The String Worldsheet as One Candidate Bulk Description of SYK Model

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Based on: arXiv:1709.06297
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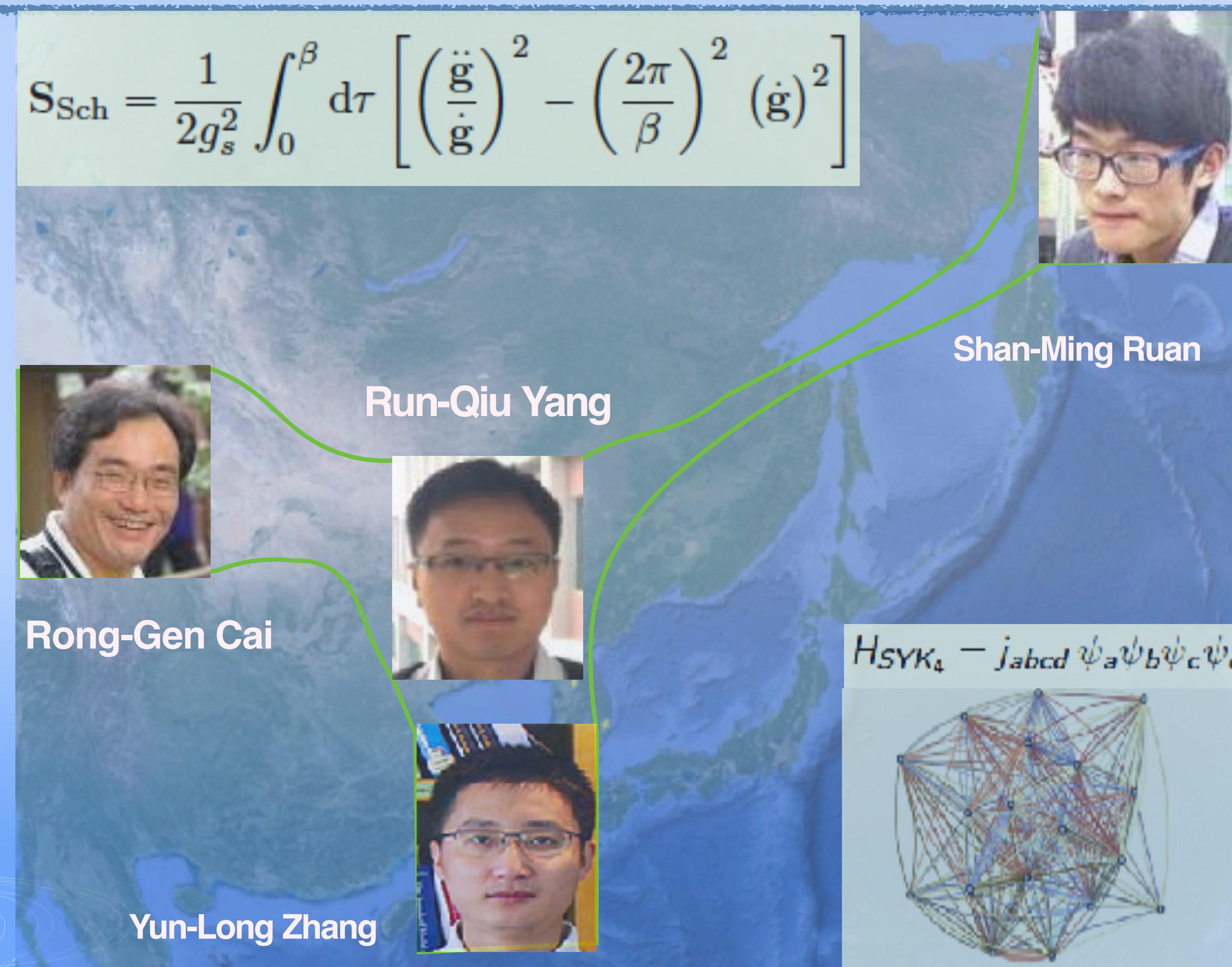
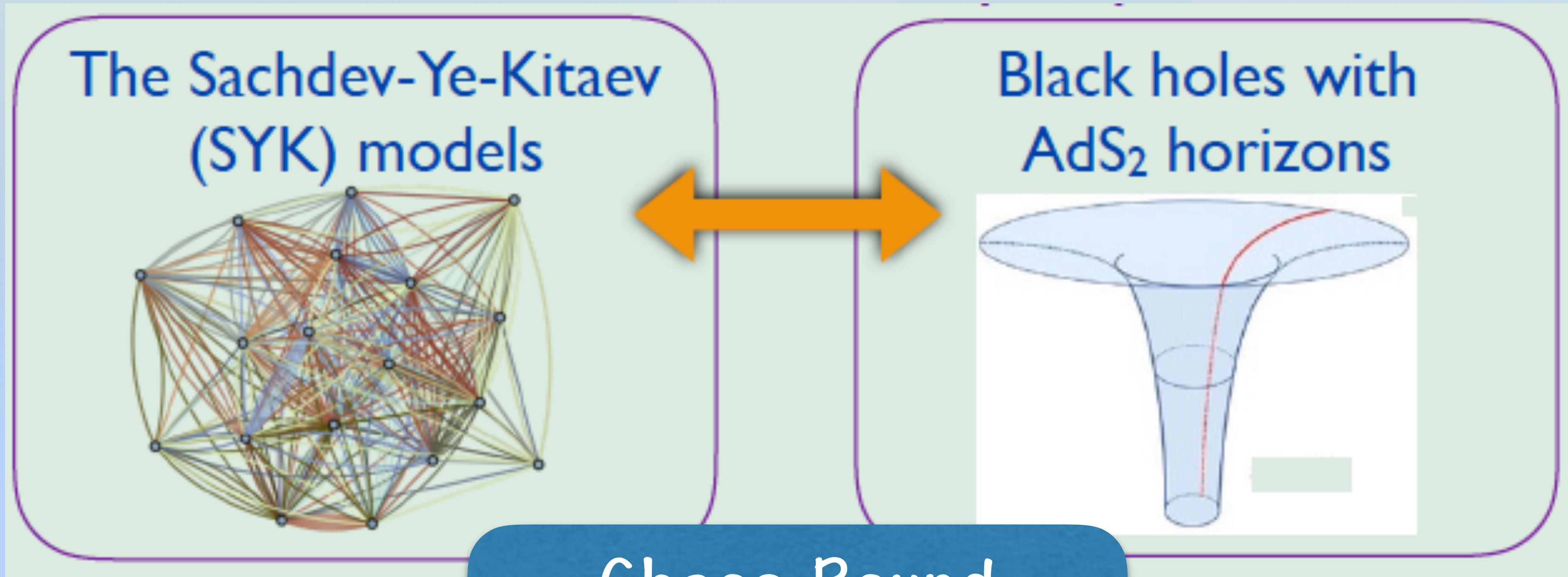
2018.06.20@APCTP

$$S_{Sch} = \frac{1}{2g_s^2}\int_0^\beta d\tau \left[\left(\frac{\ddot{g}}{\dot{g}}\right)^2 - \left(\frac{2\pi}{\beta}\right)^2 (\dot{g})^2\right]$$



Motivations



Chaos Bound

Effective Actions

Symmetries

Schwarzian Action

Effective theory of SYK

Einstein-Hilbert Action

2D Dilaton Gravity

Figure Credit: Sachdev & Balents

Logo of New Horizon Prize

Nambu-Goto Action?

Worldsheet Horizon in AdS

The SYK Model (Sachdev-Ye-Kitaev)

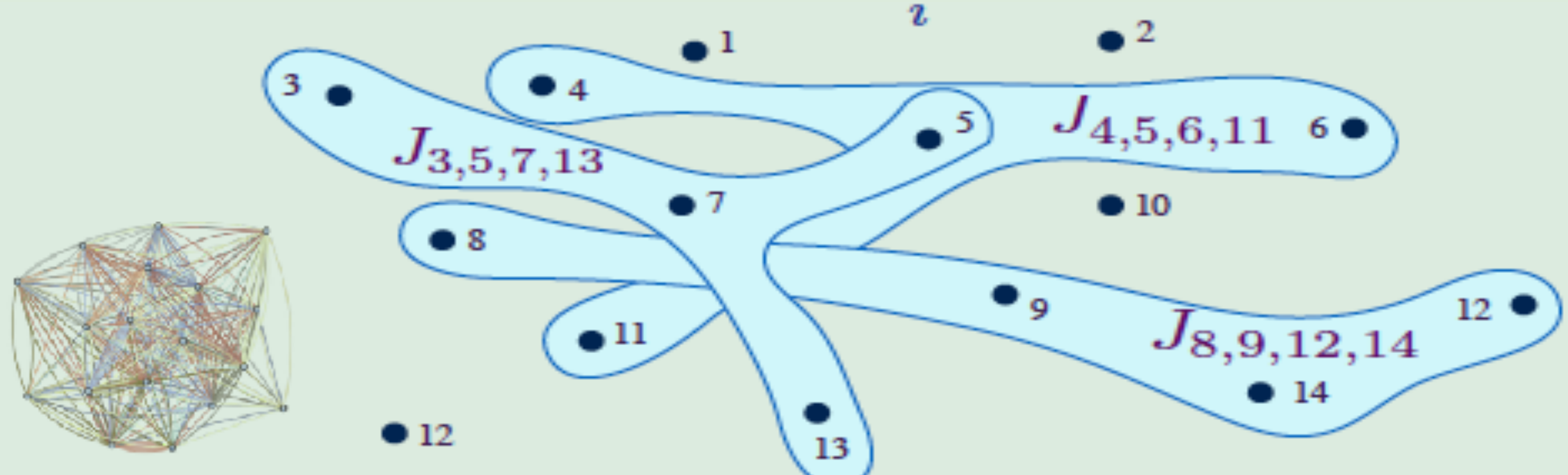
The SYK Hamiltonian is

S. Sachdev and J. Ye, PRL 70, 3339 (1993)
A. Kitaev, unpublished; S. Sachdev, PRX 5, 041025 (2015)

$$H_{SYK_4} = j_{abcd}\,\psi_a\psi_b\psi_c\psi_d \qquad \langle j_{abcd}^2\rangle = \frac{J^2}{N^3}$$

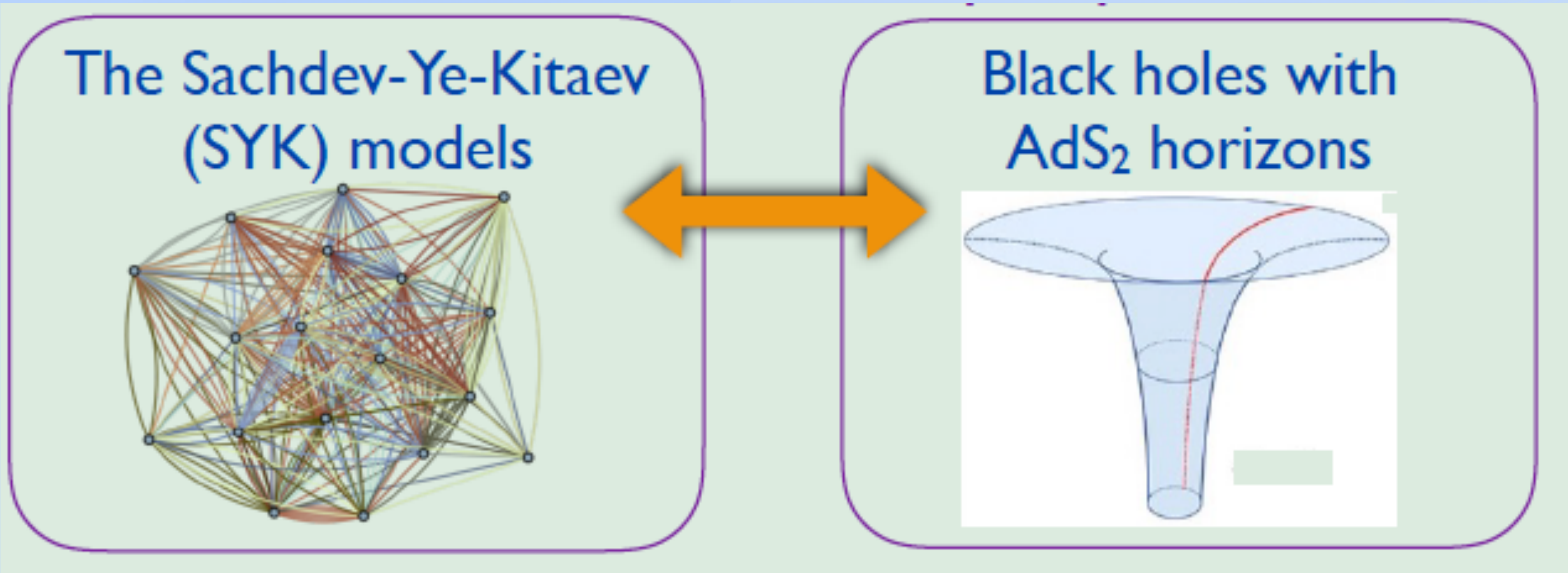
Majorana fermions in QM are matrices ψ_a satisfying

$$\{\psi_a, \psi_b\} = \delta_{ab}, \qquad a, b = 1, \dots, N$$



Credite refer to: Sachdev & Standford

Chaos Bound & Butterfly Effects



Growth of Correlator

$$\langle|\{C(x,t)C^\dagger(0,0)\}|^2\rangle \sim \exp\left[\lambda_L(t-|x|/v_B)\right]$$

Lyapunov Exponent $\lambda_L = 2\pi k_B T/\hbar$

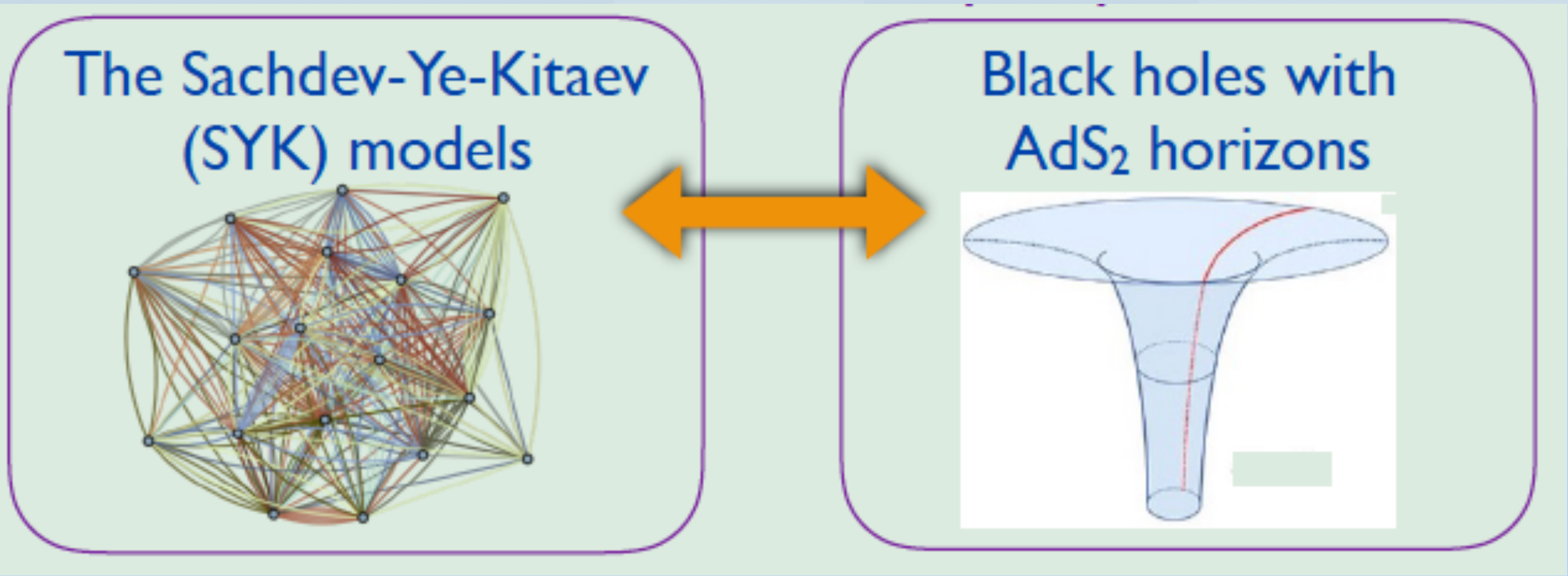
Butterfly Velocity $v_B \sim T^{1/2}$

Diffusion Constant $D_c = v_B^2/\lambda_L$

[Standford & Shenker & Susskind… '13,'14]

Figure Credit: Sachdev & Balents

Schwarzian Action



$$\langle Z_{SYK} \rangle = \int dG dX \exp[-N S_{Sch} + ...]$$

$$S_{JT} = -S_0 - \frac{1}{g}\left[\int_{Bulk} \sqrt{g}\phi(R+2) + 2\phi_b \int_{Bdy} \mathcal{K}\right]$$

Schwarzian Action

Low Energy Effective Action

Jackiw-Teitelboim Action

(2D Dilaton Gravity)

$$\{\mathbf{f}(\tau), \tau\} := \frac{\dddot{\mathbf{f}}}{\dot{\mathbf{f}}} - \frac{3}{2}\left(\frac{\ddot{\mathbf{f}}}{\dot{\mathbf{f}}}\right)^2 .$$

$$\mathbf{S}_{\mathrm{Sch}} := -\frac{1}{g_s^2}\int_0^\beta \mathrm{d}\tau \{\mathbf{f}(\tau), \tau\}, \quad \frac{1}{g_s^2} \equiv \frac{\alpha_S N}{\mathcal{J}},$$

$$S_{JT} = -S_0 - \frac{2\phi_r}{g}\int_0^\beta d\tau \{\mathbf{f}(\tau), \tau\}$$

A. Kitaev, unpublished; S. Sachdev, PRX 5, 041025 (2015)

Effective theory of SYK -> Schwarzian Action

1. Can rewrite SYK in terms of bilocal fields G, Σ

$$\langle Z \rangle_J = \int DG\, D\Sigma\, e^{-N\, I(G,\Sigma)}$$

2. In IR, $I(G,\Sigma)$ has spontaneously broken conformal symmetry. Dominant fluctuations are reparametrizations of the saddle

$$G_\phi \equiv \left(\phi'(\tau_1)\phi'(\tau_2)\right)^\Delta G_*(\phi(\tau_1), \phi(\tau_2)).$$

3. Leading action for ϕ is the "Schwarzian theory"

$$I_{Sch} = -\frac{N\alpha}{J}\int_0^\beta d\tau\, \mathrm{Sch}(\tan\phi/2, \tau) = \frac{N\alpha}{2J}\int_0^\beta \left(\frac{\phi''^2}{\phi'^2} - \phi'^2\right),$$

breaks the physical conformal symmetry.

Credit: Stanford

Gravity on Near AdS2-> Schwarzian Action

A simple theory of gravity in AdS_2 described by $(g_{\mu\nu}, f)$:

$$I_{JT} = -S_0 - \frac{1}{G}\left[\int_{bulk}\sqrt{g}\,(R+2)\,f + 2\int_{bdy} f\,K\right]$$

[Teitelboim][Jackiw][Almheiri,Polchinski]

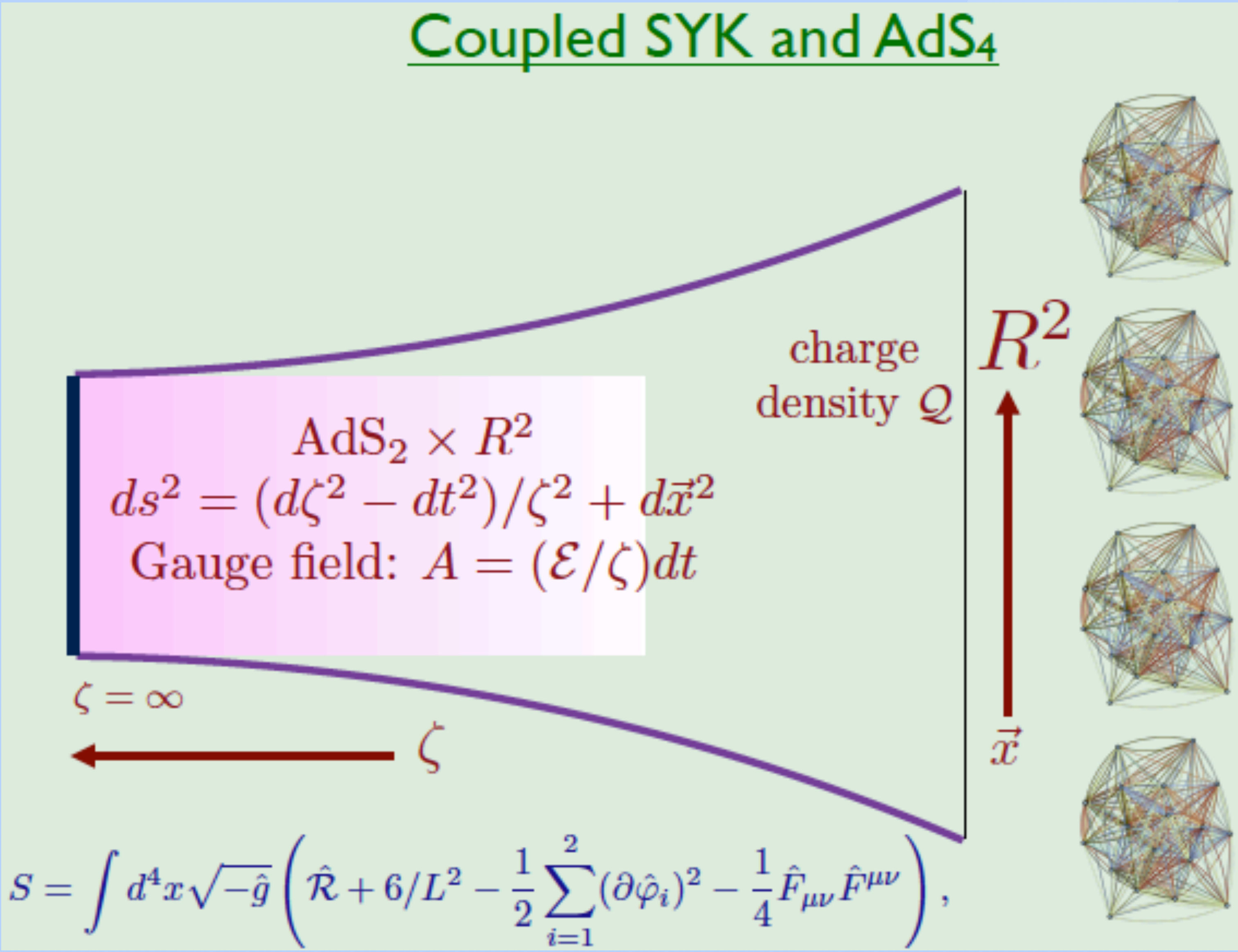
Reduces to the Schwarzian theory!

Step 1: integral over f implies $R+2=0$. Step 2: integral over metrics then reduces to cut-outs from hyperbolic disk.

$$I_{JT} = -S_0 - \frac{2}{G}\int_{bdy} f\,K \quad\longrightarrow\quad -S_0 - \frac{2f_r}{G}\int_0^\beta d\tau\,\mathrm{Sch}(\tan\phi/2, \tau)$$

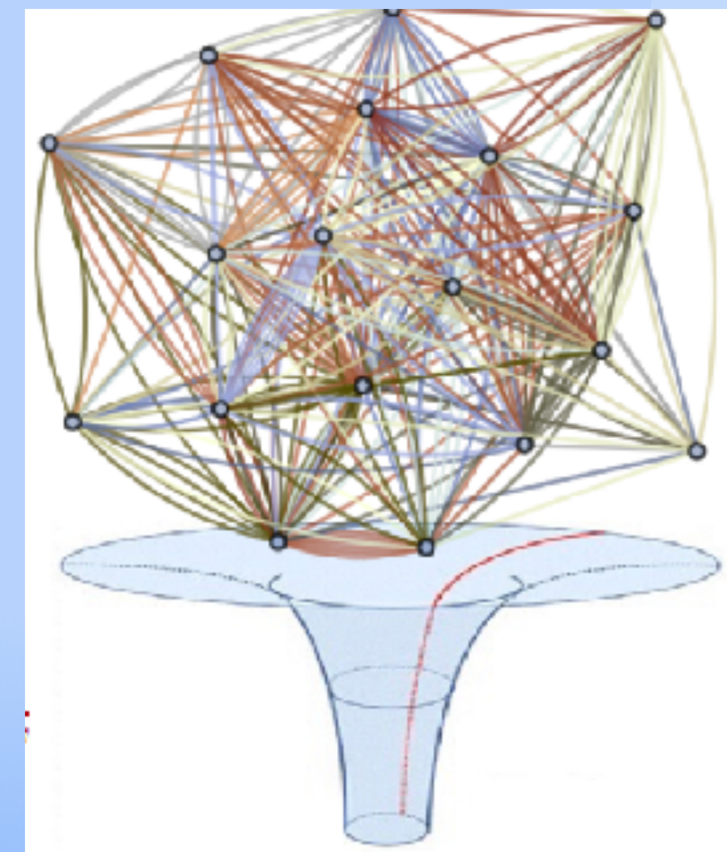
[Maldacena, DS, Z. Yang] see also [Jensen][Engelsoy, Mertens, Verlinde]

Higher Dimensional Generalization



N Majorana fermions $\{\psi_i, \psi_j\} = \delta_{ij}$

$$H = \sum_{i_1,\cdots,i_4} J_{i_1 i_2 i_3 i_4} \psi_{i_1} \psi_{i_2} \psi_{i_3} \psi_{i_4}$$



A Theory of Strange Metal

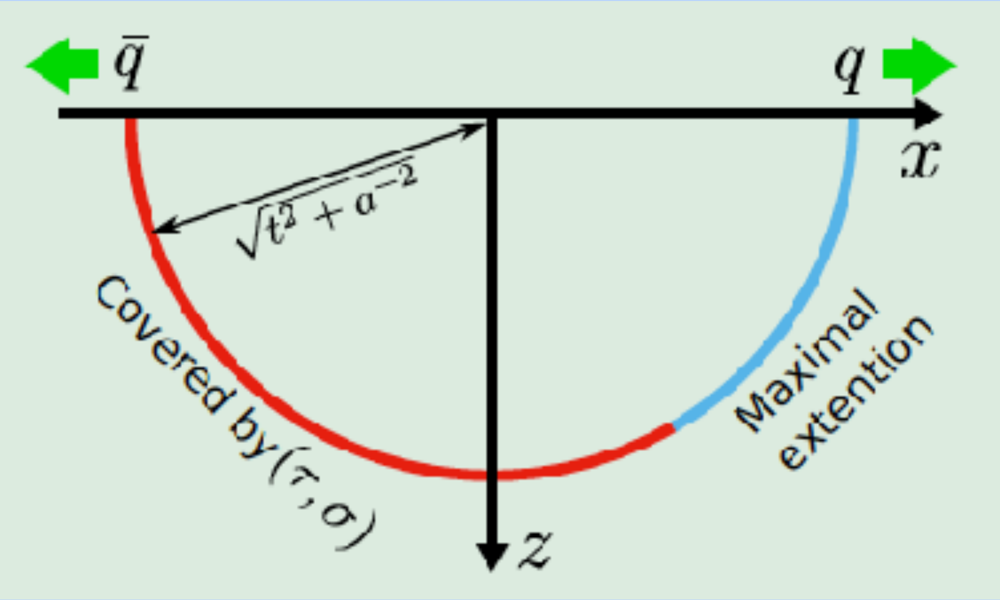
Dual Theory of Gravity on AdS2

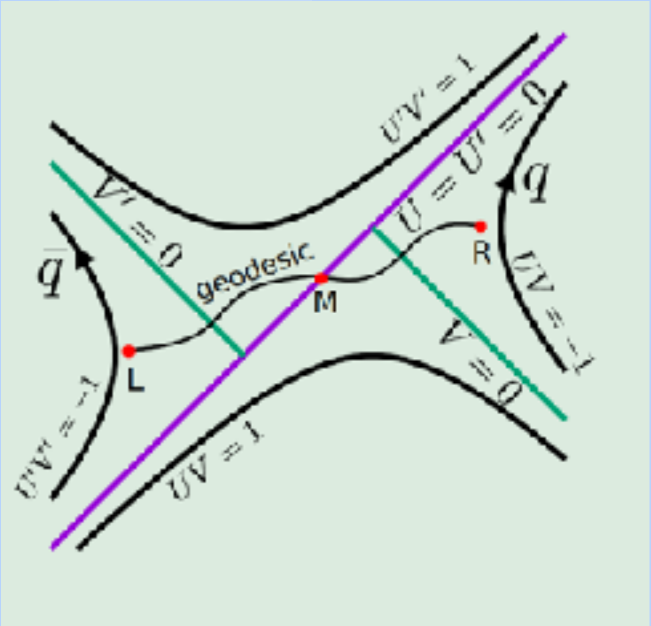
Fastest Possible Chaos

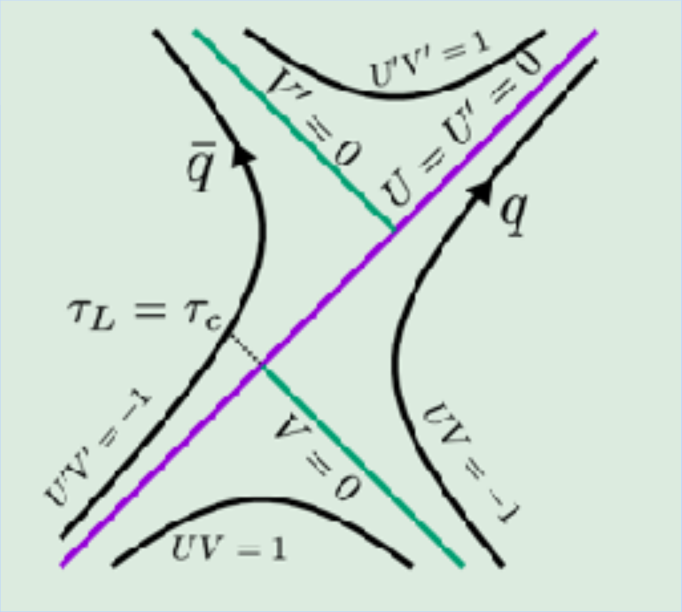
Figure Credit: Sachdev & Balents

Fast scrambling in holographic EPR pair

by **Keiju Murata** (Keio U.) 1708.09493, [Aug 30]
[JHEP 1711 (2017) 049]







4-point OTOC $\sim e^{\lambda_L t}$

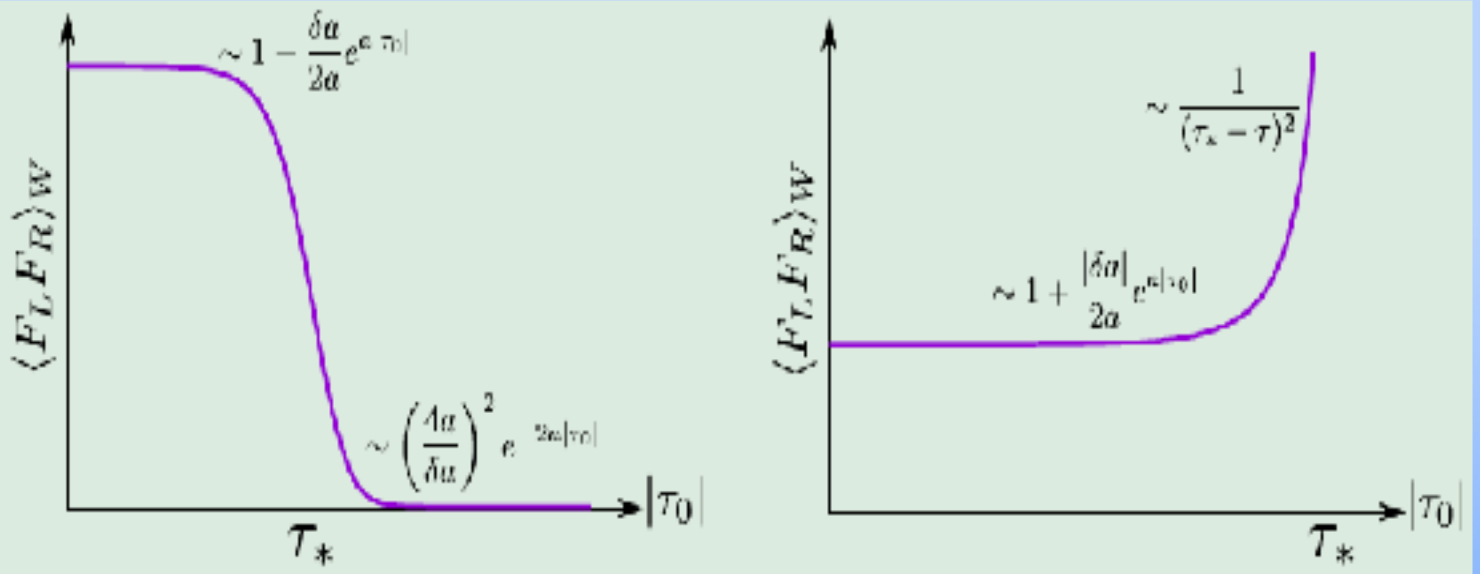
(Out-of-Time-Order Correlator)

$$\langle F_L(0) F_R(0) \rangle_W \sim \langle \Psi | W_L^\dagger(\tau_0) F_L(0) F_R(0) W_L(\tau_0) | \Psi \rangle$$

Lyapunov Exponent

$$\lambda_L = 2\pi T_U$$

Unruh Temperature



(a) $\delta a > 0$ (b) $\delta a < 0$

Chaotic strings in AdS/CFT

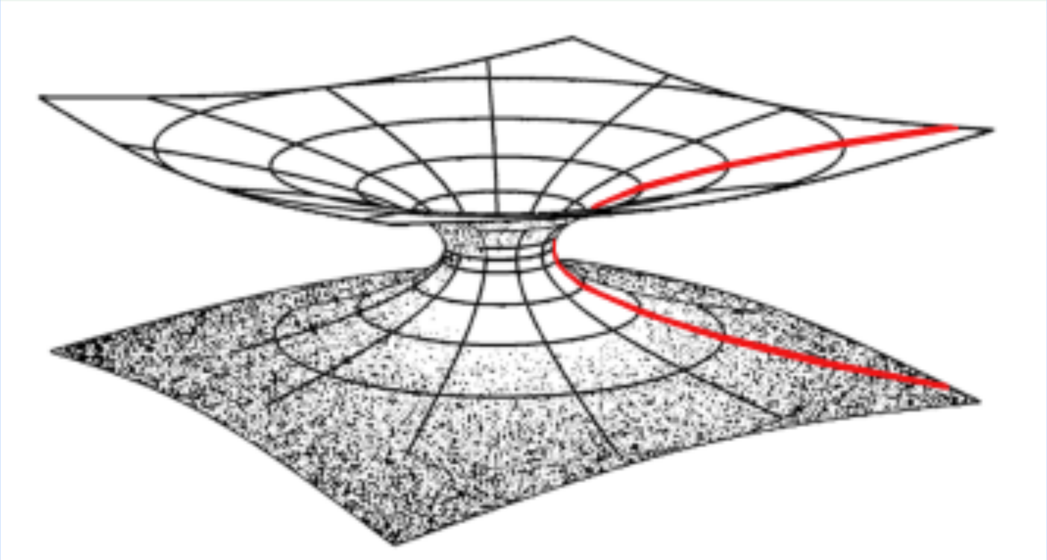
J. de Boer, E. Llabrés, J. Pedraza, D. Vegh
Amsterdam &Utrecht U. 1709.01052 [Sep.4]
[Phys.Rev.Lett. 120 (2018) no.20, 201604]

AdS Black Brane

$$ds^2 = -r^2 f(r) dt^2 + \frac{dr^2}{r^2 f(r)} + r^2 dx^2,$$
$$f(r) = 1 - \left(\frac{r_H}{r}\right)^{d-1}.$$

Induced AdS2 Wormhole

$$ds^2_{ws} = -\frac{4 du dv}{(1+uv)^2},$$



Action

$$S_{\rm NG} = -\frac{1}{\pi\alpha'} \int du dv \sqrt{\frac{1 - r_H^2 (1-uv)^2 \partial_u X \partial_v X}{(1+uv)^4}}.$$

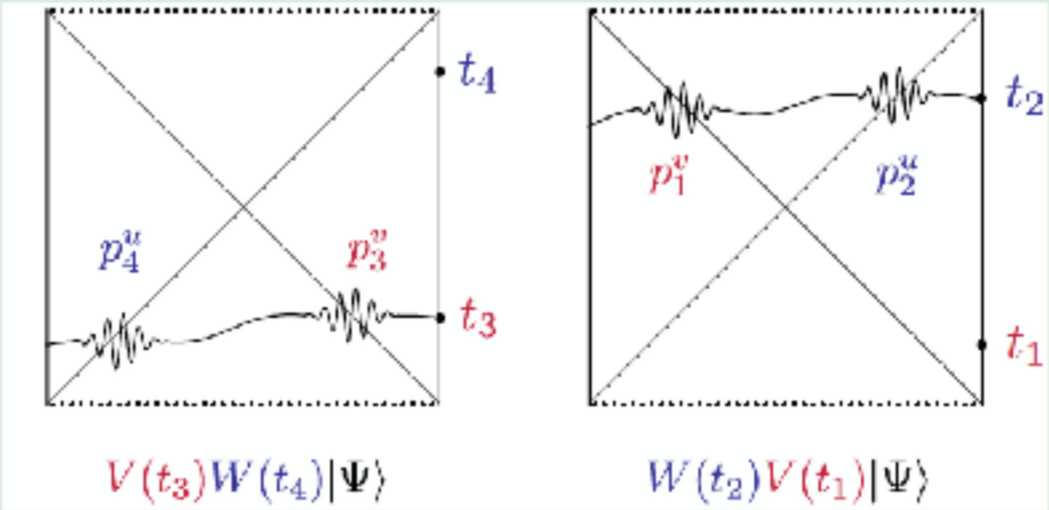
4-point OTOC

$$f(t) = \frac{\langle VW(t)VW(t)\rangle}{\langle VV\rangle\langle WW\rangle}$$

$$f(t) = 1 - \frac{f_0}{N^2} e^{\lambda_L t} + \mathcal{O}(N^{-4})$$

Lyapunov Exponent $\lambda_L = 2\pi T_H$

Hawking Temperature



String World Sheet as one Candidate Dual of Schwarzian Theory

by R.-G. Cai, S.-M. Ruan, R. -Q. Yang, Y.-L. Zhang
arXiv:1709.06297 [Sep 19]

Nambu-Goto Action

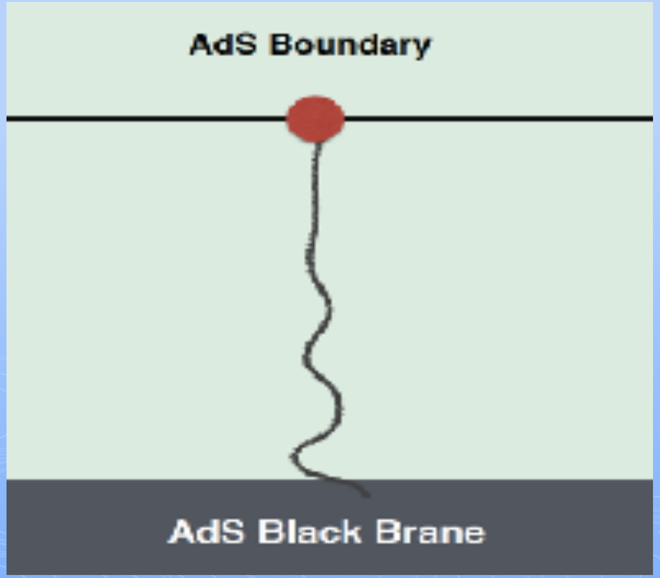
$$S_{\rm NG} = -\frac{1}{2\pi\alpha'}\int d\sigma d\tau\sqrt{-\det h_{ab}}\,,$$

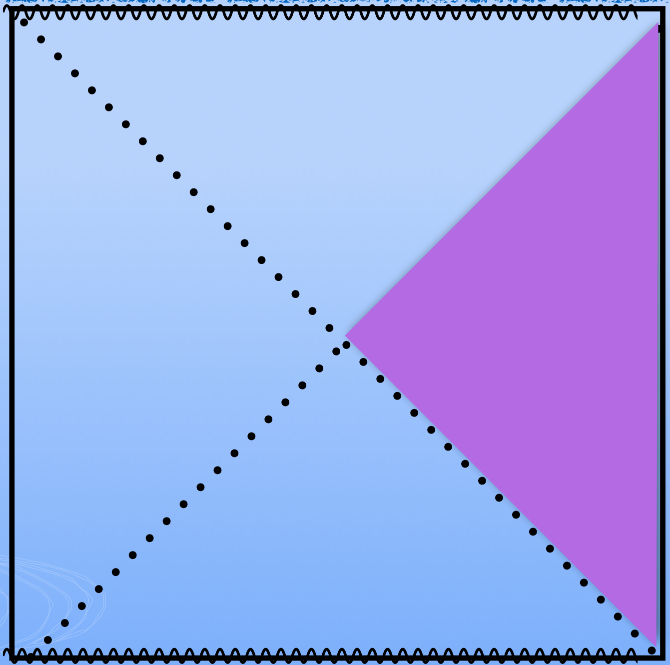
$$S^{(2)}_{\rm NG} = -\frac{1}{4\pi\alpha'}\int_{r_h}^{r_c}{\rm d}r\int_{-\frac{\Delta_0}{2}}^{\frac{\Delta_0}{2}}{\rm d}t\left[r^4 f(r)({\rm x}')^2 - \frac{1}{f(r)}(\dot{\rm x})^2\right]$$

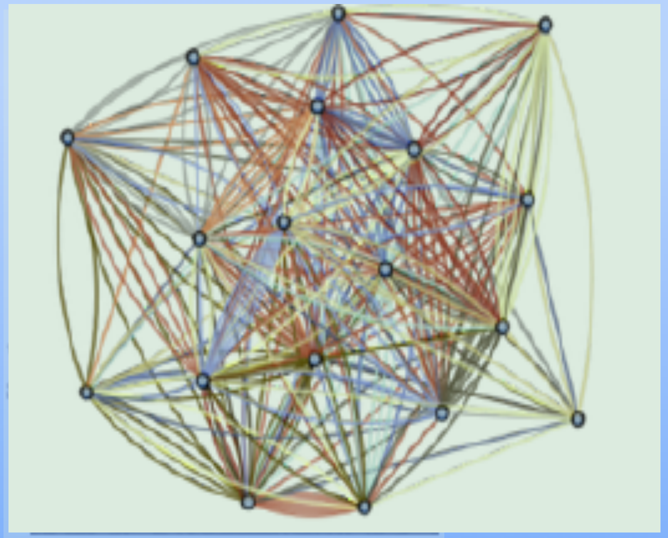
Schwarzian Action

$$\mathbf{S}_{\rm Sch} = \frac{1}{2g_s^2}\int_0^\beta {\rm d}\tau\left[\left(\frac{\ddot{\rm g}}{\dot{\rm g}}\right)^2 - \left(\frac{2\pi}{\beta}\right)^2(\dot{\rm g})^2\right]$$

$$\mathbf{S}^{(2)}_{\rm Sch} := \frac{1}{2g_s^2}\int_0^\beta {\rm d}\tau\left[(\ddot\epsilon)^2 - \left(\frac{2\pi}{\beta}\right)^2(\dot\epsilon)^2\right]$$







Renormalized Nambu-Goto Action of the WorldSheet

Black Brane Background

$$ds^2 = -r^2 f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{r^2 f(r)} + r^2 \mathrm{d}x^2,$$

$$f(r) = 1 - \frac{r_h^2}{r^2}\left[1 + \mathrm{q}^2 \ln\left(\frac{r}{r_h}\right)\right]$$

Induced Metric on WorldSheet

$$ds^2_{\mathrm{ws}} = h_{ab}\mathrm{d}\sigma^a \mathrm{d}\sigma^b = -r^2 f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{r^2 f(r)},$$

Nambu-Goto Action

$$S_{\mathrm{NG}} = -\frac{1}{2\pi\alpha'}\int d\sigma d\tau \sqrt{-\det h_{ab}}\,,$$

After Perturbations

$$S_{\mathrm{NG}} \simeq -\frac{1}{2\pi\alpha'}\int \mathrm{d}r\mathrm{d}t\Big[1 - \frac{1}{2f(r)}(\dot{\mathrm{x}})^2 + \frac{r^4 f(r)}{2}(\mathrm{x}')^2\Big],$$

Counter-term

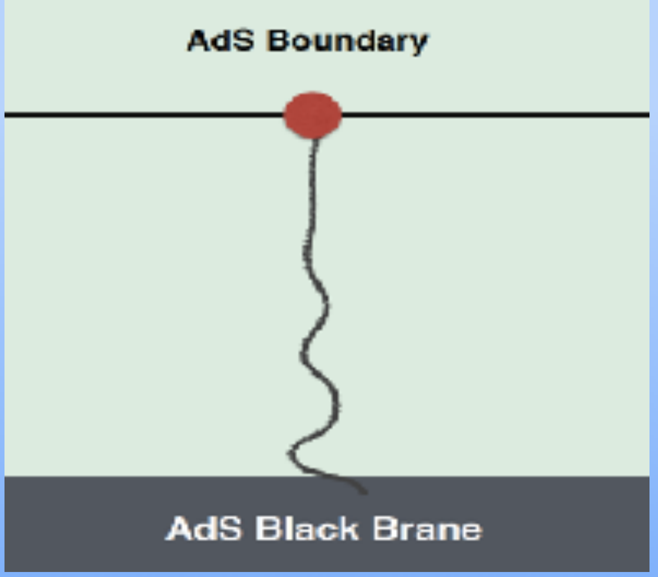
$$S_{\mathrm{ct}} := \frac{1}{2\pi\alpha'}\int_{r=r_c}\sqrt{-\gamma}\mathrm{d}t\,.$$

Renormalized action

$$S^{(2)}_{\mathrm{ren}} := S^{(2)}_{\mathrm{NG}} + S^{(2)}_{\mathrm{ct}}$$

… Final On Shell Formula

$$\mathrm{S}^{(2)}_{\mathrm{ren}} = \frac{1}{2g_s^2}\int_0^{\beta} \mathrm{d}\tilde{\tau}\left[\left(\ddot{\tilde{\varepsilon}}\right)^2 - \left(\frac{2\pi}{\beta}\right)^2\left(\dot{\tilde{\varepsilon}}\right)^2\right]$$

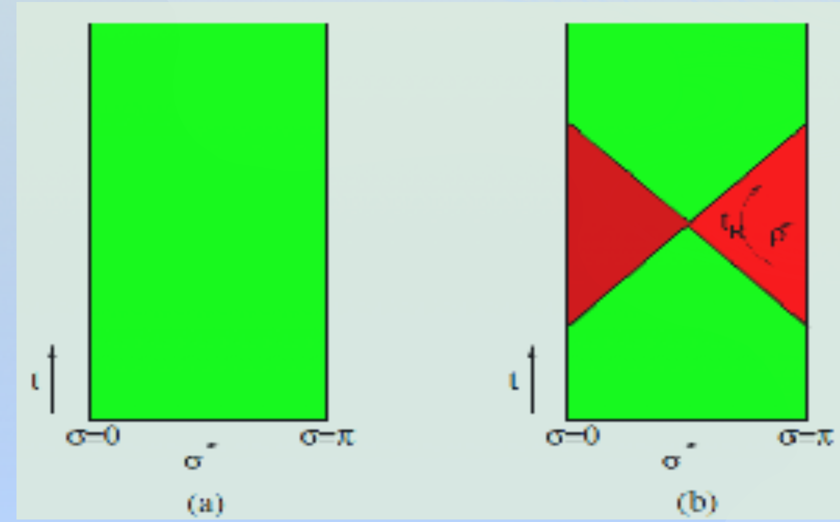


Traversable wormhole <-> Two Coupled SYK?

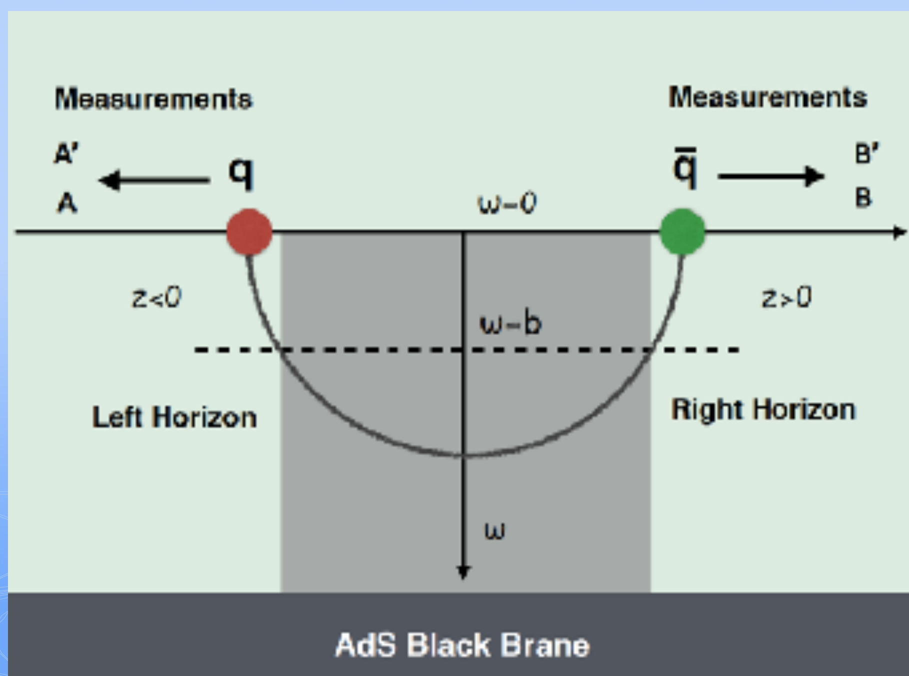
by Maldacena & Qi [1804.00491]

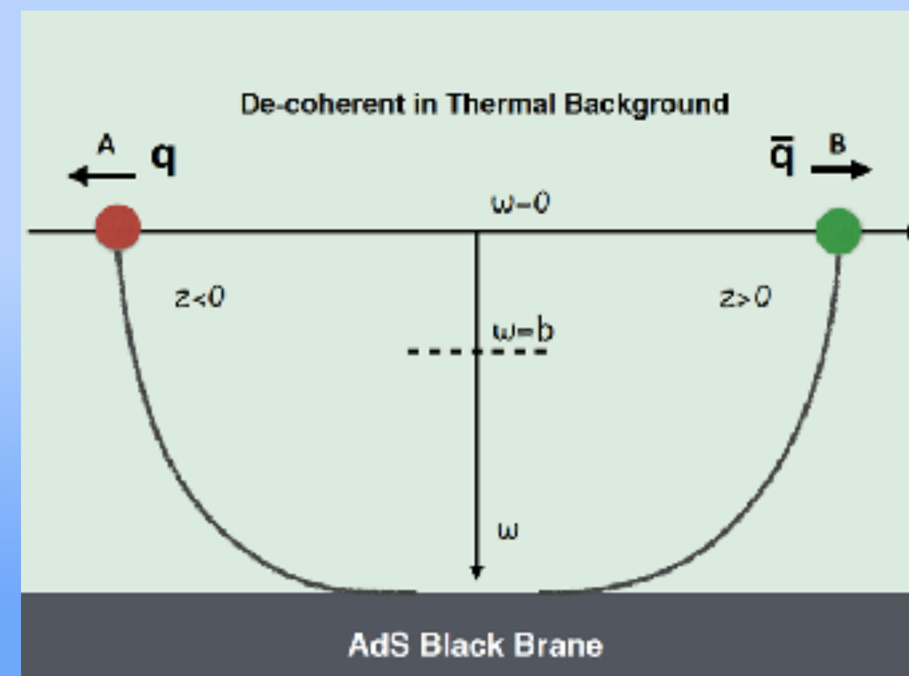
$$H_{\rm total} = H_{\rm L,SYK} + H_{\rm R,SYK} + H_{\rm int} \ , \qquad H_{\rm int} = i\mu \sum_j \psi_L^j \psi_R^j$$

$$S = N \int du \left\{ -\frac{\alpha_S}{\mathcal{J}} \left(\{\tan \frac{t_l(u)}{2}, u\} + \{\tan \frac{t_r(u)}{2}, u\} \right) + \mu \frac{c_\Delta}{(2\mathcal{J})^{2\Delta}} \left[\frac{t_l'(u) t_r'(u)}{\cos^2 \frac{t_l(u) - t_r(u)}{2}} \right]^\Delta \right\}$$

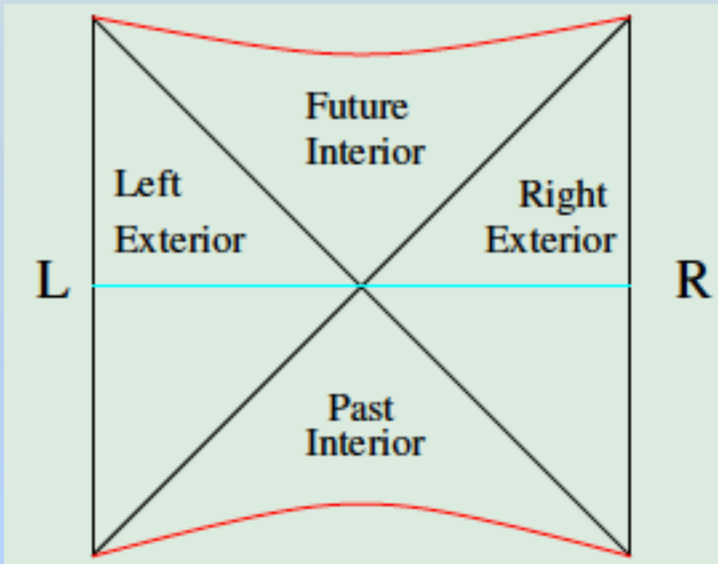


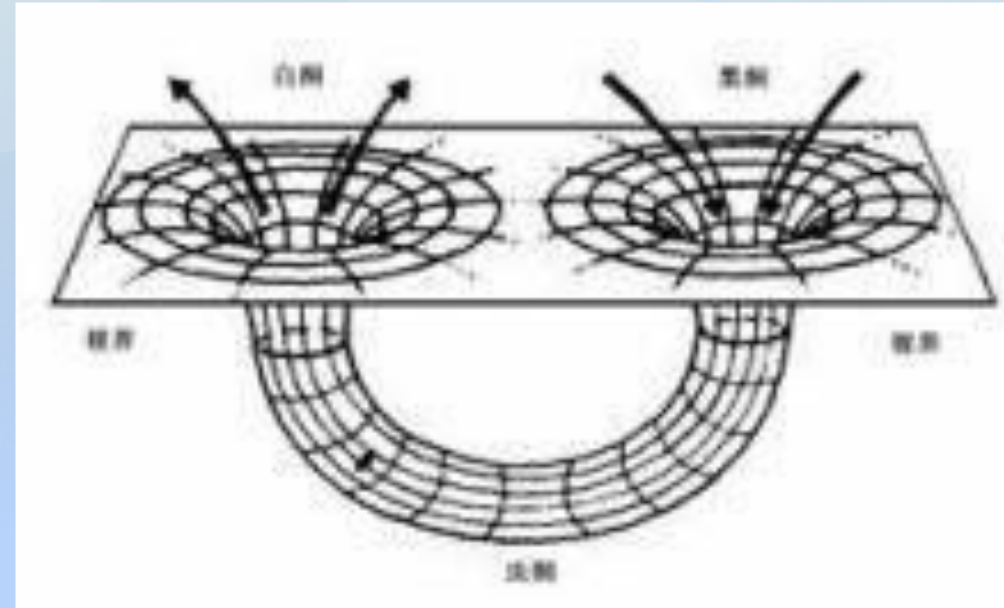
De-Coherent Phase Transition?





ER=EPR (Wormhole=Entangled Pair) ?





ER bridge(Einstein—Rosen): Non-traversable wormhole

EPR pair of maximally entangled black holes

$$H = H_R + H_L.$$

$$|\Psi(t)\rangle = \sum_n e^{-\beta E_n/2} e^{-2iE_n t} |\bar{n}, n\rangle.$$

EPR pair of two black holes in a particular entangled state

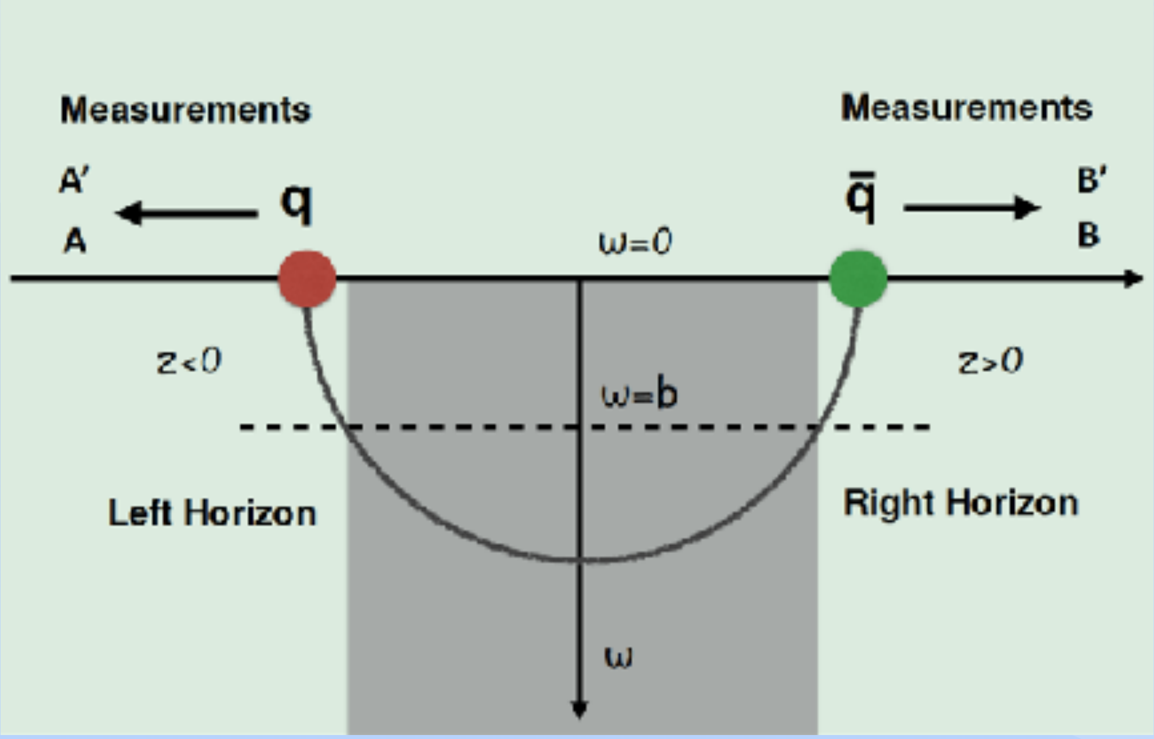
How about Traversable wormhole?

Ref: Maldacena & Susskind [1306.0533]

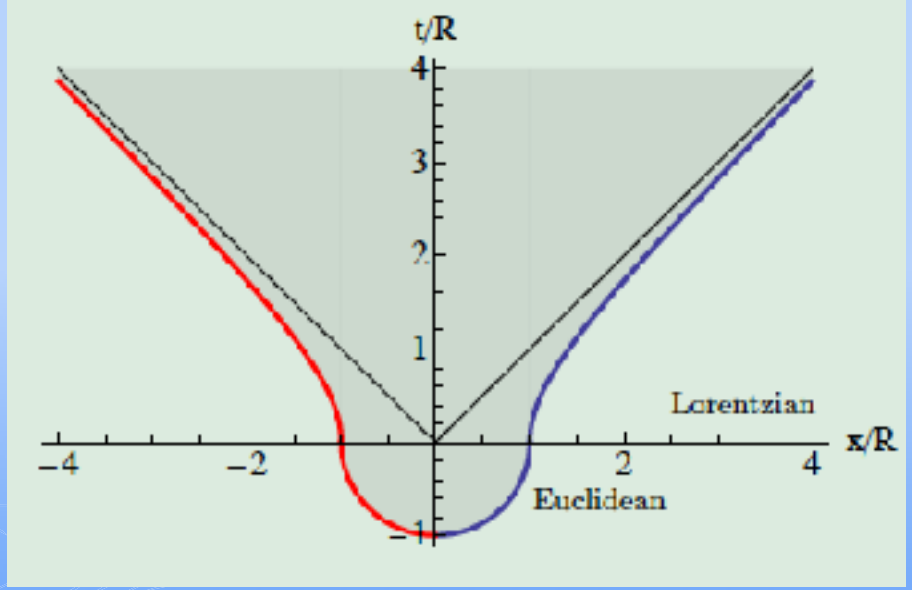
Holographic EPR Pair: Classical <-> Quantum?

$$ds^2 = \frac{R^2}{w^2}\left[-dt^2 + dw^2 + (dx^2 + dy^2 + dz^2)\right],$$

$$|z| = b\sqrt{1-\tilde{r}}e^{\tilde{z}}\cosh\tilde{\tau},$$
$$t = b\sqrt{1-\tilde{r}}e^{\tilde{z}}\sinh\tilde{\tau},$$
$$w = b\sqrt{\tilde{r}}e^{\tilde{z}}.$$

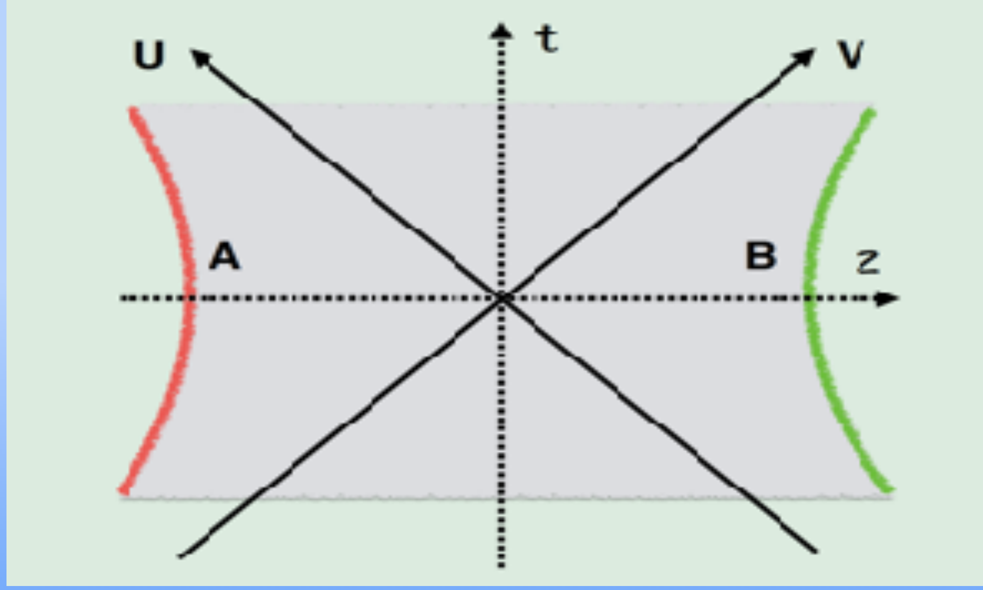


$$ds^2 = \frac{R^2}{b^2\tilde{u}}\left[-f(\tilde{u})\,d\tau^2 + \frac{b^2}{4\tilde{u}}\frac{d\tilde{u}^2}{f(\tilde{u})} + d\tilde{z}^2 + \left(dx^2 + dy^2\right)\exp\left(-2\tilde{z}/b\right)\right],$$



Holographic Schwinger effect

J.Sonner(1307.6850), PRL111.211603



Holographic EPR Pair

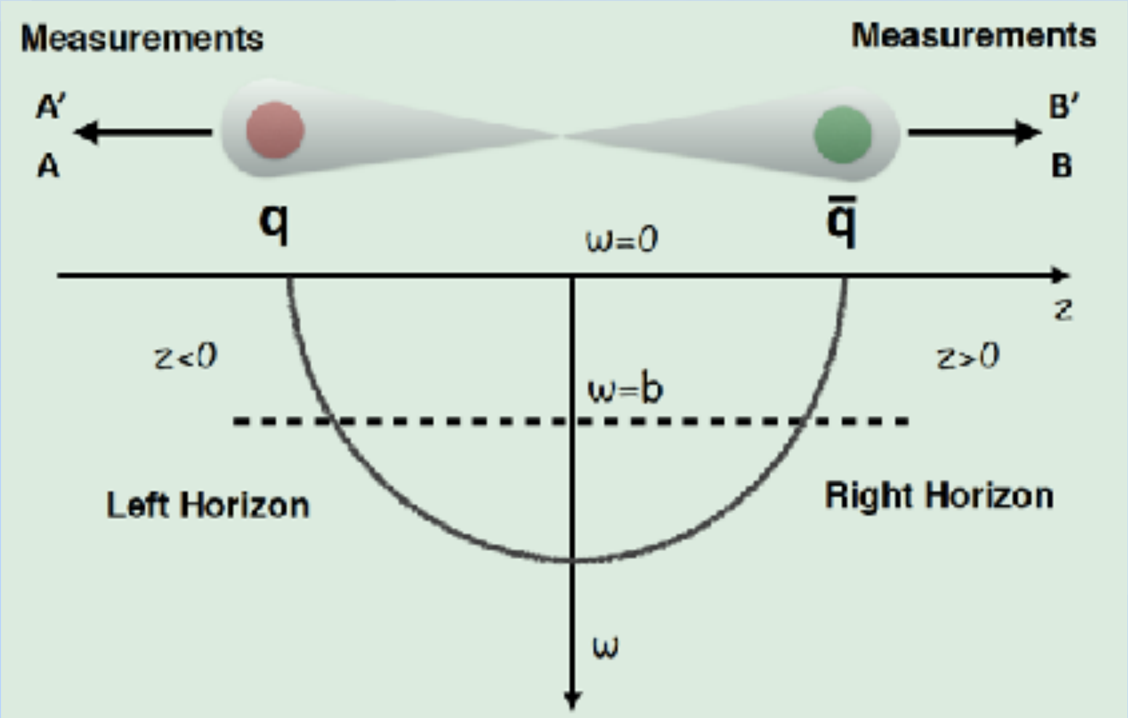
Karch & Jensen (1307.1132) PRL 111.211602

Constructing Bell inequality for Holographic EPR

J.-W. Chen, S.-C. Sun, Y.-L. Zhang [arXiv:1612.09513]

$$G_{AB}^{ij}(\omega) = \frac{2\mathrm{i}e^{-\omega/2T_U}}{1-e^{-\omega/T_U}}\mathrm{Im}G_R^{ij}(\omega)$$

$$\mathrm{i}G_{AB}^{ij}(\tau, x) = \langle \mathcal{F}_A^i(\tau, x)\mathcal{F}_B^j(0)\rangle.$$



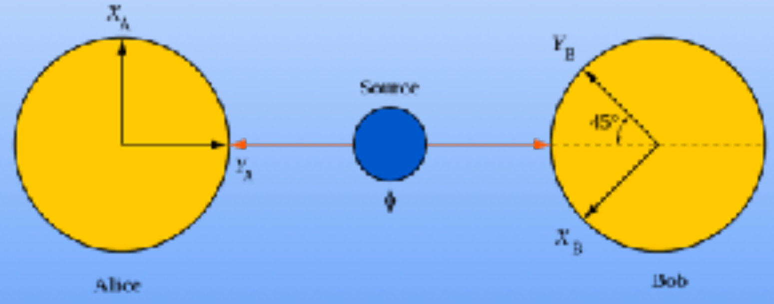
$$A_\mathcal{F} = (\cos\theta_A \mathcal{F}_A^x + \sin\theta_A \mathcal{F}_A^y)/\langle \mathcal{F}_A^x \mathcal{F}_B^x\rangle^{1/2},$$

$$B_\mathcal{F} = (\cos\theta_B \mathcal{F}_B^x + \sin\theta_B \mathcal{F}_B^y)/\langle \mathcal{F}_A^x \mathcal{F}_B^x\rangle^{1/2},$$

Bell's Theorem(CHSH formula)

$$\mathrm{i}G_{AB}^{xx} = \mathrm{i}G_{AB}^{yy} = \frac{\sqrt{\lambda}a^3}{2\pi^2}, \quad \mathrm{i}G_{AB}^{xy} = \mathrm{i}G_{AB}^{yx} = 0.$$

$$\langle C_\mathcal{F}\rangle = \langle A_\mathcal{F} B_\mathcal{F}\rangle + \langle A_\mathcal{F} B'_\mathcal{F}\rangle + \langle A'_\mathcal{F} B_\mathcal{F}\rangle - \langle A'_\mathcal{F} B'_\mathcal{F}\rangle$$
$$= \cos\theta_{AB} + \cos\theta_{AB'} + \cos\theta_{A'B} - \cos\theta_{A'B'}.$$



$$\langle A_\mathcal{F} B_\mathcal{F}\rangle = \cos(\theta_A - \theta_B) \equiv \cos\theta_{AB}.$$

$$\theta_{AB} = \theta_{AB'} = \theta_{A'B} = \pi/4, \quad \theta_{A'B'} = 3\pi/4$$

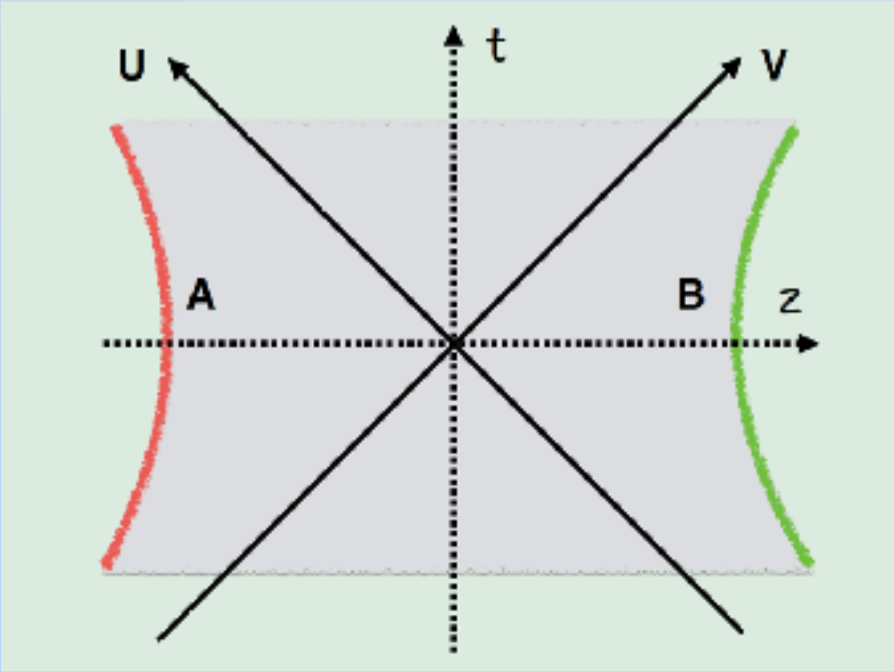
$$\langle C_\mathcal{F}\rangle = 2\sqrt{2}.$$

Violate The Bound for local system $|\langle C\rangle| \le 2$

Holographic SK Correlators from String Worldsheet

$$Z_{EPR} \equiv \langle e^{\frac{i}{\hbar} S_{EPR}} \rangle \overset{AdS/CFT}{\simeq} e^{\frac{i}{\hbar} S_{NG}[\tilde{q}_i^I, \tilde{q}_j^J]}.$$

$$\mathrm{i}G_{IJ}^{ij} \equiv \frac{\hbar^2}{\mathrm{i}^2} \frac{\delta^2 \ln Z_{EPR}}{\delta(\tilde{q}_i^I)\delta(\tilde{q}_j^J)} \simeq \frac{\delta^2 S_{NG}[\tilde{q}_i^I, \tilde{q}_j^J]}{\delta(\tilde{q}_i^I)\delta(\tilde{q}_j^J)},$$



$$S_{NG} \simeq -\frac{\sqrt{\lambda}}{2\pi} \int \frac{\mathrm{d}\tilde{\tau}\mathrm{d}\tilde{r}}{2\tilde{r}^{3/2}} \left\{ 1 + \left[2\tilde{r} f(\tilde{r}) \tilde{q}_i' \tilde{q}_j' - \frac{1}{2f(\tilde{r})} \dot{\tilde{q}}_i \dot{\tilde{q}}_j \right] h^{ij} \right\},$$

EOMs

$$\partial_{\tilde{r}} \left(\frac{2f\tilde{q}_i'}{\tilde{r}^{1/2}} \right) - \partial_{\tilde{\tau}} \left(\frac{\dot{\tilde{q}}_i}{2f\tilde{r}^{3/2}} \right) = 0.$$

$$\begin{aligned} S_{NG}[\tilde{q}_i^I, \tilde{q}_j^J] = -\frac{1}{2} \int \frac{d\omega}{2\pi} \Big\{ & \left[\tilde{q}_i^A(-\omega)\tilde{q}_j^B(\omega) + \tilde{q}_i^B(-\omega)\tilde{q}_j^A(\omega) \right] \\ & \times \sqrt{n_\omega(1+n_\omega)} \left[G_{\mathcal{A}}^{ij}(\omega) - G_{\mathcal{R}}^{ij}(\omega) \right] \\ & + \tilde{q}_i^A(-\omega)\tilde{q}_j^A(\omega) \left[(1+n) G_{\mathcal{R}}^{ij}(\omega) - n G_{\mathcal{A}}^{ij}(\omega) \right] \\ & + \tilde{q}_i^B(-\omega)\tilde{q}_j^B(\omega) \left[n G_{\mathcal{R}}^{ij}(\omega) - (1+n) G_{\mathcal{A}}^{ij}(\omega) \right] \Big\}, \end{aligned}$$

SK Correlators

$$\mathrm{i}G_{AB}^{ij}(\omega) \equiv \frac{S_{NG}[\tilde{q}_i^I, \tilde{q}_j^J]}{\delta(\tilde{q}_i^A)\delta(\tilde{q}_j^B)} = \frac{-2e^{-\omega/(2T_a)}}{1 - e^{-\omega/T_a}} \mathrm{Im} G_{\mathcal{R}}^{ij}(\omega)$$

$$\begin{aligned} G_R^{ij}(\omega) &= -\frac{2T_0 L^2}{b^2 \tilde{r}^{1/2}} f(\tilde{r}) Y_{-\omega}(\tilde{r}) \partial_{\tilde{r}} Y_\omega(\tilde{r}) \delta^{ij} \Big|_{\tilde{r}\to 0} \\ &= -\frac{a^2\sqrt{\lambda}}{2\pi} \mathrm{i}\omega\delta^{ij} + O(\omega^2), \end{aligned}$$

$$T_a = \frac{\hbar a}{2\pi k_B c} \qquad \exp\left[-\frac{\hbar\omega}{k_B T_a}\right]$$

Ref: J.-W. Chen, S.-C. Sun, Y.-L. Zhang [arXiv:1612.09513]

Traversable Wormholes or Black Holes?

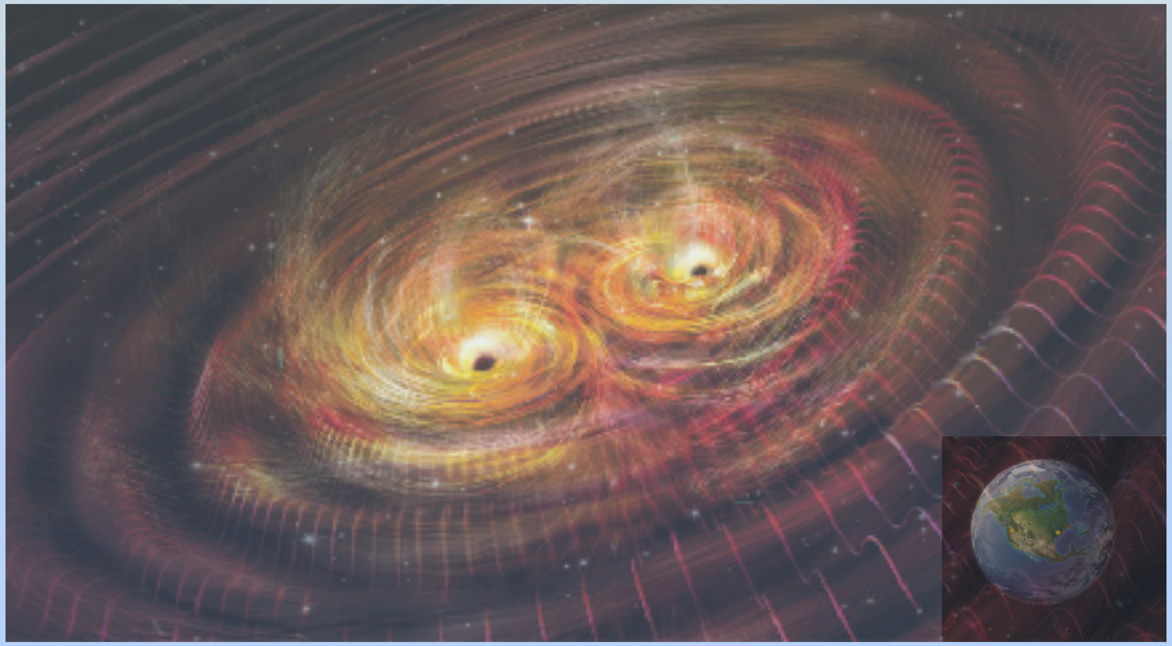
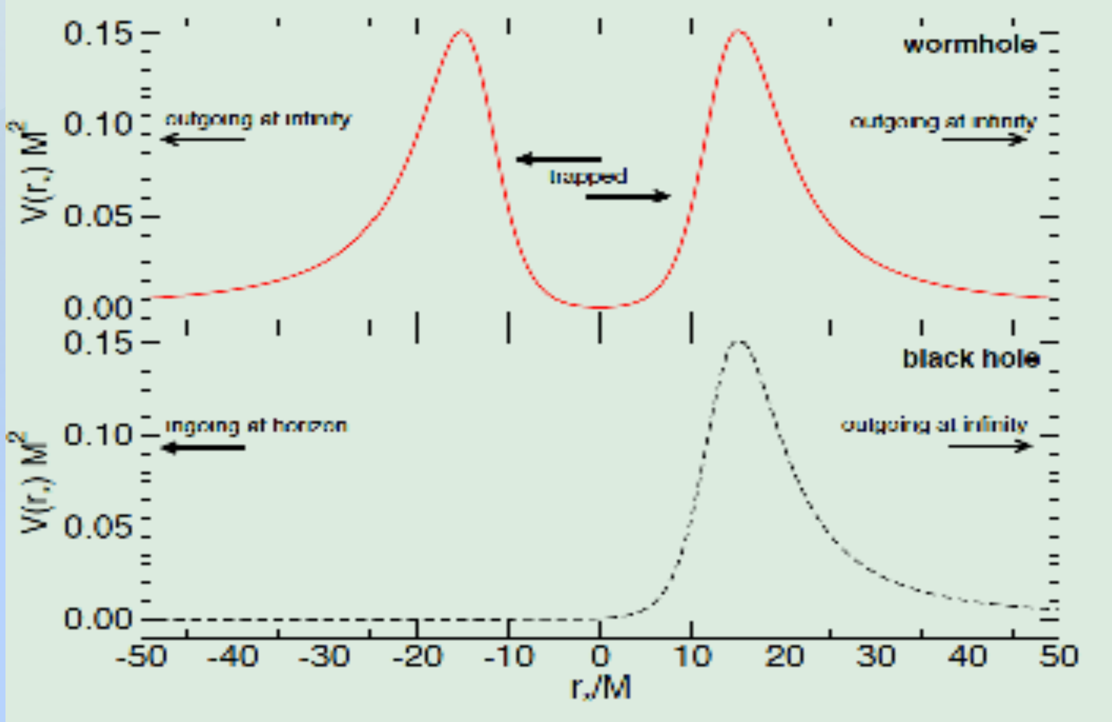
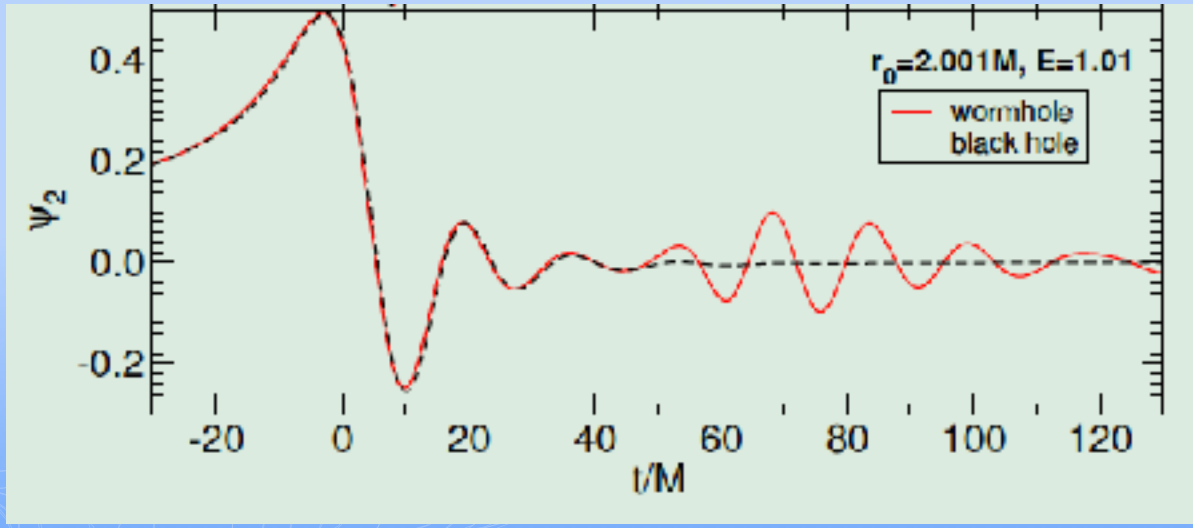
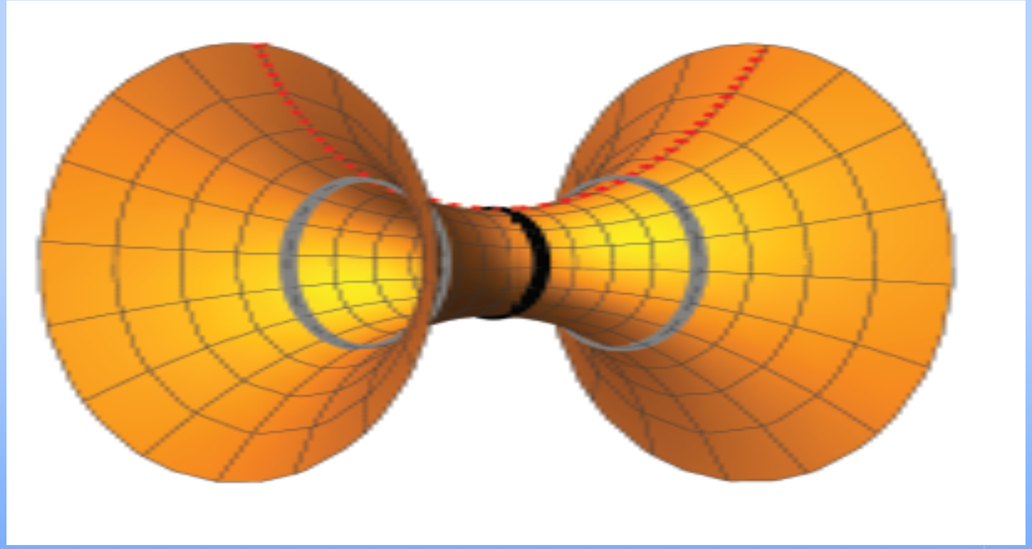


Figure Credit: ScienceNews







Is the Gravitational-Wave Ringdown a Probe of the Event Horizon?

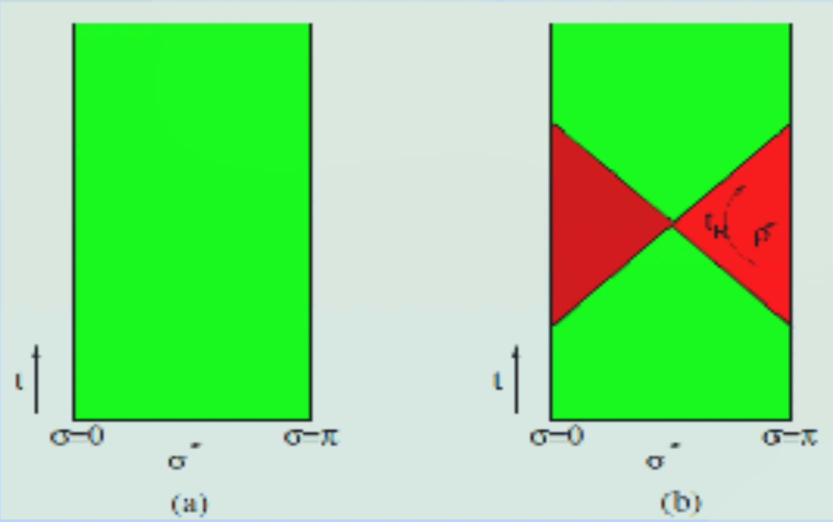
V. Cardoso, E. Franzin, P. Pani [PRL. 116, 171101 (2016)]

Traversable wormhole <-> Two Coupled SYK?

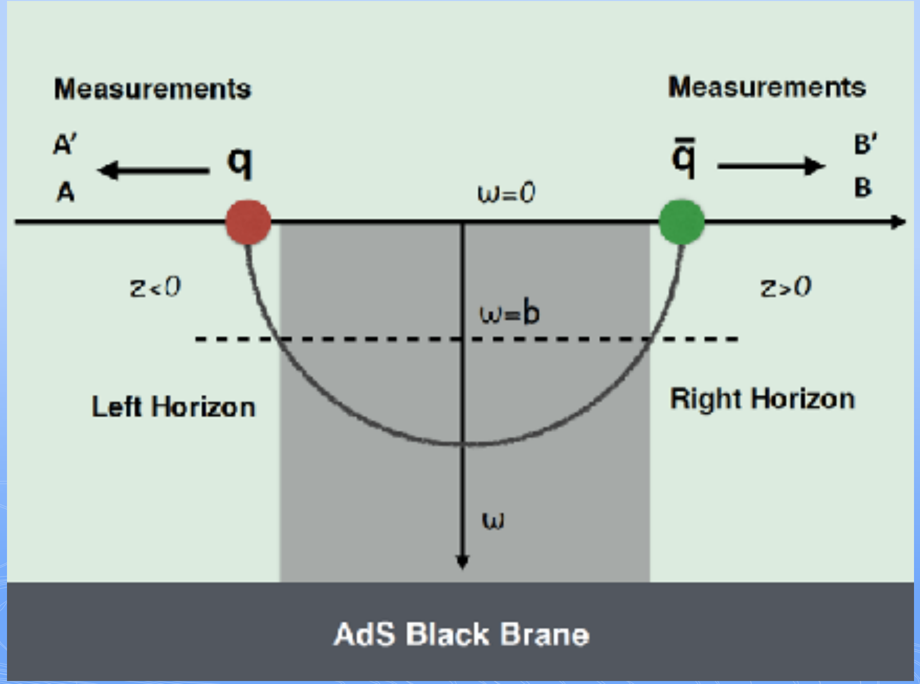
by Maldacena & Qi [1804.00491]

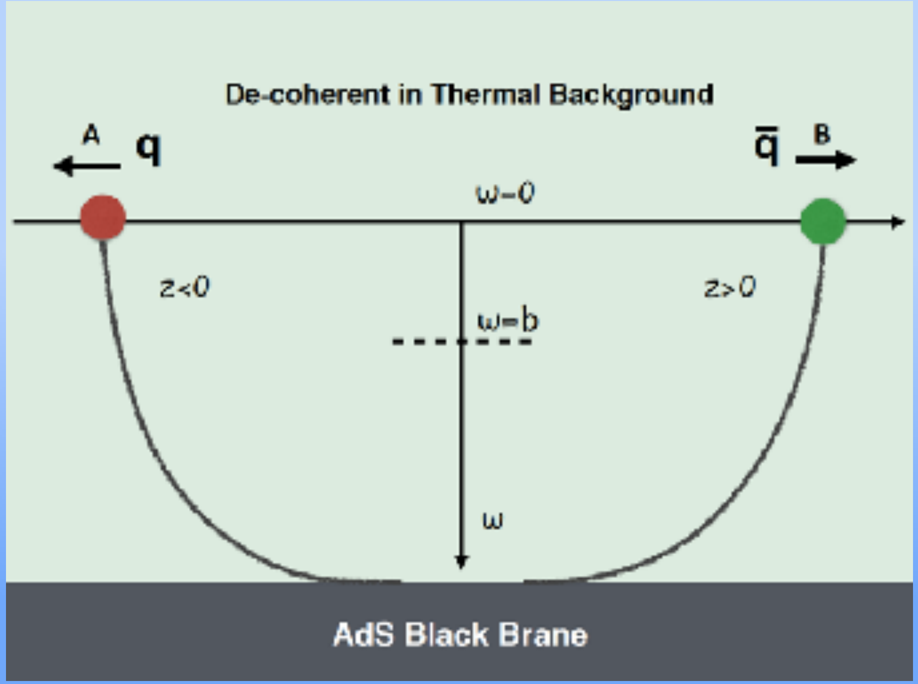
$$H_{\text{total}} = H_{\text{L,SYK}} + H_{\text{R,SYK}} + H_{\text{int}} \ , \qquad H_{\text{int}} = i\mu \sum_j \psi_L^j \psi_R^j$$

$$S \quad N \int du \left\{ \frac{\alpha_S}{\mathcal{J}} \left(\{\tan \frac{t_l(u)}{2}, u\} + \{\tan \frac{t_r(u)}{2}, u\} \right) + \mu \frac{c_\Delta}{(2\mathcal{J})^{2\Delta}} \left[\frac{t_l'(u) t_r'(u)}{\cos^2 \frac{t_l(u) - t_r(u)}{2}} \right]^\Delta \right\}$$

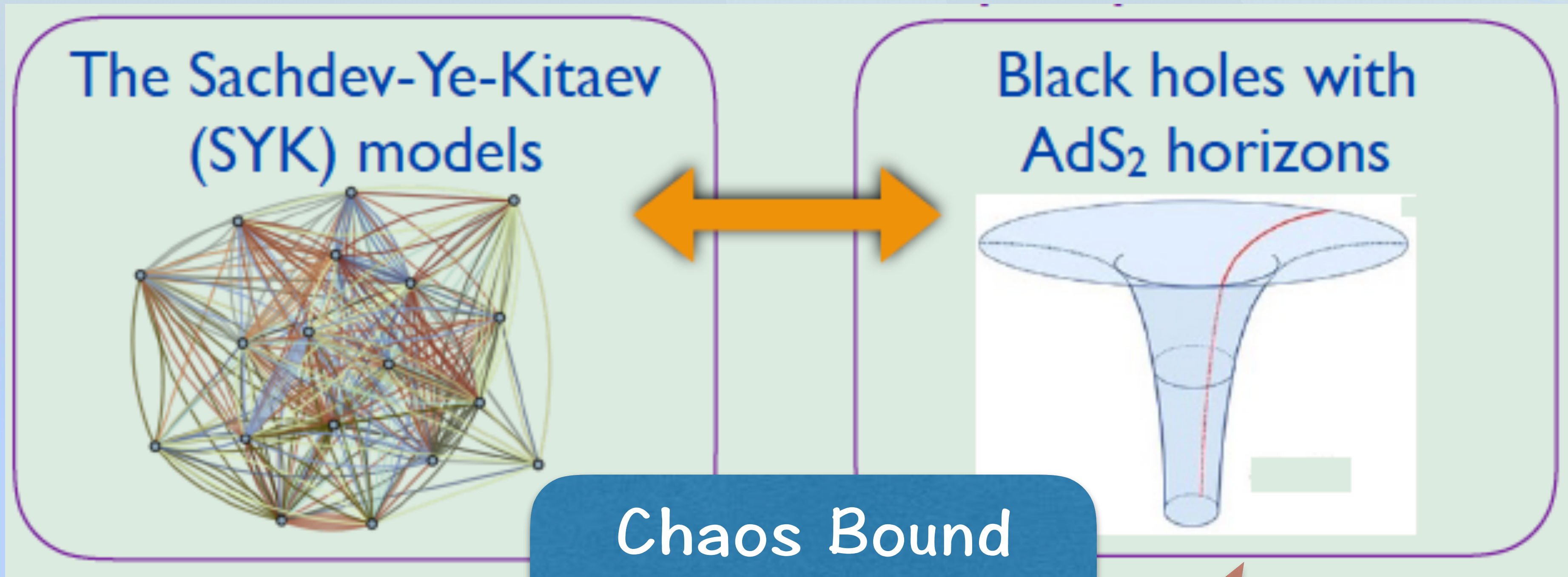


De-Coherent Phase Transition?





Summary



Chaos Bound
Effective Actions
Symmetries

Schwarzian Action
Effective theory of SYK

Einstein-Hilbert Action
2D Dilaton Gravity

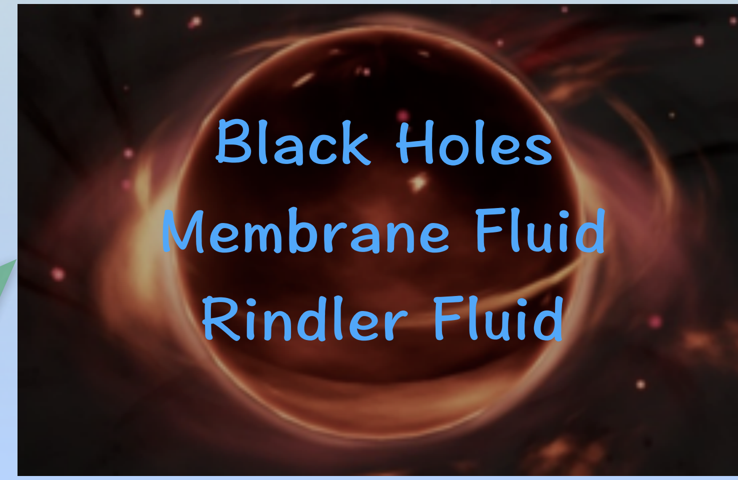
Nambu-Goto Action?
Worldsheet Horizon in AdS

Ref: 1709.06297 [Cai, Ruan, Yang, Zhang]

Summary & Outlook

$$\frac{\eta}{s} \simeq \frac{1}{4\pi}\frac{\hbar}{k_B}$$

$$\tau_c^{-1} \simeq \frac{k^2}{4\pi T_c}$$



$$\Omega_D^2 \simeq \frac{1}{2}\Omega_\Lambda(\Omega_D - \Omega_B)$$

$$\frac{H^2}{H_0^2} \simeq \frac{\Omega_B}{a^3} + \sqrt{\Omega_\Lambda\left(\frac{H^2}{H_0^2} + \frac{\Omega_I}{a^4}\right)}$$

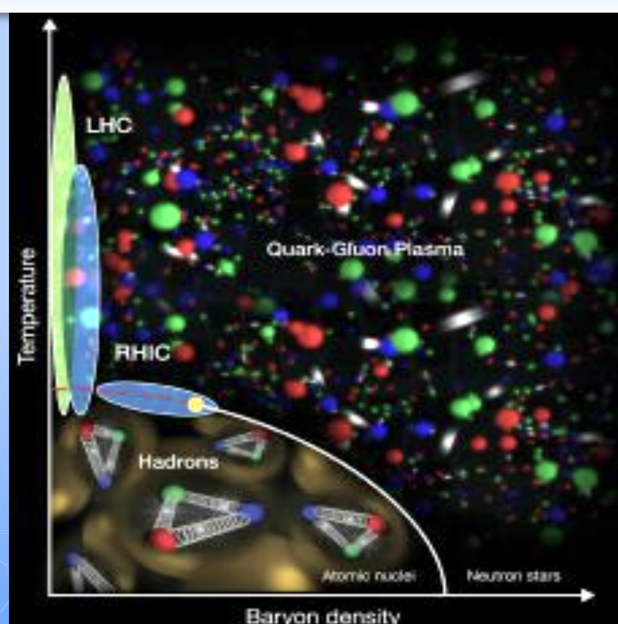
Quark Critical Liquid

QGP in RHIC ['08] & LHC ['16]

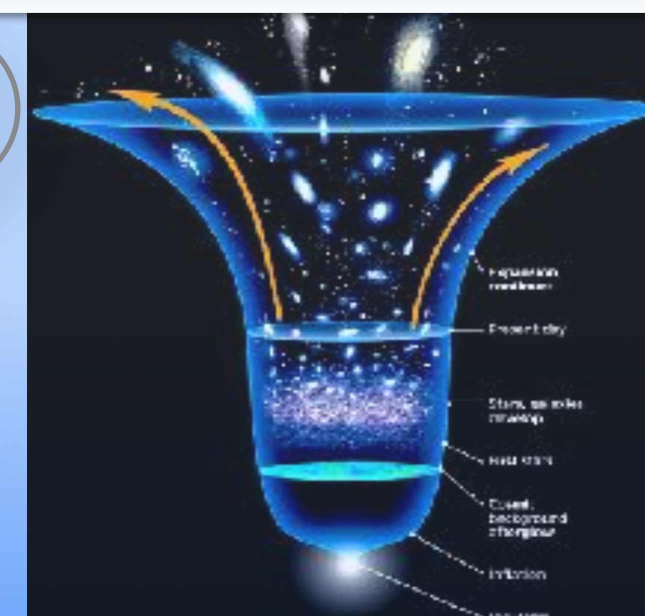
?

Cosmological Fluid

Dark Matter['70s] & Energy['90s]



$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \frac{1}{L}\left(\mathcal{K}_{\mu\nu} - \mathcal{K}g_{\mu\nu}\right) = \kappa_4 T_{\mu\nu}$$



Thanks for All Your Attention!

Ref: 1712.09326 [Cai, Sun, Zhang]

Figures Credit: Nature & Wiki