# New developments in 5d SCFTs and 5-brane webs

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Based on works with Hirotaka Hayashi, Kimyeong Lee, Futoshi Yagi, and Hee-Cheol Kim

# Plan of talk

- Review of 5d N=1 superconformal theories (SCFTs)
   Patrick's talk
- String theory construction of 5d SCFTs: 5-brane webs
- Recent development:

# 5d superconformal theories (SCFTs)

## 5d N=1 Supersymmetric gauge theory

5d N=1 supersymmetric gauge theories with gauge group G: 8 supercharges Spacetime symmetry:

 $SO(4,1) \times SU(2)_R$ 

Particle contents for 5d N=1 theories with gauge group G

Matter content

Vector multiplet: $A_{\mu}, \phi; \lambda$ Hypermultiplet: $q^A; \psi$ (A=1,2 SU(2)\_R index)

Scalar VEVs parametrize

Coulomb branch is parametrized by vev of vec. mult. scalar  $\phi$ 

Higgs branch is parametrized by vev of hypermultiplet

Hanany, Sperling, Cabrera, Zafrir, Mekareeya, Yagi, ...

## 5d Superconformal theories (SCFTs)

Properties:

- only 8 supersymmetries: N=1
- Superconformal algebra F(4)
- Bosonic subalgebra: SO(5,2) x SU(2)<sub>R</sub>
- It is strongly coupled at the conformal fixed points
- No marginal deformation
- Relevant deformation with mass parameters

$$\delta \mathcal{L} = -\frac{1}{4g_{YM}^2} F_{\mu\nu} F^{\mu\nu}$$

yielding gauge theory descriptions at low energy (can flows to gauge theory in IR)

All are associated with global symmetries





- some admits different relevant deformations: duality at the UV fixed point

## 5d Superconformal theories (SCFTs)

IR theory's point of view:

as the infinite coupling limit (of low E effective theory), a SCFT arises at UV fixed point.

In fact, a large class of 5d N=1 theories have **non-trivial UV fixed point** 

[96 Seiberg] [96 Morrison-Seiberg] [97 Intriligator-Morrison-Seiberg]

At UV fixed point, global symmetry is enhanced

$$G_{\text{Hypers}} \times U(1)_{\text{Instantons}} \subset G_{\text{Global}}$$

massless hypers + massless instanton

An example: SU(2) theory with  $N_f \le 7$  hypermultiplets in fund. representation (flavors)

$$\frac{SO(2N_f) \times U(1)_I \subset E_{N_f+1}}{\uparrow}$$

flavor symmetry Instanton number

## **Prepotential**

- Along the **Coulomb branch** (G -> U(1)<sup>rank(G)</sup>), the theory is described by abelian low energy effective theory which is characterized by prepotential.
- Prepotential is at most cubic and 1-loop exact:

$$\mathcal{F} = \frac{1}{2g_0^2} h_{ij} \phi^i \phi^j + \frac{\kappa}{6} d_{ijk} \phi^i \phi^j \phi^k + \frac{1}{12} \Big( \sum_{\text{Roots}} |R \cdot \phi|^3 - \sum_f \sum_{w \in W_f} |w \cdot \phi + m_f|^3 \Big)$$
$$h_{ij} = \text{Tr}(T_i T_j), \ d_{abc} = \frac{1}{2} \text{Tr} T_a(T_b T_c + T_c T_b), \quad W_f = \text{Weight of } G \text{ in the rep. } r_f \qquad \text{[96 Morrison-Seiberg]}$$
$$[97 \text{Intriligator-Morrison-Seiberg]}$$

Magnetic monopole string tension:  $\phi_{D_i} = \partial_i \mathcal{F}$ 

Effective coupling:  $au_{ij} = \partial_i \partial_j \mathcal{F}$ Coulomb branch metric:  $d^2s = au_{ij} \, d\phi^i d\phi^j$ 

## An example: SU(2) Prepotential

With massless hypers

$$\mathcal{F}_{SU(2)} = \frac{1}{g_0^2}a^2 + \frac{1}{6}(8 - N_f)a^3$$

Magnetic monopole string tension:  $\phi_{D_i} = \partial_i \mathcal{F}$   $\frac{a}{2} \left( \frac{4}{q_0^2} + (8 - N_f)a \right)$ 

Effective coupling: 
$$\tau_{ij} = \partial_i \partial_j \mathcal{F} \qquad \quad \frac{2}{g_0^2} + (8 - N_f) a$$

#### Notice that $N_f > 8$ , Coulomb branch is not well defined -> theory is not well defined.

Leading to a way to classifying theories based on effective coupling [97 Intriligator-Morrison-Seiberg]

## Classifications

Coulomb branch metric (or effective coupling) is non-negative everywhere

[Intriligator-Morrison-Seiberg]

: Intriligator-Morrison-Seiberg (IMS) classification / bound However, many theories beyond the IMS bound were found

[Bergman, Zafrir, ....]

 Subsector where monopole tensions are non-negative: Physical Coulomb branch.

Coulomb branch metric (or effective coupling) is non-negative in physical Coulomb brach

[Jafferson, HC Kim, Vafa, Zafrir]

- Geometric classification
- Classifying KK theories; combined fiber diagram

[Jafferson, Katz, HC Kim, Vafa] [Del Zotto, Heckman Morrison] [Bhardwaj, Jefferson]

[Apruzzi, Lawrie, Lin, Schafer-Nameki, Wang]

# **Type IIB 5-branes**

## **M-theory/String constructions**

5d SCFTs can be engineered by M-theory or string theory.

#### M- theory on non-compact Calabi-Yau 3 fold (CY3)

[Witten'96],[Morrison,Seiberg'96], [Douglas,Katz,Vafa'96] [Katz,Klemm,Vafa'96],

M2 wrapping compact 2-cycles <==> BPS particle mass = vol(2-cycles)

M5 wrapping compact 4-cycles <==> monopole string tension = vol(4-cycles)

**Type IIB 5-brane configuration** 

[Aharony,Hanany'97] [Aharony,Hanany,Kol'97], [DeWolfe,Hanany,Iqbal,Katz'99]

BPS configuration with D5 and NS5 branes with their bound states

Two descriptions are equivalent.

[Leung,Vafa'97]

## Type IIB (p,q) 5-brane

D5 and NS5 make a configuration looks like a web: (p,q) 5-brane web junctions with fixed angles and conserved charges





circumference / perimeter

(if the brane configuration is known) prepotential can be reconstructed from 5-brane webs

## **Rank 1 theories**

SU(2) theory with  $N_f \leq 7$  flavors



For other cases, a bit more complicated but possible, by introducing 7-branes (where 5-branes end)

## **Rank 1 theories**

SU(2) theory with  $N_f \leq 7$  flavors



## **CY geometry**

SU(2) theory with  $N_f \leq 7$  flavors

All rank 1 SCFTs are engineers by CY3s of Hirzebruch / del Pezzo surfaces



Triple intersection number of four cycles in CY geometry —> prepotential

5d pure SU(3) gauge theory with κ=0,1,2,3



**CS** levels are confirmed by comparing areas with monopole string tensions

• 5d pure SU(3) gauge theory with  $\kappa$ =0,1,2,3

Rank 2 or higher can be obtained by gluing rank 1 geometries



#### **5-branes provide a powerful tool to study 5d theories:**

Especially, non-perturbative aspects can be seen.

A window to reveal rich physics

- Enhanced global symmetries
- Can compute Partition function using topological vertex
- various duality: S-duality, UV duality
- KK theories (marginal): 6d N=1 theories on a circle (with T-dual)

#### **Classification of Rank 2 theories:**

from **geometry** and also from **5-brane webs** 

[Jefferson, Katz, Kim, Vafa `18] [Hayashi, SSK, Lee, Yagi '19]

## Marginal theories and 5-brane web

6d theory on a circle

• It is known that

SU(2) theory with  $N_f = 8$  flavors <==> 6d **E-string** theory on a circle



## generalization and 5d/6d

A straightforward generalization is possible:



#### 5d SU(N) theory with $N_f=2N+4 <=> 6d Sp(N-2)$ with $N_f=2N+4$ and Tensor

[15 Hayashi-SSK-Lee-Taki-Yagi]

- It is an example that is out of IMS classification [97 Intriligator-Morrison-Seiberg]
- consistent with instanton operator analysis and new classification.

[15 Yonekura]

[17 Jefferson-HC Kim-Vafa-Zafrir]

## generalization and 5d/6d

 T-dual picture of 6d Sp(N-2) theory with Nf= 2N+4 suggests that three different theories, in fact, have the same UV fixed point in 6d



O7 can be resolved into [1,1] and [1,-1] 7-branes

## **UV duality**

 T-dual picture of 6d Sp(N-2) theory with Nf= 2N+4 suggests that three different theories, in fact, have the same UV fixed point in 6d



Elliptic genus and partition functions agree.

# New developments

- Topological Vertex Formulation w/ O5
- Exceptional gauge group G2 and duality
- theories of higher CS level
- hypermultiplet in rank-3 antisymmetric representation

## **Topological vertex with O5-plane**

5d N=1 pure Sp(1) gauge theory



- Non-toric
- O5: branes and their mirrors are discontinuous.

### One can still compute Partition function ...

[SSK, Yagi '17]

Using DIM [Bourgine, Fukuda, Matsuo, Zhu]

### Use "flopped" diagram



### mirror image



### **"Fundamental region"**



#### Young diagram assigned to the cut edge



#### This is strip diagram



### "Glue" the cut edge



$$Z = \sum_{\mu,
u} Z_{\mu
u}^{
m strip} E_{\mu} E_{
u}'$$
 [SSK, Yagi '17]  
Agreed with the known results.

## **Application: E-string theory**





#### **T-dual picture:**



### **Use flop transition**



### **Mirror images**





### The same type of strip that appeared in M-string!



### **E-string Partition Function**

$$\begin{split} Z &= \sum_{\mu_1,\mu_2} \left( \frac{\mathfrak{q}^2 x_2}{x_1^3} \right)^{|\mu_1|} \left( \frac{\prod_{i=1}^8 y_i}{x_1 x_2^5} \right)^{|\mu_2|} f_{\mu_1}^{-4} f_{\mu_2}^{-4} \\ &\times \prod_{I=1}^2 \left( \prod_{i=1}^8 \frac{\Theta_{\mu_I \varnothing}(x_I y_i^{-1})}{\Theta_{\mu_I \varnothing}(x_I y_i)} \prod_{J=1}^2 \frac{\Theta_{\mu_I \mu_J}(x_I x_J)}{\Theta_{\mu_I \mu_J}(x_I x_J^{-1})} \right), \\ &\Theta_{\mu\nu}(Q) \equiv \prod_{n=0}^\infty R_{\mu\nu}(Q \mathfrak{q}^{2n}) R_{\mu^{i\nu^i}}(Q^{-1} \mathfrak{q}^{2n+2}), \qquad R_{\lambda\mu}(Q) = \prod_{i,j=1}^\infty (1 - Q g^{i+j-\mu_i-\lambda_j-1}). \\ &x_1 = \mathfrak{q}^{\frac{1}{2}} e^{-a} \prod_{i=1}^8 e^{-\frac{1}{4}m_i} \qquad x_2 = \mathfrak{q}^{\frac{1}{2}} e^{+a} \prod_{i=1}^8 e^{-\frac{1}{4}m_i} \qquad y_i = \mathfrak{q}^{\frac{1}{2}} e^{-m_i} \prod_{j=1}^8 e^{-\frac{1}{4}m_j} \\ &a: \text{ Coulomb moduli, } \mathfrak{q}: \text{ Instanton Factor, } m_i: \text{ mass, } g = e^{-\epsilon_1} = e^{+\epsilon_1} \end{split}$$

#### Checked the consistency up to 3-instanton

[C. Hwang, J. Kim, S. Kim, and J. Park '14] [SSK, Taki, Yagi '15]

## 5 brane webs for 5d G2 theories

## 5d SO(7) gauge theories

$$\widetilde{O5}^{-} = \mathbf{O5}^{-} + \mathbf{1/2} \, \mathbf{D5} + \mathbf{1/2} \, \mathbf{D7} \, \mathbf{cut}$$
 [Zafrir'15]  
 $\widetilde{O5}^{+} = \mathbf{O5}^{+} + \mathbf{1/2} \, \mathbf{D7} \, \mathbf{cut}$ 



**Sp(0)** instantons=spinors

## **G**<sub>2</sub> = Higgsing of SO(7) with spinor

#### Ms -> 0 : Higgs branch

[Hayashi-SSK-Lee-Yagi'17]



## Pure G<sub>2</sub> gauge theory



Areas = monopole string tension from the prepotential  $\mathcal{F}$ 

$$2\phi_1 - 3\phi_2 = a_2 - a_1, \quad -\phi_1 + 2\phi_2 = a_1$$
  
$$\mathcal{F}_{G_2} = m_0(\phi_1^2 - 3\phi_1\phi_2 + 3\phi_2^2) + \frac{4}{3}\phi_1^3 - 4\phi_1^2\phi_2 + 3\phi_1\phi_2^2 + \frac{4}{3}\phi_2^3$$

## Pure G<sub>2</sub> gauge theory



Areas = monopole string tension from the prepotential  $\mathcal{F}$ 

## S-Duality -> SU(3)7

[Jafferson, Katz, HC Kim, Vafa]



**3 color D5 branes!**  $\mathscr{F}_{mu2} = m_0(\phi_1^2 - \phi_1\phi_2 + \phi_2^2) + \frac{4}{3}\phi_1^3 + 3\phi_1^2\phi_2 - 4\phi_1\phi_2^2 + \frac{4}{3}\phi_2^3$ 

$$2\phi_1 - 3\phi_2 = a_2 - a_1, \ -\phi_1 + 2\phi_2 = a_1$$

$$\mathcal{F}_{G_2} = m_0(\phi_1^2 - 3\phi_1\phi_2 + 3\phi_2^2) + \frac{4}{3}\phi_1^3 - 4\phi_1^2\phi_2 + 3\phi_1\phi_2^2 + \frac{4}{3}\phi_2^3$$

**Parameter map:** 

$$egin{aligned} m_0^{SU(3)} &= -rac{m_0^{G_2}}{3}, \ \phi_1^{SU(3)} &= \phi_2^{G_2} + rac{1}{3}m_0^{G_2}, \ \phi_2^{SU(3)} &= \phi_1^{G_2} + rac{2}{3}m_0^{G_2}, \end{aligned}$$

5 brane webs with hypermultiplets in the Rank 3 antisymmetric representation

# SU(6) with hypermultiplet in the rank-3 antisymm. repre (TAS)

Hypermultiplets possible to describe on a 5-brane web are

Fundamental, Antisymmetric, Symmetric

with/without orientifolds

[Bergman, Zafrir, Rodriguez-Gomez]

#### Recent classification suggests SU(6) + TAS

[Jafferson, HC Kim, Vafa, Zafrir]

$N_{\mathbf{TAS}}$	$N_{\mathbf{Sym}}$	$N_{\mathbf{AS}}$	$N_{\mathbf{F}}$	CS
2	•	•	•	0
3/2	•	•	5	0
3/2	•	•	3	2
3/2	•	•	•	9/2
1	•	1	4	0
1	•	1	3	3/2
1	•	1	•	4
1	•	•	10	0
1	•		9	3/2
1/2	1	•	1	0
1/2	1	•	•	3/2
1/2	•	2	2	3/2
1/2	•	2	2	1/2
1/2	•	2	•	7/2
1/2	•	1	9	0
1/2	•	1	8	3/2
1/2	•		13	0
1/2	•	•	9	3

## SU(6) with hypermultiplet in the rank-3 antisymm. repre (TAS)

**5-brane webs for TAS:** 

Use the decomposition

[Tachikawa, Terashima]

 $SO(12) \supset SU(6) \times U(1)$   $32' = 20_0 \oplus 6_{-2} \oplus \overline{6}_{2},$  $66 = 1_0 \oplus 15_2 + \overline{15}_{-2} + 35_0.$ 

As 5-brane web for SO(12) + 1 conj spinor is known

Decoupling of U(1) d.o.f. leads to SU(6) with TAS (20 dim'l)

SO(12) with 1/2 hyper in conj. spinor repre.





$$\mathcal{F}_{N_{\mathbf{TAS}}=\frac{1}{2}}^{SU(6)_{\kappa}} = \frac{1}{2}m_0\sum_{i=1}^{6}a_i^2 + \frac{\kappa}{6}\sum_{i=1}^{6}a_i^3 + \frac{1}{12}\left(2\sum_{1\leq i< j\leq 6}(a_i - a_j)^3 - \sum_{2\leq i< j\leq 6}(a_1 + a_i + a_j)^3\right),$$



SU(6)<sub>5/2</sub> + 1/2 TAS Partition function:

$$Z_{
m Nek} = Z_{
m pert} \left( 1 + \sum_{k=1}^{\infty} q^k Z_k 
ight),$$

$$Z_{\text{pert}} = \text{PE}\left[\frac{g}{(1-g)^2} \left(2\sum_{1 \le i < j \le 6} A_i A_j^{-1} - \sum_{1=i < j < k \le 6} A_i A_j A_k + \mathcal{O}(A_1^{-8})\right)\right]$$

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$$Z_1 = -\sum_{\ell=1}^6 \frac{g}{(1-g)^2} \frac{A_\ell^5}{\prod_{i \neq \ell} (A_i - A_\ell)^2} \Big[ 1 - \sum_{i \neq \ell} A_i^{-1} A_\ell + \sum_{i \neq \ell} A_i A_\ell^2 - A_\ell^3 \Big]$$

SO(12) with 1/2 hyper in conj. spinor representation



There is another way: SU(6)- SU(3) quiver and its Higgsing



In a similar way, a Higgsing of SU(3)-SU(6)-SU(3) quiver lead to SU(6)+ 1TAS





SU(6)<sub>3</sub> + 1 TAS. It has a particular shape





 $SU(6)_0 + 1$ **TAS** 

pure  $SU(3)_0$ 

Earlier:  $SU(3)_0+10F$  is a marginal theory as 6d Sp(1) + 10F on a circle



SU(6)<sub>0</sub>+1**TAS**+10**F** is also a marginal theory

 $5d \quad [1\mathbf{TAS}] - SU(6)_0 - [10\mathbf{F}] \quad \xrightarrow{\mathrm{UV \ completion}} \quad 6d \quad [10\mathbf{F}] - Sp(2) - SU(2)_2$ 

## Conclusion

#### 5d SCFTs

- Rich physics: dualities, global symmetry enhancement, non-Lagrangian theories, non-perturbative aspects, ...

- 5-brane webs for higher CS and other hypermultiplets are obtained
- Based on web diagram, the Partition functions are doable.
- But still more to be discovered (Type IIB 5-brane web and interplay between 6d)
- 5-Brane configuration suggests various a way of writing the prepotential F which makes enhanced global symmetry manifest [work in progress]
- Geometry for 5-brane webs with orientifolds [work in progress]