigma}(q_1q_2) \cdot W_1 + \varepsilon^{q_1\mu\rho\sigma}(q_1q_2) \cdot W_5 \\ + \varepsilon^{q_1q_2\rho\sigma} q_1^\mu \cdot W_2 + \varepsilon^{q_1q_2\mu\sigma} q_2^\rho \cdot W_3 + \varepsilon^{q_1q_2\mu\rho} q_1^\sigma \cdot W_4$$

Decay width:

$$\Gamma(X \rightarrow \gamma J/\psi) = \frac{1}{12\pi} \frac{|\vec{q}_2|}{m_X^2} (|H_L|^2 + |H_T|^2)$$

Helicity amplitudes:

$$H_L = i \frac{m_X^2}{m_{J/\psi}} |\vec{q}_2|^2 \left[W_1 + W_3 - \frac{m_{J/\psi}^2}{m_X |\vec{q}_2|} W_4 \right], \\ H_T = -im_X |\vec{q}_2|^2 \left[W_1 + W_2 - \left(1 + \frac{m_{J/\psi}^2}{m_X |\vec{q}_2|} \right) W_4 \right], \\ |\vec{q}_2| = \frac{m_X^2 - m_{J/\psi}^2}{2m_X}.$$

Tensor Excitation: $J^{PC} = 0^{++}$

$X = \chi_{c2}$

Matrix element:

$$M(\chi_{c2} \rightarrow J/\Psi + \gamma) \sim A \cdot \left\{ \begin{array}{l} g^{\mu\rho} \left[g^{\sigma\nu} (q_1 q_2) - q_1^\sigma q_2^\nu \right] \\ + g^{\nu\rho} \left[g^{\sigma\mu} (q_1 q_2) - q_1^\sigma q_2^\mu \right] \end{array} \right\} \\ + B \cdot \left\{ \begin{array}{l} g^{\sigma\rho} \left[q_1^\mu q_2^\nu + q_1^\nu q_2^\mu \right] \\ - g^{\mu\sigma} q_1^\nu q_2^\rho - g^{\nu\sigma} q_1^\mu q_2^\rho \end{array} \right\}$$

Decay width:

$$\Gamma(\chi_{c2} \rightarrow J/\Psi + \gamma) = \frac{\alpha}{20} M_{\chi_{c2}}^3 \left(1 - \frac{M_{J/\Psi}^2}{M_{\chi_{c2}}^2} \right)^2 \cdot g_{\chi_{c2} \rightarrow J/\Psi + \gamma}^2(A, B)$$

Charmonium excitations: One-photon decay Widths

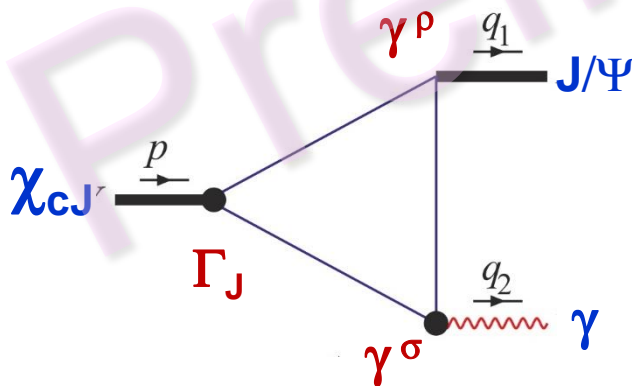
		$\Lambda_{\chi_{cJ}}$	Mass	$\Gamma^{theor}(\chi_{cJ} \rightarrow J/\Psi + \gamma)$	$\Gamma^{experim}(\chi_{cJ} \rightarrow J/\Psi + \gamma)$
J/Ψ	1^{--}	1.63 GeV	2983.4 MeV	$0.163 \cdot 10^{-5}$ GeV	$0.158 \cdot 10^{-5}$ GeV
χ_{c0}	0^{++}	1.98 GeV	3096.90 MeV	$0.129 \cdot 10^{-3}$ GeV	$0.133 \cdot 10^{-3}$ GeV
χ_{c1}	1^{++}	2.41 GeV	3414.75 MeV	$0.273 \cdot 10^{-3}$ GeV	$0.285 \cdot 10^{-3}$ GeV
χ_{c2}	2^{++}	2.84 GeV	3525.38 MeV	$0.385 \cdot 10^{-3}$ GeV	$0.371 \cdot 10^{-3}$ GeV

$$\lambda = 0.181 \text{ MeV}$$

$$m_c = 1.68 \text{ GeV}$$

PDG-2018

Dominant (one-photon) transitions:



$$\Gamma_0 = I,$$

$$\Gamma_1 = \gamma_\mu \gamma_5,$$

$$\Gamma_2 = \frac{i}{2} \left(\gamma_\mu \overleftrightarrow{\partial}_\nu + \gamma_\nu \overleftrightarrow{\partial}_\mu \right).$$

Summary:

- ♣ **Analytic (Infrared) Confinement** conception combined with QFT methods may serve reasonable frameworks to address simultaneously **different sectors in particle physics, such as:**
 - meson ground state spectrum (except light mesons: π , K)
 - weak (leptonic) decay constants of conventional mesons
 - running (mass dependent) strong effective coupling
 - the lowest (scalar) glueball mass and radius
 - the gluon condensate
 - Fermi coupling's smooth dependence on mass scale
 - radiative transition parameters and widths of excited charmonia

Outlook:

Our models can be extended to study:

- ♣ light mesons (scalar, isoscalar, ...).
- ♣ higher glueball states (0^{-+} , 2^{--} , ...)
- ♣ admixture states ($qq + gg$, ...)
- ♣ radial excitations of mesons (charmonia and bottomia).
- ♣ exotic mesons (tetraquark, $X(3872)$ and $Z(4430)$...)
- ♣ heavy baryon decays ($\Lambda_b \rightarrow \Lambda^* + \chi_{cJ}$)