Extremization principles from geometry

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Strings, Branes and Gauge Theories

In collaboration with A. Zaffaroni; 1901.05977, 1904.04269

Motivations

- The R-symmetry current is *not* unique!

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Extremization principles

How to determine the *exact* R-symmetry?

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Extremization principles

How to determine the *exact* R-symmetry?

- a-maximization in 4D $\mathcal{N} = 1$ gauge theories: $\left.\frac{\partial a(\Delta)}{\partial \Delta}\right|_{\bar{\Delta}} = 0\,.$

  [Intriligator, Wecht'03]

- F_{S^3}-maximization in 3D $\mathcal{N} = 2$ gauge theories: $\left.\frac{\partial F_{S^3}(\Delta)}{\partial \Delta}\right|_{\bar{\Delta}} = 0\,.$

  [Jafferis'10]

- Δ: R-charges of the fields
- $\bar{\Delta}$: exact R-charges of the fields
- a: trial central charge
- $F_{S^3} \equiv -\log|Z_{S^3}|$

Motivations

- c_r-extremization in 2D $\mathcal{N} = (0,2)$ gauge theories: $\left.\dfrac{\partial c_r(\Delta)}{\partial \Delta}\right|_{\bar{\Delta}} = 0\,.$

  [Benini, Bobev'12]

- $\mathcal{I}$-extremization in $\mathcal{N} = 2$ quantum mechanics: $\left.\dfrac{\partial \mathcal{I}(\Delta)}{\partial \Delta}\right|_{\bar{\Delta}} = 0\,.$

  [Benini, Hristov, Zaffaroni'15]

- c_r: trial right-moving central charge
- $\mathcal{I} \equiv -\log |Z_{S^1}|$

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Geometric perspective

- Volume minimization [Martelli, Sparks, Yau'05]
- S_{susy}-extremization [Couzens, Gauntlett, Martelli, Sparks'18; Gauntlett, Martelli, Sparks18]

a-maximization = volume minimization

AdS/CFT correspondence

$\mathrm{AdS}_5 \times Y_5 \leftrightarrow \mathcal{N} = 1$ SCFT on N D3-branes at the tip of $\mathrm{CY}_3 = C(Y_5)$.

- Y_5: Sasaki-Einstein five-manifold.

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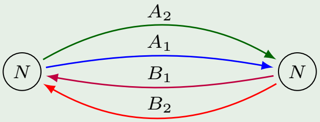
Klebanov-Witten theory ($Y_5 = T^{1,1}$)



$$W = \mathrm{Tr}\left(A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1\right),$$

$$\Delta_{A_1} + \Delta_{A_2} + \Delta_{B_1} + \Delta_{B_2} = 2,$$

$$\mathrm{U}(1)_R \times \mathrm{SU}(2)_1 \times \mathrm{SU}(2)_2 \times \mathrm{U}(1)_B\,.$$

- The toric cone $C(T^{1,1})$ is determined by the vectors

$$\vec{v}_1 = (0,0)\,, \qquad \vec{v}_2 = (1,0)\,, \qquad \vec{v}_3 = (1,1)\,, \qquad \vec{v}_4 = (0,1)\,.$$

- Toric R-charges:

$$\Delta_1 = \Delta_{A_1}\,, \qquad \Delta_2 = \Delta_{A_2}\,, \qquad \Delta_3 = \Delta_{B_1}\,, \qquad \Delta_4 = \Delta_{B_2}\,.$$

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$\mathcal{N}=1$ fields theories

- The theory has an R-symmetry and $d-1$ global symmetries.
- $d-3$ baryonic symmetries $\equiv$ nontrivial 3-cycles $S_a \subset Y_5$.
- 3 mesonic symmetries (one is the R-symmetry).

Supergravity language

- $d-3$ gauge fields from reducing of IIB 4-form potential on S_a.
- 3 gauge fields associated w/ the isometries of Y_5.

- $d-1$ g-symmetries can be parameterized by $(F_a \in \mathbb{R}) \to v_a$ w/

$$\sum_{a=1}^{d} F_a = 0\,.$$

- There are $|G|$ SU(N) gauge groups $= 2$ Area (toric diagram).

a-maximization = volume minimization

- Define $w_a \equiv v_{a+1} - v_a$ (note that $v_{d+1} \equiv v_1$).
- $|(e_1, w_a, w_b)|$ bi-fundamental chiral fields Φ_{ab} w/ charge

$$F_{a+1} + F_{a+2} + \ldots + F_b \,,$$

  for each pair (a, b).

- Baryonic symmetries: $\displaystyle\sum_{a=1}^{d} B_a v_a = 0 \,.$
- R-charges of the fields: $\Delta_{a+1} + \Delta_{a+2} + \ldots + \Delta_b \,, \quad \displaystyle\sum_{a=1}^{d} \Delta_a = 2 \,.$

Trial $a(\Delta)$ central charge — large N

$$a(\Delta) \equiv \frac{9}{32} \operatorname{Tr} R(\Delta_a)^3 = \frac{9}{32} N^2 \left(|G| + \sum_{\Phi_{ab}} \text{mult}(\Phi_{ab}) \left(\Delta_{\Phi_{ab}} - 1 \right)^3 \right) .$$

a-maximization = volume minimization

An equivalent formula

$$a(\Delta) = \frac{27}{32} N^2 \sum_{1 \le a < b < c \le d} (v_a, v_b, v_c) \Delta_a \Delta_b \Delta_c \,.$$

[Benvenuti, Pando Zayas, Tachikawa'06]

- t' Hooft anomaly coefficient: $c_{abc} = \frac{N^2}{2} (v_a, v_b, v_c) > 0.$
- a-maximization: $\frac{\partial a(\Delta)}{\partial \Delta_a} \Big|_{\bar{\Delta}_a} = 0 \,.$

AdS/CFT dictionary

$$\bar{\Delta}_a = \frac{\pi}{3} \frac{\mathrm{Vol}(S_a)}{\mathrm{Vol}(Y_5)} \,, \qquad a(\bar{\Delta}_a) = \frac{\pi^3 N^2}{4 \mathrm{Vol}(Y_5)} \,.$$

[Gubser, Klebanov'98; Gubser'98]

a-maximization = volume minimization

off-shell backgrounds

Preserve supersymmetry but relax the equations of motion.

[Martelli, Sparks, Yau'05]

- The SE metric on Y_5 is replaced with a general Sasaki metric.
- The metric depends on a Reeb vector $b = (b_1, b_2, b_3)$.

$$\zeta = \sum_{i=1}^{3} b_i \partial_{\phi_i} \,, \qquad \text{susy requires } b_1 = 3 \,.$$

- ∂_{ϕ_i}: vector fields generating the toric $\mathrm{U}(1)^3$ action.
- Volumes are now functions of the Reeb vector, *i.e.*

$$\mathrm{Vol}_{\mathrm{S}}(Y_5) = \frac{\pi^3}{b_1} \sum_{a=1}^{d} \frac{(v_{a-1}, v_a, v_{a+1})}{(v_{a-1}, v_a, b)(v_a, v_{a+1}, b)} \,,$$

$$\mathrm{Vol}_{\mathrm{S}}(S_a) = 2\pi^2 \frac{(v_{a-1}, v_a, v_{a+1})}{(v_{a-1}, v_a, b)(v_a, v_{a+1}, b)} \,.$$

a-maximization = volume minimization

a-functional

$$a(b_i) \equiv \frac{\pi^3 N^2}{4\mathrm{Vol}_{\mathrm{S}}(Y_5(b_i))} \, .$$

[Martelli, Sparks, Yau'05]

Volume minimization

It reproduces the Reeb vector $\bar{b}$ and the $\mathrm{Vol}(Y_5(\bar{b}_i))$.

- $a(\bar{b}_i) \equiv a(\bar{\Delta}_a)$ by construction.

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- $a(\bar{b}_i) \equiv a(\bar{\Delta}_a)$ by construction.
- R-charges parameterization:

$$\Delta_a(b_i) = \frac{\pi \mathrm{Vol}_{\mathrm{S}}(S_a(b_i))}{b_1 \mathrm{Vol}_{\mathrm{S}}(Y_5(b_i))}\,, \qquad \sum_{a=1}^{d} \Delta_a = 2\,.$$

- One then proves that

$$a(b_i) \equiv a(\Delta_a)\Big|_{\Delta_a(b_i)} \equiv \frac{27}{32} N^2 \sum_{1\leq a<b<c\leq d} (v_a, v_b, v_c)\Delta_a\Delta_b\Delta_c\Big|_{\Delta_a(b_i)}\,.$$

[Butti, Zaffaroni'05]

a-maximization = volume minimization

A puzzle!

- $a(\Delta)$-maximization is performed over $d-1$ ind. parameters.
- $a(b_i)$-maximization is performed over 2 ind. parameters.

Question: is $a(\Delta)$-maximization $= a(b_i)$-maximization?

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Key point

$a(\Delta_a)$ is *automatically extremized* wrt the baryonic directions.

[Butti, Zaffaroni'05]

- Baryon decoupling:

$$\sum_{a=1}^{d} B_a \frac{\partial a(\Delta)}{\partial \Delta_a}\bigg|_{\Delta_a(b)} \equiv 0\,.$$

# of ind. parameters: $(d-1)-(d-3)=2$.

Outline

- c_r-extremization equals its gravitational dual
- $\mathcal{I}$-extremization from geometry
- Outlook

c_r-extremization equals its gravitational dual

Twisted compactification of 4D $\mathcal{N} = 1$ theories

4D $\mathcal{N} = 1$ field theories $\xrightarrow{\Sigma_{\mathfrak{g}}}$ 2D $\mathcal{N} = (0, 2)$ field theories labeled by $\mathfrak{n}_a$.

- Supersymmetry is preserved by a *topological twist.*
- $\mathfrak{n}_a$: magnetic fluxes associated w/ v_a.
- Twisting condition: $\sum_{a=1}^{d} \mathfrak{n}_a = 2 - 2\mathfrak{g}$.

Gravitational dual: $\mathrm{AdS}_5 \times Y_5 \longrightarrow \mathrm{AdS}_3 \times_W Y_7$

- Y_7 is topologically a fibration of Y_5 over $\Sigma_{\mathfrak{g}}$.

Trial right-moving central charge [Benini, Bobev'12]

$$c_r(\Delta_a, \mathfrak{n}_a) = 3 \operatorname{Tr} \gamma_3 R(\Delta_a)^2 \,.$$

- γ_3: chirality operator in two dimensions.
- Trace runs over all the two-dimensional fermions.

c_r-extremization equals its gravitational dual

An index theorem — large N [SMH, Nedelin, Zaffaroni'16]

$$c_r(\Delta, \mathfrak{n}) = -\frac{32}{9} \sum_{a=1}^{d} \mathfrak{n}_a \frac{\partial a(\Delta)}{\partial \Delta_a} = -3 \sum_{a,b,c=1}^{d} c_{abc} \mathfrak{n}_a \Delta_b \Delta_c$$

$$= 3N^2 \sum_{1 \leq a < b < c \leq d} (v_a, v_b, v_c)(\mathfrak{n}_a \Delta_b \Delta_c + \mathfrak{n}_b \Delta_a \Delta_c + \mathfrak{n}_c \Delta_a \Delta_b) \, .$$

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***off-shell* backgrounds**

Preserve supersymmetry but relax the equations of motion.

[Couzens, Gauntlett, Martelli, Sparks'18; Gauntlett, Martelli, Sparks'18]

- a Reeb vector $b = (b_1, b_2, b_3)$ — susy requires $b_1 = 2$.
- d parameters $\lambda_a \to D_a$ — determine the Kähler class.
- d fluxes $\mathfrak{n}_a \to D_a$.

c_r-extremization equals its gravitational dual

Quasi-regular case

The quotient $V = Y_5/\mathrm{U}(1)_{\mathrm{Reeb}}$ is a 4D compact toric orbifold.

- The Kähler class of V is given by $\omega = -2\pi \sum_{a=1}^{d} \lambda_a c_a$.
- c_a: Poincaré dual of the restriction of D_a to V.
- Only $d-2$ λ_a are independent (# of ind. 2-cycles in V).
- $d\,\mathfrak{n}_a \to (d-3)\,\mathfrak{n}_\alpha + 3n^i$.

Remark

- $n^i = \sum_{a=1}^{d} v_a^i \mathfrak{n}_a$ are associated w/ the isometries → mesonic.
- $\mathfrak{n}_\alpha$ enter in the supergravity five-form flux → baryonic.

c_r-extremization equals its gravitational dual

Master volume [Gauntlett, Martelli, Sparks'18]

$$\mathcal{V} = 4\pi^3 \sum_{a=1}^{d} \lambda_a \frac{\lambda_{a-1}(v_a, v_{a+1}, b) - \lambda_a(v_{a-1}, v_{a+1}, b) + \lambda_{a+1}(v_{a-1}, v_a, b)}{(v_{a-1}, v_a, b)(v_a, v_{a+1}, b)} \, .$$

susy + flux quantization conditions:

$$N = - \sum_{a=1}^{d} \frac{\partial \mathcal{V}}{\partial \lambda_a} \, ,$$

$$\mathfrak{n}_a N = - \frac{A}{2\pi} \sum_{b=1}^{d} \frac{\partial^2 \mathcal{V}}{\partial \lambda_a \partial \lambda_b} - b_1 \sum_{i=1}^{3} n^i \frac{\partial^2 \mathcal{V}}{\partial \lambda_a \partial b_i} \, ,$$

$$A \sum_{a,b=1}^{d} \frac{\partial^2 \mathcal{V}}{\partial \lambda_a \partial \lambda_b} = 2\pi n^1 \sum_{a=1}^{d} \frac{\partial \mathcal{V}}{\partial \lambda_a} - 2\pi b_1 \sum_{i=1}^{3} n^i \sum_{a=1}^{d} \frac{\partial^2 \mathcal{V}}{\partial \lambda_a \partial b_i} \, .$$

- A is a constant parameterizing the Kähler class of $\Sigma_{\mathfrak{g}}$.
- $(d-1)$ ind. equations.
- $(d-2)\,\lambda_a + A \to (d-1)$ ind. parameters.

c_r-extremization equals its gravitational dual

c-functional [Gauntlett, Martelli, Sparks'18]

$$c(b_i, \mathfrak{n}_a) = -48\pi^2 \left(A \sum_{a=1}^{d} \frac{\partial \mathcal{V}}{\partial \lambda_a} + 2\pi b_1 \sum_{i=1}^{3} n^i \frac{\partial \mathcal{V}}{\partial b_i} \right) \Bigg|_{\lambda_a(b,\mathfrak{n}),\, A(b,\mathfrak{n})} .$$

- R-charges parameterization: [SMH, Zaffaroni'19]

$$\Delta_a(b_i, \mathfrak{n}_a) = -\frac{2}{N} \frac{\partial \mathcal{V}}{\partial \lambda_a} \Bigg|_{\lambda_a(b,\mathfrak{n}),\, A(b,\mathfrak{n})} , \qquad \sum_{a=1}^{d} \Delta_a = 2 .$$

- One then proves that

$$c(b_i, \mathfrak{n}_a) \equiv c_r(\Delta_a, \mathfrak{n}_a) \Big|_{\Delta_a(b,\mathfrak{n})} \equiv -\frac{32}{9} \sum_{a=1}^{d} \mathfrak{n}_a \frac{\partial a(\Delta_a)}{\partial \Delta_a} \Bigg|_{\Delta_a(b,\mathfrak{n})} .$$

c_r-extremization equals its gravitational dual

A puzzle!

- $c_r(\Delta_a, \mathfrak{n}_a)$-extremization is performed over $d-1$ ind. parameters.
- $c(b_i, \mathfrak{n}_a)$-extremization is performed over 2 ind. parameters.

Question: is $c_r(\Delta_a, \mathfrak{n}_a)$-extremization $= c(b_i, \mathfrak{n}_a)$-extremization?

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$c_r(\Delta_a, \mathfrak{n}_a)$ is *automatically extremized* wrt the baryonic directions.

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- Baryon decoupling:

$$\sum_{a=1}^{d} B_a \frac{\partial c_r(\Delta, \mathfrak{n})}{\partial \Delta_a}\Big|_{\Delta_a(b,\mathfrak{n})} = \sum_{a,b=1}^{d} B_a \mathfrak{n}_b \frac{\partial^2 a(\Delta)}{\partial \Delta_a \partial \Delta_b}\Big|_{\Delta_a(b,\mathfrak{n})} \equiv 0\,.$$

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$\mathcal{I}$-extremization from geometry

AdS/CFT correspondence

$\mathrm{AdS}_4 \times Y_7 \leftrightarrow \mathcal{N} = 2$ SCFT on N M2-branes at the tip of $\mathrm{CY}_4 = C(Y_7)$.

- Y_7: Sasaki-Einstein seven-manifold.

[Herzog, Klebanov, Pufu, Tesileanu'10]

Holographic dictionary:

$$F_{S^3} = N^{3/2} \sqrt{\frac{2\pi^6}{27 \mathrm{Vol}(Y_7)}} \,.$$

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[Martelli, Sparks, Yau'05'06]

Preserve supersymmetry but relax the equations of motion.

- The SE metric on Y_7 is replaced with a general Sasaki metric.
- The metric depends on a Reeb vector $b = (b_1, b_2, b_3, b_4)$.

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- ∂_{ϕ_i}: vector fields generating the toric $\mathrm{U}(1)^4$ action.

$\mathcal{I}$-extremization from geometry

- Volumes are now functions of the Reeb vector.

F_{S^3}-maximization = Vol(Y_7)-minimization

$$F_{S^3}(\Delta_a) = N^{3/2}\sqrt{\frac{2\pi^6}{27\text{Vol}_{\text{S}}(Y_7(b_i))}}\,.$$

[Jafferis, Klebanov, Pufu, Safdi'11; Martelli, Sparks'11]

- $\text{Vol}_{\text{S}}(Y_7(b_i))$ reproduces the Reeb vector $\bar{b}$ and the Vol(Y_7).
- R-charge parameterization: $\Delta_a(b_i) \equiv \dfrac{2\pi}{3b_1}\dfrac{\text{Vol}_{\text{S}}(S_a(b_i))}{\text{Vol}_{\text{S}}(Y_7(b_i))}\,.$
- No proof for a generic Y_7!

Remark [Jafferis, Klebanov, Pufu, Safdi'11]

F_{S^3}, at large N, does *not* depend on baryonic symmetries.

$\mathcal{I}$-extremization from geometry

Twisted compactification of 3D $\mathcal{N}=2$ theories

3D $\mathcal{N}=2$ field theories $\xrightarrow{\Sigma_{\mathfrak{g}}}$ $\mathcal{N}=2$ QM labeled by $\mathfrak{n}_a$.

- Supersymmetry is preserved by a *topological twist.*
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Gravitational dual: $\mathrm{AdS}_4 \times Y_7 \longrightarrow \mathrm{AdS}_2 \times_W Y_9$ [Benini, Hristov, Zaffaroni'15]

- Y_9 is topologically a fibration of Y_7 over $\Sigma_{\mathfrak{g}}$.
- Black holes territory!

Topologically twisted index (TTI) [Benini, Zaffaroni'15]

$$\mathcal{I}(\Delta_a, \mathfrak{n}_a) = \mathrm{Tr}_{\mathcal{H}_{\Sigma_{\mathfrak{g}}}} (-1)^F e^{-\beta H} e^{i J_a \Delta_a} \,.$$

$\mathcal{I}$-extremization from geometry

An index theorem — large N [SMH, Zaffaroni'16]

$$\mathcal{I}(\Delta_a, \mathfrak{n}_a) = -\frac{1}{2} \sum_{a=1}^{d} \mathfrak{n}_a \frac{\partial F_{S^3}(\Delta_a)}{\partial \Delta_a} \, .$$

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- d fluxes $\mathfrak{n}_a \to D_a$.

Entropy functional

[Gauntlett, Martelli, Sparks'18]

$$S(b_i, \mathfrak{n}_a) = -8\pi^2 \left(A \sum_{a=1}^{d} \frac{\partial \mathcal{V}}{\partial \lambda_a} + 2\pi b_1 \sum_{i=1}^{4} n_i \frac{\partial \mathcal{V}}{\partial b_i} \right) \Bigg|_{\lambda_a(b,\mathfrak{n}),\, A(b,\mathfrak{n})} \, .$$

$\mathcal{I}$-extremization from geometry

Mesonic twist [SMH, Zaffaroni'19]

$$\sum_{a=1}^{d} B_a^{(i)} \lambda_a = 0 \,, \qquad \forall i = 1, \dots, d-4 \,.$$

- $B_a^{(i)}$: baryonic symmetries that can be defined by

$$\sum_{a=1}^{d} B_a^{(i)} v_a = 0 \,, \qquad \forall i = 1, \dots, d-4 \,.$$

- It depends on 3 ind. R-charges and 3 ind. magnetic fluxes, *i.e.*

$$\mathfrak{n}_a = \frac{1}{2} \nabla (b_1 \Delta_a) \,, \qquad \forall a = 1, \dots, d \,,$$

leaving only the $n_i = \sum_{a=1}^{d} v_a^i \mathfrak{n}_a$ as independent fluxes.

$\mathcal{I}$-extremization = its gravitational dual

$$S(b_i, n_i) \Big|_{b_1 = 1} = \mathcal{I}(\Delta_a, \mathfrak{n}_a) \,.$$

[see also Gauntlett, Martelli, Sparks'19; Hyojoong Kim, Nakwoo Kim'19]

Outlook

- Define [Amariti, Klare, Siani'11; Amariti, Franco'12]

$$a_{3\mathrm{d}}(\Delta_a) \equiv \frac{1}{24} \sum_{a,b,c,e=1}^{d} |(v_a, v_b, v_c, v_e)| \, \Delta_a \Delta_b \Delta_c \Delta_e$$
$$+ \text{quartic corrections}\,.$$

- $a_{3\mathrm{d}}(\Delta_a) = F_{S^3}^2(\Delta_a)$ for Sasakian parameterization.

Constraints on R-charges and fluxes

$$\sum_{a=1}^{d} B_a^{(i)} \frac{\partial a_{3\mathrm{d}}(\Delta)}{\partial \Delta_a} = 0\,, \qquad \forall i = 1, \ldots, d-4\,,$$
$$\sum_{a,b=1}^{d} B_a^{(i)} \mathfrak{n}_b \frac{\partial^2 a_{3\mathrm{d}}(\Delta)}{\partial \Delta_a \partial \Delta_b} = 0\,, \qquad \forall i = 1, \ldots, d-4\,.$$

[SMH, Zaffaroni'19]

- $a_{3\mathrm{d}}(\Delta_a)$ sees what $F_{S^3}(\Delta_a)$ is seemingly blind to!

Outlook

Puzzle!

Baryonic symmetries disappear in the large N limit of the TTI.

[SMH, Zaffaroni'16; SMH, Mekareeya'16; Azzurli, Bobev, Crichigno, Min, Zaffaroni'17]

- There are black holes w/ *only* baryonic charges...

  [Halmagyi, Petrini, Zaffaroni"13; see Hyojoong's talk.]

- Prove

  $$\mathcal{I}\text{-extremization} = \text{its gravitational dual}$$

  for Y_9 w/ a generic twist.

- Find explicit examples of *magnetic* BPS black holes.

- ...

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Thank you for your attention!