Scale Invariance in Newton-Cartan and Horava-Lifshitz Gravity

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Outline

- Motivation, uses of Newton-Cartan (NC) structures
  —holography
  —field theory
  —gravity

- Basic structures in NC and Torsional Newton-Cartan (TNC) from the
  —Geometric view
  —Gauging of the algebra
  —Large c-expansion

- Making TNC dynamical
  —Scale invariant Horava-Lifshitz gravity (HLG)
  —Schrodinger invariant HLG

- Outlook

- NC introduced in problem of FQH [Son 13]
- TNC first observed as boundary geometry in z=2 Lifshitz holography [Christensen,Hartong,Rollier,Obers 13, Hartong,Kiritsis,Ober (1409)]

- TTNC introduced in FQH [Geracie,Son,Wu,Wu 14]
- TNC from gauging Schrödinger algebra [Bergshoeff,Hartong,Rosseel 14]
- TNC from gauging Bargmann (with torsion) [Hartong,Obers 15]
  - coupling of non-relativistic field theories to TNC [Jensen 14]
  (independent of holography) [Hartong,Kiritsis, Obers 14]

- TNC related and 2D WCFT [Hofmann,Rollier 14]
- other approaches (c-> inf limit, affine spaces)

[Banerjee,Mitra,Mukherjee 14, Bekaert,Morand 14, Van den Bleeken 17]

NC Geometry

- Newton-Cartan gravity, is originally developed as the generally covariant description of the Newtonian gravity [E.Cartan 1923,1924]

$$\nabla^2\phi = 4\pi G\rho\,, \qquad \frac{d^2x^a(t)}{dt^2} + \frac{\partial\phi(x)}{dx^a} = 0\,,$$

- Newton-Cartan geometry is described by a degenerate spatial metric $h^{\mu\nu}$ of rank-d and a temporal vielbein τ_μ of rank-1, together with a $\Gamma^\mu_{\nu\rho}$ connection on an orientable manifold M

$$h^{\mu\nu}\tau_\nu = 0\,.$$

Torsionless case i.e. $\Gamma^\mu_{\nu\rho} = \Gamma^\mu_{\rho\nu}$

$$\begin{aligned}\nabla_\mu\tau_\nu = \partial_\mu\tau_\nu - \Gamma^\rho_{\mu\nu}\tau_\rho &= 0\,,\\ \nabla_\mu h^{\nu\rho} = \partial_\mu h^{\nu\rho} + \Gamma^\nu_{\sigma\mu}h^{\sigma\rho} + \Gamma^\rho_{\sigma\mu}h^{\sigma\nu} &= 0\,,\end{aligned} \longrightarrow \begin{aligned}\partial_{[\mu}\tau_{\nu]} &= 0\\ \tau_\mu &= \partial_\mu f\end{aligned}$$

Given any function t the temporal length of a curve $\gamma : [s_1, s_2] \to \mathcal{M}$

$$\int_{s_1}^{s_2} t_\mu \xi^\mu ds = \int_{s_1}^{s_2} \nabla_\mu t \, \xi^\mu ds$$
$$= \int_{s_1}^{s_2} \frac{d}{ds} [t(\gamma(s))] \, ds = t(\gamma(s_2)) - t(\gamma(s_1)),$$

$$\nabla_\mu \tau_\nu = \partial_\mu \tau_\nu - \Gamma^\rho_{\mu\nu} \tau_\rho \quad = \quad 0 \, , \longrightarrow \quad \tau_\rho \Gamma^\rho_{\mu\nu} = \partial_\mu \tau_\nu \, .$$ Temporal part

For the spatial part introduce the following spatial inverse metric $h_{\mu\nu}$ and temporal inverse vielbein τ^μ

$$h^{\mu\sigma} h_{\nu\sigma} = P^\mu_\nu = \delta^\mu_\nu - \tau^\mu \tau_\nu \, , \qquad \tau^\mu \tau_\mu = 1 \, , \qquad h^{\mu\nu} \tau_\nu = 0 \, , \qquad h_{\mu\nu} \tau^\nu = 0 \, .$$

Solution to the compatibility conditions then

$$\Gamma^\rho_{\mu\nu} = \tau^\rho \partial_\mu \tau_\nu + \tfrac{1}{2} h^{\rho\sigma} \Big(\partial_\nu h_{\sigma\mu} + \partial_\mu h_{\sigma\nu} - \partial_\sigma h_{\mu\nu} \Big) - h^{\rho\sigma} \tau_{(\mu} F_{\nu)\sigma} \, ,$$

Impose Ehlers and Trautman conditions for Newtonian gravity

$$h^{\sigma[\lambda}R^{\mu]}{}_{(\nu\rho)\sigma}(\Gamma) = 0\,,$$

$$h^{\rho\lambda}R^{\mu}{}_{\nu\rho\sigma}R^{\nu}{}_{\mu\lambda\alpha}(\Gamma) = 0 \quad \text{or} \quad \tau_{[\lambda}R^{\mu}{}_{\nu]\rho\sigma}(\Gamma) = 0 \quad \text{or} \quad h^{\sigma[\lambda}R^{\mu]}{}_{\nu\rho\sigma}(\Gamma) = 0\,,$$

$$\Gamma^{a}_{00} = \delta^{ab}\partial_b\phi\,, \qquad R^{a}{}_{0a0}(\Gamma) = \nabla^2\phi = 4\pi G\rho\,, \qquad F_{\mu\nu} = 2\partial_{[\mu}m_{\nu]}\,,$$

[J. Ehlers 81, A. Trautman 63, G. Dautcourt 93, R. Andringa, E. Bergshoeff, S. Panda and M. de Roo 11]

There is a caveat here ! The inverse metrics we introduce τ^{μ}, $h_{\mu\nu}$ are not unique i.e. given 1-form ψ_μ [K. Jensen, 14]

$$\tau'^{\mu} = \tau^{\mu} + h^{\mu\nu}\psi_\nu\,, \qquad h'_{\mu\nu} = h_{\mu\nu} - (\tau_\mu P^{\rho}_{\nu} + \tau_\nu P^{\rho}_{\mu})\psi_\rho + \tau_\mu\tau_\nu h^{\rho\sigma}\psi_\rho\psi_\sigma\,,$$

Moreover,

$$m'_\mu = m_\mu - P^{\nu}_{\mu}\psi_\nu + \tfrac{1}{2}\tau_\mu h^{\nu\rho}\psi_\nu\psi_\rho\,,$$

Twistless torsional NC (TTNC) geometry

$$(h^{\mu\nu}\,,\tau_\mu\,,b_\mu\,,M_\mu)\,,$$

From the compatibility condition temporal part is fixed

$$\tau_\rho \Gamma^\rho_{[\mu\nu]} = \partial_{[\mu}\tau_{\nu]}\,.$$

Impose twistless condition

$$\tau_\lambda \tau_{[\rho}\Gamma^\lambda_{\mu\nu]} = \tau_{[\rho}\partial_\mu \tau_{\nu]} = 0\,,$$

By Frobenius theorem

$$\partial_{[\mu}\tau_{\nu]} = z b_{[\mu}\tau_{\nu]}\,,$$

Solution to the compatibility conditions is

spatial contortion

$$\Gamma^\rho_{\mu\nu} \;=\; \tau^\rho \partial_\mu \tau_\nu + \tfrac{1}{2} h^{\rho\sigma}\Big(\partial_\nu h_{\sigma\mu} + \partial_\mu h_{\sigma\nu} - \partial_\sigma h_{\mu\nu}\Big) - h^{\rho\sigma}\tau_{(\mu}F_{\nu)\sigma} - K_{\mu\nu}{}^\rho\,,$$

Transformation under Milne boost

$$\delta_M \Gamma^\rho_{\mu\nu} \;=\; h^{\rho\sigma}\Big\{(P^\alpha_\sigma \partial_{[\mu}\tau_{\nu]} + P^\alpha_\mu \partial_{[\sigma}\tau_{\nu]} + P^\alpha_\nu \partial_{[\sigma}\tau_{\mu]})\psi_\alpha + \tfrac{1}{2} h^{\alpha\beta}\psi_\alpha\psi_\beta(\tau_\nu \partial_{[\mu}\tau_{\sigma]} + \tau_\mu \partial_{[\nu}\tau_{\sigma]})\Big\} - \delta_M K_{\mu\nu}{}^\rho . \quad (2$$

- If $\delta_M K_{\mu\nu}{}^{\rho} = 0$, then we have to achieve Milne invariance by considering combinations of $\tau^{\mu}, h_{\mu\nu}$ and m_{μ}. However m_{μ} is a U(1) connection therefore such combinations will fail U(1) invariance. So we will also introduce a scalar field χ such that,

$$\delta_{\mathrm{U}(1)}\chi = \sigma \qquad\qquad M_{\mu} = m_{\mu} - \partial_{\mu}\chi\,,$$

And the following definitions will still satisfy orthogonality conditions and they are Milne+U(1) invariant

$$\hat{\tau}^{\mu} \;=\; \tau^{\mu} + h^{\mu\nu}M_{\nu}\,, \qquad \hat{h}_{\mu\nu} = h_{\mu\nu} - \tau_{\mu}M_{\nu} - \tau_{\nu}M_{\mu} + 2\tau_{\mu}\tau_{\nu}\Phi\,,$$

$$\Phi = \tau^{\sigma}M_{\sigma} + \tfrac{1}{2}h^{\rho\sigma}M_{\rho}M_{\sigma}\,.$$

Therefore, the connection for TTNC can be written as

$$\boxed{\widehat{\Gamma}^{\rho}_{\mu\nu} \;=\; \hat{\tau}^{\rho}\partial_{\mu}\tau_{\nu} + \tfrac{1}{2}h^{\rho\sigma}\Big(\partial_{\nu}\hat{h}_{\sigma\mu} + \partial_{\mu}\hat{h}_{\sigma\nu} - \partial_{\sigma}\hat{h}_{\mu\nu}\Big) + h^{\rho\sigma}\tau_{\mu}\tau_{\nu}\partial_{\sigma}\Phi - K_{\mu\nu}{}^{\rho}\,,}$$

Non-Relativistic scale symmetry and NC

- The defining property of the scale symmetry is via breaking of the compatibility condition by a particular non-metricity tensor

$$\nabla_\mu \tau_\nu = z b_\mu \tau_\nu \,, \qquad \nabla_\mu h^{\nu\rho} = -2 b_\mu h^{\nu\rho} \,,$$

which is preserved by the following transformations

$$\tau_\mu \to e^{z\Lambda_D(x)} \tau_\mu \,, \qquad h^{\mu\nu} \to e^{-2\Lambda_D(x)} h^{\mu\nu} \,, \qquad b_\mu \to b_\mu + \partial_\mu \Lambda_D(x) \,, \qquad \Gamma^\rho_{\mu\nu} \to \Gamma^\rho_{\mu\nu} \,, \qquad \delta_M b_\mu = 0 \,.$$

- Unlike the relativistic scenarios, the inclusion of the non-metricity modifies the anti-symmetric part of the connection

impose tt condition

$$\tau_\rho \Gamma^\rho_{[\mu\nu]} = \partial_{[\mu} \tau_{\nu]} - z b_{[\mu} \tau_{\nu]} \,. \longrightarrow \partial_{[\mu} \tau_{\nu]} = z b_{[\mu} \tau_{\nu]} \,,$$

- Again, one can introduce Milne invariant hatted quantities

$$\hat{\tau}^\mu \to e^{-z\Lambda_D(x)} \hat{\tau}^\mu \,, \quad \hat{h}_{\mu\nu} \to e^{2\Lambda_D(x)} \hat{h}_{\mu\nu} \,, \quad M_\mu \to e^{-(z-2)\Lambda_D(x)} M_\mu \,.$$

$$M_\mu = m_\mu - \partial_\mu \chi - (z-2) b_\mu \chi \,,$$

$$\hat{\Gamma}^\rho_{\mu\nu} \;=\; \hat{\tau}^\rho \mathcal{D}_\mu \tau_\nu + \tfrac{1}{2} h^{\rho\sigma} \left(\mathcal{D}_\nu \hat{h}_{\sigma\mu} + \mathcal{D}_\mu \hat{h}_{\sigma\nu} - \mathcal{D}_\sigma \hat{h}_{\mu\nu} \right) + h^{\rho\sigma} \tau_\mu \tau_\nu \mathcal{D}_\sigma \Phi - K_{\mu\nu}{}^\rho \,,$$

Non-relativistic algebras (not all)

$H, P_a, J_{ab}, G_a \quad N$

Bargmann

Galilean

$$[H, G_a] = P_a \qquad [P_a, G_b] = 0$$

$$[P_a, G_b] = N\delta_{ab}$$

$H, P_a, J_{ab}, D, \quad G_a, N, K(z=2)$

Lifshitz

Schrodinger

$$[D, H] = zH \qquad [D, P_a] = P_a$$

$$[D, N] = (2-z)N$$

$z \neq 2$ Scale Invariant Hořava-Lifshitz Gravity

Elements of the scale extended Bargmann algebra are

$$[P_a, G_b] = \delta_{ab} N\,, \qquad [D, G_a] = (z-1) G_a\,, \qquad [D, N] = (z-2) N\,,$$
$$[J_{ab}, P_c] = 2\delta_{c[a} P_{b]}\,, \qquad [J_{ab}, G_c] = 2\delta_{c[a} G_{b]}\,, \qquad [J_{ab}, J_{cd}] = 4\delta_{[a[c}\, J_{b]d]}\,.$$

$$[D, P_a] = -P_a\,, \qquad [D, H] = -zH\,, \qquad [H, G_a] = P_a\,,$$

Following the algebra, transformation rules are given as

$$\begin{aligned}
\delta\tau_\mu &= \partial_\mu \xi - z\xi b_\mu + z\Lambda_D \tau_\mu\,,\\
\delta e_\mu{}^a &= \partial_\mu \xi^a - \omega_\mu{}^{ab}\xi_b - b_\mu \xi^a + \lambda^a{}_b e_\mu{}^b + \lambda^a \tau_\mu - \omega_\mu{}^a \xi + \Lambda_D e_\mu{}^a\,,\\
\delta\omega_\mu{}^{ab} &= \partial_\mu \lambda^{ab} + 2\lambda^{c[a}\omega_\mu{}^{b]}{}_c\,,\\
\delta\omega_\mu{}^a &= \partial_\mu \lambda^a - \omega_\mu{}^{ab}\lambda_b + \lambda^a{}_b\, \omega_\mu{}^b + (z-1)\lambda^a b_\mu - (z-1)\Lambda_D \omega_\mu{}^a\,,\\
\delta m_\mu &= \partial_\mu \sigma - \xi^a \omega_{\mu a} + \lambda^a e_{\mu a} + (z-2)\sigma b_\mu - (z-2)\Lambda_D m_\mu\,,\\
\delta b_\mu &= \partial_\mu \Lambda_D\,,
\end{aligned}$$

Also the curvatures follows easily

$$
\begin{aligned}
R_{\mu\nu}(H) &= 2\partial_{[\mu}\tau_{\nu]} - 2zb_{[\mu}\tau_{\nu]}\,, \\
R_{\mu\nu}{}^{a}(P) &= 2\partial_{[\mu}e_{\nu]}{}^{a} - 2\omega_{[\mu}{}^{ab}e_{\nu]b} - 2\omega_{[\mu}{}^{a}\tau_{\nu]} - 2b_{[\mu}e_{\nu]}{}^{a}\,, \\
R_{\mu\nu}{}^{ab}(J) &= 2\partial_{[\mu}\omega_{\nu]}{}^{ab} - 2\omega_{[\mu}{}^{c[a}\omega_{\nu]}{}^{b]}{}_{c}\,, \\
R_{\mu\nu}{}^{a}(G) &= 2\partial_{[\mu}\omega_{\nu]}{}^{a} + 2\omega_{[\mu}{}^{b}\omega_{\nu]}{}^{a}{}_{b} - 2(z-1)\omega_{[\mu}{}^{a}b_{\nu]}\,, \\
R_{\mu\nu}(D) &= 2\partial_{[\mu}b_{\nu]}\,, \\
R_{\mu\nu}(N) &= 2\partial_{[\mu}m_{\nu]} - 2\omega_{[\mu}{}^{a}e_{\nu]a} + 2(z-2)b_{[\mu}m_{\nu]}\,.
\end{aligned}
$$

In order to make contact with the NC geometry we have discussed, impose the following set of constraints

$$
R_{\mu\nu}(H) = 0\,, \qquad R_{\mu\nu}{}^{a}(P) = 0\,, \qquad R_{\mu\nu}(N) = 0\,, \qquad R_{\mu\nu}(D) = 0\,,
$$

$$
e_{[\mu}{}^{b}R_{\nu\rho]}{}^{a}{}_{b}(J) + \tau_{[\mu}R_{\nu\rho]}{}^{a}(G) = 0\,, \qquad e_{[\mu}{}^{a}R_{\nu\rho]a}(G) = 0\,.
$$

The twistlessness condition can be seen from the curvature constraint

$$
R_{\mu\nu}(H) = 0 \qquad \Rightarrow \qquad \partial_{[\mu}\tau_{\nu]} = zb_{[\mu}\tau_{\nu]}\,,
$$

Also the following composite gauge fields are solved

$$\begin{aligned}\omega_\mu{}^{ab} &= -2e^{\nu[a}\partial_{[\mu}e_{\nu]}{}^{b]} + e^{\nu[a}e^{\ b]\rho}\partial_\nu e_\rho{}^c e_{\mu c} + 2e_\mu{}^{[a}e_\nu{}^{b]}b^\nu \\ &\quad -e^{\nu a}e^{\rho b}\tau_\mu(\partial_{[\nu}m_{\rho]} + (z-2)b_{[\nu}m_{\rho]})\,, \\ \omega_\mu{}^a &= \tau^\nu\partial_{[\mu}e_{\nu]}{}^a + e^{\nu a}\tau^\rho e_{\mu b}\partial_{[\nu}e_{\rho]}{}^b + e_\mu{}^a\tau^\nu b_\nu + e^{\nu a}(\partial_{[\mu}m_{\nu]} + (z-2)b_{[\mu}m_{\nu]}) \\ &\quad +\tau_\mu\tau^\rho e^{\nu a}(\partial_{[\rho}m_{\nu]} + (z-2)b_{[\rho}m_{\nu]})\,.\end{aligned}$$

Scale-extended Bargmann algebra does not allow the existence of a scalar field with a homogeneous dilatation and U(1) transformation due to the commutation relation.

Therefore we will work with the following set of compensating multiplets. Note that the derivative of χ is the very definition of M_μ

$$\delta\phi = \omega\Lambda_D\phi\,, \qquad \delta\chi = \sigma + (2-z)\Lambda_D\chi\,.$$

$$\mathcal{D}_\mu\chi = \partial_\mu\chi - (2-z)b_\mu\chi - m_\mu\,.$$

In order to discuss the construction of kinetic term in HLG let us consider the following

$$\mathcal{D}_\mu M_a = \partial_\mu M_a + (z-1)b_\mu M_a - \omega_\mu{}^{ab}M_b - \omega_{\mu a}\,.$$

$$\delta\mathcal{D}_a M_b = -z\Lambda_D\mathcal{D}_a M_b\,. \qquad \delta_G M_a = \lambda_a\,.$$

$$
\begin{aligned}
S^{(2)}_{z\neq 2} &= \int dt\, d^dx\, e\, \phi\, (\mathcal{D}_a M^a)^2\,, \\
S^{(3)}_{z\neq 2} &= \int dt\, d^dx\, e\, \phi\, (\mathcal{D}_a M_b)^2\,, \\
S^{(4)}_{z\neq 2} &= \int dt\, d^dx\, e\, \phi\, \triangle \mathcal{D}_a M^a\,,
\end{aligned}
$$

$$\delta_D e = (d+z)\Lambda_D\, e\,.$$

Here, for $S^{(2)}_{z\neq 2}$ and $S^{(3)}_{z\neq 2}$, the scaling dimension of ϕ is given by $\omega = z - d$ while for the $S^{(4)}_{z\neq 2}$, the scaling dimension of ϕ is $\omega = 2 - d$.

- $n_t = 1$: When we have a single time derivative acting on ϕ or χ, we first consider the models with no spatial derivatives

$$
\begin{aligned}
S^{(5)}_{z\neq 2} &= \int dt\, d^dx\, e\, \phi(\mathcal{D}_0\phi + M^a \mathcal{D}_a \phi)\,, \\
S^{(6)}_{z\neq 2} &= \int dt\, d^dx\, e\, \phi^2 (M_0 + \tfrac{1}{2} M_a M^a)\,.
\end{aligned}
$$

- $n_t = 2$: When we have two time derivative acting on ϕ or χ, the possible $z \neq 2$ scale-invariant actions are

$$
\begin{aligned}
S^{(13)}_{z\neq 2} &= \int dt\, d^dx\, e\, \phi\, (\mathcal{D}_0^2 \phi + 2M^a \mathcal{D}_0 \mathcal{D}_a \phi + M^a M^b \mathcal{D}_a \mathcal{D}_b \phi)\,, \\
S^{(14)}_{z\neq 2} &= \int dt\, d^dx\, e\, \phi^2\, (\mathcal{D}_0 M_0 + 2M^a \mathcal{D}_0 M_a + M^a M^b \mathcal{D}_a M_b)\,, \\
S^{(15)}_{z\neq 2} &= \int dt\, d^dx\, e\, \phi^2\, (M_0 + \tfrac{1}{2} M_a M^a)^2\,.
\end{aligned}
$$

NC and HLG connection [Hartong, Obers 2015]

Milne invariant Riemannian tensor can be defined as

$$g_{\mu\nu} = \hat{h}_{\mu\nu} + \tau_\mu \tau_\nu \,, \qquad g^{\mu\nu} = h^{\mu\nu} + \hat{\tau}^\mu \hat{\tau}^\nu \,,$$

From this definition, ADM decomposition of the metric and the twistless torsion condition one can show the following

$$\tau_\mu = \psi \partial_\mu \tau \,. \qquad \tau = t$$

$$\tau_t = N \,, \quad \tau_i = 0 \,,$$

$$h^{tt} = h^{ti} = h^{it} = 0 \,, \quad h^{ij} = \gamma^{ij} \,,$$

$$\hat{\tau}^t = N^{-1} \,, \quad \hat{\tau}^i = -N^{-1} N^i \,,$$

$$\hat{h}_{tt} = \gamma_{ij} N^i N^j \,, \quad \hat{h}_{ti} = \hat{h}_{it} = \gamma_{ij} N^j \,, \quad \hat{h}_{ij} = \gamma_{ij} \,,$$

Using these relations U(1) invariant vector field reads

$$M_t = -\frac{1}{2N} \gamma_{ij} N^i N^j + \Phi N \,, \qquad M_i = -\frac{1}{N} \gamma_{ij} N^j \,.$$

The dilatation gauge field can also be written in terms of HLG variables through the TT condition

$$\partial_{[\mu}\tau_{\nu]} = z b_{[\mu}\tau_{\nu]} = z b_a e_{[\mu}{}^a\tau_{\nu]} \,.$$

It is useful to identify a vector

$$a_\mu = \mathcal{L}_{\hat{\tau}}\tau_\mu = \hat{\tau}^\nu(\partial_\nu\tau_\mu - \partial_\mu\tau_\nu) = -z e_\mu{}^a b_a \,,$$

$$a_t = N^i a_i \,, \qquad a_i = -N^{-1}\partial_i N \,.$$

Finally the extrinsic curvature (kinetic term of HLG)

$$K_{ij} \;=\; \frac{1}{2N}\Big(\partial_t\gamma_{ij} - \bar{\nabla}_i N_j - \bar{\nabla}_j N_i\Big) \,,$$

Can be identified from the following tensor

$$K'_{ab} = \mathcal{D}_a M_b = \widetilde{\nabla}_{(a}M_{b)} + z b_{(a}M_{b)} - \delta_{ab} b^c M_c - \delta_{ab} b_0 \,.$$

Gathering up all these pieces the scale extended HLG is

$$S^{\mathrm{HL}}_{z\neq 2} \;=\; S^{(3)}_{z\neq 2} - \lambda\, S^{(2)}_{z\neq 2} + S_{\mathcal{V}} \,,$$

Outlook

- The scale or Schrodinger symmetry corresponds to a special choice of non-metricity in the compatibility equation, and it is possible to have a more general classification of non-relativistic geometries by imposing a more general non-metricity. This classification has been done for the relativistic scenarios.
- The true non-relativistic analogue of the relativistic conformal symmetry, which leaves the action of a massless non-relativistic particle is invariant is called the Galilean conformal algebra.
- Is it possible to get HLG from the large c expansion by [vDB17]
- Generating solutions for HLG from the known relativistic ones.

Large c expansion [VdB17]

We assume an expansion of the metric ($D = d + 1$, Lorentzian) in even powers[2] of a variable c (thought of physically as the speed of light):

$$g_{\mu\nu} = \sum_{i=-1}^{\infty} \overset{(2i)}{g}_{\mu\nu} c^{-2i} \qquad g^{\mu\nu} = \sum_{i=0}^{\infty} \overset{(2i)}{g}{}^{\mu\nu} c^{-2i} \tag{1}$$

We furthermore assume that $\overset{(-2)}{g}_{\mu\nu}$ is of rank 1 and negative, so we can write

$$\overset{(-2)}{g}_{\mu\nu} = -\tau_\mu \tau_\nu \tag{2}$$

The two expansions (1) are of course related by the condition that one series provides the inverse of the other. We can expand this condition $g_{\mu\rho} g^{\rho\nu} = \delta_\mu^\nu$ order by order and solve the resulting equations explicitely. As a first step one obtains from the leading equation (order c^2) the result that

$$\overset{(0)}{g}{}^{\mu\nu} = h^{\mu\nu} \qquad \text{with} \qquad h^{\mu\nu}\tau_\nu = 0 \tag{6}$$

$$\delta_\chi \tau^\mu = -h^{\mu\rho}\chi_\rho \qquad \delta_\chi h_{\mu\nu} = \tau_\mu \chi_\nu + \tau_\nu \chi_\mu$$

$$\begin{aligned}
\hat{\tau}^\mu &= \tau^\mu - h^{\mu\nu}C_\nu \\
\hat{h}_{\mu\nu} &= h_{\mu\nu} + 2\tau_{(\mu}C_{\nu)} + 2\hat{\Phi}\tau_\mu\tau_\nu \\
\hat{\Phi} &= -\tau^\rho C_\rho + \frac{1}{2}h^{\rho\sigma}C_\rho C_\sigma
\end{aligned}$$

$$\begin{aligned}
\hat{\beta}^{\mu\nu} &= \beta^{\mu\nu} + h^{\mu\rho}h^{\mu\sigma}C_\rho C_\sigma \\
\hat{B}_\mu &= B_\mu + h_{\mu\rho}\beta^{\rho\sigma}C_\sigma + \frac{1}{2}\tau_\mu\left(\beta^{\rho\sigma}C_\rho C_\sigma + (\tau^\rho C_\rho - h^{\rho\sigma}C_\rho C_\sigma)^2\right) \\
&\quad -C_\mu(\tau^\rho C_\rho - h^{\rho\sigma}C_\rho C_\sigma) \\
\hat{\gamma}^{\mu\nu} &= \gamma^{\mu\nu} + 2h^{\rho(\mu}h^{\nu)\sigma}C_\rho\left(\hat{B}_\sigma - 2\hat{\Phi}C_\sigma\right)
\end{aligned}$$

For the compatibility condition assume the connection expansion

$$\Gamma^\lambda_{\mu\nu} = \sum_{i=-1}^{\infty} \overset{(2i)}{\Gamma}{}^\lambda_{\mu\nu} c^{-2i}$$

Then the same structure for TTNC is obtained.