Exotic scalar glueballs in QCD Sum Rules

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in collaboration with
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based on PRD95, PRL119, PRD96 (2017)

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Outline

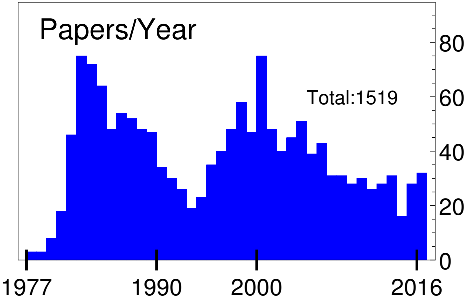
1. QCD Sum Rules (SR)
2. Constructing current for $0^{--}-$glueball [PRD95, PRL119 (2017)]
   - ill-defined current
   - Dimension-12 current
3. QCD Sum Rules for $0^{\pm -}-$glueballs [PRD96 (2017)]
   - Dimention-9 current
   - Results for $0^{\pm -}-$glueballs
4. Summary

Glueballs were studied within:

- Flux tube model
- AdS/QCD
- Constituent models
- Bethe-Salpeter equation
- Lattice QCD
- QCD Sum Rules

For review see

- Mathieu, Kochelev, Vento, IJMP 2009
- Ochs, J. Phys. 2013



Papers with title containing: glueball, gluonium, gluonic state, gluonia, boxiton.

Approaches to glueball

Glueballs were studied within:

- Flux tube model
- AdS/QCD
- Constituent models
- Bethe-Salpeter equation
- Lattice QCD
- QCD Sum Rules

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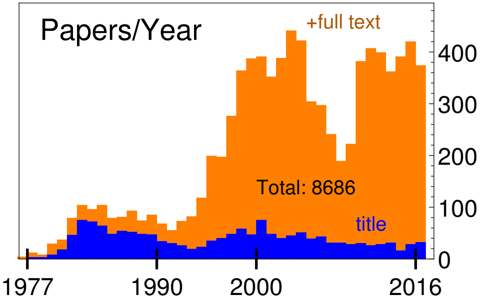

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Experiments

Running: Belle, BaBar, LHCb, BESIII, J-Park.
Planned: NICA, GlueX, PANDA, HIAF.

- J/Ψ decays (NICA), Parganlija D. EPJA52 2016
- heavy quarkonium radiative decays (BESIII,Belle, BaBar, LHCb)
- in photon-photon fusion (BESIII, Belle, BaBar)
- in the central meson production (RHIC, LHC)
- in proton-antiproton annihilation (PANDA)
- γ-proton collision (GlueX, JLab), Lyubovitskij et.al, PRD94 2016
- B meson weak decay (LHCb,Belle-II,BaBar), He&Yuan, EPJC75 2015

Determination of spectrum parameters from requirement of agreement between two ways to correlator as proposed by Shifman&VZ (1979)

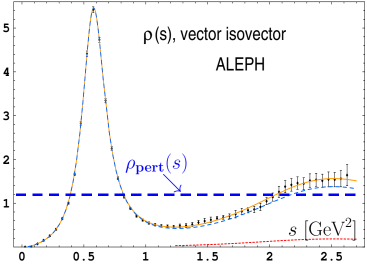
$$\Pi(Q^2) = i\int d^4x\, e^{iqx}\langle J(0)J^\dagger(x)\rangle\,,$$

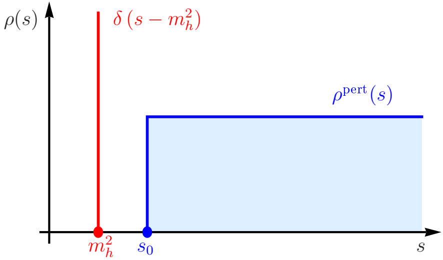
where current J_h describes the state $|h\rangle$: $\langle 0|J_h|h\rangle = f_h\,.$

- 1st way — Dispersion relation: decay constants f_h, masses m_h and others,

$$\Pi_{\text{had}}\left(Q^2\right) = \int_0^\infty \frac{\rho_{\text{had}}(s)\; ds}{s+Q^2} + \text{subtractions}\,.$$

  - model spectral density: $\rho_{\text{had}}(s) = f_h^2\,\delta\left(s-m_h^2\right) + \rho_{\text{pert}}(s)\,\theta\left(s-s_0\right)\,.$



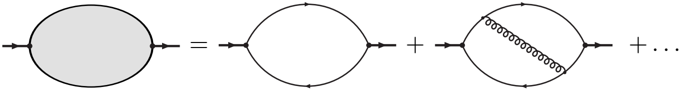


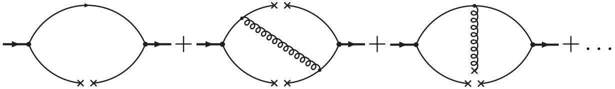
Theoretical part of QCD SR

- 2nd way — Operator product expansion:

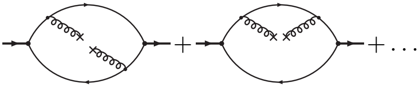
$$\Pi_{\text{OPE}}\left(Q^2\right) = \Pi_{\text{pert}}\left(Q^2\right) + \sum_n C_n \frac{\langle 0| : O_n : |0\rangle}{Q^{2n}} .$$

  - Condensates $\langle 0| : O_n : |0\rangle \equiv \langle O_n \rangle$.





$\sim \alpha_s \langle \bar{q} q \rangle^2 / Q^4$



$\sim \langle \frac{\alpha_s}{\pi} GG \rangle / Q^2$

QCD SR Approach

Determination of spectrum parameters from requirement of agreement between two ways for correlator:

- 1st way — Dispersion relation: decay constants f_h and masses m_h,

$$\Pi_{\rm had}\left(Q^2\right) = \int_0^\infty \frac{\rho_{\rm had}(s)\ ds}{s+Q^2} + \text{subtractions}\,.$$

  - model spectral density: $\rho_{\rm had}(s) = f_h^2\,\delta\left(s - m_h^2\right) + \rho_{\rm pert}(s)\,\theta\left(s - s_0\right)$.

- 2nd way — Operator product expansion:

$$\Pi_{\rm OPE}\left(Q^2\right) = \Pi_{\rm pert}\left(Q^2\right) + \sum_n C_n \frac{\langle 0|\,:O_n:\,|0\rangle}{Q^{2n}}\,.$$

  - Condensates $\langle 0|\,:O_n:\,|0\rangle \equiv \langle O_n\rangle$.

QCD SR reads:

$$\Pi_{\rm had}\left(Q^2, m_h, f_h\right) = \Pi_{\rm OPE}\left(Q^2\right)\,.$$

Borel Transform

$$\Phi(M^2) = \hat{B}_{Q^2 \to M^2}\left[\Pi(Q^2)\right] = \lim_{n\to\infty} \frac{(-Q^2)^n}{\Gamma(n)} \left[\frac{d^n}{dQ^{2n}}\Pi(Q^2)\right]_{Q^2=nM^2}.$$

$\Pi(Q^2)$	$C = \text{const}$	Q^{2n}	$1/Q^{2n}$	$1/\left(s+Q^2\right)$
$\Phi(M^2)$	0	0	$1/\left(\Gamma(n)\,M^{2n}\right)$	$e^{-s/M^2}/M^2$

- Elimination of subtractions in dispersion relation
- Exponential suppression of higher states contribution
- Factorial suppression of condensate terms

$$f_h^2\, e^{-m_h^2/M^2} + \int_{s_0}^{\infty} \rho_{\text{pert}}(s)\, e^{-s/M^2} ds$$

$$= \int_0^{\infty} \rho_{\text{pert}}(s)\, e^{-s/M^2} ds + \frac{C_G}{M^2} \langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu}\rangle + \frac{C_{\bar{q}q}}{M^4}\, \alpha_s \langle \bar{q}q\rangle^2 .$$

Borel Transform

$$\Phi(M^2) = \hat{B}_{Q^2\to M^2}\left[\Pi(Q^2)\right] = \lim_{n\to\infty} \frac{(-Q^2)^n}{\Gamma(n)} \left[\frac{d^n}{dQ^{2n}}\Pi(Q^2)\right]_{Q^2=nM^2} .$$

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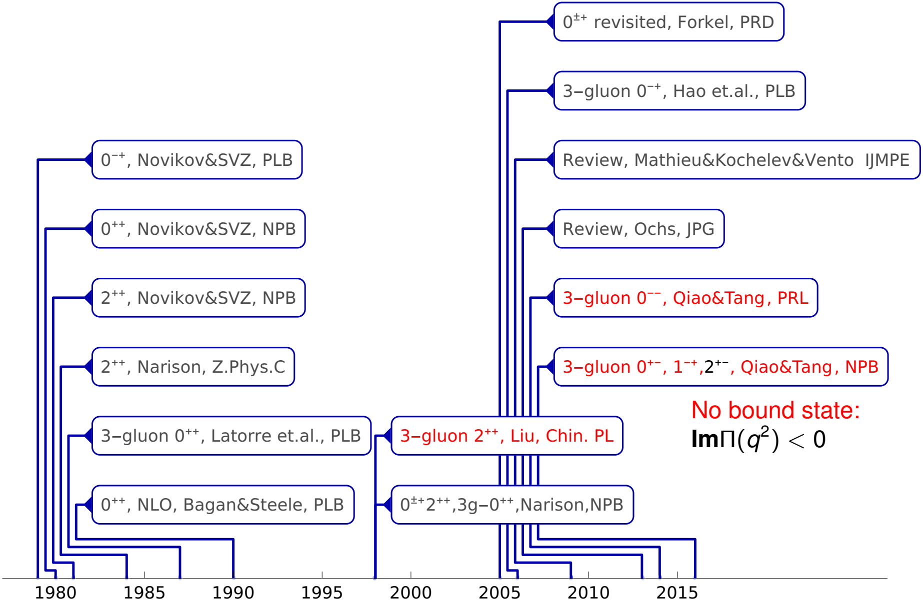
- Elimination of subtractions in dispersion relation
- Exponential suppression of higher states contribution
- Factorial suppression of condensate terms

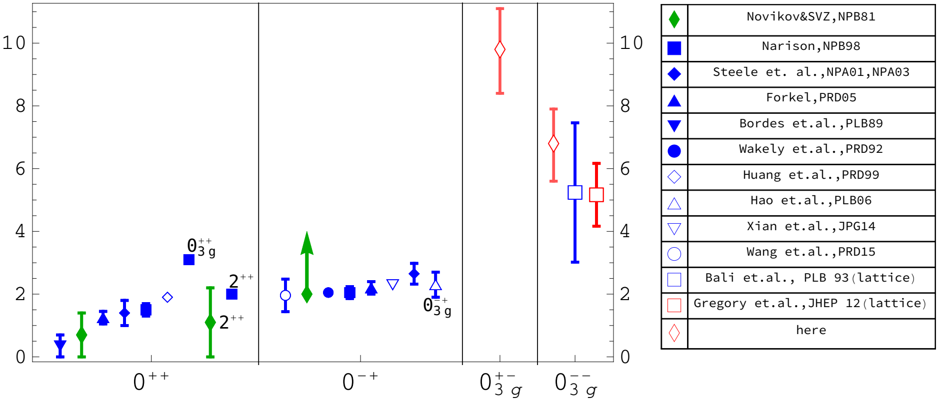
$$f_h^2\, e^{-m_h^2/M^2} = \int_0^{s_0} \rho_{\text{pert}}(s)\, e^{-s/M^2} ds + \frac{c_G}{M^2}\, \langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu}\rangle + \frac{c_{\bar{q}q}}{M^4}\, \alpha_s \langle \bar{q}q\rangle^2 .$$

Key works on Glueballs in QCD SR



No bound state:
$\mathbf{Im}\Pi(q^2) < 0$

QCD SR results



C parity

Charge conjugation

$$A^a_\mu t^a_{ij} \to -A^a_\mu t^a_{ji}$$

	∂_μ , D_μ	$\epsilon_{\mu\nu\alpha\beta}$	A^a_μ , $G^a_{\mu\nu}$	$d^{abc}G^aG^bG^c$	$f^{abc}G^aG^bG^c$	G^aG^a
C	$+1$	$+1$	$+1$ for $a = 2, 5, 7$ -1 for $a = 1, 3, 4, 6, 8$	-1	$+1$	$+1$

The first try to construct C-odd current:

$$d^{abc}G^a_{\mu\nu}(x)G^b_{\nu\rho}(x)G^c_{\rho\mu}(x) = 0\,.$$

General form for C-odd current:

$$J^{\pm-} = d^{abc}\hat{O}_1 G^a_{\mu\nu}(x)\hat{O}_2 G^b_{\nu\rho}(x)\hat{O}_3 G^c_{\rho\mu}(x)\,.$$

ill-defined current for 0^{--} glueball state

First attempt to study 0^{--} state in QCD SR [Qiao&Tang, PRL-2014] proposes current

$$J(x) = g_s^3 d^{abc} \left(g_{\alpha\beta} - \frac{\partial_\alpha \partial_\beta}{\partial^2} \right) \tilde{G}^a_{\mu\nu}(x) \partial_\alpha \partial_\beta G^b_{\nu\rho}(x) G^c_{\rho\mu}(x) \,.$$

QCD SR is based on the correlator:

$$\Pi^{PC}(q) = i \int d^4x \, e^{iqx} \langle J(x) J^\dagger(0) \rangle \,.$$

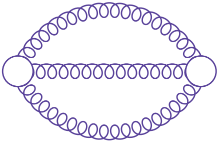
In this work **Im** $\Pi < 0$, so no bound state [Kochelev et.al.,PRL 2017], but QCD SR for mass was constructed and the resulting mass was published.
We found differ result for LO term (log-part only):

$$\Pi^{\text{cor.}}_{\text{LO}} = \frac{31}{2^6 3^3 7\pi} \alpha_S^3 Q^{12} \ln\left(\frac{Q^2}{\mu^2}\right) \quad vs. \quad \Pi^{\text{orig.}}_{\text{LO}} = \frac{487}{2^6 3^3 11 \cdot 13\pi} \alpha_S^3 Q^{12} \ln\left(\frac{Q^2}{\mu^2}\right)$$

The largest prime divisor of denominator (7) must be less than dimension (8) of the current (sunrise diagrams).

Defects:

- No bound state **Im** $\Pi < 0$
- No gauge invariance
- other



Dimension-12 current for 0^{--} glueball state

We propose the dimension-12 current of 0^{--} glueball state [PRD95(2017)]:

$$J_3^{--}(x) = \frac{2}{3} g_s^3 \epsilon^{ijk} Tr\left((O_i G_{\mu\nu}(x))(O_j G_{\nu\rho}(x))(O_k G_{\rho\mu}(x))\right) ,$$

where generally O_m is product of covariant derivatives $O_m = D_{\alpha_1} \cdots D_{\alpha_n}$:

$$\begin{aligned} O_1 G_{\mu\nu}(x) &= D_{\alpha_1} D_{\alpha_2} D_{\alpha_3} \tilde{G}_{\mu\nu}(x) , \\ O_2 G_{\mu\nu}(x) &= D_{\alpha_1} D_{\alpha_2} G_{\mu\nu}(x) , \\ O_3 G_{\mu\nu}(x) &= D_{\alpha_3} G_{\mu\nu}(x) . \end{aligned}$$

The LO part of current:

$$J_{3,\mathrm{LO}}^{--}(x) = g_s^3 d^{abc} (\partial_{\tau_1} \partial_{\tau_2} \partial_{\tau_3} \tilde{G}^a_{\mu\nu}(x))(\partial_{\tau_1} \partial_{\tau_2} G^b_{\nu\rho}(x))(\partial_{\tau_3} G^c_{\rho\mu}(x)) .$$

NLO part of the current has 9 sources for extra 4th gluon.

For 0^{--} glueball we get [PRD95(2017)]:

$$m = 6.3^{+12\%}_{-17\%} \pm 0.5\% \text{ GeV}$$

Are there any alternative currents?

Desired properties of current are

- gauge invariance
- 3-gluon state
- with nonzero LO perturbative term
- low dimension

In first work [PRD95(2017)] we consider only the scalar current in the form:

$$J \sim Tr\left((O_i G_{\mu\nu}(x))(O_j G_{\nu\rho}(x))(O_k G_{\rho\mu}(x))\right) ,$$

Any considered current in this form with dimension lower than 12 has some of the unfavorable properties:

- 4 gluon state
- Quark-gluon state
- zero LO perturbative term

Later we found that there is dimension-9 vector current of different form.

Current of minimal dimension from helicity formalism.

- The gluon field tensor $G_{\mu\nu}$ corresponds to $(1,0) \oplus (0,1)$ representation of the Lorentz group and can be decomposed to positive and negative helicity parts $G_{\mu\nu} = G^{+}_{\mu\nu} + G^{-}_{\mu\nu}$, where $G^{\mp}_{\mu\nu} = (G_{\mu\nu} \pm \tilde{G}_{\mu\nu})/2$. We used helicity strength tensor $G^{\pm}_{\mu\nu}$ as building blocks for currents $J \sim f(G^{\pm}, G^{\pm}, G^{\pm})$.

- New dimension-9 unconserved vector currents have been constracted:

$$J^{+-}_{\alpha} = g_s^3 \,\mathrm{Tr}(\{(D_\tau G_{\mu\nu}), (D_\tau G_{\rho\nu})\}(D_\mu G_{\rho\alpha}))\,,$$
$$J^{--}_{\alpha} = g_s^3 \,\mathrm{Tr}(\{(D_\tau G_{\mu\nu}), (D_\tau G_{\rho\nu})\}(D_\mu \tilde{G}_{\rho\alpha}))\,.$$

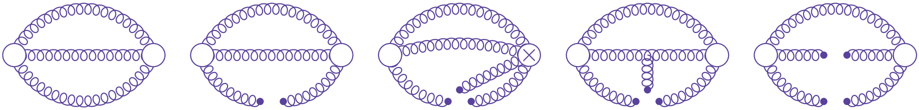
Advantages:

- Better convergence of correlator OPE is observed:
  dim-12: $\mathrm{OPE} = \mathrm{LO} + \quad 0 \quad + \langle G^4 \rangle + \cdot$
  dim-9: $\mathrm{OPE} = \mathrm{LO} + \langle G^3 \rangle + \langle G^4 \rangle + \cdots$

- The resonance contribution is larger.

- The QCD SR result for mass is comparable with dimension-12 current estimation: $m_{\text{dim-9}} \sim m_{\text{dim-12}}$, while decay constant is significantly larger: $f_{\text{dim-9}} \gg f_{\text{dim-12}}$.

OPE of correlator. Diagrams.



We depict gluon condensates in nonlocal condensate form.

$$i\int d^4x\, e^{iqx}\langle J^{\pm -}_{\mu}(x) J^{\pm -}_{\nu}(0)^{\dagger}\rangle = \Pi^{\pm}_{(1)}(q^2)(q_\mu q_\nu - q^2 g_{\mu\nu}) + \Pi^{\pm}_{(0)}(q^2) q_\mu q_\nu\,,$$

OPE of spin-0 part: $\Pi^{\pm}_{(0)} = \Pi^{\pm}_{\text{pert}} + \Pi^{\pm}_{G2} + \Pi^{\pm}_{G3} + \Pi^{\pm}_{G4} + \cdots\,,$ with **Im** $\Pi^{\pm}_{(0)} > 0\,,$

$$\Pi^{\pm}_{(\text{pert})} = \frac{-5\alpha_s^3}{9!8\pi} q^{12} \ln\frac{-q^2}{\mu^2}\,, \quad \Pi^{\pm}_{(\text{G2})} = 0\,,$$

$$\Pi^{\pm}_{(\text{G3})} = \pm\frac{5\alpha_s^2}{2^8 3}\langle g^3 G^3\rangle \cdot q^6 \ln\frac{-q^2}{\mu^2}\,, \quad \Pi^{\pm}_{(\text{G4})} = \mp\frac{\alpha_s^2\pi^2}{2^6 3^3}\langle \alpha_s^2 G^4\rangle_{\pm} \cdot q^4 \ln\frac{-q^2}{\mu^2}\,,$$

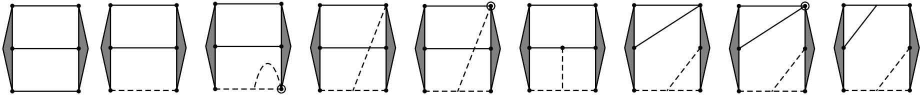
Quark-gluon condensates are omitted. Only log-terms.

$$\langle \alpha_s^2 G^4\rangle_{\pm} \overset{\text{HVD}}{=} c_{\pm}\langle \alpha_s G^2\rangle^2\,.$$

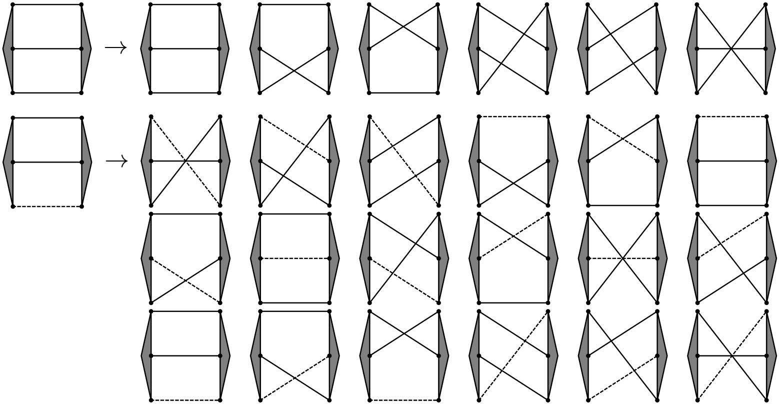
HVD – Hypothesis of Vacuum Dominance.

Diagrammatics

6 nonzero topologies, 150 diagrams, $\sim$1000 (local condensate) terms



Pert. $\langle G^3 \rangle + \ldots$ $\langle G^4 \rangle + \ldots$. Dashed lines denote soft part (condensates).



QCD SR

New current of 0^{--} state has dimension $[J] = 9$ (f -decay constant, m - mass):

$$\langle 0 | J^{\pm -}_{\alpha} | G(0^{\pm -}) \rangle = p_{\alpha} f_{\pm} m^{6}_{\pm} \,.$$

$$\Pi^{\text{OPE}} = \Pi^{\text{pert}} + \Pi^{G2} + \Pi^{G3} + \Pi^{G4} + \cdots \,, \quad \text{with} \quad \Pi^{G2} = 0 \,.$$

One resonance model with continuum modeled by Im-part of pert. term:

$$\begin{aligned} \mathbf{Im}\Pi^{(\text{ph})}(-s) &= \pi m^{12} f^{2} \delta(s - m^{2}) + \Theta(s - s_{0}) \mathbf{Im}\Pi^{(\text{OPE})}(-s) \,, \\ \mathcal{R}^{t}(s_{0}) &= M^{2} \hat{B}_{Q^{2} \to M^{2}} \Pi^{t}(Q^{2}) = \frac{1}{\pi} \int_{0}^{s_{0}} ds \, \mathbf{Im}\Pi^{t}(-s) \, e^{-s/M^{2}} \,. \end{aligned}$$

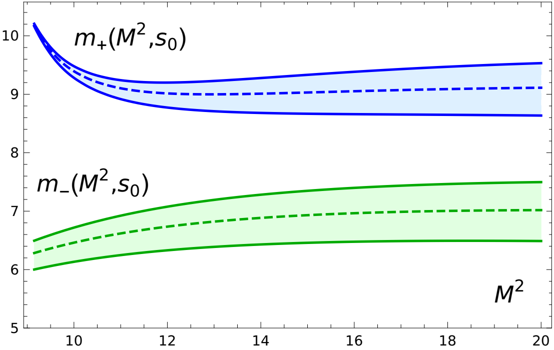
QCD SR reads: $\quad \mathcal{R}^{(\text{OPE})}(s_{0}) = \mathcal{R}^{(\text{res})}(s_{0}) \equiv m^{12} f^{2} e^{-m^{2}/M^{2}} \,.$

Fiducial window $M^{2}_{\text{min}} < M^{2} < M^{2}_{\text{max}}$ is limited by the conditions:

$$|\mathcal{R}^{(\text{G4})}(\infty)| / \mathcal{R}^{(\text{OPE})}(\infty) \; < \; 1/3 \,, \quad \mathcal{R}^{(\text{res})}(s_{0}) / \mathcal{R}^{(\text{OPE})}(\infty) > 1/10 \,.$$

The unconventional choice of latter condition is motivated by significance of the continuum term thanks the large dimension of the current : $\mathbf{Im}\Pi^{\text{pert}} \sim s^{6}$

QCD SR analysis



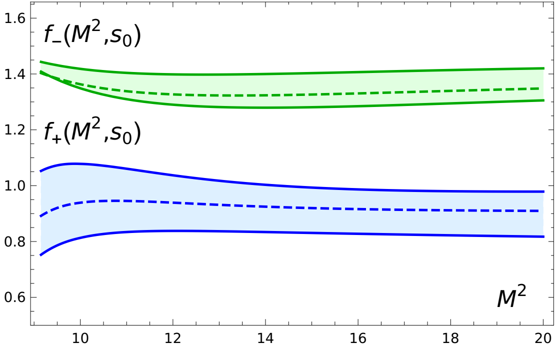
Masses of $0^{\pm-}$ glueballs:

$m_- = 6.84 \pm 0.36\ ^{+0.44}_{-0.47}\ ^{+0.26}_{-0.37}$ GeV

$m_+ = 9.23 \pm 0.56 \pm 0.38^{+0.40}_{-0.47}$ GeV

Uncertainties:

- Gluon condensate
- SR stability
- OPE truncation



Decay constants of of $0^{\pm-}$ states:

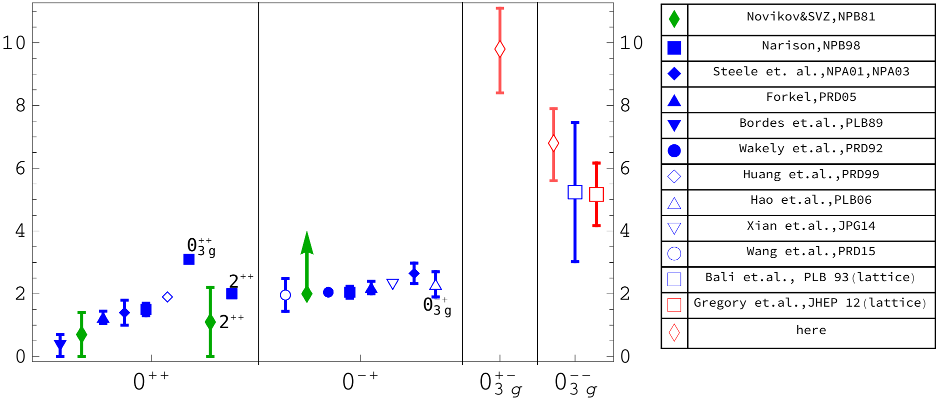
$f_- = 1.34 \pm 0.04 \,^{+0.07}_{-0.03} \pm 0.02$ MeV

$f_+ = 0.93 \pm 0.02 \,^{+0.08}_{-0.10} \pm 0.02$ MeV

Uncertainties:

- Gluon condensate
- SR stability
- OPE truncation

QCD SR results



No significant 0^{--} glueball signals are observed [Belle, PRD95, 012001 (2017)] at theoretically predicted masses: 2.8, 3.81, and 4.33 GeV.

Summary

- Dimension-9 currents for three gluon $0^{\pm-}$ state have been constructed.
- OPE of $\langle JJ^\dagger \rangle$-correlators have been calculated up to dimension 8 gluon condensates
- The QCD SR predictions for masses of $0^{\pm-}$ glueballs:

$$m_- = 6.8^{+1.1}_{-1.2}\ \text{GeV}\,, \qquad m_+ = 9.2^{+1.3}_{-1.4}\ \text{GeV}\,.$$

Thank you for attention!

Extra slides

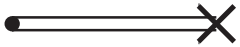
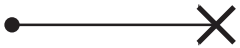
Gluon condensates

It is convenient to calculate soft part in the fixed–point (Fock–Schwinger) gauge

$$x_\mu A^a_\mu(x) = 0.$$

The Taylor expansions for $A^a_\mu(x)$ has gauge–covariant form:

$$G^a_{\mu\nu}(x) = G^a_{\mu\nu}(0) + \frac{1}{1!} x_\alpha D_\alpha G^a_{\mu\nu}(0) + \frac{1}{2!} x_\beta x_\alpha D_\beta D_\alpha G^a_{\mu\nu}(0) + O(x^3),$$

$z,\ \mu\nu,\ a$ 	$G^a_{\mu\nu}(z)$
$z,\ \nu,\ a$ 	$A^a_\nu(z) = z_\mu \int_0^1 dt\ t\ G^a_{\mu\nu}(tz)$

The Taylor expansions for condensate

$$\langle \mathrm{Tr}\, G_{\mu_1\mu_2}(0) G_{\mu_3\mu_4}(x) \rangle = \langle \mathrm{Tr}\, G_{\mu_1\mu_2}(0) G_{\mu_3\mu_4}(0) \rangle + \cdots .$$

Teylor expansion of nonlocal gluon condensates

We have reproduced result [Grozin,1994]. Here we used different tensor basis:

$$g^2\langle G^a_{\mu_1\nu_1}(x)G^b_{\mu_2\nu_2}(y)\rangle \simeq \frac{\delta^{ab}}{N_c C_F d(d-1)}\left(\prod_j^2 \mathbb{A}_{\mu_j\nu_j}\right)\sum_{k=0}^{6}\Gamma_{2k}(x,y)M_{2k}(x,y)+\dots\,,$$

where

$$\Gamma_{20} \quad = \quad g_{\mu_1\mu_2}g_{\nu_1\nu_2}/2\,,$$

$$M_{20}(x,y,z) = G^4_1 + \frac{(x-y)^2}{(d-2)(d+2)}\left(G^6_{20}+\frac{G^6_{21}}{d+4}+\left(G^8_{20}+\frac{G^8_{211}}{2(d+4)}\right)\frac{(x-y)^2}{3!4}\right)+\dots\,,$$