# Universality in Open-Evolving Systems 

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## Statistics of open-evolving systems

- lifetime distribution of each element
- extinction-size distribution
- intermittency in time series
- structure of the emergent system
- ....



## Outline of the Talk

- Review on theories for lifetime distributions
- Population dynamics models
- -> A skewed species lifetime distribution is robustly found.
- propose a minimal model to understand the underlying mechanisms
- comparison with empirical data


## Theories for lifetime distributions

- Poisson process
- age-independent mortality (constant failure rate) $\exp (-t / \tau)$
- a simple exponential function
- known as "Red-Queen" hypothesis in ecology
- age-dependent mortality
- stretched exponential distribution (Weibull)

$$
t^{\beta-1} \exp \left[-(t / \tau)^{\beta}\right]
$$

- q-exponential distribution

$$
[1-(1-q)(t / \tau)]^{1 /(1-q)}
$$

$$
m(t)=\frac{p(t)}{1-\int_{0}^{t} p(t) d t}
$$




- return time distribution

Pigolotti et al. PNAS (2005)

- 1-d random walk $t^{-3 / 2}$
- 1-d random walk $t^{-3 / 2}$
- critical branching process $t^{-2}$



What about the lifetime distribution of mutually interacting systems?

## Population dynamics models

- population dynamics models
- Models with two time scales:


## population dynamics model

(or individual based model)
addition of new species extinction of species

$$
\dot{x_{i}}=f\left(\left\{x_{j}\right\}, \ldots\right)
$$




## various forms of population dynamics

Since we do not know an established model, we tried to find a universality shared for various models.
Y. Murase et al., J.Theor.Biol., 264, 663 (2010) Y.Murase et al.,Phys. Rev. E, 81, 041908 (2010)

Scale-invariant model

$$
\dot{x}_{i}=-b_{i} x_{i}+\sum_{a_{i j}<0} a_{i j} x_{i}^{\lambda} x_{j}^{(1-\lambda)}+\sum_{a_{i j}>0} a_{i j} x_{i}^{(1-\lambda)} x_{j}^{\lambda}
$$

Tangled-Nature model A

$$
\Delta_{I}\left(R,\left\{n_{J}(t)\right\}\right)=\sum_{J_{\text {interaction with J'th species }}}^{M_{I J} n_{J}(t) / N_{t o t}(t)}-\underbrace{N_{t o t}(t) / N_{0}}_{\text {globally applied suppression }}
$$

Tangled-Nature model B

$$
\Delta_{I}\left(R,\left\{n_{J}(t)\right\}\right)=-b_{I}+\eta_{\text {birth costcoupling to external resource }}^{\eta_{I} R / N_{t o t}(t)}+\sum_{\text {interaction with J'th species }}
$$



## Commonly observed pattern

- For all models
- 1/f² fluctuations of N
- Skewed species-lifetime distribution
- exponential extinction-size distribution
- log-normal like population distribution




## Lifetime distribution for population dynamics models

Skewed profile is universal for various population dynamics models




## Dynamical Graph Model

- System is represented by a weighted and directed graph.
- Interspecies interaction is denoted by $\mathrm{a}_{\mathrm{ij}}$.
- If $\sum a_{i j} \geqq 0$, species / can survive. ( $\sum a_{i j}=$ fitness of $i^{\prime \prime}$ th species: $f_{i}$ )
- $a_{i j}$ takes a random number drawn from a Gaussian distribution with probability c. (With $1-\mathrm{c}, \mathrm{a}_{\mathrm{ij}}$ is zero.)

(iii)

Y. Murase et al., New J. Phys., 12, 063021 (2010)


## Lifetime Distribution of DG model

- neither simple exponential or simple power law


$$
\begin{aligned}
& P(t) \propto \exp \left(-(t / \tau)^{\beta}\right) \\
& \beta=0.54
\end{aligned}
$$


well fitted by a stretched exponential function with exponent $1 / 2$

## What is the origin of the profile?

- If we assume the mortality function $t^{-1 / 2}$, we get a stretched exponential distribution.

$$
m(t)=\frac{p(t)}{1-\int_{0}^{t} p(t) d t}
$$



Does long-living species have advantages to survive??

## Mortality is age-independent

Mortality is not dependent on age, but N .


## Origin of $1 / \mathrm{N}$ dependence of mortality

- fitness changes by immigration and extinction
- changes in $f_{i}$ caused by immigrant is neutral
- $\rightarrow$ this yields $t^{-3 / 2}$ (does not have a time scale)
- changes in $f_{i}$ caused by extinction is negative
- this is because $f_{i}$ is positive by model definition
- $\rightarrow$ negative drift is proportional to $1 / \mathrm{k}(\sim 1 / \mathrm{cN})$




## Modified Red-Queen Hypothesis

- Assumption: random walk in N space + "Red-Queen" hypothesis


$$
\begin{aligned}
& p(N) \propto \exp (-b N) \\
& e^{-t / \tau} \\
& \text { mortality } \propto 1 / N \quad \tau \propto N \\
& \begin{aligned}
p(t) & =\int_{0}^{\infty} \frac{\exp (-t / \tau)}{\tau} b \exp (-b \tau) d \tau \\
& =2 b K_{0}(2 \sqrt{b t}) \\
& \approx \sqrt{\pi}(b t)^{-1 / 4} \exp (-2 \sqrt{b t}) \quad(t \gg 1)
\end{aligned}
\end{aligned}
$$

Stretched exponential function with exponent $1 / 2$ is obtained by modified Red-Queen hypothesis

# Comparison with Empirical Data 

Ecosystem : lifetime of families


Product lifecycle of convenience stores

lifetime distribution of bankrupted firms


Fig. 17.1 Cumulative age distribution $N_{>}(T)$ of firms in Japan included in database ORBIS. Horizontal axis is firm age $T$, and vertical axis is number of firms whose age exceeds $T$

J. Roy. Soc. Interface(2015)



We found a few example showing the "skewed" lifetime distribution although exponential distribution is also common.

## Conclusions

- Stretched exponential function with exponent $1 / 2$ is universally observed for various multi-species models.
- We proposed a new theory, modified Red-Queen hypothesis, to interpret the skewed lifetime distribution.
- Age-independent mortality is not excluded if a lifetime distribution has a heavier tail than exponential.


## Peferences


Y. Murase et al., J. Theor. Biol., 264, 663 (2010)

Random walk in genome space: A key ingredient of intermittent dynamics of community assembly on evolutionary time scales
Yohsuke Murase ${ }^{2, *}$, Takashi Shimada ${ }^{\text {a }}$, Nobuyasu Ito ${ }^{\text {a }}$, Per Arne Rikvold ${ }^{\text {b }}$ ${ }^{-}$Department of Applied Physics, School of Engineering The University of Tolyo, 7.3-1 Hongo, Bunkyo-ka, Tokyo 113-8656, Japan "Center for Materials Research and Technology and Department of Physics, Forida Stote Universig. Tallahassee, F 32306 -4350, USA
https://github.com/yohm/dynamical_graph_model


Simulation code of the dynamical graph model proposed in the paper Y. Murase et al., New J. Phys. (2010), - Edit


