Gregory-Laflamme instability of black hole in Einstein-scalar-Gauss-Bonnet(ESGB) theory

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Outline of the Talk:

1. Motivation

2. Instability of black hole without scalar hair in ESGB gravity

3. Static scalar perturbation solution in ESGB gravity

4. Summary and future work

1. Motivation to learn

1.1 String Theory

- unification of all fundamental interactions

- dimensions reduction to lower spacetime dimensions:

  $\rightarrow$ low energy effective theories

- Einstein Gravity + higher curvature terms

  $\rightarrow$ Gauss-Bonnet term is the simplest leading term.

$$\mathcal{R}^2_{\rm GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

- EOM contains no more than second derivatives of metric functions
- free of ghosts when expanding about the flat space, evading any problems with unitarity

Among the gravity theories with higher derivative curvature terms——Lovelock gravity

D. L. Wiltshire, Phys. Rev. D 38, 2445 (1988) ; R. G. Cai, Phys. Rev. D 65, 084014 (2002)

1.2 No-Hair Theorem of Black Holes

Israel, Penrose, Wheeler, …

- Stationary black holes (in 4-dim Einstein Gravity) are completely described by 3 parameters of the Kerr-Newman metric :

  mass, charge, and angular momentum (M, Q, J)

- Novel "no-scalar-hair" theorem for black holes

  action

$$S_\psi = -\frac{1}{2}\int [\psi,_\alpha \psi,^\alpha + V(\psi^2)](-g)^{1/2}\, d^4x$$

  action

$$S_{\psi,\chi,\ldots} = -\int \mathscr{E}(\mathscr{I},\mathscr{J},\mathscr{K},\ldots,\psi,\chi,\ldots)(-g)^{1/2}\, d^4x.$$

multiplet of scalar fields $\psi,\chi,\ldots,$ here $\mathscr{E}$ is a function,

$$\mathscr{I} \equiv g^{\alpha\beta}\psi,_\alpha\psi,_\beta, \quad \mathscr{J} \equiv g^{\alpha\beta}\chi,_\alpha\chi,_\beta, \quad \mathscr{K} \equiv g^{\alpha\beta}\chi,_\alpha\psi,_\beta$$

energy-momentum tensor
$$T_\alpha{}^\beta = -\mathscr{E}\delta_\alpha{}^\beta + 2(\partial\mathscr{E}/\partial\mathscr{I})\psi,_\alpha\psi,^\beta + 2(\partial\mathscr{E}/\partial\mathscr{J})\chi,_\alpha\chi,^\beta + (\partial\mathscr{E}/\partial\mathscr{K})(\chi,_\alpha\psi,^\beta + \psi,_\alpha\chi,^\beta).$$

J.D.Bekenstein, Phys.Rev.D.51, no. 12, R6608 (1995).

◆Metric

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta = -e^\nu dt^2 + e^\lambda dr^2 + r^2(d\theta^2 + \sin\theta^2 d\varphi^2),$$

with ν and λ depending on r.

Considering the conservation law $T_\mu{}^\nu{}_{;\nu} = 0,$

r component $[(-g)^{1/2} T_r{}^r]' - (1/2)(-g)^{1/2}(\partial g_{\alpha\beta}/\partial r) T^{\alpha\beta} = 0$

angular components $T_\theta{}^\theta = T_\varphi{}^\varphi$

$$T_r{}^r(r) = -\frac{e^{-\nu/2}}{r^2} \int_{r_h}^{r} (r^2 e^{\nu/2})' \, \mathscr{E} dr.$$

$$T_t{}^t = T_\theta{}^\theta = -\mathscr{E}.$$

a) Near horizon $r = r_h$

e^ν vanishes at $r = r_h$, and must be positive outside it

$r^2 e^{\nu/2}$ grows with r near the horizon.

$\mathscr{E} > 0$

$\Rightarrow T_r{}^r < 0.$

$$(T_r{}^r)' = -e^{-\nu/2} r^{-2} (r^2 e^{\nu/2})' (\mathscr{E} + T_r{}^r)$$

$$\mathscr{E} + T_r{}^r = 2e^{-\lambda}[(\partial\mathscr{E}/\partial\mathscr{T})\psi_{,r}^2 + (\partial\mathscr{E}/\partial\mathscr{Y})\chi_{,r}^2 + (\partial\mathscr{E}/\partial\mathscr{K})\chi_{,r}\psi_{,r}].$$

$\Rightarrow$ $(T_r{}^r)' < 0$

b) At infinity $r \to \infty$

$$(T_r{}^r)' = -e^{-\nu/2} r^{-2} (r^2 e^{\nu/2})' (\mathscr{E} + T_r{}^r) \Rightarrow (T_r{}^r)' < 0 \quad T_r{}^r > 0.$$

No-Hair Theorem!!

◆ Einstein-dilaton-Gauss-Bonnet gravity

P. Kanti et al (1996)

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[R - \frac{1}{2}(\partial_\mu \phi)^2 + \frac{\alpha}{4} e^{\phi} R^2_{\rm GB} \right] ,$$

- The presence of curvature-squared terms can drastically change the situation
- Hairy black hole solution is possible in the Einstein-dilaton-Gauss-Bonnet theory.

1.3 Einstein-dilaton-Gauss-Bonnet(EdGB) gravity

- equations of motions are still of second order and this theory is ghost-free
- One of the consistent modifications of GR
- Compatible with all solar system tests

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Dilatonic black holes in higher curvature string gravity

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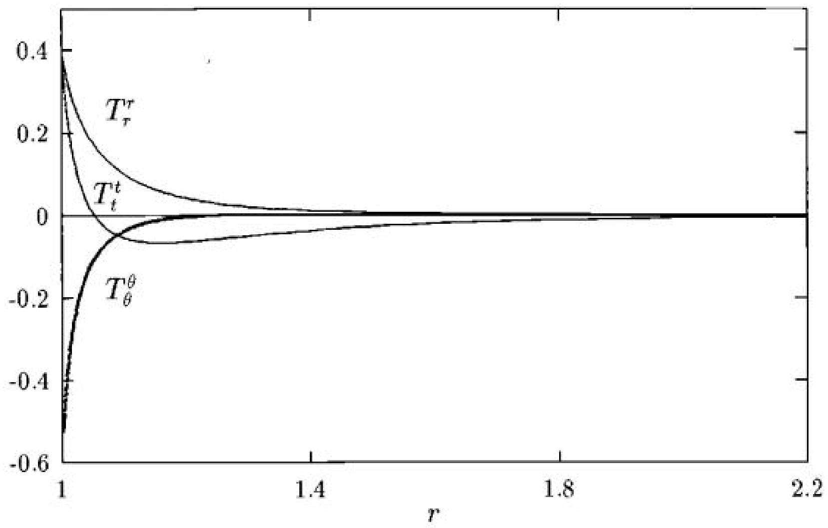
(Received 10 November 1995)

We give analytical arguments and demonstrate numerically the existence of black hole solutions of the 4D effective superstring action in the presence of Gauss-Bonnet quadratic curvature terms. The solutions possess nontrivial dilaton hair. The hair, however, is of ''secondary type,'' in the sense that the dilaton charge is expressed in terms of the black hole mass. Our solutions are not covered by the assumptions of existing proofs of the ''no-hair'' theorem. We also find some alternative solutions with singular metric behavior, but finite energy. The absence of naked singularities in this system is pointed out. [S0556-2821(96)01920-0]

PACS number(s): 04.70.Bw, 04.20.Jb, 04.50.+h, 11.25.Mj

- these BHs contain classical non-trivial dilaton fields so that evade the “no-scalar-hair” theorem.



- static hairy BH solutions in this model were extensively studied perturbatively, and numerically.

S. Mignemi and N. R. Stewart, Phys. Rev. D 47 (1993) 5259 [arXiv:hep-th/9212146].
S. Mignemi, Phys. Rev. D 51 (1995) 934 [arXiv:hep-th/9303102].
T. Torii, H. Yajima and K. i. Maeda, Phys. Rev. D 55 (1997) 739 [arXiv:gr-qc/9606034].
S. O. Alexeev and M. V. Pomazanov, Phys. Rev. D 55 (1997) 2110 [arXiv:hep-th/9605106].
J.L.Blázquez-Salcedo et al., IAU Symp. 324, 265 (2016)arXiv:1610.09214

- Einstein-dilaton-Gauss-Bonnet gravity
  A particular model of Einstein-Scalar-Tensor Gauss-Bonnet gravity

1.4 Einstein-scalar-Gauss Bonnet theory

◆**Action**

$$S=\frac{1}{16\pi}\int d^4x\sqrt{-g}\left[R-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi+f(\phi)R^2_{\rm GB}\right]$$

here a coupling function *f*(ϕ), $\mathcal{R}^2_{\rm GB}=R^2-4R_{\mu\nu}R^{\mu\nu}+R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$

◆ The energy-momentum tensor $T_{\mu\nu}$, conservation law

$D_\mu T^\mu_\nu=0$ ⟹

- r component: $(T^r_r)'=\frac{A'}{2}(T^t_t-T^r_r)+\frac{2}{r}(T^\theta_\theta-T^r_r)$
- angular components: $T^\theta_\theta=T^\varphi_\varphi$

$$T^t_t=-\frac{e^{-2B}}{4r^2}\{\phi'^2[r^2e^B+16\ddot{f}(e^B-1)]-8\dot{f}[B'\phi'(e^B-3)-2\phi''(e^B-1)]\},$$

$$T^r_r=\frac{e^{-B}\phi'}{4}\left[\phi'-\frac{8e^{-B}(e^B-3)\dot{f}A'}{r^2}\right],$$

$$T^\theta_\theta=-\frac{e^{-2B}}{4r}\{\phi'^2(re^B-8\ddot{f}A')-4\dot{f}[\phi'(A'^2+2A'')+A'(2\phi''-3B'\phi')]\}.$$

G. Antoniou, A. Bakopoulos and P. Kanti, Phys. Rev. Lett. 120, no. 13, 131102 (2018), [arXiv:1711.03390 [hep-th]].

- at infinity $r \to \infty$

$$-T^t_t \simeq -T^\theta_\theta \simeq T^r_r \simeq \phi'^2/4 + \mathcal{O}(1/r^6) \quad > 0$$

$$(T^r_r)' \simeq \frac{2}{r}(T^\theta_\theta - T^r_r) \simeq -\frac{1}{r}\phi'^2 \quad < 0$$

- near-horizon regime $r \to r_h$

$$T^r_r = -\frac{2e^{-B}}{r^2}A'\phi'\dot{f} + \mathcal{O}(r - r_h) \quad > 0$$

$$(T^r_r)' = e^{-B}A'\left[-\frac{r\phi'^2}{4Z} - \frac{2(\ddot{f}\phi'^2 + \dot{f}\phi'')}{rZ} + \frac{4\dot{f}\phi'}{r^2}\left(\frac{1}{r} - e^{-B}B'\right)\right] + \mathcal{O}(r - r_h) \quad < 0$$

➢As a result, the no-hair theorem can be evaded.

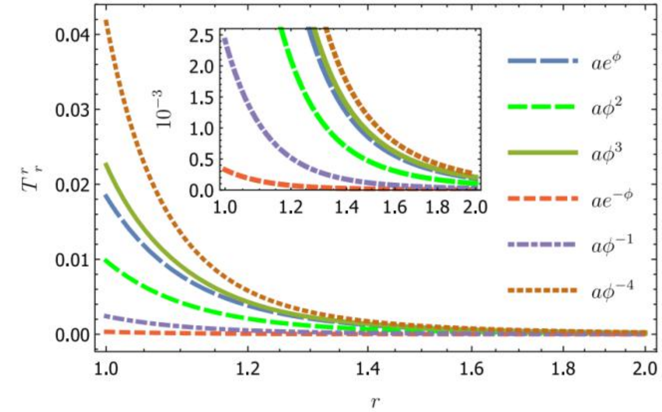


FIG. 2. The T^r_r component for different coupling functions $f(\phi)$, for $a = 0.01$ and $\phi_h = 1$.

➢Numerical Black hole solutions with scalar hair were found from Einstein-scalar-Gauss Bonnet theories.

D. D. Doneva and S. S. Yazadjiev, Phys. Rev. Lett. 120, no. 13, 131103 (2018), [arXiv:1711.01187 [gr-qc]].
H. O. Silva, J. Sakstein, L. Gualtieri, T. P. Sotiriou and E. Berti, Phys. Rev. Lett. 120, no. 13, 131104 (2018) [arXiv:1711.02080 [gr-qc]].

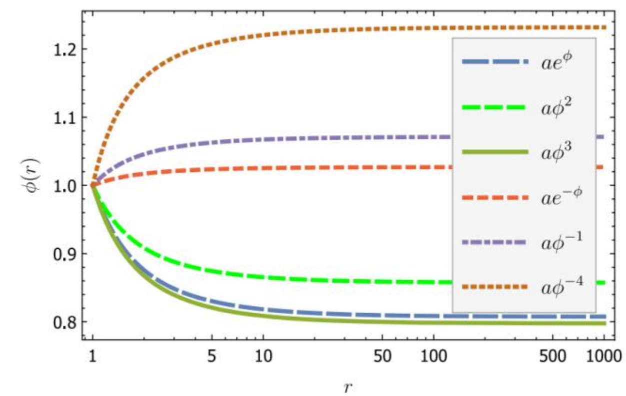


FIG. 1. The scalar field ϕ for different coupling functions $f(\phi)$, for $a = 0.01$ and $\phi_h = 1$.

2. Instability of black hole without scalar hair in ESGB gravity

◆**Action**

$$S_{\rm ESGB} = \frac{1}{16\pi}\int d^4x\sqrt{-g}\Big[R - 2\partial_\mu\phi\partial^\mu\phi - V_\phi + \lambda^2 f(\phi)\mathcal{R}^2_{\rm GB}\Big],$$

where ϕ is the scalar field with a potential V_ϕ *and* we choose $V_\phi = 0$.

◆**Equation of motion**

$$G_{\mu\nu} = 2\partial_\mu\phi\partial_\nu\phi - (\partial\phi)^2 g_{\mu\nu} + \Gamma_{\mu\nu},$$

where $G_{\mu\nu} = R_{\mu\nu} - (R/2)g_{\mu\nu}$ is the Einstein tensor and $\Gamma_{\mu\nu}$ is given by

$$\begin{aligned}\Gamma_{\mu\nu} &= 2R\nabla_{(\mu}\Psi_{\nu)} + 4\nabla^\alpha\Psi_\alpha G_{\mu\nu} - 8R_{(\mu|\alpha|}\nabla^\alpha\Psi_{\nu)} \\ &+ 4R^{\alpha\beta}\nabla_\alpha\Psi_\beta g_{\mu\nu} - 4R^\beta{}_{\mu\alpha\nu}\nabla^\alpha\Psi_\beta\end{aligned}$$

with

$$\Psi_\mu = \lambda^2\frac{df(\phi)}{d\phi}\partial_\mu\phi = \lambda^2 f'(\phi)\partial_\mu\phi.$$

Y. S. Myung and D. C.Zou, arXiv:1805.05023 [gr-qc].

◆**Scalar field equation**

$$\Box\phi + \frac{\lambda^2}{4} f'(\phi)\mathcal{R}_{\rm GB}^2 = 0.$$

◆**Choosing ϕ = 0 and $f'(\phi)|_{\phi=0} = 0$**

$$ds^2 = \bar{g}_{\mu\nu}dx^\mu dx^\nu = -\left(1 - \frac{r_+}{r}\right)dt^2 + \frac{dr^2}{\left(1 - \frac{r_+}{r}\right)} + r^2 d\Omega_2^2$$

with $r_+ = 2M$

Schwarzschild black hole solution

◆ **stability analysis**

the metric perturbation $h_{\mu\nu}$ and scalar perturbation $\delta\phi$ propagating around Schwarzschild solution

$$\begin{aligned} \delta R_{\mu\nu}(h) &= 0, \\ \left(\bar{\Box} + \frac{\lambda^2}{4}\bar{\mathcal{R}}_{\rm GB}^2\right)\delta\phi &= 0. \end{aligned}$$

overbar(‾) denotes computation based on the Schwarzchild solution

"$-\frac{\lambda^2}{4}\bar{\mathcal{R}}_{\rm GB}^2$" plays a role of not a mass $\tilde{m}^2$ but an effective mass $\tilde{m}_{\rm eff}^2$ for $\delta\phi$

◆Linearized scalar field equation

Considering

$$\delta\phi(t,r,\theta,\varphi) = \frac{u(r)}{r} e^{-i\omega t} Y_{lm}(\theta,\varphi),$$

introducing a tortoise coordinate $r_* = r + r_+ \ln(r/r_+ - 1)$

defined by $dr_* = dr/(1 - r_+/r)$

- the radial equation of perturbation scalar field equation

$$\frac{d^2 u}{dr_*^2} + \left[\omega^2 - V(r)\right] u(r) = 0,$$

where the potential *V* (*r*) is $V(r) = \left(1 - \frac{2M}{r}\right)\left[\frac{2M}{r^3} + \frac{l(l+1)}{r^2} - \frac{12\lambda^2 M^2}{r^6}\right]$

- a sufficient condition of an unstable bound

W. Buell et al (1995)

$$\int_{2M}^{\infty} dr V(r)/(1 - r_+/r) < 0 \quad \Longrightarrow \quad \frac{M^2}{\lambda^2} < \frac{3}{10} \Rightarrow 0 < \frac{r_+}{\lambda} < 1.095$$

W. Buell and B. Shadwick, Am. J. Phys. 63, 256 (1995)

◆threshold of instability

- the second-order differential equation

$\omega = i\Omega$ →

$$\frac{d^2u}{dr_*^2} - \left[\Omega^2 + V(r)\right]u(r) = 0,$$

- **sufficient condition** of an unstable bound

$$\frac{M^2}{\lambda^2} < \frac{3}{10} \Rightarrow 0 < \frac{r_+}{\lambda} < 1.095$$

not a necessary and sufficient condition

- two boundary conditions:

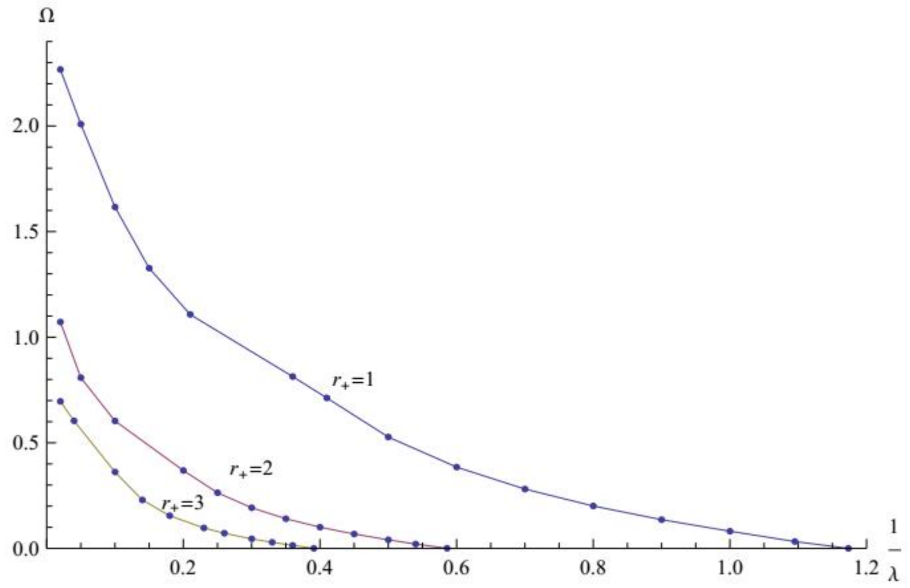
at infinity $u(\infty) \sim e^{-\Omega r_*}$

near the horizon $u(r_+) \sim (r - r_+)^{\Omega r_+}$

we read off the unstable bound for scalar mass parameter (1/λ)

$$0 < \frac{1}{\lambda} < \left(\frac{1}{\lambda}\right)^{\rm th} \approx \frac{1.174}{r_+}$$

the threshold of instability is located at $r_+ = r_c \approx 1.174$ which is greater than 1.095 (sufficient condition for instability)



Ω graphs as function of mass parameter $1/\lambda$ for small black holes of $r_+ = 1, 2, 3$.

◆ For comparison, we would like to mention the Lichnerowicz-Ricci tensor equation around the Schwarzschild black hole in the Einstein-Weyl gravity

$$S_{\mathrm{EW}} = \int d^4x\sqrt{-g}\Big[\gamma R - \alpha C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\Big]$$

: $C_{\mu\nu\rho\sigma}$ is the Weyl tensor

The Lichnerowicz-Ricci tensor equation for the traceless and transverse Ricci tensor $\delta R_{\mu\nu}$

$$\left(\Delta_{\mathrm{L}} + m^2\right)\delta R_{\mu\nu} = 0, \quad m^2 = \frac{\gamma}{2\alpha},$$

$$\Delta_{\mathrm{L}}\delta R_{\mu\nu} = -\bar{\Box}\delta R_{\mu\nu} - 2\bar{R}_{\mu\rho\nu\sigma}\delta R^{\rho\sigma}.$$

Which describes a massive spin-2 mode $(\delta R_{\mu\nu})$ with mass m propagating on the black hole background

the limit of $\alpha \to 0$

$$(2\alpha\Delta_{\mathrm{L}} + \gamma)\delta R_{\mu\nu} = 0. \Longrightarrow \delta R_{\mu\nu} = 0$$

the Gregory-Laflamme instability mass bound for the *s*(*l* = 0)-mode

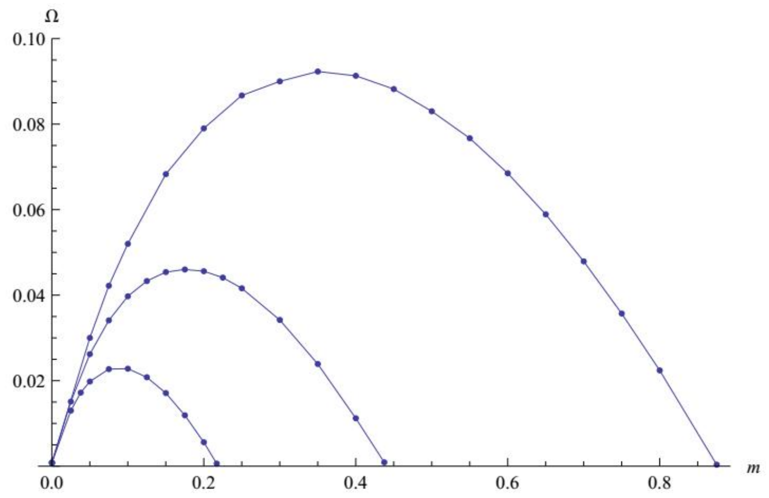
of linearized Ricci tensor δR_{tr}

$$0 < m < m^{\rm th} \approx \frac{0.876}{r_+}$$

selecting $m = 1$

→ the bound for unstable (small) black holes

$$r_+ < r_c \approx 0.876.$$



Plots of unstable modes (•) on three curves with the horizon radii $r_+ = 1, 2, 4$

Y.~S.~Myung, Phys. Rev. D,88, no. 2, 024039 (2013) [arXiv:1306.3725 [gr-qc]]

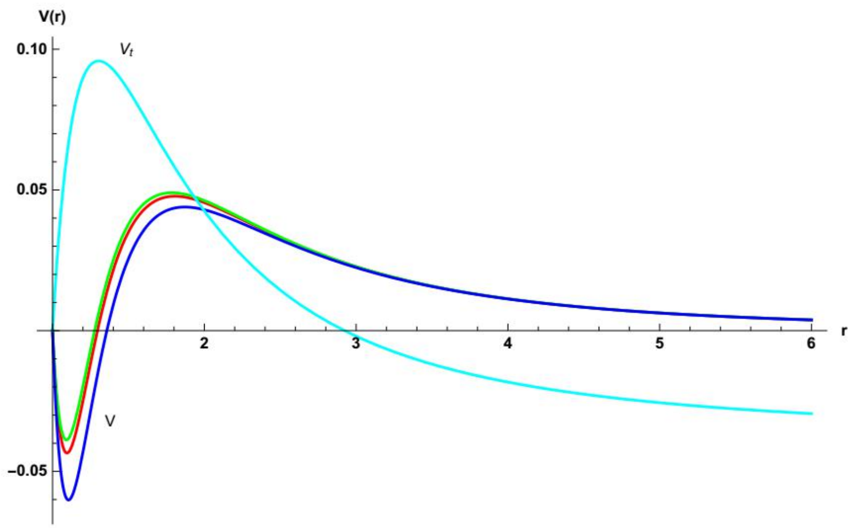
➢ introducing the negative scalar potential $V_\phi = -m_{\rm T}^2\phi^2/2$ instead of $-\lambda^2 f(\phi)\mathcal{R}^2_{\rm GB}$

the tachyonic potential takes the form

$$V_{\rm t}(r) = \left(1-\frac{2M}{r}\right)\left[\frac{2M}{r^3}+\frac{l(l+1)}{r^2}-m_{\rm T}^2\right]$$

not types of **Regge-Wheeler potentials** which are positive definite outside the horizon

➢Zerlli-type potential for GL instability



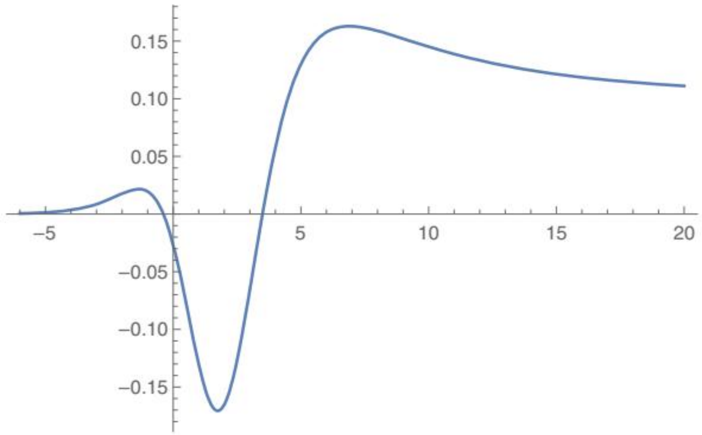


FIG. 2. Zerilli-type potential $V(r_*)$ in $n = 4$ dimensions for the Schrödinger problem.

- blue (bottom) curve: 1/*λ* = 1.095 (**sufficient condition for instability**),
- red (middle) curve: 1/*λ* = 1.174 (**threshold of instability**),
- green (top) curve : 1/*λ* =1.2 (**stable case**)

H. L, et al, Phys. Rev. D **96**, no. 4, 046006 (2017) [arXiv:1704.05493 [hep-th]]

3. Static scalar perturbation solution

- Considering $\omega = 0(\Omega = 0)$, the radial equation of perturbation scalar field

$$\frac{r^5(r_+ - r)}{3r_+^2}u''(r) + \frac{r^4}{3r_+}u'(r) - \left[\frac{r^3}{3r_+} + \frac{l(l+1)r^4}{3r_+^2}\right]u(r) = \lambda^2 u(r)$$

a new coordinate $z = \frac{r}{r_+}[z \in [1, \infty)]$

a new parameter $\lambda_s = \frac{\lambda}{r_+}$

$$\frac{z^5(z-1)}{3}u''(z) + \frac{z^4}{3}u'(z) - \frac{z^4}{3}\left[\frac{1}{z} + l(l+1)\right]u(z) = \lambda_s^2 u(z)$$

- numerical solution

consider the near-horizon Taylor expansion for *u*(*z*)

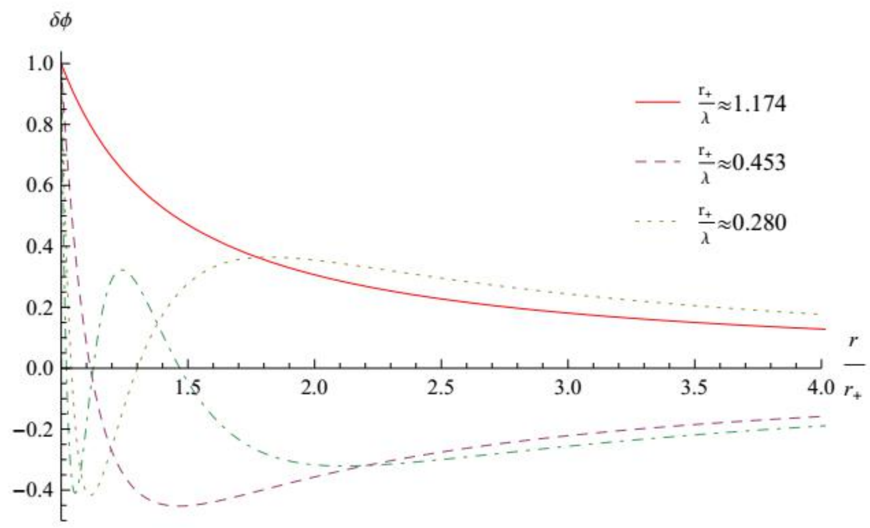
$$u(z) = u_+ + u'_+(z-1) + \frac{u''_+}{2}(z-1)^2 + \cdots$$

$$u'_+ = (1 - 3\lambda_s^2)u_+ \text{ and } u''_+ = \frac{3(3\lambda_s^2+2)}{2\lambda_s^6}u_+^2$$

an asymptotic form of $u(z)$ near $z = \infty$

$$u(z) = u_\infty + \frac{u^{(1)}}{z} + \frac{u^{(2)}}{z^2} + \cdots \qquad u^{(1)} = u_\infty/2 \text{ and } u^{(2)} = u_\infty/3.$$

➢The numerical solutions



➢We obtain a discrete spectrum of parameter:

$$1/\lambda_s = r_+/\lambda \in [1.174, 0.453, 0.280, 0.202, \cdots]$$

- these solutions are classified by order number $n = 0, 1, 2, 3, \cdots$
  which is identified with the number of nodes for $\delta\phi(z) = u(z)/z$

- the n = 0 scalar mode without zero represents a stable black hole, while the n = 1, 2 scalar modes with zero denote unstable black holes.

- a regular solution to perturbation equation with Ω = 0 is found only when the parameter λ takes a specific value $r_+/\lambda \approx 1.174$ (threshold of instability=the edge of domain of instability).

- In the small mass regime of $1/\lambda < 1.174/r_+$, the Schwarzschild solution becomes unstable and a new branch of solution with scalar hair bifurcates from the Schwarzschild one.

4. Summary and future work

- the instability of the Schwarzschild black hole in ESGB theory can be interpreted as a scalar theory version of the GL instability for a small black hole in the tensor theory of Einstein-Weyl gravity.

Theory	ESGB theory	Einstein-Weyl gravity
Action	$S_{\rm ESGB}$ in (1)	$S_{\rm EW}$ in (10)
BH without hair	SBH with $\bar{\phi}(r) = \bar{R}_{\mu\nu} = 0$	SBH with $\bar{R}_{\mu\nu} = 0$
Linearized equation	scalar equation (9)	tensor equation (11)
GL instability mode	s($l = 0$)-mode of $\delta\phi$	s($l = 0$)-mode of $\delta R_{\mu\nu}$
Unstable mass bound	$0 < \frac{1}{\lambda} < \frac{1.174}{r_+}$	$0 < m < \frac{0.876}{r_+}$
Bifurcation point	1.174	0.876
Potential	$V(r)$ in (15)	$V_z(r)$ (18) in Ref.[19]
Small unstable SBH	$r_+ < r_c \approx 1.174$ for $\frac{1}{\lambda} = 1$	$r_+ < r_c \approx 0.876$ for $m = 1$
BH with hair	scalar hair in Refs.[2, 3]	Ricci-tensor hair in Ref.[13]

Table 1: Similar Properties for Schwarzschild black hole (SBH) in ESGB theory and Einstein-Weyl gravity.

- Future work
  - instability of the full numerical solutions in ESGB theory
  - C.A R.Herdeiro, E.Radu, N. Sanchis-Gual and J.A. Font, ``**Spontaneous scalarisation of charged black holes**,' ‘ arXiv:1806.05190 [gr-qc].

Thank you!