SUSY enhancement from T-branes

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Based on work with

F. Carta, S. Giacomelli

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SUSY enhancement

- Remarkable phenomenon along certain RG flows of 4d SFT's.
  [Maruyoshi, Song `16]; [Agarwal, Maruyoshi, Song `16,`18]

- 4d N=2 SCFT w/ non-Abelian flavor G_F + free chiral field $M \in \mathrm{Adj}(G_F)$.

- Deform with $\delta W = \mathrm{Tr}(\mu M)$: $\mathcal{N}=2 \to \mathcal{N}=1$ μ is the flavor moment map.

- Give vev $\langle M \rangle = \rho(\sigma_+)$ $\rho: su(2) \to G_F$ is a nilpotent orbit.

- Depending on ρ, δW may trigger RG flow w/ $\mathcal{N}=2$ fixed point!

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- Very interesting, but also particularly useful:
  - IR is non-Lagrangian, but UV may have Lagrangian.
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- Very interesting, but also particularly useful:
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  - Use it to compute RG-invariant quantities for IR theory!
- How was it discovered? a-maximization: [Intriligator, Wecht `03]
  - $<M>$: Adj(G_F) → $\oplus_j V_j$ V_j spin-j irrep. of su(2).
  - Only M w/ spin ($j, j_3 = -j$) stay coupled, i.e. $[M, \rho(\sigma_-)] = 0$.
  - Maximize $32a = 9\mathrm{Tr}R^3 - 3\mathrm{Tr}R$ w.r.t. trial R-charge $R = R_0 + \epsilon F$.
  - Re-iterate until all fields have $D_* = 3/2\, R_* > 1$.

SUSY enhancement

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  - Engineer rank-1 SCFT's by probing F-theory singularities.
  - Interpret deformations as T-brane backgrounds.
  - Get simple algebraic criterion to exclude enhancement.

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    - Engineer rank-1 SCFT's by probing F-theory singularities.
    - Interpret deformations as T-brane backgrounds.
    - Get simple algebraic criterion to exclude enhancement.
- Bonus: in case of enhancement, IR conformal dimensions are derived algebraically!

Warm-up: 3d SQED

- Easier setting, but key geometric features already apparent.

- Single D2 probing a stack of *N* D6-branes in flat space:

  ➡ 3d $\mathcal{N}$=4 U(1) gauge theory w/ flavors in *fund* of $G_F = U(N)$.

  ➡ IR dynamics described by an M2 probing ALE $\mathbb{C}^2/\mathbb{Z}_N$.

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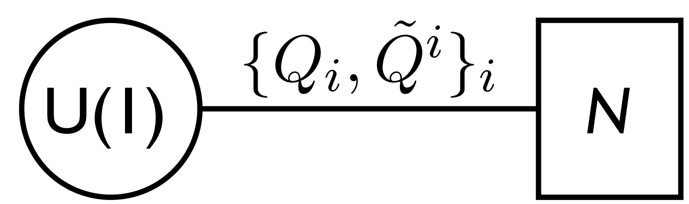
[Seiberg `96]

s_1 (directions 4, 5), s_2 (directions 6, 7) ⟶ *free hyper!*

Type IIA	0	1	2	3	4	5	6	7	8	9
D2	×	×	×						ϕ	
N D6	×	×	×		×	×	×	×		



$$W = \sum_{i=1}^{N} \tilde{Q}^i \phi Q_i$$

M-theory	0	1	2	3	4	5	6	7	8	9	10
M2	×	×	×								
ALE				×					×	×	×

⟶ $V_+ V_- = \phi^N$ CB

Deformations

- Add the IR relevant coupling $\delta W = \mathrm{Tr}(\mu M)$:
  - $\mu_i^j \equiv Q_i \tilde{Q}^j$ is the meson matrix.
  - M=<Φ> is vev of scalar controlling D6's position in 8,9 plane.
  - M constant & $[M,M^\dagger]=0$ ➠ $\mathcal{N}=4$ preserving cplx masses.
  - M 3d field ➠ D6's coordinate dependence <Φ>(s_1,s_2).

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- Give M nilpotent vev ➠ $[M,M^\dagger] \neq 0$ & $\mathcal{N}=4 \rightarrow \mathcal{N}=2$:
  - Probing a T6-brane background. [Cecotti, Cordova, Heckman, Vafa `10]
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su(3) principal orbit

- Consider $$M = \underbrace{\begin{pmatrix} 0 & m & 0 \\ 0 & 0 & m \\ 0 & 0 & 0 \end{pmatrix}}_{\langle M \rangle} + \underbrace{\begin{pmatrix} 0 & 0 & 0 \\ s_2 & 0 & 0 \\ s_1 & s_2 & 0 \end{pmatrix}}_{\delta M}$$

- Then $$W = \phi \sum_{i=1}^{3} Q_i \tilde{Q}^i + m(Q_2 \tilde{Q}^1 + Q_3 \tilde{Q}^2) + s_2(Q_1 \tilde{Q}^2 + Q_2 \tilde{Q}^3) + s_1 Q_1 \tilde{Q}^3$$

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- This is U(1) w/ 1 flavor $\Longleftrightarrow$ theory of 1 free hyper.

su(3) principal orbit

- Geometrically: M2 probing the fourfold (CY_4).

$$V_+V_- = \phi^3 - 2m\phi s_2 + m^2 s_1$$

field-dependent
deformation of $\mathbb{C}^2/\mathbb{Z}_3$

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➡ $CY_4 \quad \leftrightsquigarrow \quad \{ V_+V_- = s_1 \} \times \mathbb{C}^2 \quad \Rrightarrow \quad \mathcal{N}=2 \rightarrow \mathcal{N}=4.$

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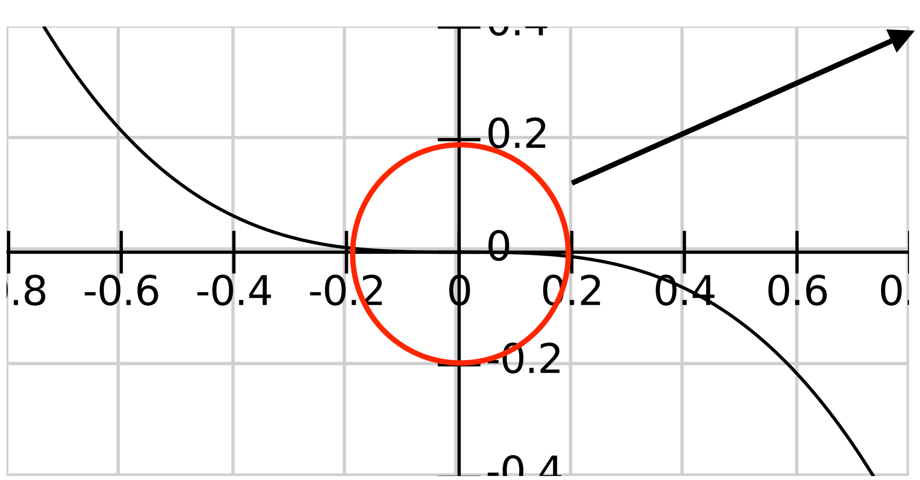


IR region

D6 @ $\quad s_1 - s_2\phi + \phi^3 = 0$

IR

D6 @ $\quad s_1 = 0$

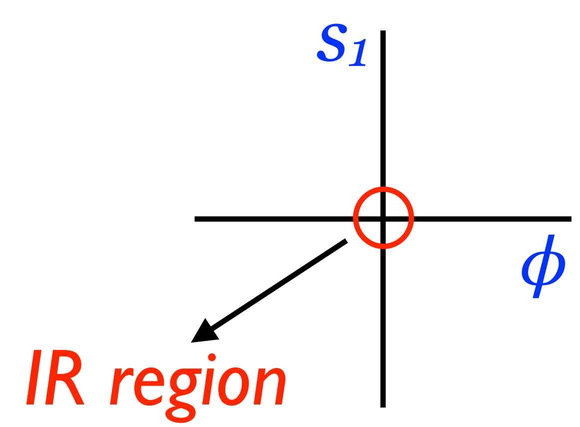
su(3) minimal orbit

- Consider instead $$M = \underbrace{\begin{pmatrix} 0 & m & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{\langle M \rangle} + \underbrace{\begin{pmatrix} 0 & 0 & 0 \\ s_1 & 0 & 0 \\ s_2 & 0 & 0 \end{pmatrix}}_{\delta M}$$

- $\mathbb{C}^2/\mathbb{Z}_3$ gets deformed to $V_+ V_- = \phi^3 - 2m\phi s_1$

- In the IR, this is equivalent to a conifold! $V_+ V_- = \phi s_1$
  - No factorized twofold ➠ no enhancement.

- D2 sees two intersecting D6-branes:



Rank-1 4d SCFT's & F-theory

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- Single D3 probing a stack of 7-branes (possibly non-perturb.):

  ➡ 4d $\mathcal{N}$=2 rank-1 theory (possibly non-Lagrangian).

  ➡ IR physics given by D3 prob. loc. elliptic K3 w/ Kodaira sing.

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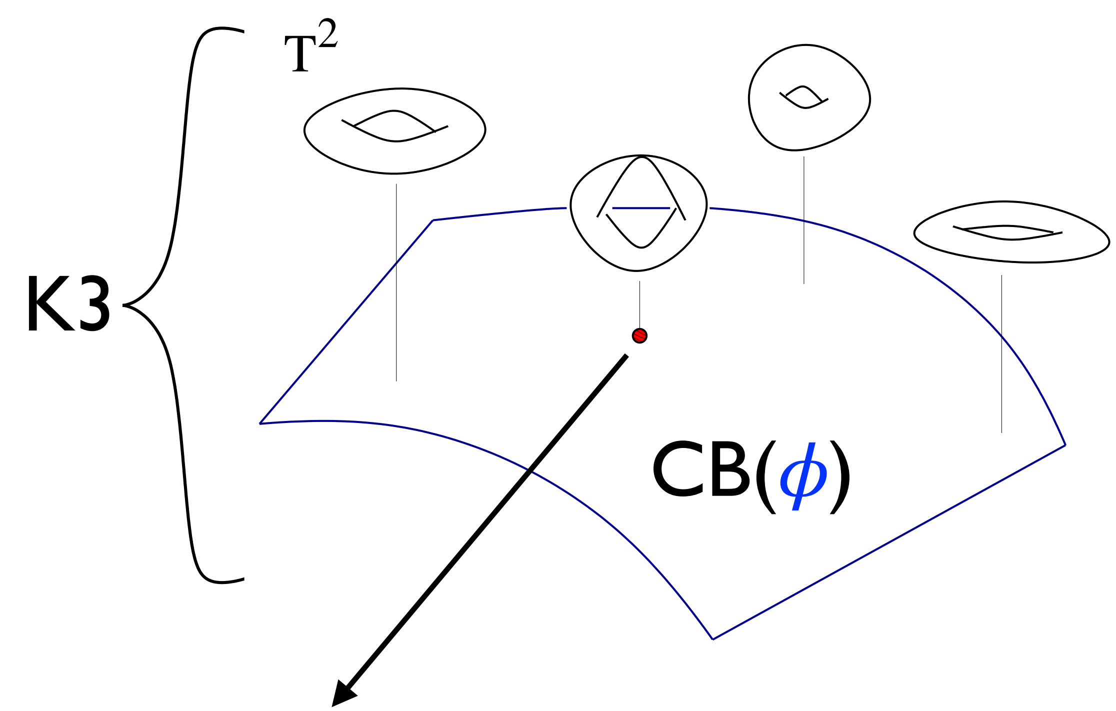

T^2 ⇹ SW curve

K3 is NOT a moduli space

singularity ⇹ flavor structure

Deformations

- Add the $\mathcal{N}=2 \to \mathcal{N}=1$ coupling $\delta W = \mathrm{Tr}(\mu M)$:
  - $M = \langle \Phi \rangle$ is vev of 7-brane worldvolume scalar.
  - $M = \rho(\sigma_+) + \sum_{j=s_1,s_2} \delta M_{(j,-j)}$
  - $[M, M^\dagger] \neq 0$ ➠ D3 probes a T7-brane background.
  - $D^{UV}(s_i) = \mathrm{spin}(s_i) + 1.$
- $\delta W \Rightarrow$ field-dependent deformations of K3 :
  - Casimir invariants of M ⇜⇝ Versal deform'ns of singularity.

[Katz, Morrison `92]

Enhancement criterion

- For any RG flow & any Energy scale, F-theory geometry is:

$$y^2 = x^3 + f(\phi, s_1, s_2)x + g(\phi, s_1, s_2)$$ Elliptic CY_4 in Weierstrass form

  - f & g depend on UV theory & choice of nilpotent orbit.
  - Original K3 retrieved by $s_1 \equiv s_2 \equiv 0$.

- If SUSY enhances in IR, then:

$$\mathrm{K3}^{\mathrm{UV}} \xrightarrow{\mathrm{Def}} \mathrm{CY}_4 \xrightarrow{\mathrm{IR}} \mathrm{K3}^{\mathrm{IR}} \times \mathbb{C}^2$$

  - $\mathrm{K3}^{\mathrm{UV}} \equiv \mathrm{K3}^{\mathrm{IR}} \iff$ trivial orbit.
  - Educated guess: $\mathrm{K3}^{\mathrm{IR}} = \mathrm{CY}_4|_{\phi \equiv s_2 \equiv 0}$ ➠ s_1 new CB in IR.

Enhancement criterion

- Conformal dim. of operators $\Longleftrightarrow$ $\mathbb{C}^*$-action on coordinates:
  - Promote affine variables to projective ones.
- Relative scalings of operators are RG-flow invariants:
  - Impose homogeneity of CY_4 polynomial.
- Ansatz for IR geom. consistent $\Longleftrightarrow$ $D^{IR}(\phi) \leq 1$ & $D^{IR}(s_2) \leq 1$.

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Example: $D_4 \rightsquigarrow \mathscr{H}_2$

- $D^{UV}(s_1) \leq D^{UV}(\phi)$ ➠ no enhancement!
- Highest spin multiply populated ➠ no enhancement!
- Consider the Lagrangian th. SU(2) w/ 4 flavors ➠ G_F = SO(8).
  - From D3 probing $K3^{UV}$: $y^2 = x^3 + \tau\phi^2 x + \phi^3$.

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$$P_M(z) = z^8 - 4\sqrt{2}s_2 z^6 + 8s_2^2 z^4 - 4s_1^2 z^2$$

⇩

$$C_{(2)} = -4\sqrt{2}s_2\,; \quad C_{(4)} = 8s_2^2\,; \quad C_{(6)} = -4s_1^2$$

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➡ $y^2 = x^3 +$ ~~$\tau\phi^2 x + 2s_2\phi x + s_2^2 x + \phi^3$~~ $+ s_1^2$ ⇨ $\mathscr{H}_2$

- $D^{IR}(y) = D^{IR}(s_1) = 3/2$; $D^{IR}(x) = D^{IR}(s_2) = D^{IR}(\phi) = 1$.

Example: No enhancement

- Consider E_8 Minahan-Nemeschanski.
  - From D3 probing $K3^{UV}$: $y^2 = x^3 + \phi^5$.
- Deform by $E_8(a_2)$ orbit: $V_1 \oplus V_3 \oplus V_5 \oplus V_7 \oplus V_8 \oplus V_9 \oplus 2V_{11} \oplus V_{13} \oplus V_{14} \oplus V_{17} \oplus V_{19}$

➡ s_1: 20th-order Casimir ; s_2: 18th-order Casimir.

➡ $y^2 = x^3 + \phi^2 s_2 + s_1 x + \phi^5$.

- $D^{IR}(y) = 1$; $D^{IR}(x) = 2/3$; $D^{IR}(s_1) = 4/3$.
- $D^{IR}(\phi) = 2/5$ ➠ decouples!
- BUT $D^{IR}(s_2) = 6/5$ ➠ prevents enhancement.
- Incorrect conformal dimensions… a-maximization needed!

Lessons & Outlook

✓ Geometry of IR SUSY enhancement involves local structure of some algebraic space around (singular) point.

✓ Branch of moduli space for 3d case, but auxiliary space for 4d case (rank-1 ➠ F-theory fibration).

✓ Enhancement only arises when non-trivial factorization occurs ➠ holonomy reduction in M/F-theory.

◉ Higher-rank theories ? In class-*S*, factorization ➠ hyperkähler structure of moduli space of sol'ns of Hitchin system.
[in progress with Carta, Giacomelli, Hayashi]

◉ Geometry behind factorization still obscure… RG-flow changes singularity structure, reminiscent of simultaneous resolutions!