Viscous HydHSD: proton and pion transverse momentum and rapidity spectra

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Contents

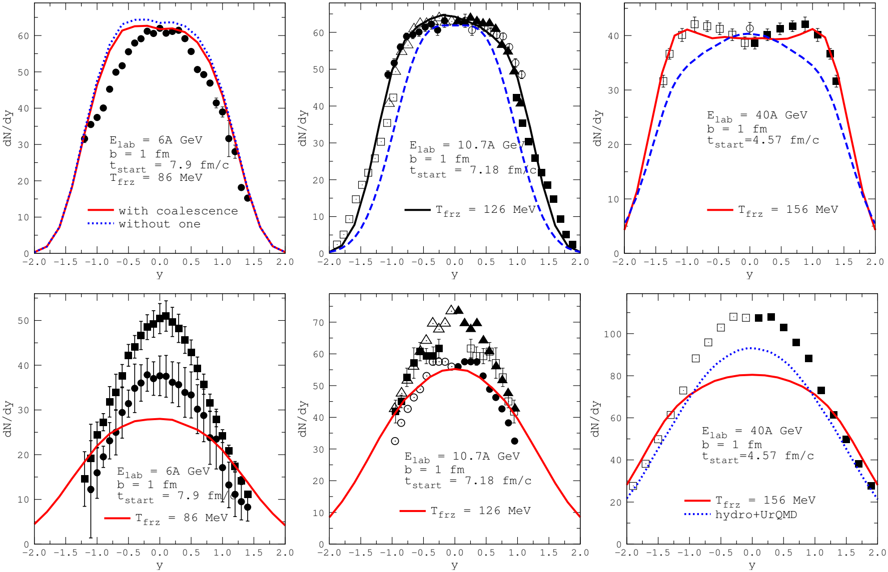
- The search for QGP in heavy ion collisions $\rightarrow$ it is necessary to connect observables with medium properties
- Experimental data $\rightarrow$ we need to take into account collective effects

$\Rightarrow$ Simplest way is hydrodynamics.

- Hydrodynamics applicability conditions $\Rightarrow$ It cannot be applied at the initial stage of a collision $\Rightarrow$ We need, e.g., a kinetic model. Our choice is HSD/PHSD (Parton Hadron String Dynamics, PHSD 1.0).

  W. Cassing and E. L. Bratkovskaya, Phys. Rept. **308**, 65 (1999)
- HSD/PHSD describes many experimental data in the energy range $E_{\rm lab} = 2 - 50\ A \cdot \mathrm{GeV}$ (NICA, FAIR)
- Hydrodynamics must be stopped when medium become nonequilibrium $\Rightarrow$ "freeze-out"

HydHSD (ideal hydrodynamics)



The lack of pions is due to the absence in our model of dissipative effects which increase the entropy.

Hydrodynamics: equations, parameters, and numerical algorithm

Just conservation laws of energy-momentum and baryon charge in the differential form:

$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu J^\mu = 0 \tag{1}$$

When dissipation processes are included, the energy-momentum tensor and the baryon current can be expanded as:

$$T^{\mu\nu} = T^{\mu\nu}_{\mathrm{id}} + \Pi\Delta^{\mu\nu} + \pi^{\mu\nu}, \quad J^\mu = n\, u^\mu + V^\mu\,,$$

$$T^{\mu\nu}_{\mathrm{id}} = \varepsilon\, u^\mu u^\nu - P\Delta^{\mu\nu}\,, \quad \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu\,,$$

$$u^\mu = \gamma(1, \mathbf{v}), \quad \gamma = (1 - v^2)^{-1/2}, \quad g^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$$

$\pi^{\mu\nu}$ is a traceless symmetric tensor and satisfies the orthogonality relations:

$$u_\mu \pi^{\mu\nu} = 0, \quad \pi^{\mu\nu} = \pi^{\nu\mu}, \quad \pi^\mu_\mu = 0.$$

The system of equations (1) is supplemented by an equation of state (EoS)

$$P = P(\varepsilon, n)$$

Equations of 2nd order hydrodynamics: relaxation equations

1st order (Navier-Stokes) hydrodynamics can fail relativistic constraints $\Rightarrow$
2nd order theory is necessary for modeling heavy-ion collisions
Dissipative terms $\pi^{\mu\nu}$, Π, V^{μ} are independent dynamical variables!
We consider only shear viscosity and follow the original Israel-Stewart approach

$$(u^{\lambda}\partial_{\lambda})\,\pi^{\mu\nu} = -\frac{\pi^{\mu\nu} - \eta W^{\mu\nu}}{\tau_{\pi}}\,,$$

$$W^{\mu\nu} = \Delta^{\mu\lambda}\partial_{\lambda}u^{\nu} + \Delta^{\nu\lambda}\partial_{\lambda}u^{\mu} - \frac{2}{3}\Delta^{\mu\nu}\,\theta\,,\;\; \theta = \partial_{\lambda}u^{\lambda}$$

W. Israel and J.M. Stewart, Ann. Phys. **118**, 341 (1979)

To solve the relaxation equation, one need to know τ_π and η.
We use simple relations E. Molnár *et al.*, PRC **90**, 044904 (2014)

$$\tau_\pi = \frac{5\eta}{\varepsilon + P}, \qquad \frac{\eta}{s} = \mathrm{const}$$

SHASTA (the SHarp and Smooth Transport Algorithm)
E. Molnár *et al.*, Eur. Phys. J. C **65**, 615 (2010).

$$\mathrm{d}x = 0.2 \text{ fm}, \quad \lambda = \mathrm{d}t/\mathrm{d}x = 0.4, \quad A_{ad} = 0.6$$

EoS = the hadron gas in a mean field + σ-meson

Satarov *et al.*, Phys. Atom. Nucl. **72**, 1390 (2009)

Some important features

1. We rewrite the energy-momentum equation as

$$\partial_\mu T^{\mu\nu}_{\rm id} = -\partial_\mu \pi^{\mu\nu}.$$

 Independent propagated (primary) variables are

$$T^{\mu\nu}_{\rm id},\ J^0,\ \pi^{22},\ \pi^{33},\ \pi^{12},\ \pi^{13},\ \pi^{23}$$

2. On each calculation step we have to ensure the applicability of 2nd order hydrodynamics. The theory requires that the dissipative currents give sufficiently small corrections to the local equilibrium (= ideal fluid) quantities.

$$q = \max_{\mu,\nu} \frac{|\pi^{\mu\nu}|}{|T^{\mu\nu}_{\rm id}|} < C \quad \text{(S-cond.)}$$

$$\pi^{\mu\nu} \to \pi^{\mu\nu}\frac{C}{q} \quad \text{by hand}$$

$$C = 0.3 \quad \text{by default}$$

The transition from kinetic description to the hydrodynamic one occurs at some time moment $t_{\rm start}$ when S/N_B is flattened

V. V. Skokov and V. D. Toneev, YaF **70**, 114 (2007)

$$\pi^{\mu\nu}(t_{hydro} = 0) = 0$$

The impact parameter $b = 1$ fm for all considering energies.

Only particles that have suffered interactions are included for obtaining the initial state.

Table: Starting times of hydrodynamical calculations

$E_{\rm lab}$ [$A \cdot$ GeV]	6	10.7	40	80	158
$t_{\rm start}$ [fm/c]	7.9	7.18	4.57	3.8	3.01

Observable calculation (particlization)

Hydrodynamics is finished by "instantaneous freeze-out".
A particlization procedure is applied to convert fluids to particles according to Cooper-Fry formulae:

$$E\frac{\mathrm{d}^3 N_a}{\mathrm{d}p^3} = \frac{g_a}{(2\pi)^3}\int \mathrm{d}^3\sigma n_\mu p^\mu f_a(x,p)\,, \qquad (2)$$

The hypersurface for isothermal freeze-out is determined by CORNELIUS algorithm.

P. Huovinen and H. Petersen, EPJA **48**, 171 (2012)

For viscous fluids, one has to take into account the modification of the distribution function:

$$f_a(x,p) = f_a^{(0)}(x,p)\left\{1 + [1 \mp f_a^{(0)}(x,p)]\frac{p_\mu p_\nu \pi^{\mu\nu}}{2T^2(\varepsilon+P)}\right\}\,,$$

$$f_a^{(0)}(x,p) = \left\{e^{\beta[p^\nu u_\nu(x) - \mu_a(x)]} \pm 1\right\}^{-1}, \quad \beta = 1/T\,.$$

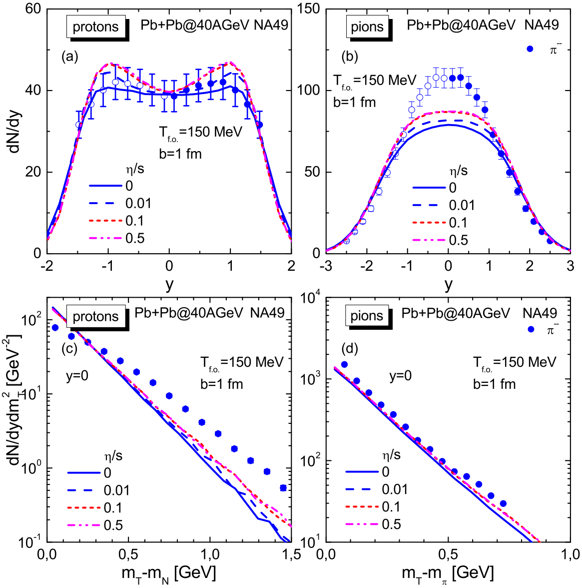
Iu. A.Karpenko *et al.*, PRC **91**, 064901 (2015)

+ resonance decays

Response of our model on parameter changing

The dependence on η/s ratio

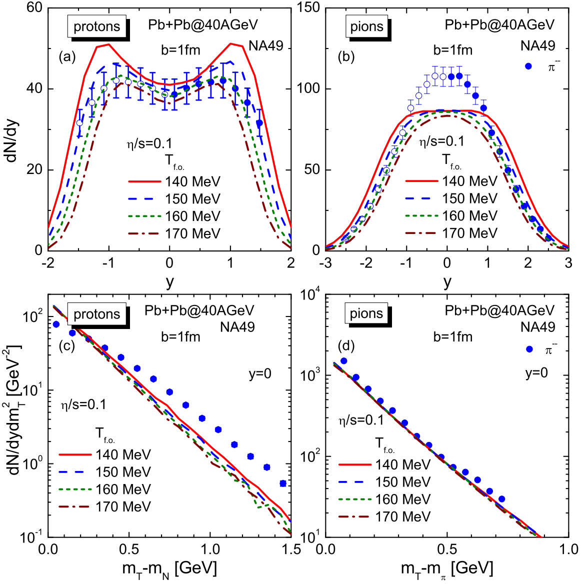
$E_{\mathrm{lab}} = 40\ A \cdot \mathrm{GeV}$ as a reference point



- Calculations with the very small value of $\eta/s = 0.01$ demonstrate that already such small viscosity changes visibly rapidity distributions comparing to the non-viscous case.
- An increase of the viscosity leads to sizable changes in the rapidity distributions. The two-hump structure in the proton rapidity distribution becomes much more pronounced (more slow expansion!). The pion rapidity distributions becomes higher (the temperature decreases more slow).
- We observe saturation with an increase of the η/s ratio.
- The transverse momentum spectra are very weak dependent on η/s.

The dependence on $T_{\rm frz}$

$\eta/s = 0.1$



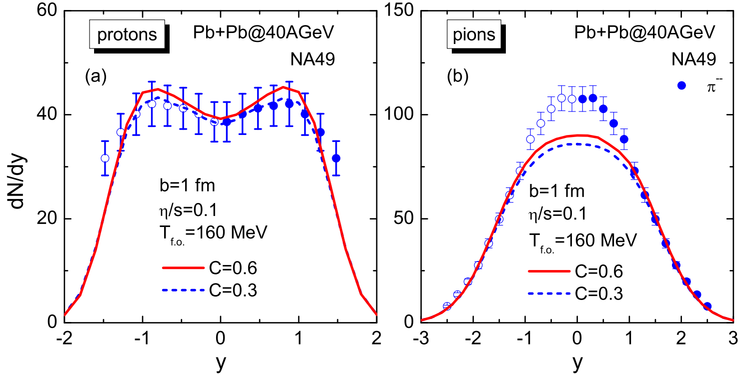
- The proton rapidity distribution becomes higher if the freeze-out temperature $T_{\rm frz}$ is lower.
- We observe similarity in effects caused by $T_{\rm frz}$ decreasing and η/s increasing, both lead to a growth of the humps in dN_p/dy. The reason of that an increase of the evolution duration in both cases.
- Rapidity distributions becomes wider with increasing $T_{\rm frz}$.
- The height of dN_π/dy has a limit! It results in that we are able to reproduce the proton rapidity distribution but not the pion one.
- The slope of the pion transverse momentum spectra is almost insensitive to the variation of $T_{\rm frz}$ and the proton spectra demonstrates very weak dependence.

- We are able to reproduce the proton rapidity distribution but not the pion one. The technical reason of this failure in reproducing both distributions is the mentioned insensitivity of the distributions to $\eta/s > 0.1$. We can obtain only the correct width of the dN_π/dy.
- To approach experimental data for the proton transverse momentum spectrum, we need $T_{\rm frz} < 120\,\mathrm{MeV}$, whereas $T_{\rm frz} \sim 150 - 160$ MeV is necessary for the description of the dN_p/dy. This problem occurs for all energies $E_{\rm lab} \geq 10.7\ A \cdot \mathrm{GeV}$ and may be caused by an inappropriate EoS and/or an initial state.
- The slope of pion $m_{\rm T}$ spectrum is well described.

A question:

- Why are viscous effects in our hybrid model quite small ($\sim$ 10%) for pion rapidity distribution while results of authors [Iu.A. Karpenko *et al.*, PRC **91**, 064901 (2015)] within the vHLLE+UrQMD model demonstrate that the response is large (about 20%)?

The viscous response to the constant C



If C is larger, the viscous effects are expected to be more pronounced. That is indeed the case:

- The two-hump structure in proton rapidity distributions is more pronounced for larger values of C-parameter
- Simultaneously pion rapidity distribution is getting a bit higher but the gain at mid-rapidity too small to improve the agreement with the experiment.

Constraints on the $\pi^{\mu\nu}$ tensor in vHLLE and MUSIC

The vHLLE model uses another constraint on the $\pi^{\mu\nu}$ tensor magnitude using the criterion: the quantity q is calculated as

$$q = q_{\rm V} \equiv \frac{\max_{\mu,\nu} |\pi^{\mu\nu}|}{\max_{\mu,\nu} \left| T_{\rm id}^{\mu\nu} \right|} . \quad \text{(V-cond.)}$$

It clearly results in a weaker condition than our definition.
Let us also consider the condition which is applied in the MUSIC model

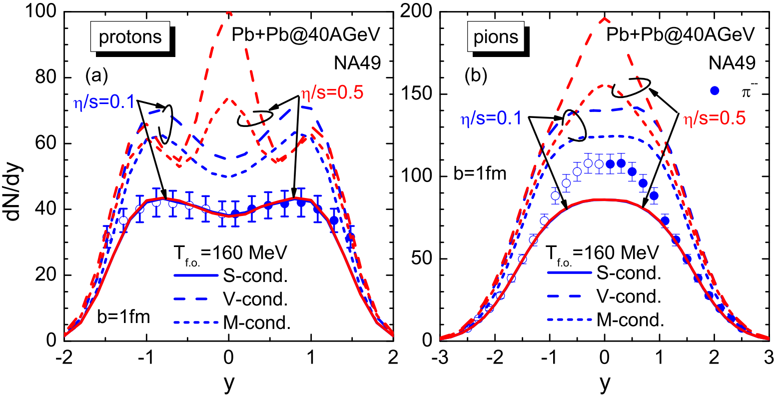
B. Schenke *et al.*, PRL **106**, 042301 (2011)

$$q = q_{\rm M} \equiv \sqrt{\frac{\pi^{\mu\nu}\pi_{\mu\nu}}{T_{\rm id}^{\mu\nu} T_{{\rm id},\mu\nu}}} . \quad \text{(M-cond.)}$$

Remember that we applied

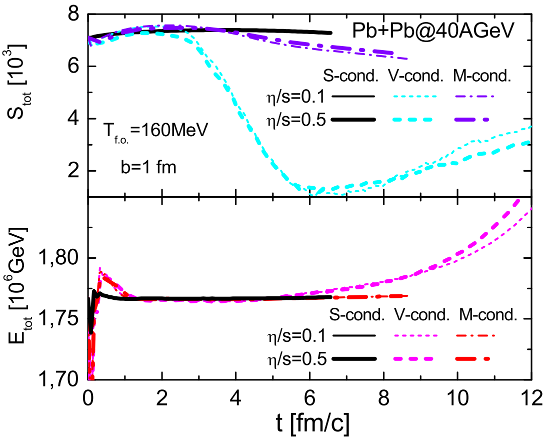
$$q = \max_{\mu,\nu} \frac{|\pi^{\mu\nu}|}{\left| T_{\rm id}^{\mu\nu} \right|} < C \quad \text{(S-cond.)}$$

Rapidity distributions for different conditions



- Applying weaker constraints leads to dramatically change in the rapidity distributions. The height of the proton humps and of the pion distribution increase sizably compared to the calculations with the stricter constraint.
- Sensitivity of rapidity spectra to the η/s value are much larger for the V- and M-conditions than for the S-condition.
- Applying V- and M-conditions, we obtain qualitatively similar results.

Evolution of the total energy and entropy



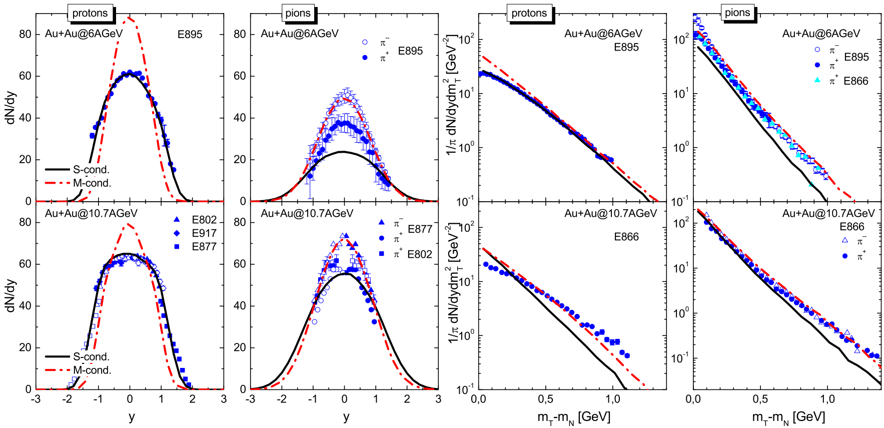
A weaker constraint leads to

- longer evolution time;
- a larger fluctuations of the total energy;
- a larger decrease of S_{tot}.

Disadvantages of the V-condition:

- E_{tot} starts to increase after the first 5 fm/c;
- longest evolution (much longer than for two other constraints, $\sim$ 20 fm/c for the considered case);
- most large fluctuations and a strongest decrease of the total entropy.

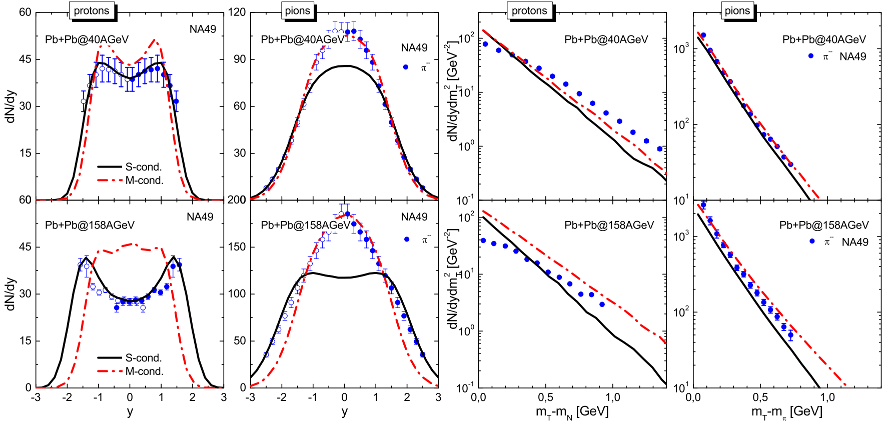
Best fit: AGS



At $E_{\rm lab} = 6\ \ A \cdot \mathrm{GeV}$, the proton m_T-spectra and the rapidity distribution can be reproduced reasonably with S-cond.

At higher energies, proton m_T-spectra have too steep slope.

Best fit: SPS



Using the M-constraint, it is possible reproduce pion rapidity distribution and m_T-spectra.

We cannot simultaneously reproduce pion and proton distributions using any condition.

The fitted parameters

$E_{\rm lab}$ [$A \cdot$ GeV]	S-condition		(π)M-condition	
	$T_{\rm frz}$ [MeV]	η/s	$T_{\rm frz}$ [MeV]	η/s
6	78	0.03	120	0.07
10.7	125	0.01	143	0.24
40	155	0.05	170	0.08
158	160	0	205	0.1

- The fit of proton rapidity distribution needs small values of η/s (in agreement with our ideal-hydro calculations!). As a result, the height of pion distribution is also close to non-viscous case.
- The fit of pions with a weaker condition leads to systematically large values of $T_{\rm frz}$ and η/s. $\eta/s \sim 0.1$ at all energies, excluding $E_{\rm lab} = 10.7\ A \cdot \mathrm{GeV}$.
- We see an extremum for η/s at $E_{\rm lab} = 10.7\ A \cdot \mathrm{GeV}$ with M-condition and may be with S-condition.

Conclusions

- We extended the earlier developed HydHSD model by inclusion shear viscosity within Israel-Stewart hydrodynamics.
- It is shown that the form of $\pi^{\mu\nu}$-constraints plays a crucial role for sensitivity to the η/s value. A numerical algorithm with a weaker condition, like used in MUSIC or vHLLE codes, is more responsive and leads to a higher pion rapidity distribution.
- Any considered condition does not allow to reproduce simultaneously pion and proton experimental data.
- We assume that the reason is an inappropriate EoS and/or initial conditions and also a non-zero bulk viscosity has to be taken into account.

Thank you for attention!