Universal Features of Pre-inflationary Universe in Loop Quantum Cosmology and Gravity

Anzhong Wang

Zhejiang University of Technology
&
Physics Department, Baylor University

Based on arXiv:1607.06329; 1705.07544; 1709.07479; 1710.09845; 1801.07313

International workshop for String theory, Gravitation and Cosmology (STGCOS2018)
June 18 (Mon), 2018 ~ June 20 (Wed), 2018

APCTP
Asia Pacific Center for Theoretical Physics

Pohang, Korea

Collaborators

G. Cleaver (Phys/BU)

B.-F. Li (CASPER/BU)

K. Kirsten (Math/BU)

Q. Sheng (Math/BU)

P. Singh (Phys/LSU)

M. Sharma (Phys/ZJUT)

M. Shahalam (Phys/ZJUT)

Q. Wu (Phys/ZJUT)

T. Zhu (Phys/ZJUT)

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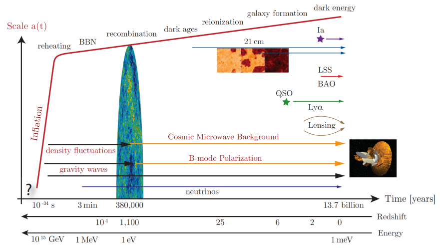
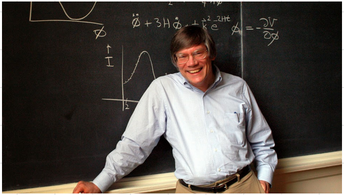
1.1 Inflation

- Inflation is an extremely rapid (exponentially) expansion of the universe after its creation ($t \simeq 10^{-34}$ s):

$$a(t) = a_i e^{H(t-t_i)}$$

$a(t)$: the expansion factor; H: Hubble constant





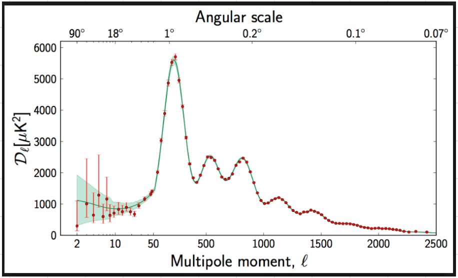
(A. Guth, 1981)

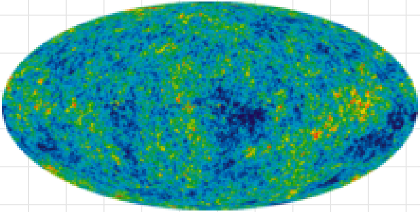
1.1 Inflation (Cont.)

- Inflation solves several Big Bang Puzzles:
  - The Cauchy Problem
  - The Horizon Problem
  - The Monopole Problem
  - The Flatness Problem
  - ...

1.1 Inflation (Cont.)

- All observations carried so far support Inflation
  - CMB measured by Planck 2015 (arXiv:1502.02114):





1.2 Initial Singularity Problem

However, inflation is also facing various challenging (theoretical) problems

- Initial Singularity Problem:
  - General relativity (GR) inevitably leads inflation to an initial singularity [1], with which in principle it is not clear how to impose the initial conditions.

[1] A. Borde and A. Vilenkin, PRL72 (1994) 3305; A. Borde, A. H. Guth, and A. Vilenkin, PRL90 (2003) 151301.

1.3 The Problem of Initial Conditions

- The Problem of Initial Conditions:
  - Many inflationary scenarios only work if the fields are initially *very homogeneous and/or start with precise initial positions and velocities*.
  - Any physical understanding of this “fine-tuning” requires a more complete formulation with ever-higher energies, such as string theory.
- ...

1.4 Inflation is sensitive to Planckian physics

- Therefore:

Inflation is very sensitive to Planck-scale physics, and effects of quantum gravity in the early universe are important and need to be taken into account [2].

[2] D. Baumann, TASI Lectures on Inflation, arXiv:0907.5424
C.P. Burgess, M. Cicoli, F. Quevedo, JCAP 1311 (2013) 003
D. Baumann and L. McAllister, Inflation and String Theory (Cambridge Monographs on Mathematical Physics, Cambridge University Press, 2015)
E. Silverstein, TASI lectures on cosmological observables and string theory, arXiv:1606.03640.

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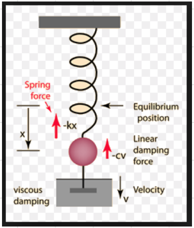
2. A Brief Introduction to LQC

- Simple Harmonic Oscillator:

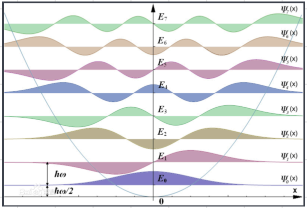


$$
\begin{aligned}
H &= \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2 = \hbar\omega \left(N + \frac{1}{2} \right) \\
N &= a^\dagger a \\
a &= \sqrt{\frac{m\omega}{2\hbar}} \left(x + \frac{ip}{m\omega} \right) \\
a^\dagger &= \sqrt{\frac{m\omega}{2\hbar}} \left(x - \frac{ip}{m\omega} \right)
\end{aligned}
$$

- Discrete eigenstate:

$$
N\,|n\rangle = n\,|n\rangle , \quad H\,|n\rangle = E_n\,|n\rangle
$$

$$
H\,|0\rangle = E_0\,|0\rangle , \quad E_n \equiv \hbar\omega \left(n + \frac{1}{2} \right)
$$



2. A Brief Introduction to LQC

- In LQC, one can naturally define operators representing geometric observables.
- The area operator $\hat{\mathrm{A}}$:

$$\hat{\mathrm{A}}\,|0\rangle = \Delta\,|0\rangle$$

$\Delta \equiv 4\sqrt{3}\pi\gamma$: the smallest nonzero eigenvalue
γ: Barbero-Immirzi parameter

- Δ represents the fundamental area gap and sets an energy scale,

$$\rho_{\mathrm{B}} \equiv 18\pi \left(\frac{\mathrm{m_{pl}}}{\Delta}\right)^{3} \mathrm{m_{pl}}$$

2. A Brief Introduction to LQC (Cont.)

- In GR, the Friedmann equation

$$H^2 = \frac{8\pi G}{3}\rho \tag{1}$$

  always leads to a singularity [3].

- For example,

$$\begin{aligned} a(t) &= a_i t^{\alpha}, \quad \alpha > 0 \\ \Rightarrow H^2 &= \left(\frac{\dot{a}}{a}\right)^2 = \frac{\alpha^2}{t^2} \\ \Rightarrow \rho &= \left(\frac{3}{8\pi G}\right)\frac{\alpha^2}{t^2} \rightarrow \infty, \quad \text{as} \, t \rightarrow 0 \end{aligned} \tag{2}$$

[3] A. Borde and A. Vilenkin, PRL72 (1994) 3305; A. Borde, A. H. Guth, and A. Vilenkin, PRL90 (2003) 151301.

2. A Brief Introduction to LQC (Cont.)

- In LQC, the matter density is bounded above [4]

$$\rho \leq \rho_{\rm B}$$

and the Friedmann equation is modified to

$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_{\rm B}}\right), \quad (\rho \leq \rho_{\rm B}) \tag{3}$$

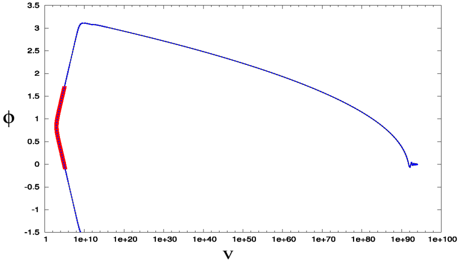
- In LQC, the modifications are purely due to quantum geometry, and matter satisfies the same field equations, as in QFTs. For example, for a scalar field, the Klein-Gordon equation still holds,

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

[4] A. Ashtekar, T. Pawlowski and P. Singh, PRL96 (2006) 141301.

2. A Brief Introduction to LQC (Cont.)

- The bib bang singularity is replaced by a quantum bounce, which naturally happens at $\rho \simeq \rho_{\mathrm{B}}$.



[$V \propto a^3$. Ashtekar & Barrau, CQG32 (2015) 234001]

2. A Brief Introduction to LQC (Cont.)

- By now, a large number of cosmological models have been studied in detail in LQC [5], including
  - $f(R)$ universe
  - the closed FLRW model
  - FLRW models with Λ with any signs
  - the Bianchi models
  - the Gowdy model, which incorporates the simplest types of inhomogeneities in full GR
  - ...
- In ALL cases, the singularity is resolved
- Therefore, the first problem is resolved!!!

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[5] A. Ashtekar and P. Singh, CQG 28 (2011) 213001; I. Agullo and A. Corichi, arXiv:1302.3833.

2. A Brief Introduction to LQC (Cont.)

- Does a slow-roll phase compatible with observations rise naturally from the quantum bounce? or is an enormous fine tuning needed?
- It was found that: the probability for the desired — i.e. in agreement with CMB measurements — slow roll inflation not to occur in an LQC solution is less than about one part in a million [6],

$$\lesssim 1.2 \times 10^{-6}$$

— Slow-roll inflation is an attractor in LQC!

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[6] P. Singh, K. Vandersloot and G. V. Vereshchagin, PRD74 (2006) 043510;
X. Zhang and Y. Ling, JCAP08 (2007) 012;
A. Ashtekar A and D. Sloan, GRG43 (2011) 3619;
A. Corichi and A. Karami PRD83 (2011) 104006;
L. Linsefors and A. Barrau, PRD87 (2013) 123509;
L. Chen and J.-Y. Zhu, PRD92 (2015) 084063.

2. A Brief Introduction to LQC (Cont.)

- Can one arrive at the BD vacuum at the onset of the slow-roll inflation?
- or is an even more elaborate fine tuning of quantum state of perturbations necessary in the Planckian regime?
- Because of the pre-inflationary dynamics, particles could be created during the Planckian regime and are carried over to the BD vacuum. This could source non-Gaussianity during inflation and give rise to potential effects in CMB.
- Can the state at the onset of the slow roll close enough to the BD vacuum in order to agree with current observations, and yet to be sufficiently different to give rise observational signatures of LQC?

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3. Universality of the Background Evolution

- In the framework of LQC, the background evolution can be divided into two classes:
  - Initially the evolution is dominated by the kinetic energy of the inflaton:

$$\frac{1}{2}\dot{\phi}^2(t_B) > V(\phi(t_B))$$

  - Initially it is dominated by the potential energy:

$$\frac{1}{2}\dot{\phi}^2(t_B) < V(\phi(t_B))$$

- However, a potential dominated bounce is either not able to produce the desired slow-roll inflation or leads to a large amount of e-folds of expansion [7].

[7] A. Ashtekar and A. Barrau, CQG32 (2015) 234001

3. Universality of the Background Evolution (Cont.)

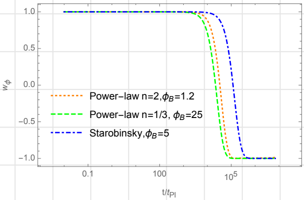
- In the kinetic energy initially dominated case, the evolution of the background can always be divided into three different phases [8]:

(a) Bouncing, (b) transition, (c) slow-roll inflation

$$\mathrm{w}(\phi) \equiv \frac{\dot{\phi}^2/2 - \mathrm{V}(\phi)}{\dot{\phi}^2/2 + \mathrm{V}(\phi)} = \begin{cases} +1, & \text{bouncing} \\ - < \mathrm{w}(\phi) < +1, & \text{transition} \\ -1, & \text{slow-roll inflation} \end{cases}$$

- The transition phase is short, during which the kinetic energy decreases dramatically:

$$\dot{\phi}^2/2 \simeq \rho_{\mathrm{B}} \to 10^{-12}\rho_{\mathrm{B}} \leq \mathrm{V}(\phi)$$



[8] Zhu, AW, Cleaver, Kirsten, Sheng, PLB773 (2017) 196; PRD96 (2017) 083520; Shahalam, Sharma, Wu, AW, PRD96 (2017) 123533.

3. Universality of the Background Evolution (Cont.)

- The equations for the evolution of the background (the zeroth-order approximations):

$$\mathrm{H}^2 = \frac{8\pi \mathrm{G}}{3}\rho\left(1 - \frac{\rho}{\rho_\mathrm{B}}\right), \quad \mathrm{H} \equiv \frac{\dot{\mathrm{a}}}{\mathrm{a}}$$

$$\ddot{\phi} + 3\mathrm{H}\dot{\phi} + \frac{\mathrm{dV}(\phi)}{\mathrm{d}\phi} = 0$$

$$\rho = \frac{1}{2}\dot{\phi}^2 + \mathrm{V}(\phi), \quad \rho_\mathrm{B} = \frac{1}{2}\dot{\phi}_\mathrm{B}^2 + \mathrm{V}(\phi_\mathrm{B})$$

- The initial conditions, $\left(\mathrm{a}_\mathrm{B}, \phi_\mathrm{B}, \dot{\phi}_\mathrm{B}\right)$, but we can always choose $\mathrm{a}_\mathrm{B} = 1$, and we also have,

$$\dot{\phi}_\mathrm{B} = \pm\sqrt{2\rho_\mathrm{B} - \mathrm{V}(\phi_\mathrm{B})}$$

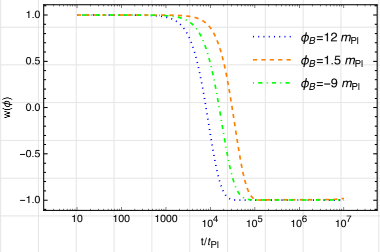
so the initial conditions are

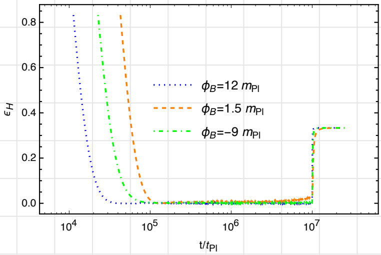
$$\left(\mathrm{a}_\mathrm{B}, \phi_\mathrm{B}, \dot{\phi}_\mathrm{B}\right) \quad \Rightarrow \quad \phi_\mathrm{B} \tag{4}$$

3. Universality of the Background Evolution (Cont.)

The three-phase division is universal:

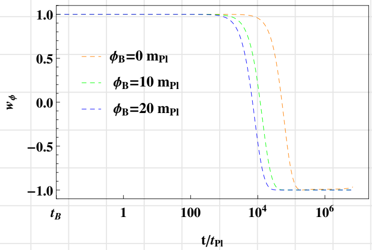
- Quadratic Potential $V(\phi) = \lambda_0 \phi^2$:

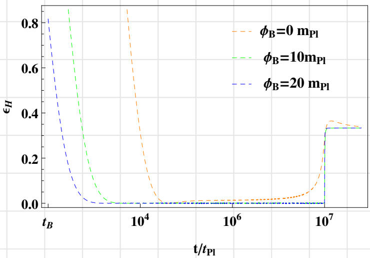




3. Universality of the Background Evolution (Cont.)

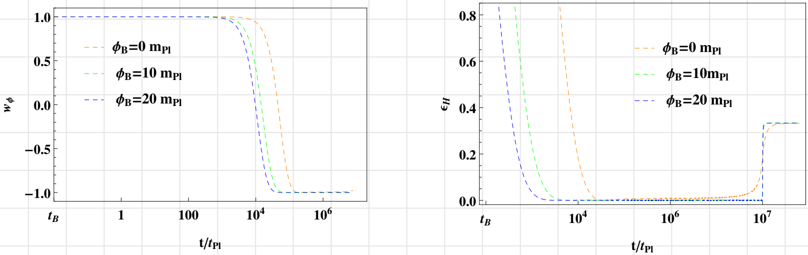
- Power-law Potential $V(\phi) = \lambda_0 \phi^{7/4}$:





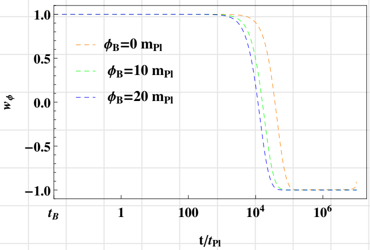
3. Universality of the Background Evolution (Cont.)

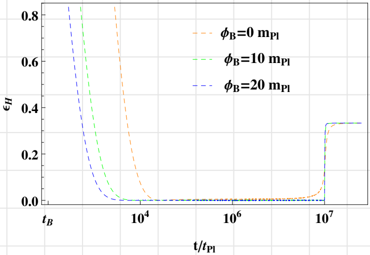
- Power-law Potential $V(\phi) = \lambda_0 \phi^{4/3}$:



3. Universality of the Background Evolution (Cont.)

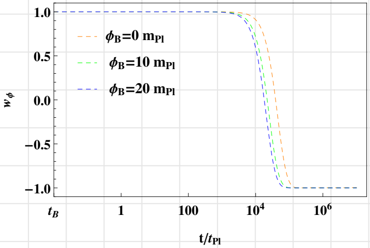
- Power-law Potential $V(\phi) = \lambda_0 \phi$:

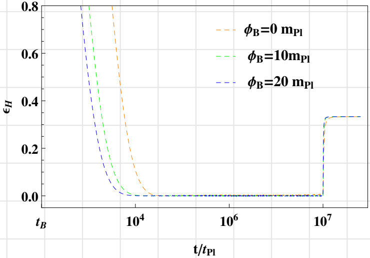




3. Universality of the Background Evolution (Cont.)

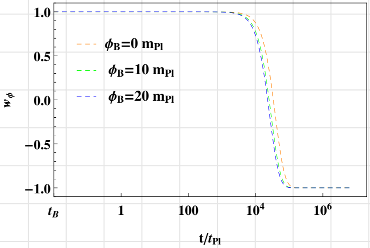
- **Power-law Potential** $V(\phi) = \lambda_0 \phi^{2/3}$:

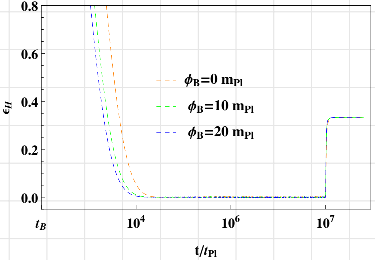




3. Universality of the Background Evolution (Cont.)

- Power-law Potential $V(\phi) = \lambda_0 \phi^{1/3}$:

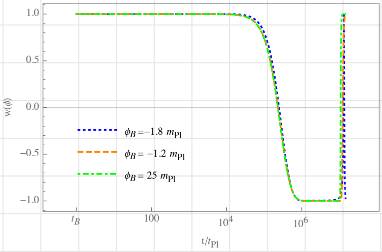


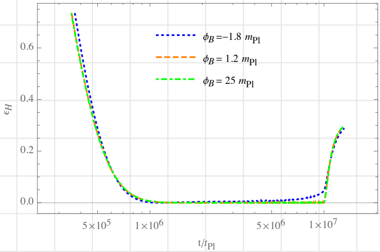


3. Universality of the Background Evolution (Cont.)

- Starobinsky Potential

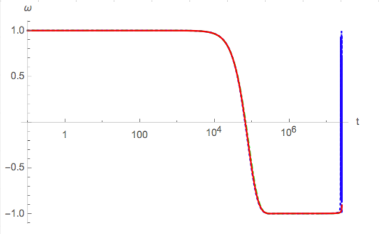
$$V(\phi) = \frac{3}{32\pi} M^2 m_{Pl}^2 \times \left(1 - e^{-\sqrt{\frac{16\pi}{3}}\frac{\phi}{m_{Pl}}}\right)^2$$

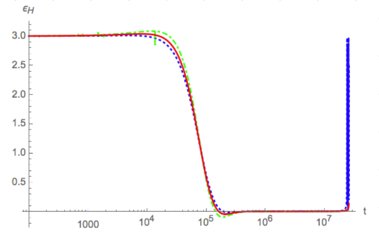




3. Universality of the Background Evolution (Cont.)

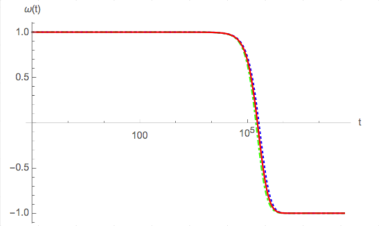
- Monodromy Potential $V(\phi) = V_0 \left| \frac{\phi}{m_{pl}} \right|^p$:

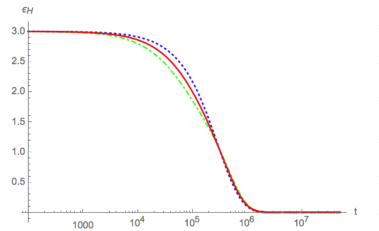




3. Universality of the Background Evolution (Cont.)

- Higgs Potential $V(\phi) = V_0\left(\phi^2 - \phi_0^2\right)^2$:



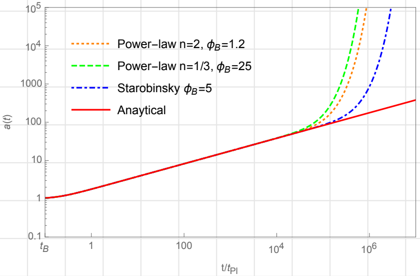


3. Universality of the Background Evolution (Cont.)

- During the bouncing phase, the evolution of $a(t)$ is independent of :

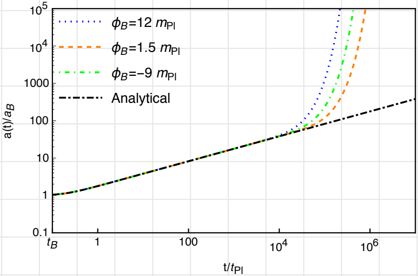
(a) the initial conditions ϕ_B; (b) the inflationary potential; and (c) is given analytically by

$$a(t) = a_B\left(1+\gamma_B\frac{t^2}{t_{Pl}^2}\right)^{1/6}, \quad \gamma_B \equiv 24\pi\rho_B/m_{Pl}^4$$

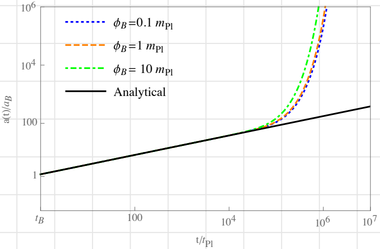


3. Universality of the Background Evolution (Cont.)

- Evolution of $a(t)$ for different potentials:

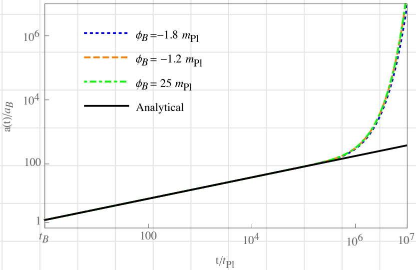


$(V = V_0\phi^2)$

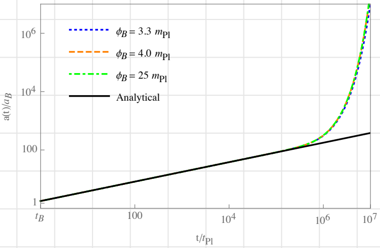


$(V = V_0\phi^{1/2})$

- Evolution of $a(t)$ for the Starobinsky Potential:



$(\dot{\phi}_B > 0)$



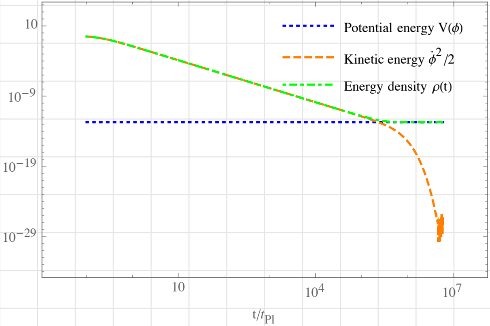
$(\dot{\phi}_B < 0)$

3. Universality of the Background Evolution (Cont.)

- The main reason is that

$$\frac{1}{2}\dot{\phi}_{\rm B}^2 \gg V(\phi_{\rm B}) \quad \Rightarrow \quad \frac{1}{2}\dot{\phi}^2 \gg V(\phi),$$

holds in the whole bouncing phase, once it holds at the bounce $t = t_{\rm B}$.



3. Universality of the Background Evolution (Cont.)

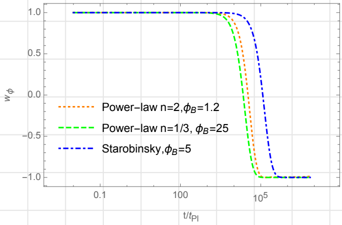
- The evolution during the transition phase is given by,

$$\phi(t) = \phi_c + t_c \dot{\phi}_c \ln \frac{t}{t_c}, \qquad a(t) = a_c \left(1 + t_c H_c \ln \frac{t}{t_c}\right),$$

H_c, a_c, ϕ_c: integration constants

- During the slow-roll inflation, we have

$$a(t) = a_i e^{H_{\text{inf.}} t}, \qquad \phi \simeq \phi_0 \tag{6}$$



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4. Universality of Linear Perturbations

- The scalar and tensor perturbations are given by [9],

$$\mu_k'' + \left(k^2 - \frac{a''}{a} + U(\eta)\right)\mu_k = 0 \tag{7}$$

where $a' \equiv da/d\eta$, $d\eta = dt/a(t)$, and

$$U(\eta) = \begin{cases} a^2\left(\mathfrak{f}^2 V(\phi) + 2\mathfrak{f} V_{,\phi}(\phi) + V_{,\phi\phi}(\phi)\right), & \text{scalar} \\ 0, & \text{tensor} \end{cases}$$

$$\mathfrak{f} \equiv \sqrt{24\pi G}\dot{\phi}/\sqrt{\rho}.$$

[9] A. Ashtekar and A. Barrau, CQG32 (2015) 234001

4. Universality of Linear Perturbations (Cont.)

- Both of the scalar and tensor perturbations are universal and independent of the slow-roll inflationary models during the bouncing phase
- This is because the potential $U(\eta)$ is very small in comparing with a''/a, so we have

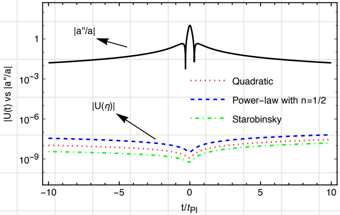
$$\Omega_k^2 = k^2 - \frac{a''}{a} + U(\eta) \simeq k^2 - \frac{a''}{a}$$

during the whole bouncing phase.

- Since $a(t)$ is universal during this phase, clearly the mode functions $\mu_k^{(s,t)}$,

$$\mu_k^{(s,t)''} + \Omega_k^2 \mu_k^{(s,t)} = 0$$

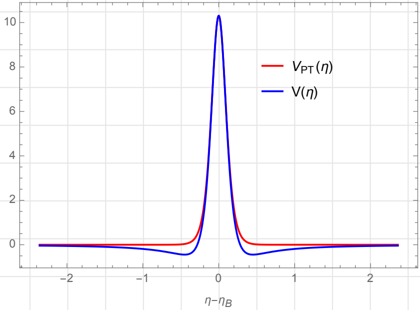
are also universal.



4. Universality of Linear Perturbations (Cont.)

- More interestingly, the term a''/a can be replaced by a Pöschl-Teller (PT) potential,

$$\mathrm{V}_{\mathsf{PT}}(\eta) = \frac{\mathscr{V}_0}{\cosh^2 \alpha(\eta - \eta_{\mathsf{B}})}, \quad \mathscr{V}_0 = \mathrm{k}_{\mathrm{B}}^2 = \frac{\alpha^2}{6}, \quad \mathrm{V}(\eta) \equiv \frac{\mathrm{a}''}{\mathrm{a}}$$



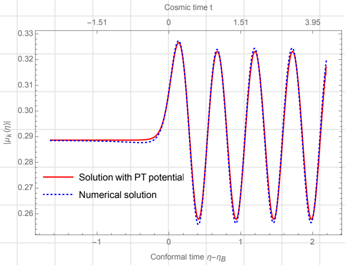
4. Universality of Linear Perturbations (Cont.)

- Then, the mode function has the analytical solution,

$$
\begin{aligned}
\mu_k^{(\mathrm{PT})}(\eta) \quad = \quad & a_k x^{ik/(2\alpha)} (1-x)^{-ik/(2\alpha)} \\
& \times {}_2F_1(a_1 - a_3 + 1, a_2 - a_3 + 1, 2 - a_3, x) \\
& + b_k [x(1-x)]^{-ik/(2\alpha)} \; {}_2F_1(a_1, a_2, a_3, x).
\end{aligned}
$$

a_k, b_k: integration constants, to be determined by initial conditions. ${}_2F_1(a, b, c, x)$: the hypergeometric function

$$
\begin{aligned}
a_1 &\equiv \frac{1}{2}\left(1 + \frac{1}{\sqrt{3}}\right) - \frac{ik}{\sqrt{6}\, k_B}, \\
a_2 &\equiv \frac{1}{2}\left(1 - \frac{1}{\sqrt{3}}\right) - \frac{ik}{\sqrt{6}\, k_B}, \\
a_3 &\equiv 1 - \frac{ik}{\sqrt{6}\, k_B}.
\end{aligned}
$$



4. Universality of Linear Perturbations (Cont.)

- In the transition phase, the mode functions are given by,

$$\mu_k(\eta) = \frac{1}{\sqrt{2k}}\left(\tilde{\alpha}_k e^{-ik\eta} + \tilde{\beta}_k e^{ik\eta}\right)$$

$\tilde{\alpha}_k, \tilde{\beta}_k$: integration constants

- In the slow-roll inflation phase, the mode functions are given by the standard forms,

$$\mu_k^{(s,t)}(\eta) \simeq \frac{\sqrt{-\pi\eta}}{2}\left[\alpha_k H^{(1)}_{\nu_{s,t}}(-k\eta) + \beta_k H^{(2)}_{\nu_{s,t}}(-k\eta)\right],$$

α_k, β_k: integration constants.

- Three sets of integration constants:

1) Bouncing: (a_k, b_k)

2) Transition: $\left(\tilde{\alpha}_k, \tilde{\beta}_k\right)$

3) Slow-roll Inflation: (α_k, β_k)

4. Universality of Linear Perturbations (Cont.)

- Matching them together, we find that the Bogoliubov coefficients, $\alpha_k,\ \beta_k$, are given by

$$\begin{aligned}\frac{\alpha_k}{\sqrt{2k}} &= \left[a_k\frac{\Gamma(2-a_3)\Gamma(a_1+a_2-a_3)}{\Gamma(a_1-a_3+1)\Gamma(a_2-a_3+1)}\right.\\ &\qquad\left.+\, b_k\frac{\Gamma(a_3)\Gamma(a_1+a_2-a_3)}{\Gamma(a_1)\Gamma(a_2)}\right]e^{ik\eta_B},\\ \frac{\beta_k}{\sqrt{2k}} &= \left[a_k\frac{\Gamma(2-a_3)\Gamma(a_3-a_1-a_2)}{\Gamma(1-a_1)\Gamma(1-a_2)}\right.\\ &\qquad\left.+\, b_k\frac{\Gamma(a_3)\Gamma(a_3-a_1-a_2)}{\Gamma(a_3-a_1)\Gamma(a_3-a_2)}\right]e^{-ik\eta_B}.\end{aligned}$$

- Since $a_i = a_i(k/k_B)$, so $\alpha_k,\ \beta_k$ are in general k-dependent.

4. Universality of Linear Perturbations (Cont.)

- In general $|\beta_k|^2 \neq 0$, so particles are *generically* created at the onset of inflation.
- In GR, we normally impose the BD vacuum at the onset of the inflation,

$$\alpha_k^{GR} = 1, \quad \beta_k^{GR} = 0$$

4. Universality of Linear Perturbations (Cont.)

- Then, the scalar and tensor power spectra are given by,

$$\mathcal{P}_{\mathcal{R}}(\mathrm{k}) = |\alpha_{\mathrm{k}} + \beta_{\mathrm{k}}|^2 \mathcal{P}_{\mathcal{R}}^{\mathsf{GR}}(\mathrm{k}),$$
$$\mathcal{P}_{\mathrm{h}}(\mathrm{k}) = |\alpha_{\mathrm{k}} + \beta_{\mathrm{k}}|^2 \, \mathcal{P}_{\mathrm{h}}^{\mathsf{GR}}(\mathrm{k}),$$

with

$$\mathcal{P}_{\mathcal{R}}^{\mathsf{GR}}(\mathrm{k}) \equiv \frac{\mathrm{k}^2}{4\pi^3}\left(\frac{\mathrm{H}}{\mathrm{a}\dot{\phi}}\right)^2 \Gamma^2(\nu_{\mathrm{s}}) \left(\frac{-\mathrm{k}\eta}{2}\right)^{1-2\nu_{\mathrm{s}}},$$
$$\mathcal{P}_{\mathrm{h}}^{\mathsf{GR}}(\mathrm{k}) \equiv \frac{\mathrm{k}^2}{\pi^3 \mathrm{M}_{\mathsf{Pl}}^2} \frac{1}{\mathrm{a}^2} \Gamma^2(\nu_{\mathrm{t}}) \left(\frac{-\mathrm{k}\eta}{2}\right)^{1-2\nu_{\mathrm{t}}}$$

4. Universality of Linear Perturbations (Cont.)

- Note that, as mentioned above, α_k, β_k are usually k-dependent, so the quantities $\mathcal{P}_{\mathcal{R}}(k)$ and $\mathcal{P}_h(k)$ now also become k-dependent.

- This provides an excellent opportunity to test LQC.

- Clearly, such dependence cannot be strong. Otherwise, it will not be consistent with current observations, which show that the power spectra are almost scale-invariant [10].

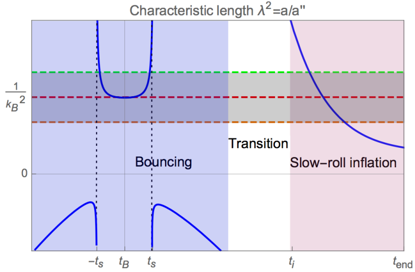
- To fix (α_k, β_k) or (a_k, b_k), one needs to impose the initial conditions, which is still a challenging question in LQC.

[10] P. Collaboration et al., Planck 2015. XX. Constraints on inflation, arXiv:1502.02114.

4. Universality of Linear Perturbations (Cont.)

- In the framework of LQC, various sets of initial conditions have been investigated. However, this is a subtle issue, because in general there is not a preferred initial state for a quantum field in arbitrarily curved space-times.
- If the universe is sufficiently spatially flat and evolves sufficiently slowly so that the characteristic scale for a perturbation mode is much larger than its wavelength, there is an approximate definition of the initial state: the Bunch-Davies vacuum.

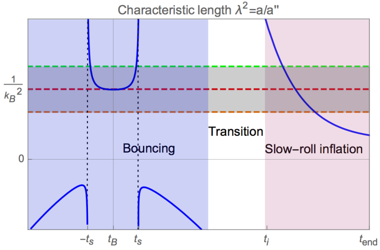
$$a''(t_s) = 0, \ \lambda^2 = 2L_H^2.$$



4. Universality of Linear Perturbations (Cont.)

- However, in the pre-inflationary phases, especially near the bounce, the wavelengths could be larger, equal, or smaller than the corresponding characteristic scale. Thus, it is in general impossible to assume that the universe is in the Bunch-Davies vacuum at the bounce.
- Recently, we considered two different kinds of initial conditions [Zhu et al, PRD96, 083520 (2017)]:
  - The fourth-order adiabatic vacuum right at the bounce
  - The BD vacuum in contracting phase
- Surprisingly, both of them lead to the same results:

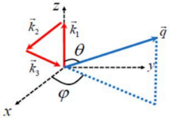
$$a_k = 0, \quad b_k = \frac{e^{ik\eta_B}}{\sqrt{2k}}$$



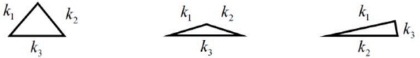
4. Universality of Linear Perturbations (Cont.)

- Recently, we also studied the non-Gaussianity and found that it is consistent with current observations [11].

- But, the non-Gaussianity in the squeezed limit can be enhanced at superhorizon scales, which can yield a large statistical anisotropy on the power spectrum.



$$\boldsymbol{k}_1 = k\hat{z}, \quad \boldsymbol{k}_2 = k\frac{\sqrt{4\alpha^2-1}}{2\alpha^2}\hat{x} + k\frac{(1-2\alpha^2)}{2\alpha^2}\hat{z}, \quad \boldsymbol{k}_3 = -\boldsymbol{k}_1 - \boldsymbol{k}_2$$



The Equilateral Limit $k_1 = k_2 = k_3 : \alpha = 1$ | The Folded Limit $k_1 = k_2 = 2k_3 : \alpha = 1/2$ | The Squeezed Limit $k_1 \simeq k_2 \gg k_3 : \alpha \gg 1$

[11] T. Zhu, AW, K. Kirsten, G. Cleaver, Q. Sheng, PRD97 (2018) 043501 [arXiv:1709.07479].

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5. Cosmology of Loop Quantum Gravity

- Loop quantum gravity (LQG):
  A background independent, nonperturbative quantization of GR by using the Ashtekar variables [12].

- Loop quantum cosmology (LQC):
  Symmetry reduced quantization of cosmology by mimicking the constructions used in LQG [13].

- LQC has not yet been rigorously derived from LQG, but an attempt to use LQG-like methods in cosmology.

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[12] C. Rovelli and F. Vidotto, Covariant Loop Quantum Gravity: An Elementary Introduction to Quantum Gravity and Spinfoam Theory (Cambridge Monographs on Mathematical Physics, Cambridge, 2015).

[13] M. Bojowald, Rep. Prog. Phys. 78 (2015) 023901;
I. Agullo and P. Singh, arXiv:1612.01236.

5. Cosmology of Loop Quantum Gravity (Cont.)

- In LQG, the elementary classical phase space variables are the SU(2) Ashtekar-Barbero connection A_a^i and the conjugate triad E_i^a.
- In the spatially flat FLRW universe, the only relevant constraint is the gravitational Hamiltonian constraint, which is a sum of the Euclidean and Lorentz terms,

$$\begin{aligned}
\mathcal{H}_{\mathrm{grav}} &= \mathcal{H}_{\mathrm{grav}}^{(E)} - (1+\gamma^2)\mathcal{H}_{\mathrm{grav}}^{(L)} \\
\mathcal{H}_{\mathrm{grav}}^{(E)} &= \frac{1}{16\pi G}\int d^3x\, \epsilon_{ijk} F_{ab}^i \frac{E^{aj}E^{bk}}{|\det(q)|} \\
\mathcal{H}_{\mathrm{grav}}^{(L)} &= \frac{1}{8\pi G}\int d^3x\, K_{[a}^j K_{b]}^k \frac{E^{aj}E^{bk}}{|\det(q)|}
\end{aligned}$$

F_{ab}: the field strength of connection A_a^i

K_a^i: the extrinsic curvature

q_{ab}: the spatial metric.

5. Cosmology of Loop Quantum Gravity (Cont.)

- For spatially flat FLRW universe, we have

$$\mathcal{H}^{(\mathrm{E})}_{\mathrm{grav}} = 2\gamma^2 \mathcal{H}^{(\mathrm{L})}_{\mathrm{grav}}$$

$$\mathcal{H}_{\mathrm{grav}} = \mathcal{H}^{(\mathrm{E})}_{\mathrm{grav}} - (1+\gamma^2)\mathcal{H}^{(\mathrm{L})}_{\mathrm{grav}} = -\frac{1}{\gamma^2}\mathcal{H}^{(\mathrm{E})}_{\mathrm{grav}}$$

In LQC, using the above relation, instead of quantizing the Euclidean and Lorentz terms separately, only the Euclidean term $\mathcal{H}^{(\mathrm{E})}_{\mathrm{grav}}$ is quantized.

- However, in LQG, these two terms are usually regularized differently.

5. Cosmology of Loop Quantum Gravity (Cont.)

- In particular, if one follows the non-graph-changing regularization [14], one finds [15]

$$\mathcal{H}_{\mathrm{LQG-I}} = \frac{3\mathrm{v}}{8\pi G\lambda^2}\left\{\sin^2(\lambda b) - \frac{(\gamma^2+1)\sin^2(2\lambda b)}{4\gamma^2}\right\} + \mathcal{H}_M$$

- Then, the Hamilton's equations for the variables v and b,

$$\begin{aligned}\dot{\mathrm{v}} &= \left\{\mathrm{v}, \mathcal{H}\right\} = \frac{3\mathrm{v}\sin(2\lambda b)}{2\gamma\lambda}\left\{(\gamma^2+1)\cos(2\lambda b) - \gamma^2\right\},\\ \dot{b} &= \left\{b, \mathcal{H}\right\} = \frac{3\sin^2(\lambda b)}{2\gamma\lambda^2}\left\{\gamma^2\sin^2(\lambda b) - \cos^2(\lambda b)\right\}\\ &\quad - 4\pi G\gamma P \end{aligned} \tag{8}$$

v: the volume; b: momentum; P: pressure

[14] T. Thiemann, CQG24 (1998) 839; 875.

[15] J. Yang, Y. Ding, Y. Ma, PLB682 (2009) 1; A. Dapor and K. Liegener, arXiv:1706.09833.

5. Cosmology of Loop Quantum Gravity (Cont.)

- On the other hand, due to the spatial homogeneity and isotropy, one can also set the spin connection to zero, and the resulted Hamiltonian takes the form [16],

$$\mathcal{H}_{\mathrm{LQG-II}} = -\frac{3\mathrm{v}}{2\pi \mathrm{G}\lambda^2\gamma^2}\sin^2\left(\frac{\lambda \mathrm{b}}{2}\right)\left\{1+\gamma^2\sin^2\left(\frac{\lambda \mathrm{b}}{2}\right)\right\} + \mathcal{H}_{\mathrm{M}}$$

- Then, the Hamilton's equations for the variables v and b,

$$\begin{aligned}\dot{\mathrm{v}} &= \left\{\mathrm{v}, \mathcal{H}\right\} = \frac{3\mathrm{v}\sin(\lambda \mathrm{b})}{\gamma\lambda}\left\{1+\gamma^2-\gamma^2\cos\left(\lambda \mathrm{b}\right)\right\},\\ \dot{\mathrm{b}} &= \left\{\mathrm{b}, \mathcal{H}\right\} = -\frac{6\sin^2\left(\frac{\lambda \mathrm{b}}{2}\right)}{\gamma\lambda^2}\left\{1+\gamma^2\sin^2\left(\frac{\lambda \mathrm{b}}{2}\right)\right\}\\ &\quad - 4\pi \mathrm{G}\gamma \mathrm{P}\end{aligned}$$

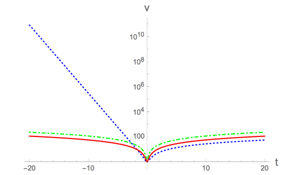
[16] J. Yang, Y. Ding, Y. Ma, PLB682 (2009) 1.

5. Cosmology of Loop Quantum Gravity (Cont.)

- Therefore, due the quantization ambiguities, so far we have three different models:

LQC LQG-I LQG-II

- However, in all three models, we find the following [17]:
  - the replacement of the big bang singularity by a quantum bounce is a robust feature against the quantization ambiguities.
    - LQC (red solid curve)
    - LQG-I (blue dotted curve)
    - LQG-II (green dot-dashed curve)

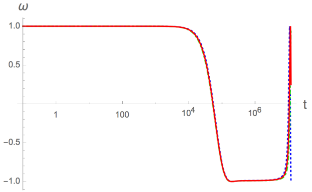


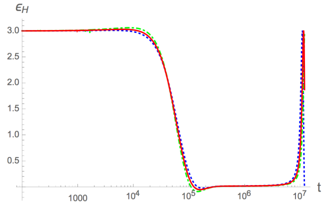
[17] B. F. Li, P. Singh, AW, PRD97 (2018) 084029; Qualitative dynamics in pre-inflationary universe from loop quantum gravity, arXiv: 1806.xxxxx

5. Cosmology of Loop Quantum Gravity (Cont.)

- In each of the three cosmological models, we find universal properties of the background evolution for the kinetic dominated bounce, irrespective of the nature of the inflationary potentials.
- In the post-bounce stage but before the Universe enters the reheating phase, three distinctive phases are identified,

bouncing, transition, slow-roll inflation





5. Cosmology of Loop Quantum Gravity (Cont.)

- The evolution of the expansion factor of the universe in the bouncing phase is independent of the inflationary potentials and initial conditions, given explicitly by,

$$a(t) = a_B \left[1 + \left(\frac{t}{t_0^A}\right)^2\right]^{1/6}, \quad t_0^A \equiv \left(24\pi G \rho_c^A\right)^{-1/2}$$

ρ_c^A: the critical energy density
$(A = 0, I, II)$

- Slow-roll inflation is a generic outcome in all these three models.

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6. Conclusions

- We study pre-inflationary dynamics in the frameworks of LQC and LQG and we find:

  - The replacement of the big bang singularity by a quantum bounce is a robust feature against the quantization ambiguities

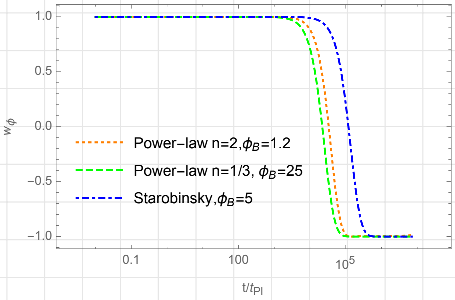
  - The slow-roll inflation is generic

6. Conclusions (Cont.)

- For initially kinetic energy dominated models, we find:

  - The evolution of the universe is always divided into three different phases:

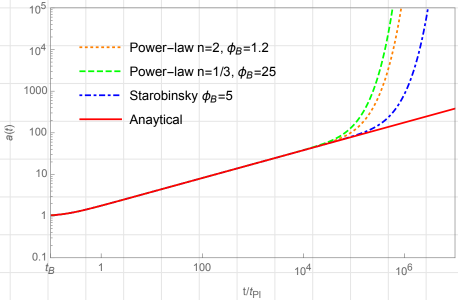
(1) Bouncing (2) transition (3) slow-roll inflation



6. Conclusions (Cont.)

- The evolution of the expansion factor is universal during the bouncing phase:

$$a(t) = a_B \left[1 + \left(\frac{t}{t_0^A}\right)^2\right]^{1/6}, \quad t_0^A \equiv \left(24\pi G \rho_c^A\right)^{-1/2}$$



6. Conclusions (Cont.)

- During the pre-inflationary phase, the evolutions of the scalar and tensor perturbations are all universal and independent of the slow-roll inflationary models.
- In this phase the potentials of the scalar and tensor perturbations can be well approximated by an effective PT potential, for which analytic solutions of the mode functions are known.

- The Bogoliubov coefficients at the onset of the slow-roll inflation are generically non-zero,

$$\beta_k \neq 0,$$

in contrast to GR where the initial conditions are normally taken as the BD vacuum,

$$\beta_k^{GR} = 0.$$

6. Conclusions (Cont.)

- The non-Gaussianity is consistent with current observations, but in the squeezed limit it can be enhanced at superhorizon scales, which can yield a large statistical anisotropy on the power spectrum.

Thank You!