Vanishing OPE coefficients in 4d N=2 SCFTs

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Strings, Branes and Gauge Theories

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- Based on arXiv: 1812.04743
- In collaboration with Jaewon Song & Sungjay Lee

Plan of the talk

- ❑ 4d N=2 theories and 2d Chiral Algebras
- ❑ Structure of the Superconformal Index of the AD theory
- ❑ OPE coefficients in generalized AD theories

4d N=2 theories & 2d Chiral Algebras

- 4d N=2 Superconformal theories have a subset of operators called Schur Operators

- Defining property: $E = 2R + j_1 + j_2 \ , \ r = j_2 - j_1$

- Their OPEs, when restricted to a plane, have a meromorphic subsector with the non-meromorphic part being cohomologically-trivial [Beem, Lemos, Liendo, Peelaers, Rastelli, van Rees '15]

$$\mathcal{O}_1(z,\bar{z})\mathcal{O}_2(0,0) = \sum_{k_{Schur}} \frac{\lambda_{123}}{z^{h_1+h_2-h_k}} \mathcal{O}_k + \{\mathfrak{Q}, \ldots]$$

$$h = \frac{E + (j_1 + j_2)}{2}$$

- Upto $\mathfrak{Q} - \mathrm{cohomology}$, the Schur operators behave like 2d meromorphic operators with holomorphic dimension h

- Also, the meromorphic part of the OPE of the 4d R-current $J^{11}_{+\dot{+}}$, has the same structure as a 2d stress-tensor

- It follows that the 2d chiral algebra must have a central charge

$$c_{2d} = -12 c_{4d}$$

- Similarly, 4d moment map operators of flavor symmetries give 2d flavor current s.t.

$$k_{2d} = -\frac{k_{4d}}{2}$$

- The torus partition function of the 2d chiral algebra corresponds to the Schur limit of the 4d Superconformal index

An Example

- Argyres-Douglas (AD) theory (A_1, A_2) has $c_{4d} = \frac{11}{30}$
- Underlying chiral algebra must have $c_{2d} = -\frac{22}{5}$
- Same as the central charge of the celebrated Lee-Yang model
- Indeed, its vacuum character matches with the Schur Index of the 4d theory [Cordova-Shao '15][Song '16]

- Notice that the vacuum module of the Lee-Yang model has a null state at level 4

$$(L_{-2}^2 - \frac{3}{5}L_{-4})|\Omega\rangle = 0$$

- What is the corresponding statement in 4d?

- The answer was provided by Liendo, Ramirez, Seo
- OPE of two 4d stress-tensors contains a multiplet $\mathcal{C}_{1(\frac{1}{2},\frac{1}{2})}$
- The interplay of 4d-2d physics implies the corresponding OPE coefficient should be

$$\lambda^2_{\mathcal{C}_{1(\frac{1}{2},\frac{1}{2})}} \propto \left(2 - \frac{11}{15 c_{4d}}\right)$$

- The OPE coefficient is exactly zero for the AD theory
- The $\mathcal{C}_{1(\frac{1}{2},\frac{1}{2})}$ multiplet is absent in the AD theory
- The 2d operator corresponding to $\mathcal{C}_{1(\frac{1}{2},\frac{1}{2})}$ has

$$h = 1 + \frac{1}{2} + \frac{1}{2} + 2 = 4$$

- Consistent with the level-4 null state in the Lee-Yang model

- Lesson: presence of null states in the underlying 2d chiral algebra indicates the absence of certain superconformal multiplets in the 4d theory.

- Caveat: holomorphic dimension of the 2d null state is not enough to identify the corresponding 4d operator

- Liendo, Ramirez, Seo had to work out the explicit OPE selection rules for 4d N=2 stress-tensors to avoid this caveat

- Claim: one can instead use superconformal indices and the representation theory of N=2 superconformal group [Song '15] [PA, Song, Lee '18]

Structure of the Supeconformal Index of AD theory

- The Macdonald index of the AD theory can be written in the following form [Song '15]

$$I_{(A_1,A_2)} = \mathrm{PE}\left[\frac{q^2T-(q^2T)^2}{1-q} + \mathcal{O}(q^7T^3)\right]$$

- The letter index (in Macdonald limit) of 4d N=2 stress tensor is

$$I_{\mathcal{C}_{0(0,0)}} = \frac{q^2T}{1-q}$$

- We identify the first term in the PE for the AD theory index as coming from the stress-tensor multiplet

$$I_{(A_1,A_2)} = \mathrm{PE}\left[\frac{q^2T-(q^2T)^2}{1-q} + \mathcal{O}(q^7T^3)\right]$$

- The minus sign in the second term might correspond to a fermionic operator
- This cannot be: the underlying Lee-Yang model is purely bosonic and has no fermionic generators
- It therefore actually indicates the absence of a superconformal multiplet that generically appears in the tensor product of two stress tensors
- Need representation theory of 4d N=2 superconformal multiplets

- Implemented the Cordova-Dumitrescu-Intriligator algorithm

- Definen the superconformal character as

$$\chi_V(q, z_1, z_2, y, s) = \mathrm{Tr}\ \mathrm{q}^{\Delta} \mathrm{z}_1^{2\mathrm{j}_1} \mathrm{z}_2^{2\mathrm{j}_2} \mathrm{y}^{2\mathrm{R}} \mathrm{s}^{-2\mathrm{r}}$$

- Trace is over all the states in the superconformal multiplet

- Use this to decompose the tensor product of two or more supermultiplets into a sum over the various possible supermultiplets

$$
\begin{aligned}
\chi_{\hat{\mathcal{C}}_{0(0,0)}} \times \chi_{\hat{\mathcal{C}}_{0(0,0)}} = {} & \chi_{\mathcal{A}^4_{0,0(0,0)}} + \chi_{\mathcal{C}_{\frac{1}{2},\frac{1}{2}(\frac{1}{2},0)}} + \chi_{\bar{\mathcal{C}}_{\frac{1}{2},-\frac{1}{2}(0,\frac{1}{2})}} + \chi_{\mathcal{C}_{0,1(1,0)}} + \chi_{\bar{\mathcal{C}}_{0,-1(0,1)}} \\
& + \chi_{\hat{\mathcal{C}}_{1(\frac{1}{2},\frac{1}{2})}} + 2\chi_{\mathcal{A}^5_{0,0(\frac{1}{2},\frac{1}{2})}} + 2\chi_{\mathcal{C}_{\frac{1}{2},\frac{1}{2}(1,\frac{1}{2})}} + 2\chi_{\bar{\mathcal{C}}_{\frac{1}{2},-\frac{1}{2}(\frac{1}{2},1)}} \\
& + \chi_{\mathcal{C}_{0,1(\frac{3}{2},\frac{1}{2})}} + \chi_{\bar{\mathcal{C}}_{0,-1(\frac{1}{2},\frac{3}{2})}} + \chi_{\hat{\mathcal{C}}_{1(1,1)}} + 3\chi_{\mathcal{A}^6_{0,0(1,1)}} + \chi_{\mathcal{A}^6_{0,0(1,0)}} \\
& + \chi_{\mathcal{A}^6_{0,0(0,1)}} + \chi_{\mathcal{A}^6_{0,0(0,0)}} + \cdots \ .
\end{aligned}
$$

- There is a unique supermultiplet appearing in this decomposition that can contribute like the term we need

$$\chi_{\hat{\mathcal{C}}_{0(0,0)}} \times \chi_{\hat{\mathcal{C}}_{0(0,0)}} = \chi_{\mathcal{A}^4_{0,0(0,0)}} + \chi_{\mathcal{C}_{\frac{1}{2},\frac{1}{2}(\frac{1}{2},0)}} + \chi_{\bar{\mathcal{C}}_{\frac{1}{2},-\frac{1}{2}(0,\frac{1}{2})}} + \chi_{\mathcal{C}_{0,1(1,0)}} + \chi_{\bar{\mathcal{C}}_{0,-1(0,1)}}$$
$$+ \chi_{\hat{\mathcal{C}}_{1(\frac{1}{2},\frac{1}{2})}} + 2\chi_{\mathcal{A}^5_{0,0(\frac{1}{2},\frac{1}{2})}} + 2\chi_{\mathcal{C}_{\frac{1}{2},\frac{1}{2}(1,\frac{1}{2})}} + 2\chi_{\bar{\mathcal{C}}_{\frac{1}{2},-\frac{1}{2}(\frac{1}{2},1)}}$$
$$+ \chi_{\mathcal{C}_{0,1(\frac{3}{2},\frac{1}{2})}} + \chi_{\bar{\mathcal{C}}_{0,-1(\frac{1}{2},\frac{3}{2})}} + \chi_{\hat{\mathcal{C}}_{1(1,1)}} + 3\chi_{\mathcal{A}^6_{0,0(1,1)}} + \chi_{\mathcal{A}^6_{0,0(1,0)}}$$
$$+ \chi_{\mathcal{A}^6_{0,0(0,1)}} + \chi_{\mathcal{A}^6_{0,0(0,0)}} + \cdots .$$

- This is agrees with the result found by Liendo, Ramirez, Seo

OPE coefficients in generalized AD theories

(A_1, A_{2n}) theory

- The Macdonald index of these theories is given by

$$I_{(A_1,A_{2n})} = \mathrm{PE}\left[\frac{\mathrm{q}^2\mathrm{T}-(\mathrm{q}^2\mathrm{T})^{\mathrm{n}+1}}{(1-\mathrm{q})} + \mathrm{O}(\mathrm{q}^{2\mathrm{n}+2})\right]$$

- The underlying chiral algebra has been conjectured to be the (2, 2n+3) Virasoro Minimal model

- This is again bosonic with no fermionic generators

- The negative term in the PE indicates the absence of a certain superconformal multiplet that appears in the tensor product of ‘n+1’ copies of stress-tensors

- By explicitly computing for some low values of n = 1, 2, 3, 4, …, we find that there is exactly one such supermultiplet that does this : $\mathcal{C}_{n(\frac{n}{2},\frac{n}{2})}$

- Moreover, the tensor product of ‘n’ copies of the stress tensor contains $\mathcal{C}_{n-1(\frac{n-1}{2},\frac{n-1}{2})}$

$$(\mathcal{C}_{0(0,0)})^n = \ldots + \mathcal{C}_{n-1(\frac{n-1}{2},\frac{n-1}{2})} + \ldots$$

- Also $\mathcal{C}_{0(0,0)} \otimes \mathcal{C}_{n-1(\frac{n-1}{2},\frac{n-1}{2})} = \ldots + \mathcal{C}_{n(\frac{n}{2},\frac{n}{2})} + \ldots$

- Now $(\mathcal{C}_{0(0,0)})^{n+1} = \mathcal{C}_{0(0,0)} \otimes (\mathcal{C}_{0(0,0)})^n$

- The vanishing multiplet in (A_1, A_{2n}) theory actually arises from the tensor product: $\mathcal{C}_{0(0,0)} \otimes \mathcal{C}_{n-1(\frac{n-1}{2},\frac{n-1}{2})}$

- The absence of $\mathcal{C}_{n(\frac{n}{2},\frac{n}{2})}$ in the (A_1, A_{2n}) theory therefore implies that the OPE coefficient

$$\lambda\left[\mathcal{T},\mathcal{C}_{n-1\left(\frac{n-1}{2},\frac{n-1}{2}\right)},\mathcal{C}_{n\left(\frac{n}{2},\frac{n}{2}\right)}\right]=0$$

- This gives us the 4d lift of the null state at level 2n+2 in the (2, 2n+3) Virasoro Minimal model

Implications for general 4d N=2 theories

- The 4d OPE coefficient is proportional to the 2d OPE coefficient

- On the other hand, Virasoro symmetry is enough to constrain the 2d OPE coefficient to vanish whenever the 2d central charge equals that of the (2, 2n+3) minimal model

- We thus expect that the 4d OPE coefficient $\lambda\left[\mathcal{T},\mathcal{C}_{n-1(\frac{n-1}{2},\frac{n-1}{2})},\mathcal{C}_{n(\frac{n}{2},\frac{n}{2})}\right]$ will vanish for all 4d N=2 theories that have the same central charge 'c' as the (A_1, A_{2n}) theory

$(G^h[n], F)$ type AD theories

- h is the dual coxeter number of the Lie algebra G
- The AD theory has a flavor symmetry G when (h,n)=1
- The Schur index can be written as

$$I_{(G^h[n],F)} = \mathrm{PE}\left[\frac{q-q^{h+n}}{(1-q)(1-q^{h+n})}\chi_{\mathrm{adj}}(\vec{z})\right]$$

- The first term in the PE corresponds to 4d flavor currents (B_1)

- The underlying chiral algebra is the G-type affine Kac-Moody with level

$$k_{2d} = -h + \frac{h}{h+n}$$

- This is also bosonic with no fermionic generators

- Thus the negative term in the 4d Schur index again indicates the absence of some operator appearing in the (h+n)-fold tensor product of flavor currents

- By using superconformal characters as before, we again find that there is a unique possibility: the B_{h+n} multiplet

- In terms of OPE, we conclude that for these theories

$$\lambda\left[B_1, B_{h+n-1}, B_{h+n}\right]\Big|_{\mathrm{adj}} = 0$$

The (A_2, A_{3k}) and (A_2, A_{3k+1}) theory

- The underlying chiral algebras are the W-minimal model $\mathcal{W}(3, 3k+4)$ and $\mathcal{W}(3, 3k+5)$ respectively

- One can apply the prescription of Macdonald grading to these to obtain the Macdonald index [Song ' 16] [Beem, Rastelli '17]

- The result is

$$I_{(A_2,A_{3k})} = \mathrm{PE}\Big[\frac{q^2T+q^3T^2-(q^3T^2)^k q^2T-(q^3T^2)^{k+1}}{1-q} + O(q^{3k+4})\Big]$$

$$I_{(A_2,A_{3k+1})} = \mathrm{PE}\Big[\frac{q^2T+q^3T^2-(q^3T^2)^{k+1}-(q^3T^2)^k(q^2T)^2}{1-q} + O(q^{3k+5})\Big]$$

- As before, the $\frac{q^2T}{1-q}$ term is the letter index for the stress-tensor $\mathcal{C}_{0(0,0)}$

- The $\frac{q^3T^2}{1-q}$ piece encapsulates the contribution of the multiplet $\mathcal{C}_{1(0,0)}$

$$I_{(A_2,A_{3k})} = \mathrm{PE}\Big[\frac{q^2T+q^3T^2-(q^3T^2)^k q^2T-(q^3T^2)^{k+1}}{1-q} + O(q^{3k+4})\Big]$$

$$I_{(A_2,A_{3k+1})} = \mathrm{PE}\Big[\frac{q^2T+q^3T^2-(q^3T^2)^{k+1}-(q^3T^2)^k(q^2T)^2}{1-q} + O(q^{3k+5})\Big]$$

- The negative terms in $I_{(A_2,A_{3k})}$ indicate absence of certain multiplets appearing in the tensor products: $(\mathcal{C}_{1(0,0)})^k\mathcal{C}_{0(0,0)}$ and $(\mathcal{C}_{1(0,0)})^{k+1}$
- The negative terms in $I_{(A_2,A_{3k+1})}$ indicate the absence of certain multiplets in the tensor products: $(\mathcal{C}_{1(0,0)})^{k+1}$ and $(\mathcal{C}_{1(0,0)})^k(\mathcal{C}_{0(0,0)})^2$

- As before, we find that there is a unique multiplet appearing in these tensor products whose absence accounts for these negative terms

$$\left(\mathcal{C}_{1(0,0)}\right)^{k+1} \ni \mathcal{C}_{2k+1(\frac{k}{2},\frac{k}{2})}$$

$$\mathcal{C}_{0(0,0)} \otimes \left(\mathcal{C}_{1(0,0)}\right)^{k} \ni \mathcal{C}_{2k(\frac{k}{2},\frac{k}{2})}$$

$$\left(\mathcal{C}_{0(0,0)}\right)^{2} \otimes \left(\mathcal{C}_{1(0,0)}\right)^{k} \ni \mathcal{C}_{2k+1(\frac{k+1}{2},\frac{k+1}{2})}$$

- By carefully looking at the possible 'channels' in which these short multiplets can appear we find that the following OPE coefficients should vanish

- In (A_2, A_{3k}) theory:

$$\lambda \left[\mathcal{C}_{1(0,0)}, \mathcal{C}_{2k-1(\frac{k-1}{2},\frac{k-1}{2})}, \mathcal{C}_{2k+1(\frac{k}{2},\frac{k}{2})}\right] = 0$$

$$\lambda \left[\mathcal{C}_{0(0,0)}, \mathcal{C}_{2k-1(\frac{k-1}{2},\frac{k-1}{2})}, \mathcal{C}_{2k(\frac{k}{2},\frac{k}{2})}\right] = 0$$

$$\lambda \left[\mathcal{C}_{1(0,0)}, \mathcal{C}_{2k-2(\frac{k-1}{2},\frac{k-1}{2})}, \mathcal{C}_{2k(\frac{k}{2},\frac{k}{2})}\right] = 0$$

- In (A_2, A_{3k+1}) theory:

$$\lambda\left[\mathcal{C}_{1(0,0)}, \mathcal{C}_{2k-1(\frac{k-1}{2},\frac{k-1}{2})}, \mathcal{C}_{2k+1(\frac{k}{2},\frac{k}{2})}\right] = 0$$

$$\lambda\left[\mathcal{C}_{1(\frac{1}{2},\frac{1}{2})}, \mathcal{C}_{2k-1(\frac{k-1}{2},\frac{k-1}{2})}, \mathcal{C}_{2k+1(\frac{k+1}{2},\frac{k+1}{2})}\right] = 0$$

$$\lambda\left[\mathcal{C}_{0(0,0)}, \mathcal{C}_{2k(\frac{k}{2},\frac{k}{2})}, \mathcal{C}_{2k+1(\frac{k+1}{2},\frac{k+1}{2})}\right] = 0$$

$$\lambda\left[\mathcal{C}_{1(0,0)}, \mathcal{C}_{2k-1(\frac{k}{2},\frac{k}{2})}, \mathcal{C}_{2k+1(\frac{k+1}{2},\frac{k+1}{2})}\right] = 0$$

Summary

- All 4d N=2 theories have an underlying chiral algebra
- The infinite conformal symmetry of this chiral algebra can be used to exactly solve for certain subsectors of 4d correlation functions and OPEs
- The null states in the 2d chiral algebra indicate the absence of a corresponding 4d operator
- However, identifying the exact 4d operator that gave rise to a particular 2d operator needs more input

- One way is to explicitly work out explicit OPE selection rules and use that to try to identify vanishing 4d operator

- This is somewhat tedious

- We propose that for certain classes of AD theories, one can use their superconformal indices and superconformal representation theory to lift the 2d null states to 4d

- The OPE coefficients involving these operators should necessarily be zero

- The underlying chiral algebras for (A_1, A_{2n}) AD theories are given by Virasoro minimal models
- The vanishing of their OPE coefficients is dictated by Virasoro symmetry alone
- Thus we expect that the corresponding OPE coefficients will vanish for all 4d N=2 theories whose 'c' coincides with an (A_1, A_{2n}) type AD theory
- Similar considerations and results also hold for $(G^h[n], F)$ type AD theories whose underlying algebras are given by affine Kac-Moody algebras

- Also described similar results for the (A_2, A_{3k}) and (A_2, A_{3k+1}) type AD theories

- Results for the more general (A_{k-1}, A_{n-1}) can be found in our paper

감사합니다

Thank You !

- we expand the product of the characters as a series in the variable q. Each monomial in this expansion represents an operator whose scaling dimension and $SU(2)_{j1}\times SU(2)_{j2}\times SU(2)_R\times U(1)_r$ charges can be read-off from the monomial. The coefficient of the monomial represents the multiplicity of such operators. The monomial with the lowest power of q in this expansion necessarily represents a superconformal primary. Thus we are guaranteed to have the corresponding superconformal multiplet. We now subtract the character of this superconformal multiplet from our product to obtain a series which start at some higher power of q. This then gives us the next supermultiplet that must also be present in the product. We can now repeat the above steps to obtain the list of superconformal multiplets that appear upon decomposing a given product.