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Jost-matrix analysis of the resonance $^5\mathrm{He}^*\left(\frac{3}{2}^+\right)$ near the dt - threshold

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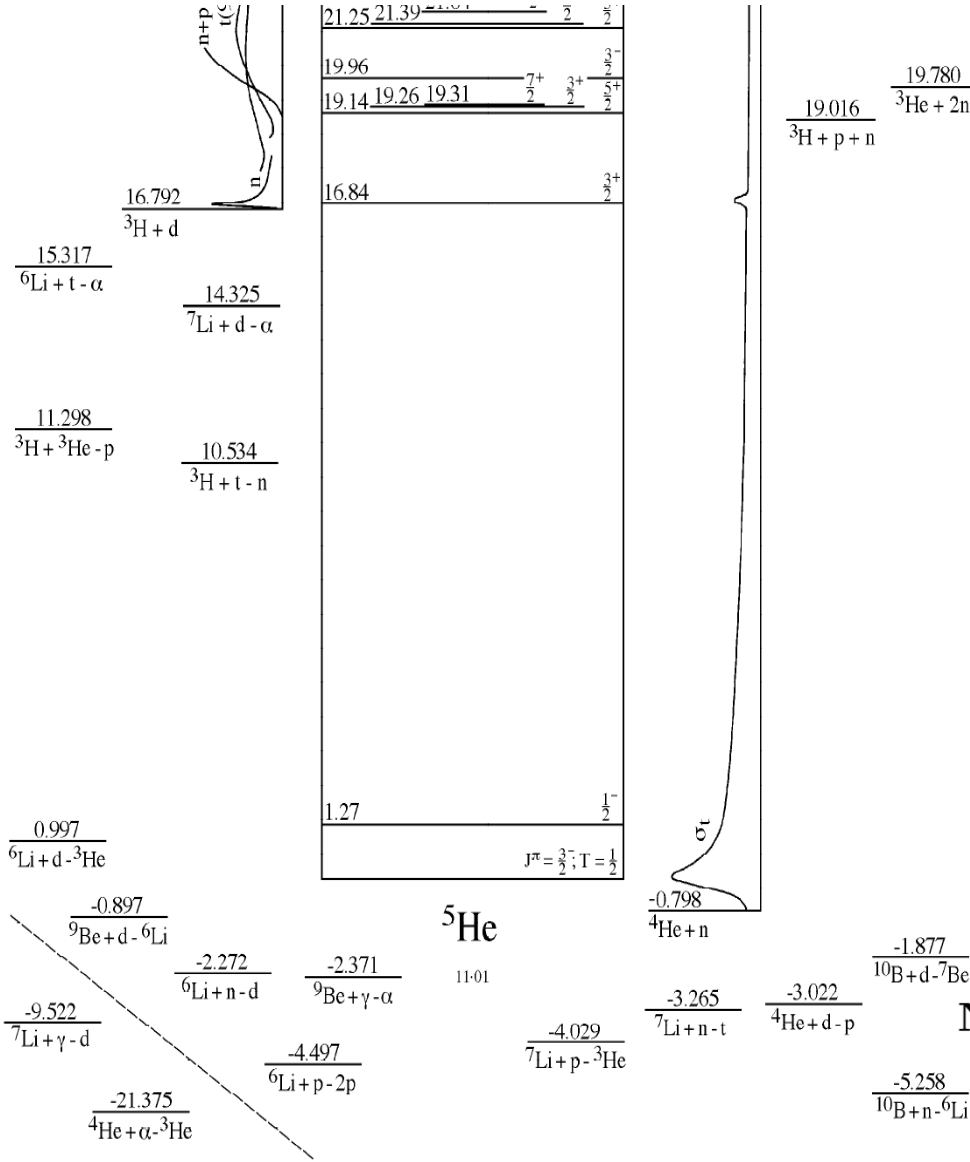
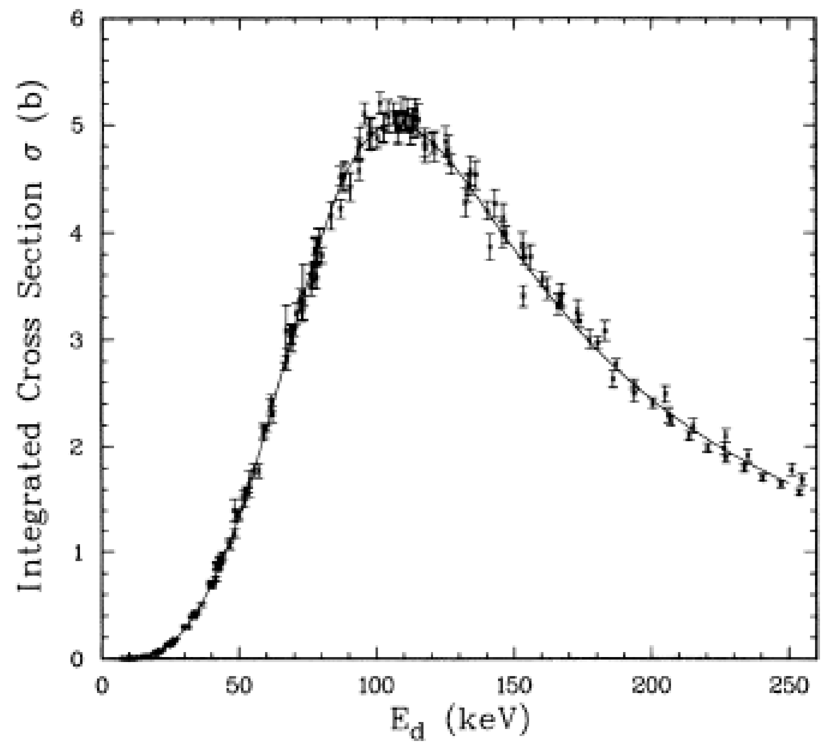
$$d + t \rightarrow {}^{5}\mathrm{He}^{*}\left(J^{\pi} = \frac{3}{2}^{+}\right) \rightarrow n + \alpha$$





N. Jarmie et al., Phys. Rev. C29 (1984) 2031

D.R. Tilley et al., Nucl. Phys. A706 (2002) 3

Binary multi-channel reactions

$^5\mathrm{He}^*$

$$d + t \rightarrow \left\{ \begin{array}{c} d + t \\ n + \alpha \end{array} \right\} \leftarrow n + \alpha$$

^{5}He resonant state above dt threshold: 4 channels

$$J^\pi = \frac{3}{2}^+ \rightarrow \quad (l, S = s_1 + s_2)$$

$$n\alpha \rightarrow \quad \left(2, \frac{1}{2}\right)$$

$$dt \rightarrow \left(0, \frac{3}{2}\right) \left(2, \frac{1}{2}\right) \left(2, \frac{3}{2}\right)$$

an accurate and efficient way of parametrizing a collection of experimental data

→

the R-matrix fit

↓

S-matrix at real energies E

the comprehensive four-level four-channel R-matrix fit:

G.M. Hale, R. E. Brown, N. Jarmie

Pole Structure of the $J^\pi = \frac{3}{2}^+$ Resonance in ^{5}He

Phys. Rev. Lett., 59, 763 (1987)

The system of the N-channel radial Schrödinger equations

$$\left[\partial_r^2 + k_n^2 - \frac{l_n(l_n+1)}{r^2} - \frac{2k_n\eta_n}{r}\right] u_n\left(E, r\right) = \sum_{n'=1}^{N} V_{nn'}(r)\, u_{n'}\left(E, r\right)$$

has $2N$ linearly independent column solutions, $(u_1, u_2, \cdots, u_N)^T$

N regular (zero at r = 0) solutions form a regular basis

$$\varphi(E, r) = \begin{bmatrix} \varphi_{11} & \cdots & \varphi_{1N} \\ \vdots & \ddots & \vdots \\ \varphi_{N1} & \cdots & \varphi_{NN} \end{bmatrix}$$

$\varphi(E, r)$ becomes a linear combination of the pure Coulomb functions

at large distances $r \to \infty$, where potentials $V_{nn'}(r) = 0$

$$\begin{aligned} H_l^{(\pm)}(\eta,\ kr) &= F_l(\eta,\ kr) \mp \imath\, G_l(\eta,\ kr) \\ &\xrightarrow{r\to\infty} \mp\, \imath \exp\left\{\pm\, \imath \left[kr - \eta \ln(2kr) - \frac{l\pi}{2} + \sigma_l\right]\right\} \end{aligned}$$

for N = 2 case at $r \to \infty$

$$\varphi(E,r) \overset{r\to\infty}{\longrightarrow} \begin{bmatrix} H_{l_1}^{(-)}(\eta_1,k_1r)e^{+\imath\sigma_{l_1}} & 0 \\ 0 & H_{l_2}^{(-)}(\eta_2,k_2r)e^{+\imath\sigma_{l_2}} \end{bmatrix} \begin{bmatrix} f_{11}^{(in)}(E) & f_{12}^{(in)}(E) \\ f_{21}^{(in)}(E) & f_{22}^{(in)}(E) \end{bmatrix}$$

$$+ \begin{bmatrix} H_{l_1}^{(+)}(\eta_1,k_1r)e^{-\imath\sigma_{l_1}} & 0 \\ 0 & H_{l_2}^{(+)}(\eta_2,k_2r)e^{-\imath\sigma_{l_2}} \end{bmatrix} \begin{bmatrix} f_{11}^{(out)}(E) & f_{12}^{(out)}(E) \\ f_{21}^{(out)}(E) & f_{22}^{(out)}(E) \end{bmatrix}$$

where $f^{(in/out)}(E)$ are the Jost matrices

physical solution is the linear combination of regular solutions

$$u(E,r) = \begin{bmatrix} u_1(E,r) \\ u_2(E,r) \end{bmatrix} = \begin{bmatrix} \varphi_{11}(E,r) \\ \varphi_{21}(E,r) \end{bmatrix} c_1 + \begin{bmatrix} \varphi_{12}(E,r) \\ \varphi_{22}(E,r) \end{bmatrix} c_2$$

$$= \begin{bmatrix} \varphi_{11}(E,r) & \varphi_{12}(E,r) \\ \varphi_{21}(E,r) & \varphi_{22}(E,r) \end{bmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \varphi(E,r)\, c$$

coefficients (c_1,c_2) are to be chosen
to satisfy certain physical boundary conditions at infinity

For a spectral point (either bound or a resonant state) the physical wave function should only have the outgoing waves in its asymptotic behaviour

$$u(E,r) \xrightarrow{r\to\infty} \begin{bmatrix} H^{(-)}_{l_1}(\eta_1,k_1 r)e^{+\imath\sigma_{l_1}} & 0 \\ 0 & H^{(-)}_{l_2}(\eta_2,k_2 r)e^{+\imath\sigma_{l_2}} \end{bmatrix} f^{\,(in)}(E)\; c$$
$$+ \begin{bmatrix} H^{(+)}_{l_1}(\eta_1,k_1 r)e^{-\imath\sigma_{l_1}} & 0 \\ 0 & H^{(+)}_{l_2}(\eta_2,k_2 r)e^{-\imath\sigma_{l_2}} \end{bmatrix} f^{\,(out)}(E)\; c$$

This can only be achieved if

$$f^{\,(in)}(E)\; c = \begin{bmatrix} f^{(in)}_{11}(E) & f^{(in)}_{12}(E) \\ f^{(in)}_{21}(E) & f^{(in)}_{22}(E) \end{bmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = 0$$

This homogeneous system of linear equations for coefficients (c_1, c_2) has a non-zero solution if and only if

$$\det \begin{bmatrix} f^{(in)}_{11}(E) & f^{(in)}_{12}(E) \\ f^{(in)}_{21}(E) & f^{(in)}_{22}(E) \end{bmatrix} = 0$$

Roots $E = \mathcal{E}_n$ at real negative energies ($\mathcal{E}_n < 0$) correspond to the bound states
Roots at complex energies ($\mathcal{E}_n = E_r$ - $\imath\Gamma/2$) correspond to the resonances

the scattering S-matrix is determined by the ratio of the amplitudes of the out-going and in-coming waves

$$S(E) = f^{(out)}(E) \left[f^{(in)}(E) \right]^{-1}$$

roots of $\det \left[f^{(in)}(E) \right] = 0$ correspond to poles of the S-matrix

Reaction amplitudes

$$f^{J}_{n'n}(E) = \frac{S^{J}_{n'n}(E) - \delta_{n'n}}{2\imath k_n} \, \imath^{l_n - l_{n'}}$$

partial cross section between any two particular channels

$$\sigma^{J}(\gamma' \, l' \, S' \leftarrow \gamma \, l \, S) = 4\pi \, \frac{\mu_{\gamma} \, k_{\gamma'}}{\mu_{\gamma'} \, k_{\gamma}} \, \frac{(2J+1)}{(2S+1)} \, \left| f^{J}_{n'n}(E) \right|^2$$

total cross section

$$\sigma(\gamma' \leftarrow \gamma) = \sum_{J l' S' l S} \sigma^{J}(\gamma' \, l' \, S' \leftarrow \gamma \, l \, S)$$

Analytic properties

The Jost matrices (and thus the S-matrix) are not single-valued functions of the energy. There are two reasons for this:

- The in-coming and out-going spherical waves (thus their amplitudes $f^{(in/out)}(E)$) depend on E via all the channel momenta k_n
- For charged particles, there is an additional complication, the in-coming and out-going spherical waves (thus their amplitudes $f^{(in/out)}(E)$) depend on $\ln(k_n)$

For the channel momenta (E_n are threshold energies)

$$k_n = \pm\sqrt{\frac{2\mu_n}{\hbar^2}(E - E_n)}, \quad n = 1, 2, \cdots, N_{ch}$$

$2^{N_{ch}}$ possible combinations of signs for each value of energy E

The complex function $\ln(k)$ has infinitely many different values

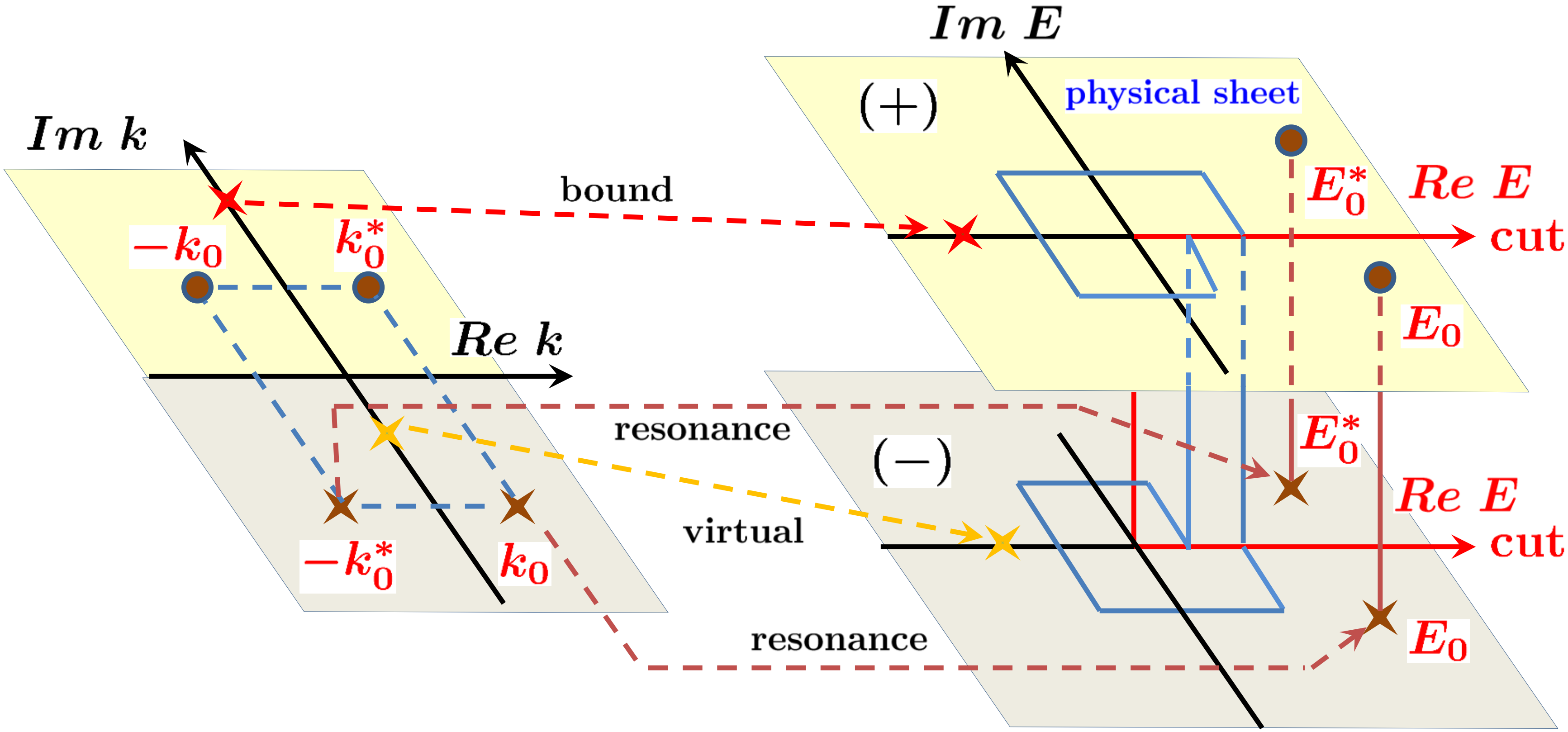
$$\ln(k) = \ln|k| + \imath\,(arg(k) + 2\pi m)$$

$$-\pi < arg(k) \leq \pi, \quad m = 0, \pm 1, \pm 2, \cdots$$

Single-channel Riemann surface: neutral particles

$$k = \pm\sqrt{\frac{2\mu}{\hbar^2}E}$$

$$S(k=k_0)=S_0,\ S(k=-k_0^*)=S_0^*,\ S(k=-k_0)=\frac{1}{S_0},\ S(k=k_0^*)=\frac{1}{S_0^*}$$



physical sheet: (+), $Im\ k > 0$

unphysical sheet: (−), $Im\ k < 0$

S - matrix pole

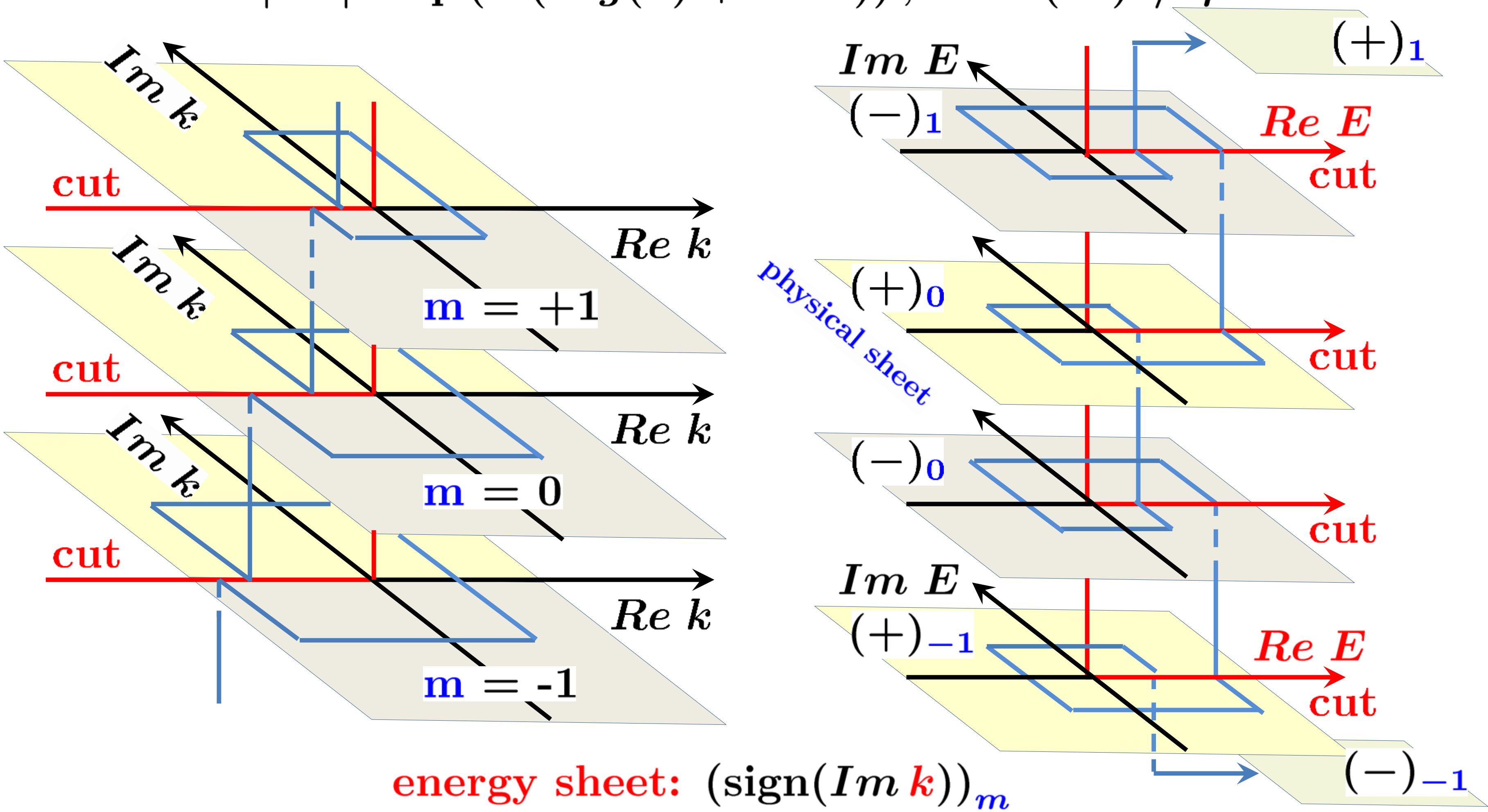
S - matrix zero

Single-channel Riemann surface: charged particles

$$\ln(k) = \ln|k| + \imath\,(arg(k) + 2\pi m)$$

$$-\pi < arg(k) \le \pi,\ m = 0, \pm 1, \pm 2, \cdots$$

$$E = |E|\ \exp\left(2\imath(arg(k) + 2\pi m)\right),\ E = (\hbar k)^2/2\mu$$

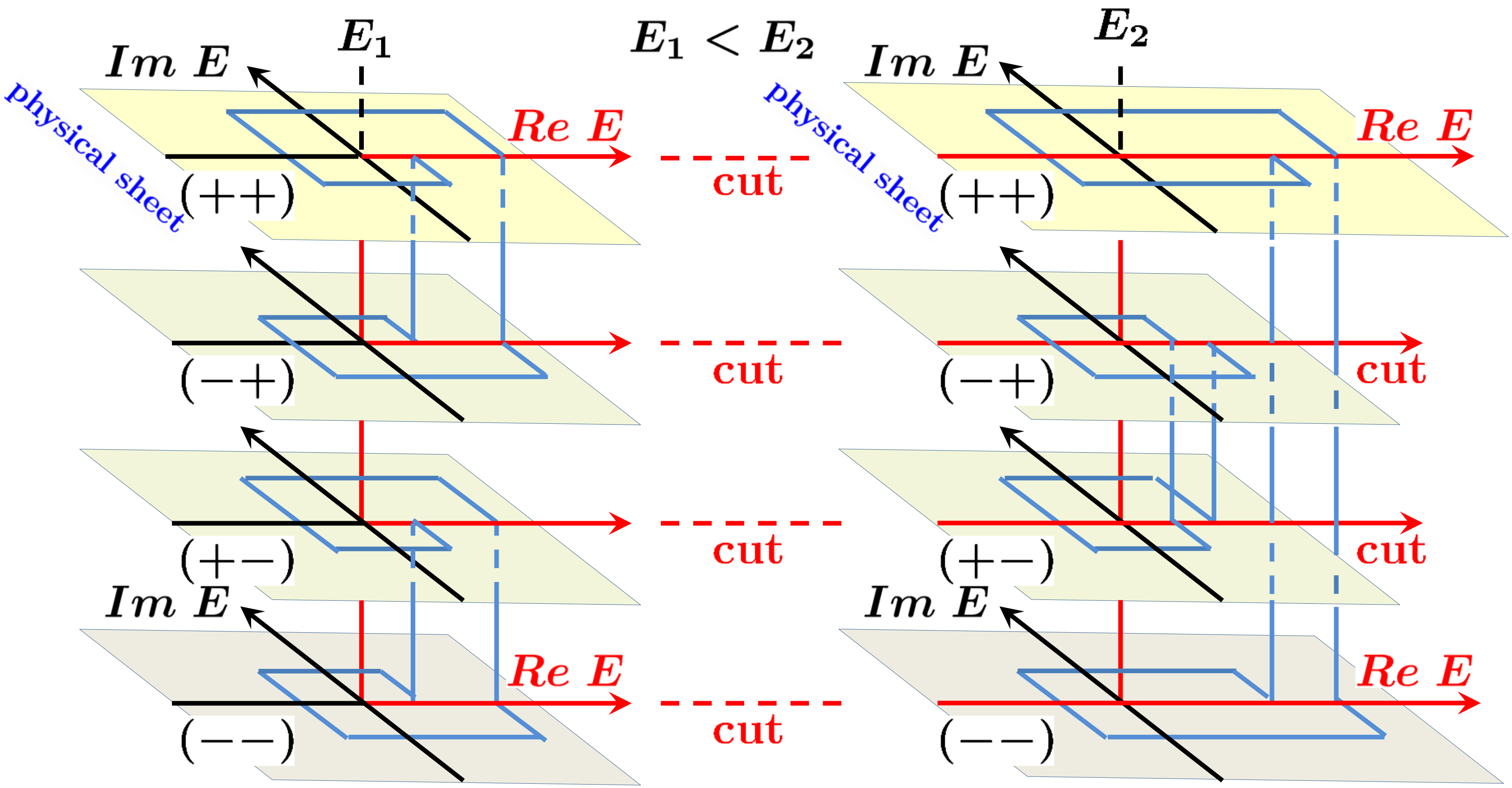


energy sheet: $(\text{sign}(Im\,k))_m$

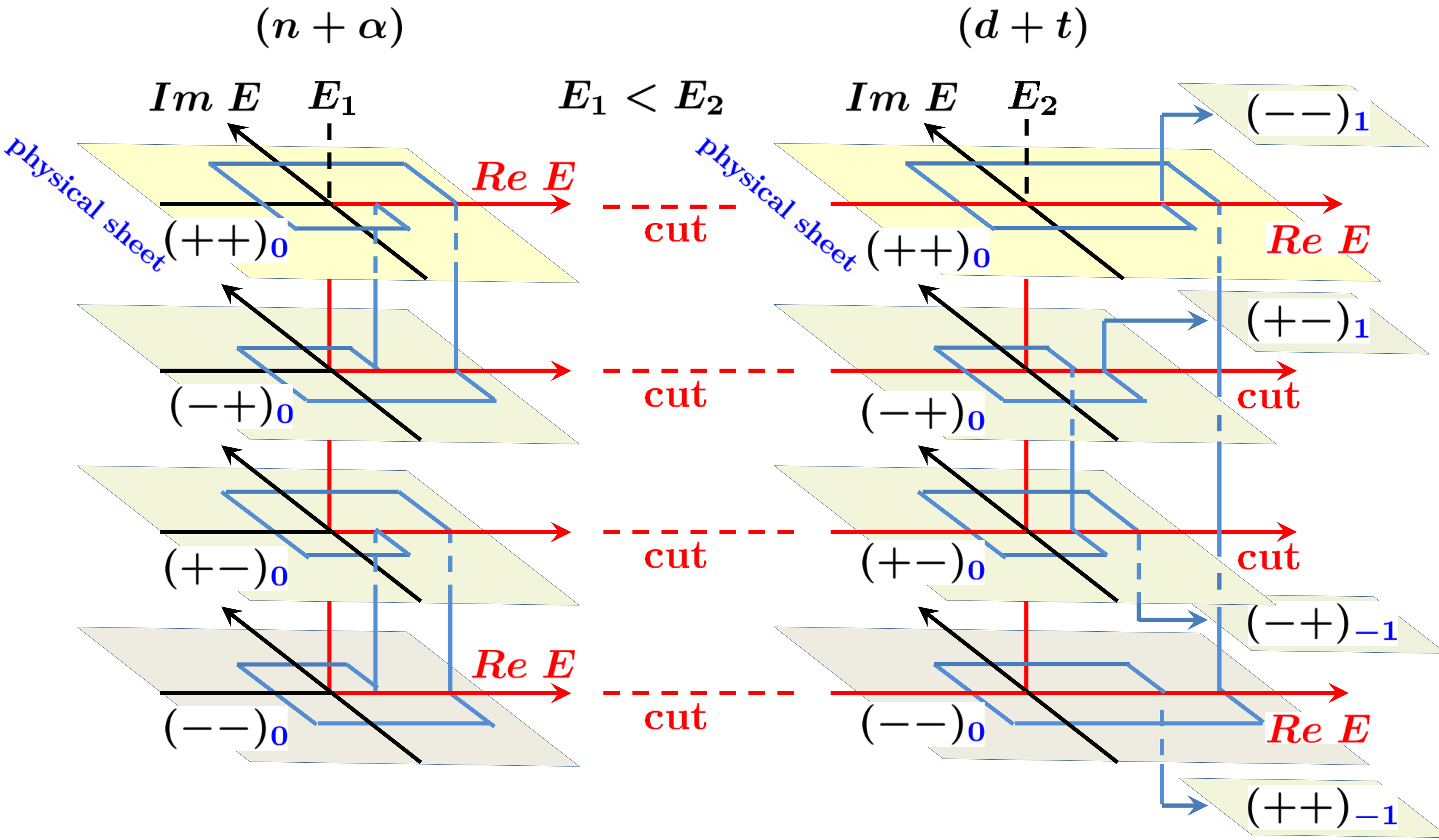
Two-channel Riemann surface: neutral particles

$$k_n = \pm\sqrt{\frac{2\mu_n}{\hbar^2}(E - E_n)},\ n = 1, 2, \cdots, N_{ch}$$

$2^{N_{ch}}$ energy sheets: $(\text{sign}(Im\,k_1), \text{sign}(Im\,k_2), \cdots)$



Two-channel Riemann surface: neutral and charged particles



Analytic structure

special semi-analytic representation of the Jost matrices is applied, where factors responsible for the branching of Riemann surface are given explicitly

S.A. Rakityansky, N. Elander, J. Phys. A 44, 115303 (2011)
S.A. Rakityansky, N. Elander, J. Math. Phys. 54, 122112 (2013)

Structure of the Jost matrices for neutral particles

$$f^{(in/out)}_{mn}(E) = \frac{k_n^{l_n+1}}{k_m^{l_m+1}}\, A_{mn}(E) \mp \imath\, k_m^{l_m}\, k_n^{l_n+1}\, B_{mn}(E)$$

matrices A(E) and B(E) are
- single-valued functions of E
- the same for both $f^{(in)}$ and $f^{(out)}$
- for real energies are real

Structure of the Jost matrices for charged particles

$$f^{(in/out)}_{mn}(E) = \frac{e^{\pi\eta_m/2}\,(l_m)!}{\Gamma(l_m+1\pm\imath\,\eta_m)}\left\{\frac{C_{l_n}(\eta_n)\,k_n^{l_n+1}}{C_{l_m}(\eta_m)\,k_m^{l_m+1}}\,A_{mn}(E) - \left[\frac{2\eta_m\,h(\eta_m)}{C_0^2(\eta_m)}\pm\imath\right] C_{l_m}(\eta_m)\,C_{l_n}(\eta_n)\,k_m^{l_m}\,k_n^{l_n+1}\,B_{mn}(E)\right\}$$

$$C_l(\eta) = \frac{2^l e^{-\pi\eta/2}}{(2l)!!}\exp\left\{\frac{1}{2}\left[\ln\Gamma(l+1+\imath\eta)+\ln\Gamma(l+1-\imath\eta)\right]\right\}\xrightarrow{\eta\to 0} 1$$

Approximation and analytic continuation

unknown matrices A(E) and B(E) are single-valued and analytic and can be expanded in Taylor series around arbitrary complex energy E_0

$$
\begin{aligned}
A(E) &= a^{(0)}(E_0) + a^{(1)}(E_0)\,(E-E_0) + a^{(2)}(E_0)(E-E_0)^2 + \cdots \\
B(E) &= b^{(0)}(E_0) + b^{(1)}(E_0)\,(E-E_0) + b^{(2)}(E_0)\,(E-E_0)^2 + \cdots
\end{aligned}
$$

$a^{(m)}(E_0)$ and $b^{(m)}(E_0)$ are the $(N_{ch} \times N_{ch})$-matrices depending on the choice of the expansion center E_0

parameters $a^{(m)}(E_0)$ and $b^{(m)}(E_0)$ are found by fitting some available experimental data

It is convenient to choose E_0 on the real axis.
Then, parameters $a^{(m)}(E_0)$ and $b^{(m)}(E_0)$ are also real.

After finding the fitting parameters $a^{(m)}(E_0)$ and $b^{(m)}(E_0)$, the analytic expression for the Jost matrix $f^{(in)}(E)$ is used to locate resonances as roots of equation $\det[f^{(in)}(E)] = 0$ at complex energies.

Symmetry properties of the Jost matrices

Matrices $f^{(in)}(E)$ and $f^{(out)}(E)$ are related to each other at different points of the Riemann surface and obey certain symmetry rules.

Since matrices $A(E)$ and $B(E)$ are the same on all sheets, symmetry relations are determined by explicitly given factors in semi-analytic representations that undergo certain changes when k_n is replaced with $-k_n$ or with k_n^*.

To specify the sheet of the Riemann surface, to which the energy point E belongs, it is convenient to replace the notation

$$f^{(in/out)}(E) \to f^{(in/out)}(k_1, k_2, \cdots, k_N)$$

Neutral particles: "vertical" symmetry of the Jost matrices

Replacing (k_1, k_2) with $(-k_1, -k_2)$ in

$$f_{mn}^{(in/out)}(E) = \frac{k_n^{l_n+1}}{k_m^{l_m+1}} A_{mn}(E) \mp \imath\, k_m^{l_m}\, k_n^{l_n+1}\, B_{mn}(E)$$

$$f_{mn}^{(in/out)}(-k_1, -k_2) = (-1)^{lm+l_n} f_{mn}^{(out/in)}(k_1, k_2)$$

the parity conservation $((-1)^{l_m+l_n} = 1) \rightarrow$ whole matrices

$$f^{(in/out)}(-k_1, -k_2) = f^{(out/in)}(k_1, k_2)$$

Change $(k_1, k_2) \rightarrow (-k_1, -k_2)$ moves to a different sheet of the Riemann surface: $(++) \leftrightarrow (--)$ or $(-+) \leftrightarrow (+-)$

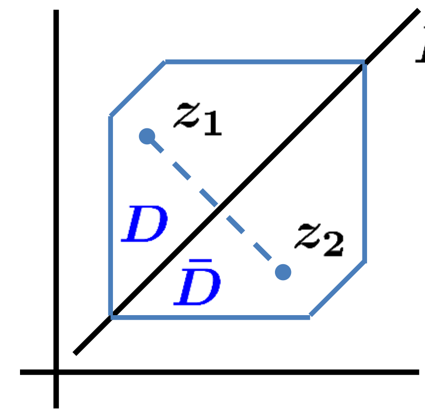
transition is to energy point above or below the initial location on a vertical line that corresponds to the same energy

vertical symmetry

Neutral particles: "diagonal" symmetry of the Jost matrices

the Schwartz reflection principle

domain $\bar{D}$ is a mirror reflection of D $(L \subset D)$
relative to the line L



in a domain D
function $f(z)$ is analytic
$f(z)$ is real on line L

⇒

analytically continued
across L into $\bar{D}$
$f(z_1) = f^*(z_2)$

Matrices $A(E)$ and $B(E)$ are real on the real axis

$$A(E^*) = A^*(E), \qquad B(E^*) = B^*(E)$$

change $E \to E^*$ is equivalent to $(k_1, k_2) \to (k_1^*, k_2^*)$

$$f_{mn}^{(in/out)}(E^*) = \left(\frac{k_n^{l_n+1}}{k_m^{l_m+1}}\right)^* A_{mn}(E^*) \mp \imath \left(k_m^{l_m}\, k_n^{l_n+1}\right)^* B_{mn}(E^*) = \left[f_{mn}^{(out/in)}(E)\right]^*$$

⇓

$$f^{(in/out)}(E^*) = \left[f^{(out/in)}(E)\right]^*$$

$E \to E^*$ change sheets : $(Im\, k_1, Im\, k_2) \to (-Im\, k_1, -Im\, k_2)$

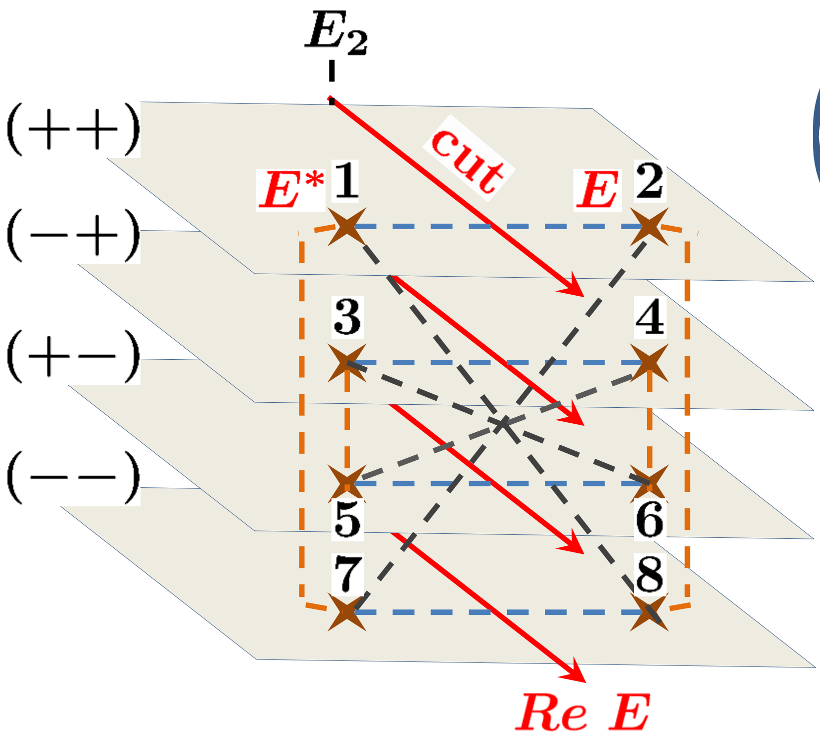
Neutral particles: "mirror" symmetry of the Jost matrices

combine the "vertical" and "diagonal" symmetries

$$(k_1, k_2) \;\rightarrow\; (-k_1^*, -k_2^*) \;\;:\Rightarrow\;\; E \;\rightarrow\; E^*$$

two points (k_1, k_2) and $(-k_1^*, -k_2^*)$ are on the same Riemann sheet, but on the opposite sides of the cut → "mirror" symmetry

$$f^{(in/out)}(k_1, k_2) = \left[f^{(in/out)}(-k_1^*, -k_2^*) \right]^*$$



Symmetry of the Jost matrices

$$S(E) = f^{(out)}(E) \left[f^{(in)}(E) \right]^{-1}$$

S-matrix symmetry

vertical	diagonal	mirror
$S(1) = S(7)^{-1}$	$S(1) = \left[S(8)^{-1}\right]^*$	$S(1) = [S(2)]^*$
$S(3) = S(5)^{-1}$	$S(3) = \left[S(6)^{-1}\right]^*$	$S(3) = [S(4)]^*$
$S(2) = S(8)^{-1}$	$S(5) = \left[S(4)^{-1}\right]^*$	$S(5) = [S(6)]^*$
$S(4) = S(6)^{-1}$	$S(7) = \left[S(2)^{-1}\right]^*$	$S(7) = [S(8)]^*$

Charged particles: Symmetry of the Jost matrices

Analytic structure of the Jost matrices

$$f^{(in/out)} = Q^{(\pm)}\left[D^{-1}AD - (M \pm \imath)\,K^{-1}DBD\right]$$

$$Q^{(\pm)} = \mathrm{diag}\left\{\frac{e^{\pi\eta_1/2}(l_1)!}{\Gamma(l_1+1\pm\imath\eta_1)},\cdots,\frac{e^{\pi\eta_N/2}(l_N)!}{\Gamma(l_N+1\pm\imath\eta_N)}\right\} \qquad K = \mathrm{diag}\left\{k_1,\cdots,k_N\right\}$$

$$D = \mathrm{diag}\left\{C_{l_1}(\eta_1)\,k_1^{l_1+1},\cdots,C_{l_N}(\eta_1)\,k_N^{l_N+1}\right\} \qquad M = \mathrm{diag}\left\{\frac{2\,\eta_1\,h(\eta_1)}{C_0(\eta_1)^2},\cdots,\frac{2\,\eta_N\,h(\eta_N)}{C_0(\eta_N)^2},\right\}$$

"vertical" symmetry $k_n \to -k_n$ is broken

$$(\ \eta_n \to -\eta_n,\ C_{l_n}(\eta_n) \to e^{\pi\eta_n}C_{l_n}(\eta_n),\ h(\eta_n) \to h(\eta_n)+\imath\pi(2j+1)\ )$$

"diagonal" symmetry $E \to E^*$ $(k_n \to k_n^*)$ remains valid

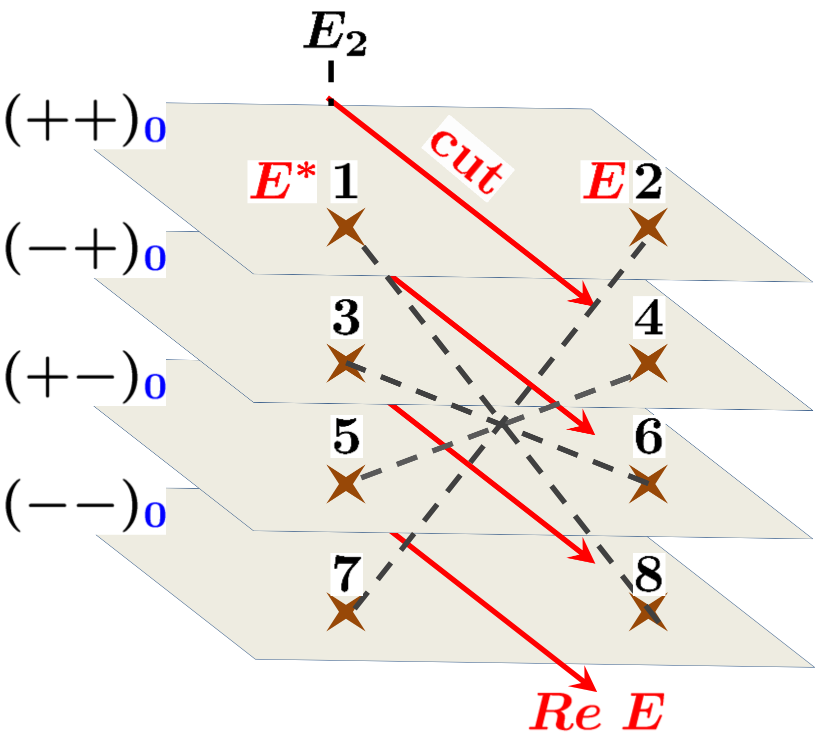
$$Q^{(\pm)}(E^*) = \left[Q^{(\mp)}(E)\right]^*,\ D(E^*) = [D(E)]^*,\ M(E^*) = [M(E)]^*,\ K(E^*) = [K(E)]^*$$

$$f^{(in/out)}(E^*_{m_1m_2\cdots}) = \left[f^{(out/in)}(E_{-m_1-m_2\cdots})\right]^*$$

$$\Downarrow$$

$$S(E^*_{m_1m_2\cdots}) = \left[S^*(E_{-m_1-m_2\cdots})\right]^{-1}$$

"mirror" symmetry $k_n \to -k_n^*$ is also broken



S-matrix symmetry

vertical	diagonal	mirror
$S(1) = S(7)^{-1}$	$S(1) = [S(8)^{-1}]^*$	$S(1) = [S(2)]^*$
$S(3) = S(5)^{-1}$	$S(3) = [S(6)^{-1}]^*$	$S(3) = [S(4)]^*$
$S(2) = S(8)^{-1}$	$S(5) = [S(4)^{-1}]^*$	$S(5) = [S(6)]^*$
$S(4) = S(6)^{-1}$	$S(7) = [S(2)^{-1}]^*$	$S(7) = [S(8)]^*$

Results

spurious solutions:
unstable and drastically change
as a result of any small changes
in parameters of the problem

pole position:
not depend on
choice of parameters

Calculations for different choices of $E_0 = 40$ keV, 50 keV, 60 keV:
fitting parameters $a^{(m)}$ and $b^{(m)}$ are different
non-spurious zeros of the Jost matrix determinant are the same

S-matrix poles (keV)			
$(++)_0$	$(-+)_0$	$(+-)_0$	$(--)_0$
9.0 - ı 4.6	9.1 - ı 4.6	9.1 - ı 4.5	9.1 - ı 4.5
43.8 - ı 33.8	57.1 - ı 26.5	47.8 - ı 38.3	50.2 - ı 23.2
55.5 - ı 23.9	73.6 - ı 26.3	51.2 - ı 22.0	62.1 - ı 46.4
9.0 + ı 4.6	9.1 + ı 4.6	33.3 + ı 22.6	43.0 + ı 57.5
43.8 + ı 33.8	57.1 + ı 26.5	42.5 + ı 57.3	48.5 + ı 33.2
55.5 + ı 23.9	73.6 + ı 26.3	62.3 + ı 19.4	

Calculations of the partial widths

The partial widths for the two-channel system are expressed in terms of matrix elements of the Jost matrices

$$\Gamma_i = \frac{\mathrm{Re}(k_i)\mid \mathcal{A}_i \mid^2 \Gamma}{\sum_{i'=1}^{N_{ch}} \frac{\mu_i}{\mu_{i'}} \mathrm{Re}(k_{i'}) \mid \mathcal{A}_i \mid^2}$$

where $i = 1, 2$ correspond to the $n\alpha$ and dt channels, $\mathcal{A}_1$ and $\mathcal{A}_2$ are the asymptotic amplitudes of channels

$$\mathcal{A}_1 = f_{11}^{(out)} - \frac{f_{11}^{(in)} f_{12}^{(out)}}{f_{12}^{(in)}}, \quad \mathcal{A}_2 = f_{21}^{(out)} - \frac{f_{11}^{(in)} f_{22}^{(out)}}{f_{12}^{(in)}}$$

the Jost matrices are taken at the complex resonant energy

S.A. Rakityansky, J. Phys. CS 915, 012008 (2017)

Contributions from individual poles

If E is a point inside contour (choose it on the real axis) then according to the Mittag-Leffler theorem (split a meromorphic function in the pole and the non-singular terms)

$$S(E) = \sum_{j=1}^{L} \frac{\mathrm{Res}(S, E_j)}{E_j - E} + \frac{1}{2\pi\imath} \oint \frac{S(\zeta)}{\zeta - E} d\zeta$$

S-matrix residues at known poles E_j can be found by numerical differentiation of the determinant of the Jost matrix

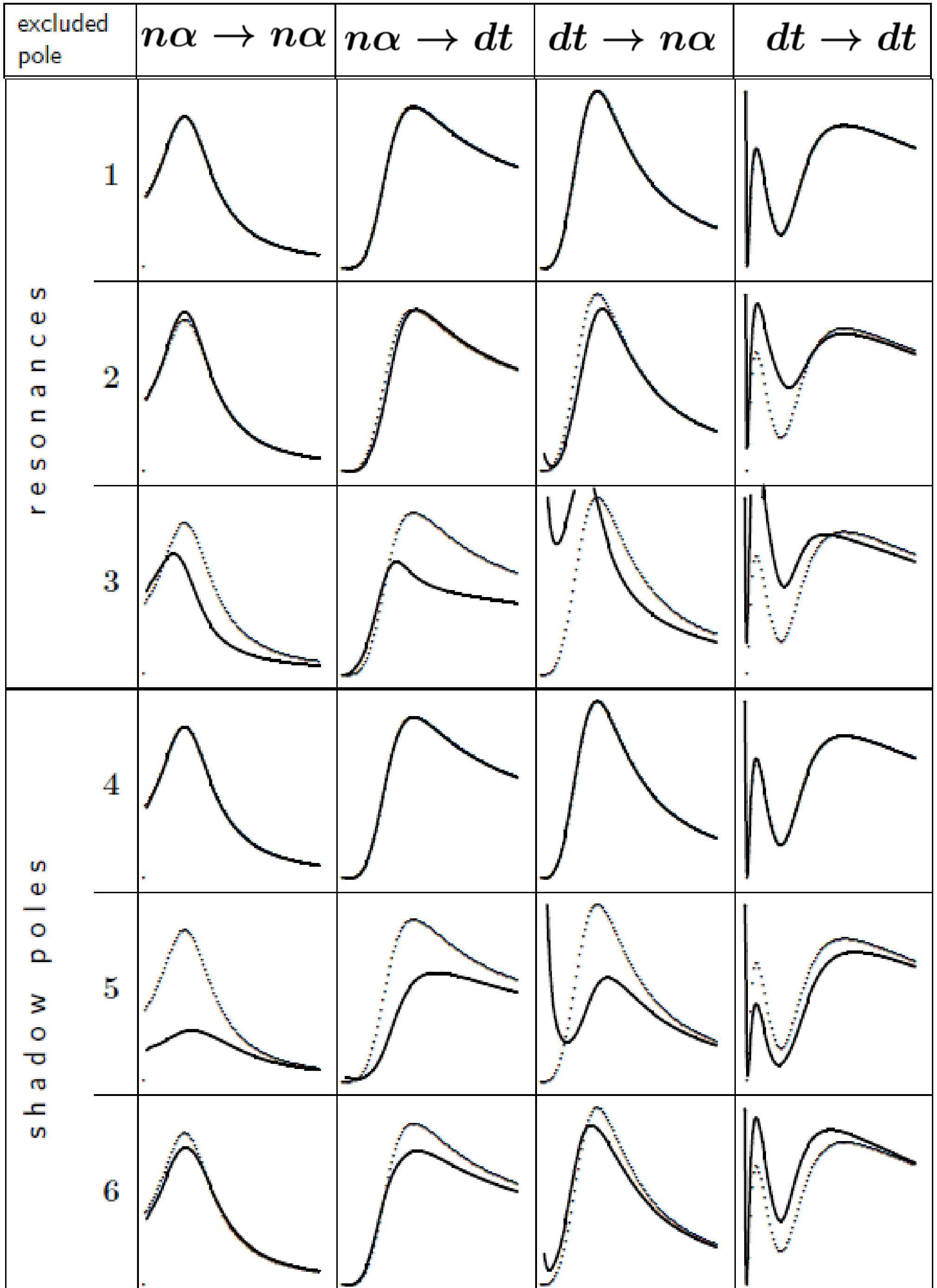
$$S(E) = \left[f^{(out)}(E) \right] \begin{bmatrix} f^{(in)}(E)_{22} & -f^{(in)}(E)_{12} \\ -f^{(in)}(E)_{21} & f^{(in)}(E)_{11} \end{bmatrix} \frac{1}{\det f^{(in)}(E)}$$

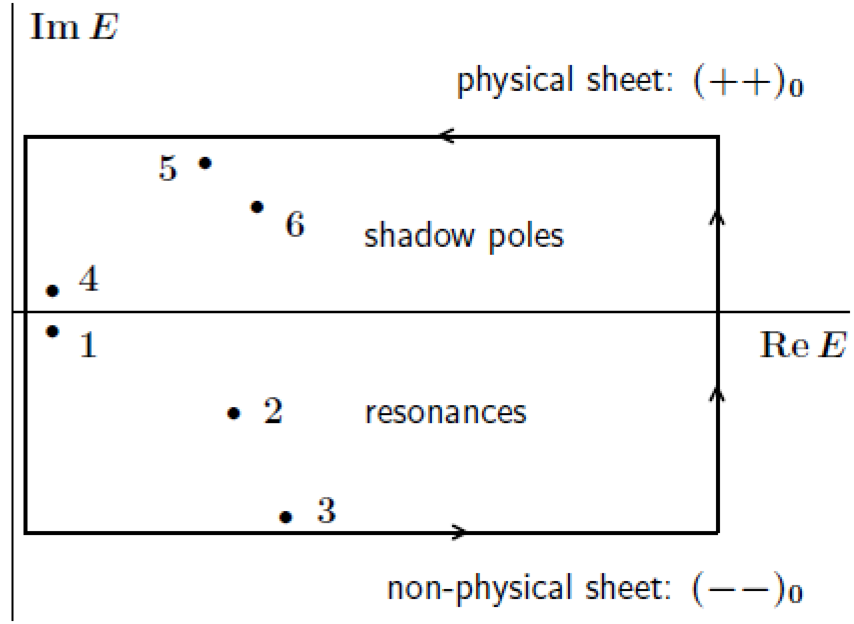
$$\mathrm{Res}\left[S, E_j\right] = \left[f^{(out)}(E_j) \right] \begin{bmatrix} f^{(in)}(E_j)_{22} & -f^{(in)}(E_j)_{12} \\ -f^{(in)}(E_j)_{21} & f^{(in)}(E_j)_{11} \end{bmatrix} \left[\frac{d}{dE} \det f^{(in)}(E) \right]_{E_j}^{-1}$$

In calculations, $\epsilon = 1$ eV gives the accuracy of at least 5 digits

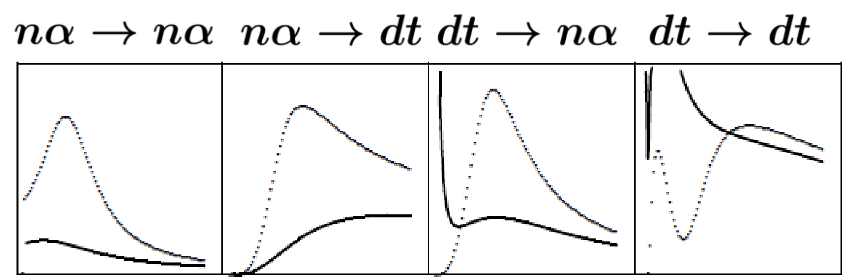
$$\frac{d}{dE} \det f^{(in)}(E) \approx \frac{\det f^{(in)}(E+\epsilon) - \det f^{(in)}(E-\epsilon)}{2\epsilon}$$

calculations: check of self-consistency





Contribution from contour integral
(all six poles are excluded)



Summary and conclusion

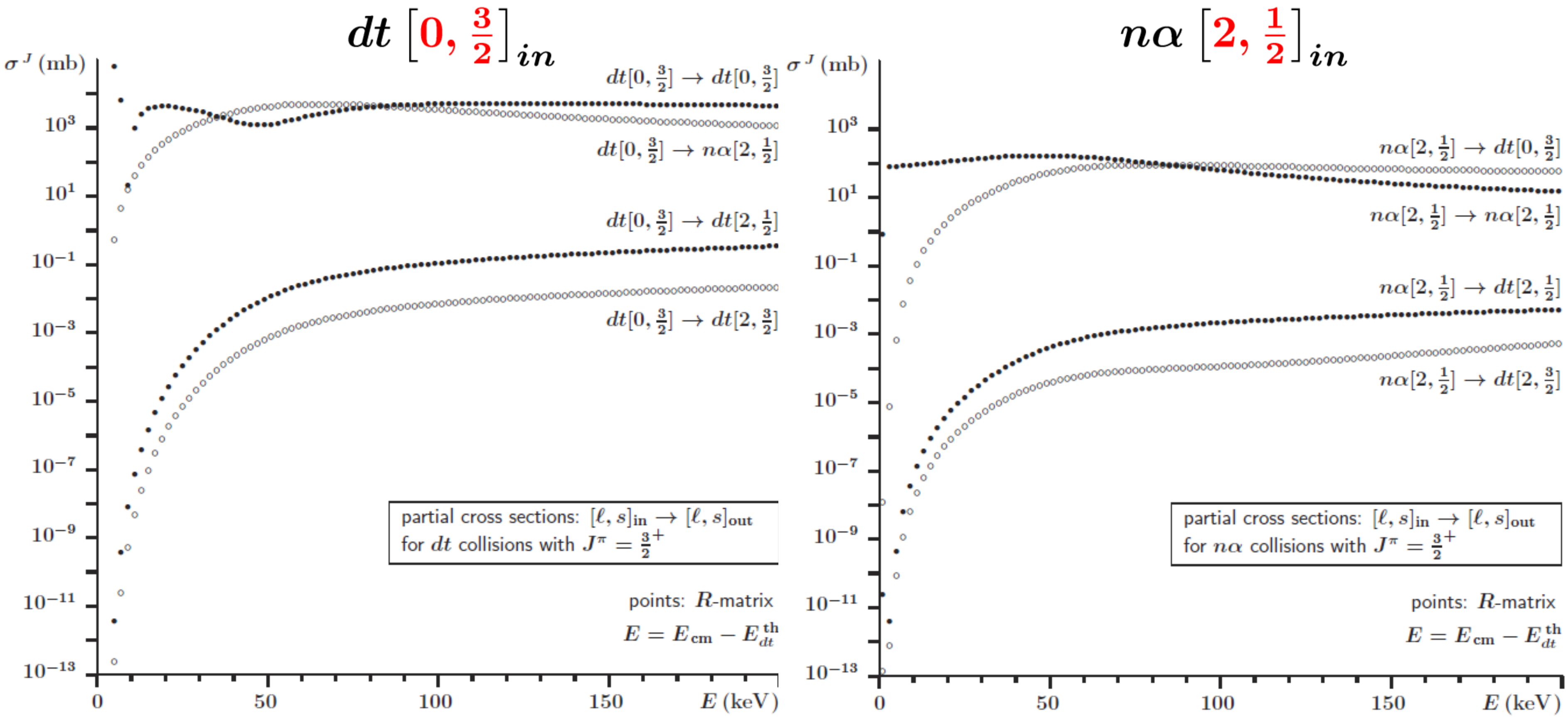
- The nature of $^5\mathrm{He}^*(\frac{3}{2}^+)$-resonance was investigated
- Using an available R-matrix fit, the Jost-matrices with proper analytic structure and proper topology of the Riemann surface (both the square-root and logarithmic branching are present) were constructed
- 23 poles of the S-matrix were located on various sheets of the Riemann surface
- Only 6 of these poles are close enough to the axis of the real scattering energies and can influence the observable quantities.
- The set consists of 3 resonances and 3 shadow poles
- Using the Mittag-Leffler representation, the individual contributions to the S-matrix from all resonances and shadow poles were estimated. Near the dt-threshold the partial cross sections are determined by the two resonant and two shadow poles

$$\text{resonant}: \left(50.2 - \frac{\imath}{2}46.3\right)\,\mathrm{keV},\quad \Gamma_{n\alpha} = 29.1\,\mathrm{keV},\quad \Gamma_{dt} = 17.2\,\mathrm{keV}$$

$$\text{resonant}: \left(62.1 - \frac{\imath}{2}92.8\right)\,\mathrm{keV},\quad \Gamma_{n\alpha} = 76.6\,\mathrm{keV},\quad \Gamma_{dt} = 16.2\,\mathrm{keV}$$

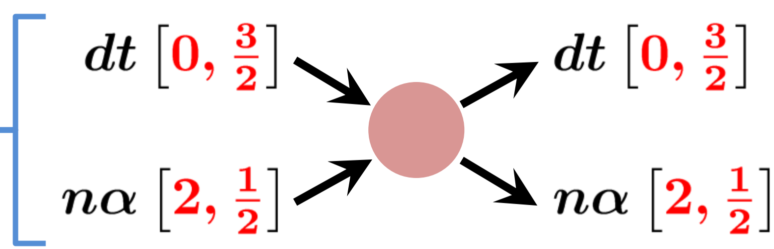
$$\text{shadow}: (43.8 + \imath\,33.8)\ \mathrm{keV},\quad (55.5 + \imath\,23.9)\ \mathrm{keV}$$

partial cross sections for $J^{\pi} = \left(\frac{3}{2}\right)^{+}$: $[l, S]_{in} \rightarrow [l, S]_{out}$



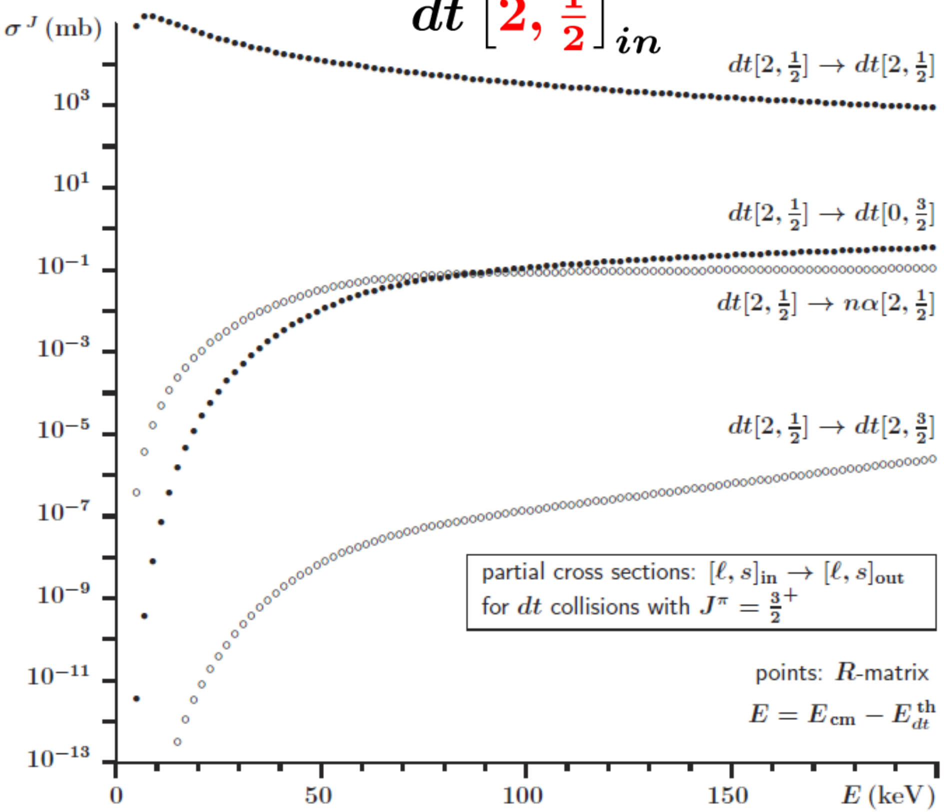
strongly coupled

the R-matrix fit

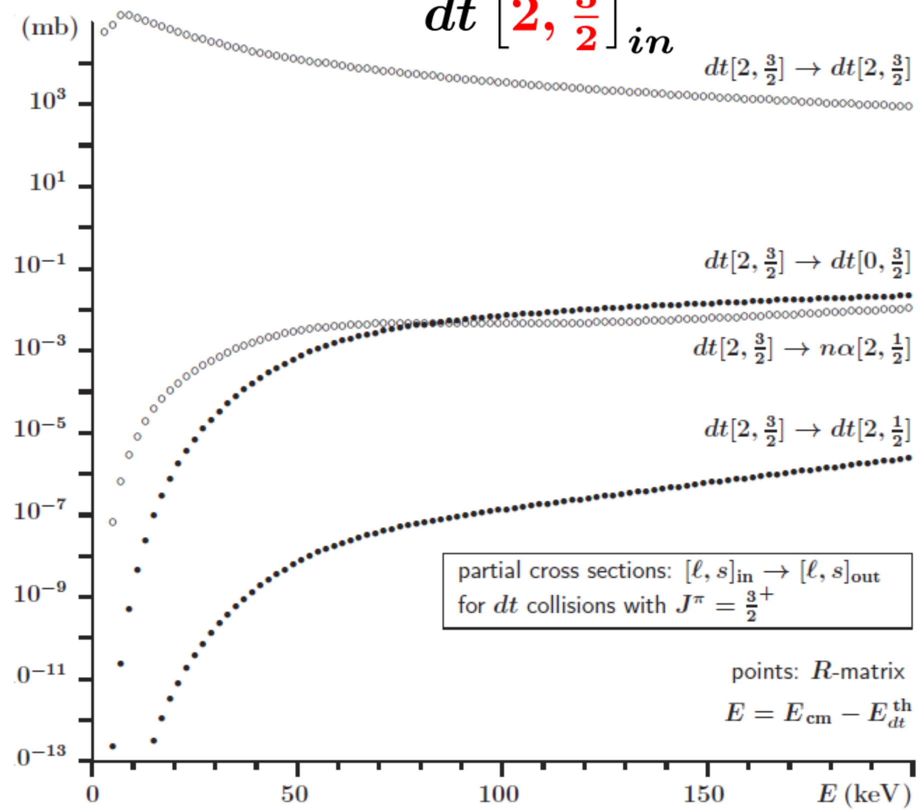


partial cross sections for $J^\pi = \left(\frac{3}{2}\right)^+$: $[l, S]_{in} \rightarrow [l, S]_{out}$

$dt\,\left[2, \frac{1}{2}\right]_{in}$



$dt\,\left[2, \frac{3}{2}\right]_{in}$



not coupled

the R-matrix fit

$$dt\,\left[2, \frac{1}{2}\right] \rightarrow dt\,\left[2, \frac{1}{2}\right]$$

$$dt\,\left[2, \frac{3}{2}\right] \rightarrow dt\,\left[2, \frac{3}{2}\right]$$