Schwinger Pair Production in Electromagnetic Field in (A)dS Space

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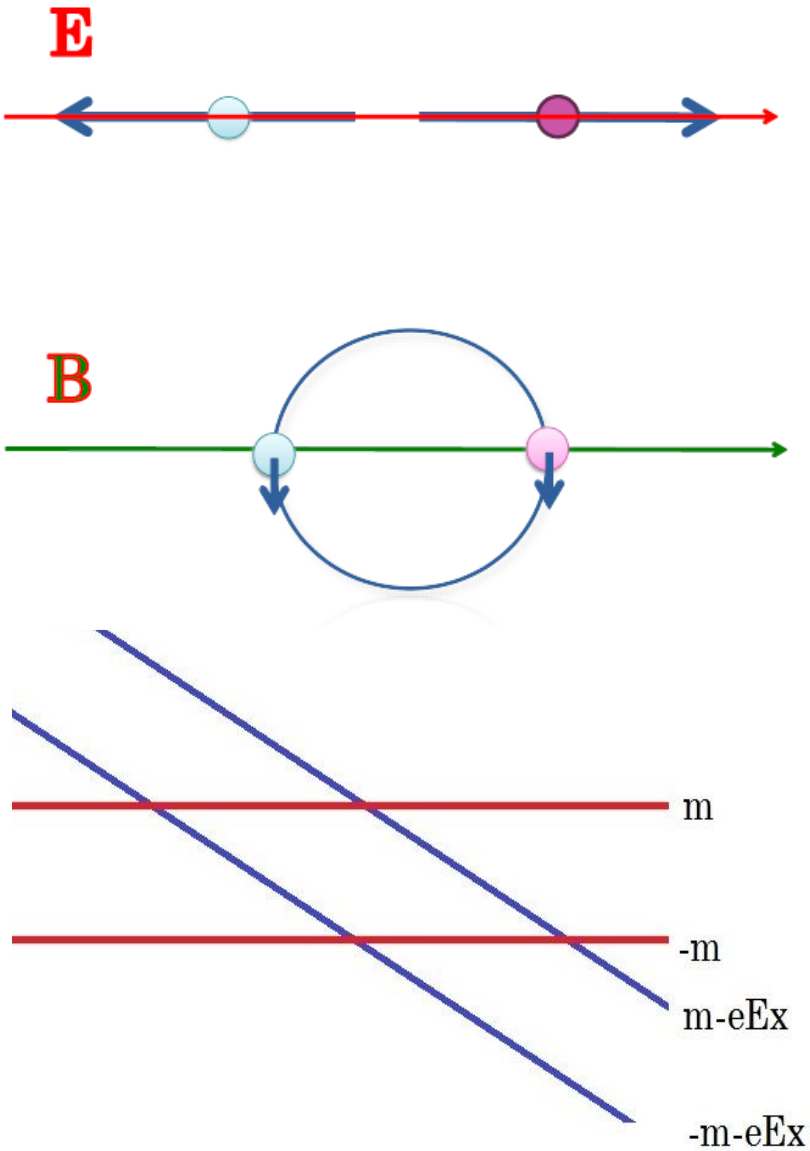
APCTP Workshop
New Perspective on Gravitation & Cosmology
*3rd LeCosPA Symposium, November 27, 2017

Outline

- Why Schwinger effect in curved spacetimes?
- Spontaneous pair production
- QED in (A)dS_4
- Schwinger effect in charged (dynonic) BHs
- QED in E-B in (A)dS_4
- Constraint on primordial magnetic field generation
- Conclusion

Why Schwinger Effect in Curved Spacetimes?

What is Schwinger Effect?



- Constant E-field changes energy spectra in Minkowski spacetime:

$$\varepsilon_{\pm} = |eE|x \pm \sqrt{\vec{p}^2 + m^2}$$

- Spontaneous creation of a particle-antiparticle pair from the Dirac sea (quantum mechanical tunneling)

$$N_S = \exp\left(-\frac{m}{T_S}\right), \quad T_S = 2 \times \frac{1}{2\pi}\left(\frac{qE}{m}\right)$$

- Critical (Schwinger) field to energetically separate the pair

$$eE_c \times \left(\frac{\hbar}{mc}\right) = mc^2$$

Hawking Radiation & Schwinger Effect

- Hawking emission formula in charged BH [CMP ('74)]

$$N_H = \frac{\Gamma_{j\omega lm}}{e^{\frac{\omega - q_j \Phi}{T_H}} \mp 1}$$

$$T_H = \frac{1}{2\pi} \frac{\sqrt{M^2 - Q^2}}{\left(M + \sqrt{M^2 - Q^2}\right)^2}$$

- No Hawking radiation when Q = M

- Schwinger emission formula in E-field [PR ('51)]

$$N_S = \exp\left(-\frac{m}{T_S}\right)$$

$$T_S = 2 \times \frac{1}{2\pi}\left(\frac{qE}{m}\right)$$

- Heisenberg-Euler, Weisskopf, Schwinger QED actions

Schwinger Effect in Curved Spacetimes

Zaumen ('74), Carter ('74), Gibbons ('75), Damour & Ruffini ('76)

⋮

Khriplovich ('99), Gabriel ('01)

SPK, Page ('04), ('05), ('08)

Ruffini, Vereshchagin, Xue ('10)

Chen, SPK, Lin, Sun, Wu ('12); Chen, Sun, Tang, Tsai ('15)

Ruffini, Wu, Xue ('13)

SPK ('13)

Cai, SPK ('14)

Fröb, Garriga, Kanno, Sasaki, Soda, Tanaka, Vilenkin ('14)

Fischler, Nguyen, Pedraza, Tangarife ('14)

SPK, Lee, Yoon ('15); SPK ('15), ('16), ('17)

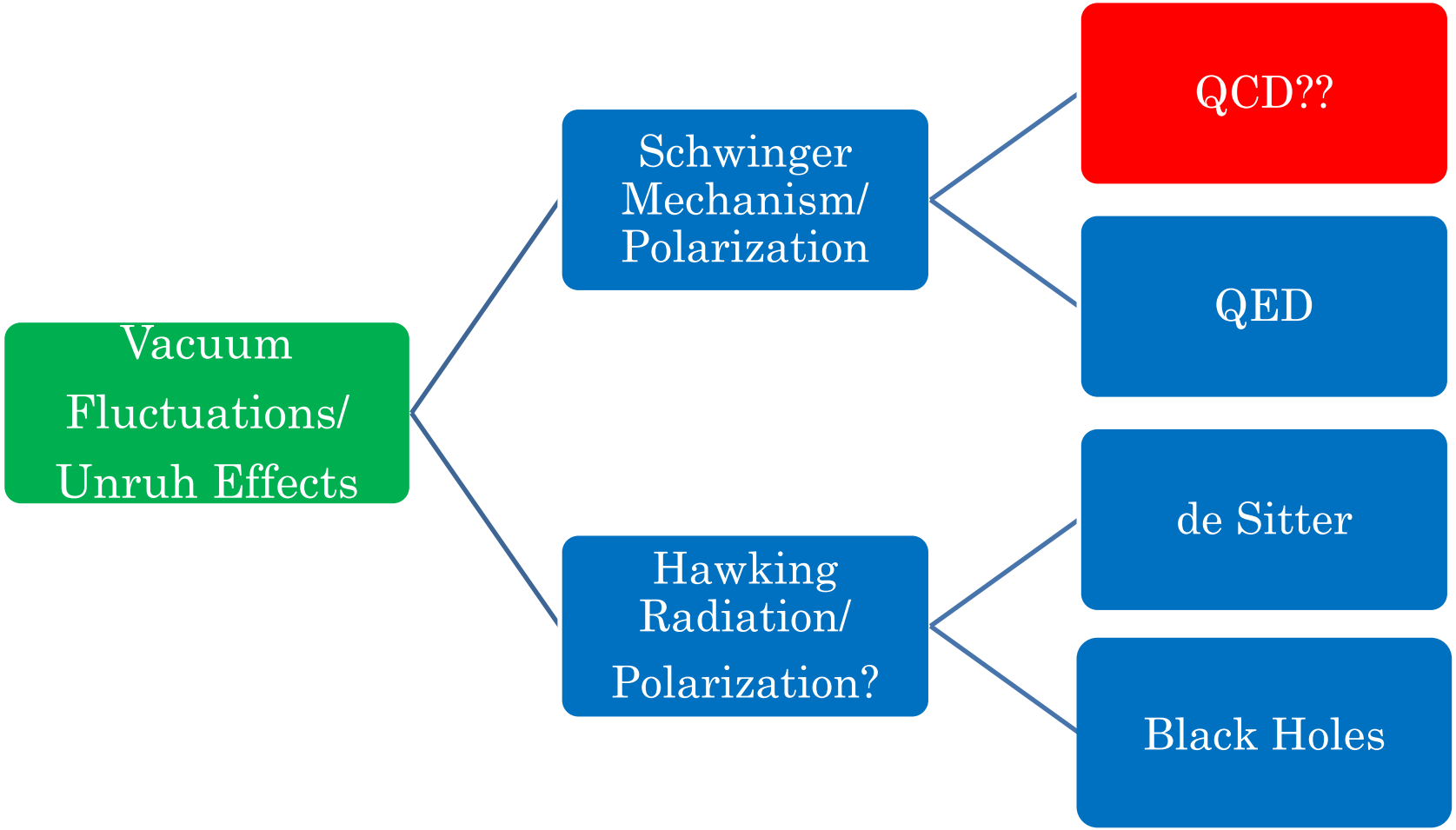
C-M. Chen, SPK, J-R. Sun, F-Y. Tang, Kerr-Newman BH ('16)

C-M. Chen, SPK, J-R. Sun, F-Y. Tang, Dyonic Kerr-Newman BH ('17)

Bavarsad, SPK, Stahl, Xue, E-B & dS ('17)

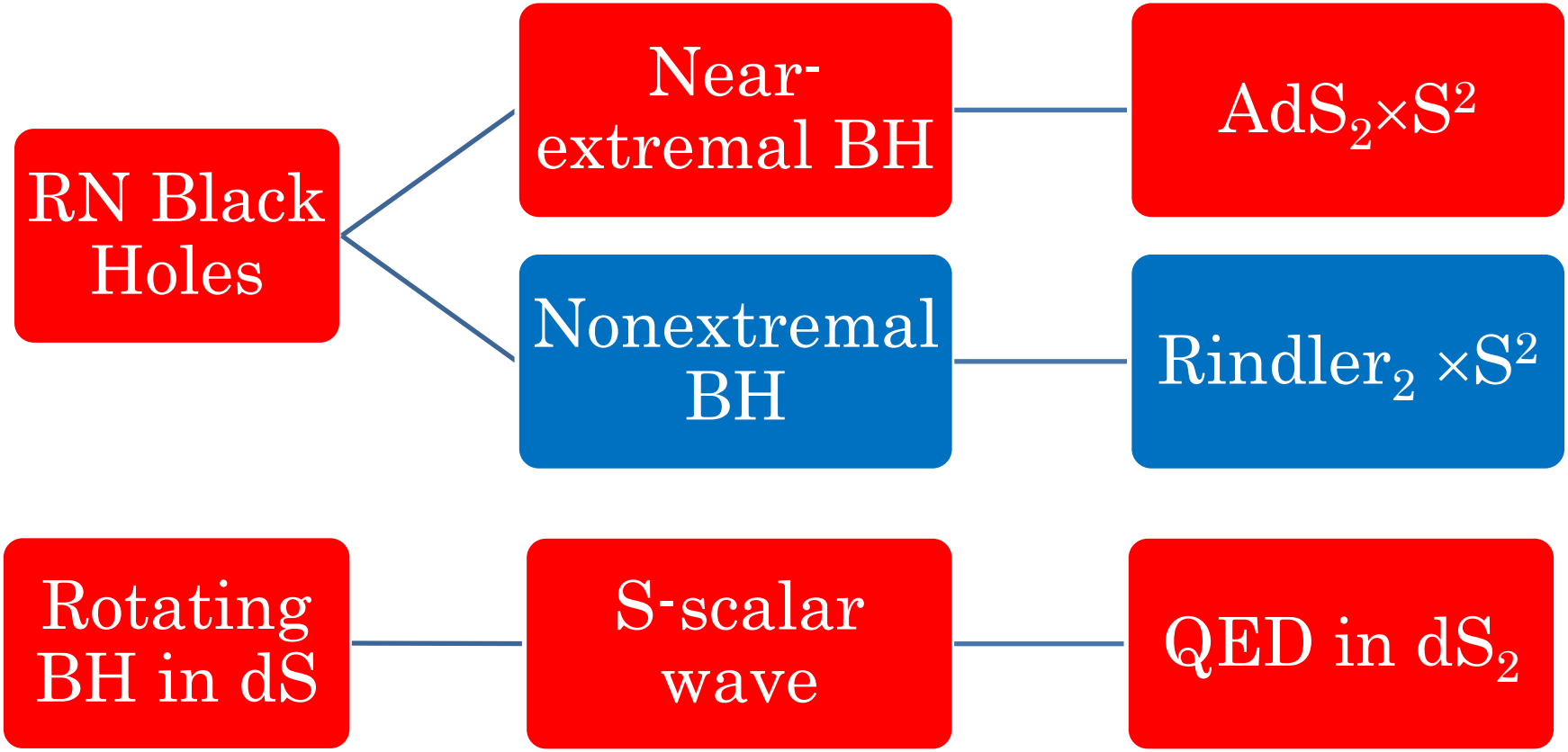
Spontaneous Pair Production
: Gauge Theory & BH or Universe

Spontaneous Pair Production: Unified Picture [SPK ('07)]

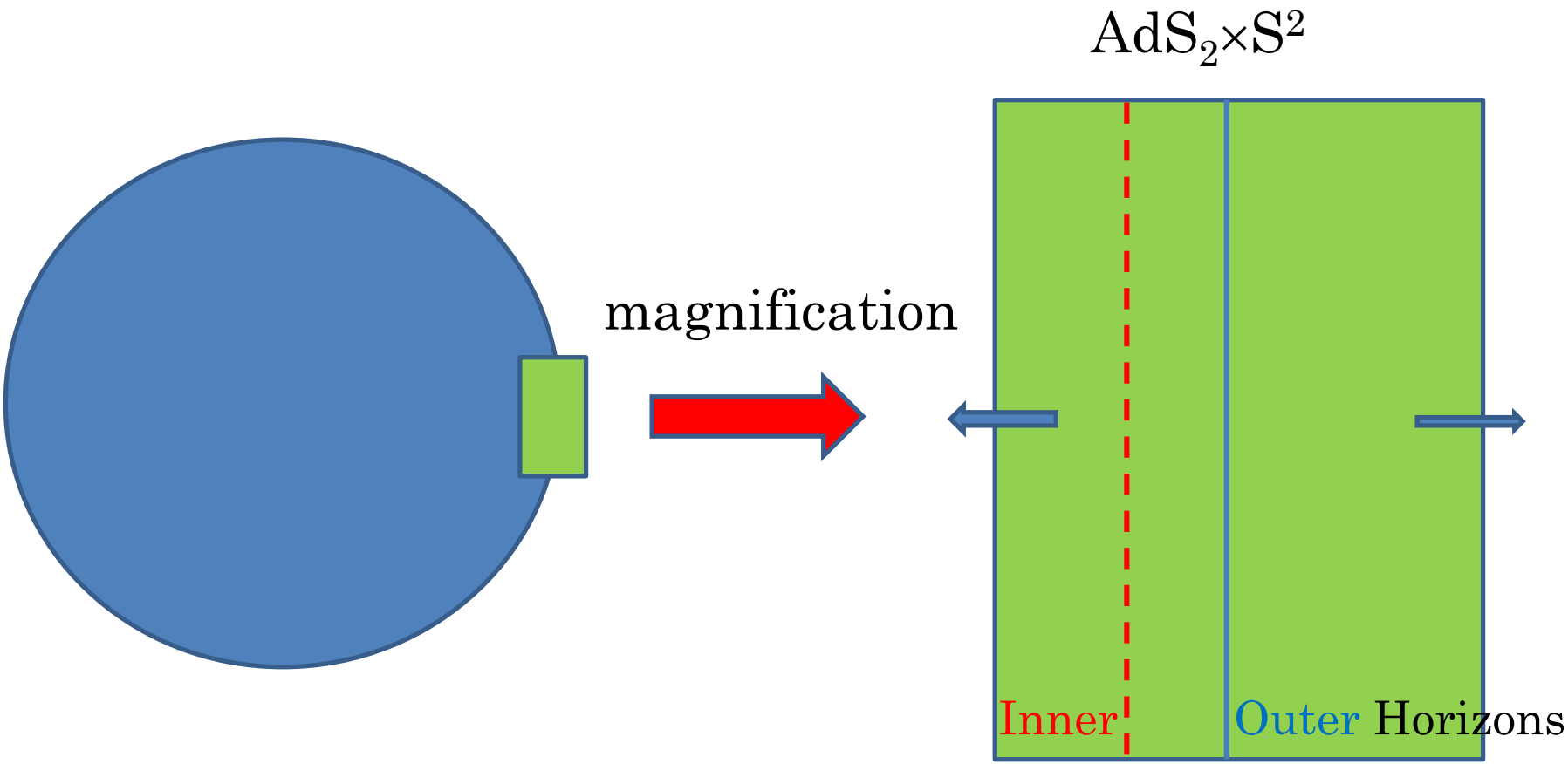


Why Schwinger Effect in (A)dS_2?

Near-Horizon Geometry of RN BHs



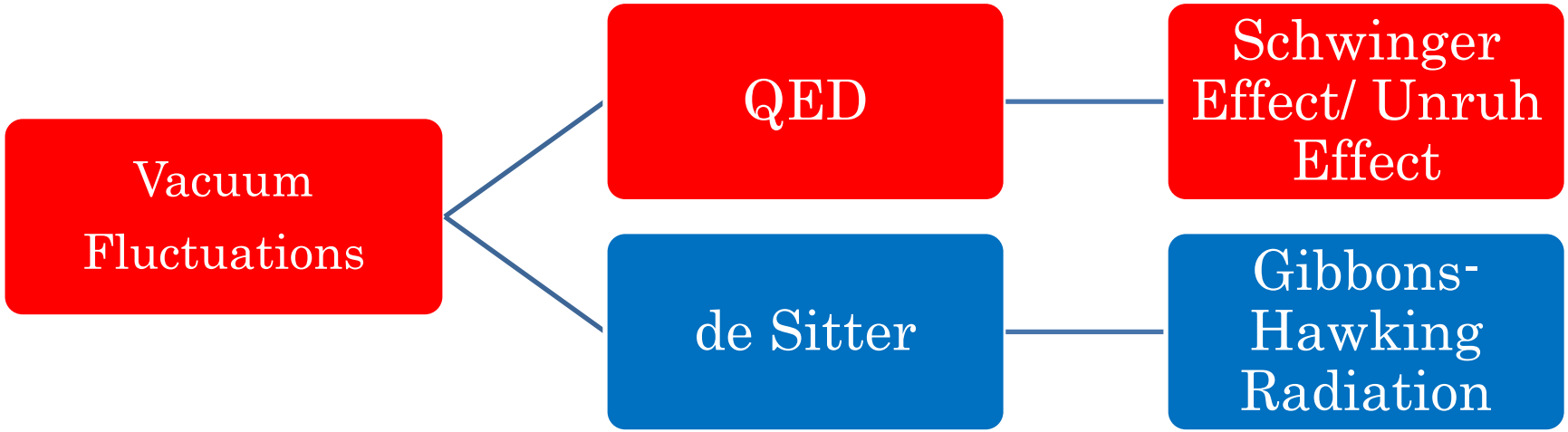
Near-horizon Geometry of Near-extremal RN BH



G. t'Hooft & A. Strominger, "conformal symmetry near the horizon of BH," MG14, July 2015.

Schwinger Effect in (A)dS$_2$

[Cai, SPK ('14)]



Effective Temperature for Unruh Effect in (A)dS$_2$

[Narnhofer, Peter, Thirring ('96); Deser, Levin ('97)]

Effective Temperature

Unruh Effect — $T_U = a/2\pi$

(A)dS — $R = 2H^2$ or $-2K^2$

$$T_{\text{eff}} = \sqrt{T_{\text{U}}^2 + \frac{R}{8\pi^2}}$$

QED in E-B & dS_4

Schwinger Effect in D-dimensional dS_D

- The Schwinger effect in a constant E in a D-dimensional dS should be independent of t and $x_{||}$ due to the symmetry of spacetime and the field, and the integration of $k_{||}$ gives the density of states D [SPK ('16)].
- dS radiation in E=0 limit and Schwinger effect in H=0 limit

$$\frac{d^D N_{\mathrm{dS}}}{dt d^{D-1}x} = \frac{(2|\sigma|+1)H^2 S_\mu}{2(2\pi)} \int \frac{d^{D-2}k_\perp}{(2\pi)^{D-2}} \left(\frac{e^{-(S_\mu - S_\lambda)} \pm e^{-2S_\mu}}{1-e^{-2S_\mu}} \right)$$

$$S_\mu = 2\pi \sqrt{\left(\frac{qE}{H^2}\right)^2 + \left(\frac{m}{H}\right)^2 - \left[\left(\frac{D-1}{2}\right)^2\right]}, \quad S_\lambda = 2\pi \frac{qE}{H^2} \left(\frac{qE/H}{\sqrt{(qE/H)^2 + \vec{k}_\perp^2}} \right)$$

Schwinger formula in E-B & dS_4

- Schwinger formula (mean number) for scalars in E and B and dS_4 [Bavarsad, SPK, Stahl, Xue ('17)].

$$\frac{d^4 N_{\mathrm{dS}}}{dt d^3 x} = \frac{H^2 S_\mu}{(2\pi)^2} \frac{qB}{2\pi} \sum_{n=0} \frac{e^{-(S_\mu - S_n)} + e^{-2S_\mu}}{1 - e^{-2S_\mu}}$$

$$S_\mu = 2\pi \sqrt{\left(\frac{qE}{H^2}\right)^2 + \left(\frac{m}{H}\right)^2 - \left(\frac{3}{2}\right)^2}\,, \quad S_n = 2\pi \frac{qE}{H^2} \left(\frac{qE/H}{\sqrt{(qE/H)^2 + qB(2n+1)}} \right)$$

- Zeta-function regularization

$$\frac{d^4 N_{\mathrm{dS}}}{dt d^3 x} = \frac{H^2 S_\mu}{(2\pi)^2} \frac{qB}{2\pi} \frac{1}{e^{2S_\mu} - 1} \left[\frac{1}{2} + \sum_{n=0} e^{S_\mu - S_n} \right]$$

Schwinger Effect in Near-extremal RN Black Hole

Interpretation of Schwinger Effect

- A thermal interpretation of the Schwinger formula for charged scalars/ fermions (upper/lower signs) in spherical harmonics [Chen, SPK, Lin, Sun, Wu ('12); Chen, Sun, Tang, Tsai ('15); SPK, Lee, Yoon ('15)]

$$N_{NBH} = \left(\frac{e^{-\frac{\bar{m}}{T_{RN}}} - e^{-\frac{\bar{m}}{\bar{T}_{RN}}}}{1 \pm e^{-\frac{\bar{m}}{\bar{T}_{RN}}}} \right) \times \left(\frac{1 \mp e^{-\frac{\omega - qA_0}{T_H}}}{1 + e^{-(\frac{\omega - qA_0}{T_H} + \frac{\bar{m}}{T_{RN}})}} \right),$$

$$T_{RN} = T_U + \sqrt{T_U^2 - (\frac{1}{2\pi Q})^2}, \quad \bar{T}_{RN} = T_U - \sqrt{T_U^2 - (\frac{1}{2\pi Q})^2}$$

$$T_U = \frac{qE_H / \bar{m}}{2\pi} = \frac{q}{2\pi \bar{m} Q}$$

Schwinger Effect and Hawking Radiation

- Thermal interpretation of Schwinger formula for charged scalars and fermions [SPK, Lee, Yoon ('15); SPK ('15)]

$$N_{NBH} = e^{\frac{\bar{m}}{T_{RN}}} \times \underbrace{\left(\frac{e^{-\frac{\bar{m}}{T_{RN}}} - e^{-\frac{\bar{m}}{\bar{T}_{RN}}}}{1 \pm e^{-\frac{\bar{m}}{\bar{T}_{RN}}}} \right)}_{\substack{\text{Schwinger Effect in AdS}_2 \\ \text{Cai \& SPK ('14)}}} \times \underbrace{\left(\frac{e^{-\frac{\bar{m}}{T_{RN}}} (1 \mp e^{-\frac{\omega - qA_0}{T_H}})}{1 + e^{-\frac{\omega - qA_0}{T_H}} e^{-\frac{\bar{m}}{T_{RN}}}} \right)}_{\substack{\text{Schwinger Effect in Rindler Space} \\ \text{Gabriel \& Spindel ('00)} \\ \text{Hawking Radiation of charges}}}$$

Schwinger Effect in (Dyonic) Kerr-Newman BH

Schwinger Effect for Charged Scalars

- Kerr-Newman (KN) black hole: $M, Q, a = \frac{J}{M}; r_0^2 \equiv Q^2 + a^2$
- Near-horizon geometry is warped AdS_2 of near-extremal KN BH leads to Schwinger formula [Chen, SPK, Sun, Tang ('17); Chen at ICGAC13-IK15]

$$N_{NKN} = \left(\frac{e^{-(S_a - S_b)} - e^{-(S_a + S_b)}}{1 + e^{-(S_a + S_b)}} \right) \times \left(\frac{1 - e^{-(S_c - S_a)}}{1 + e^{-(S_c - S_b)}} \right),$$

$$S_a = 2\pi \frac{qQ^3 - 2nar_0}{r_0^2 + a^2}, \quad S_b = 2\pi \sqrt{\left(\frac{S_a}{2\pi} \right)^2 - m^2 \left(r_0^2 + a^2 \right) - \lambda - \frac{1}{4}},$$

$$S_c = 2\pi \frac{\omega}{B}$$

Interpretation of Schwinger Effect

- Thermal interpretation of Schwinger formula for charged scalars in spheroidal harmonics in near-extremal KN BH

$$N_{NKN} = e^{\frac{\bar{m}}{T_{KN}}} \times \underbrace{\left(\frac{e^{-\frac{\bar{m}}{T_{KN}}} - e^{-\frac{\bar{m}}{\bar{T}_{KN}}}}{1 + e^{-\frac{\bar{m}}{\bar{T}_{KN}}}} \right)}_{\text{Schwinger Effect in AdS}_2} \times \underbrace{\left(\frac{e^{-\frac{\bar{m}}{T_{KN}}} \left(1 - e^{-\frac{\omega_t - q\Phi_H - n\Omega_H}{T_H}}\right)}{1 + e^{-\frac{\omega_t - q\Phi_H - n\Omega_H}{T_H}} e^{-\frac{\bar{m}}{T_{KN}}}} \right)}_{\text{Schwinger Effect with Chemical Potential in Rindler space}}$$

$$T_{KN} = T_U + \sqrt{T_U^2 + \frac{R}{8\pi^2}}, \quad \bar{T}_{KN} = T_U - \sqrt{T_U^2 + \frac{R}{8\pi^2}}$$

$$T_U = \frac{qQ^3 - 2nar_0}{2\pi\bar{m}\left(r_0^2 + a^2\right)^2}, \quad R = -\frac{2}{r_0^2 + a^2}, \quad \bar{m} = \sqrt{1 + \frac{\lambda + 1/4}{m^2\left(r_0^2 + a^2\right)}}$$

Schwinger Effect for Scalar Dyons

- Dyonic Kerr-Newman (DKN) black hole: $M, Q, P, a = \frac{J}{M}; r_0^2 \equiv Q^2 + P^2 + a^2$
- Near-horizon geometry of near-extremal DKN BH is a warped AdS_3 leads to Schwinger formula [Chen, SPK, Sun, Tang ('17); Chen at ICGAC13-IK15]

$$N_{DKN} = \left(\frac{e^{-(S_a - S_b)} - e^{-(S_a + S_b)}}{1 + e^{-(S_a + S_b)}} \right) \times \left(\frac{1 - e^{-(S_c - S_a)}}{1 + e^{-(S_c - S_b)}} \right),$$

$$S_a = 2\pi \frac{(qQ + pP)(Q^2 + P^2) - 2nar_0}{r_0^2 + a^2}$$

$$S_b = 2\pi \sqrt{\left(\frac{S_a}{2\pi} \right)^2 - m^2 \left(r_0^2 + a^2 \right) - \lambda_l - \frac{1}{4}}, \quad S_c = 2\pi \frac{\omega}{B}$$

Interpretation of Schwinger Effect

- Thermal interpretation of Schwinger formula for charged scalars in spheroidal harmonics in near-extremal DKN BH

$$N_{DKN} = e^{\frac{\overline{m}}{T_{KN}}} \times \underbrace{\left(\frac{e^{-\frac{\overline{m}}{T_{KN}}} - e^{-\frac{\overline{m}}{\overline{T}_{KN}}}}{1 + e^{-\frac{\overline{m}}{\overline{T}_{KN}}}} \right)}_{\text{Schwinger Effect in AdS}_2} \times \underbrace{\left(\frac{e^{-\frac{\overline{m}}{T_{KN}}} \left(1 - e^{-\frac{\omega_t - q\Phi_H - p\overline{\Phi}_H - n\Omega_H}{T_H}}\right)}{1 + e^{-\frac{\omega_t - q\Phi_H - p\overline{\Phi}_H - n\Omega_H}{T_H}} e^{-\frac{\overline{m}}{T_{KN}}}} \right)}_{\text{Schwinger Effect with Chemical Potential in Rindler space}}$$

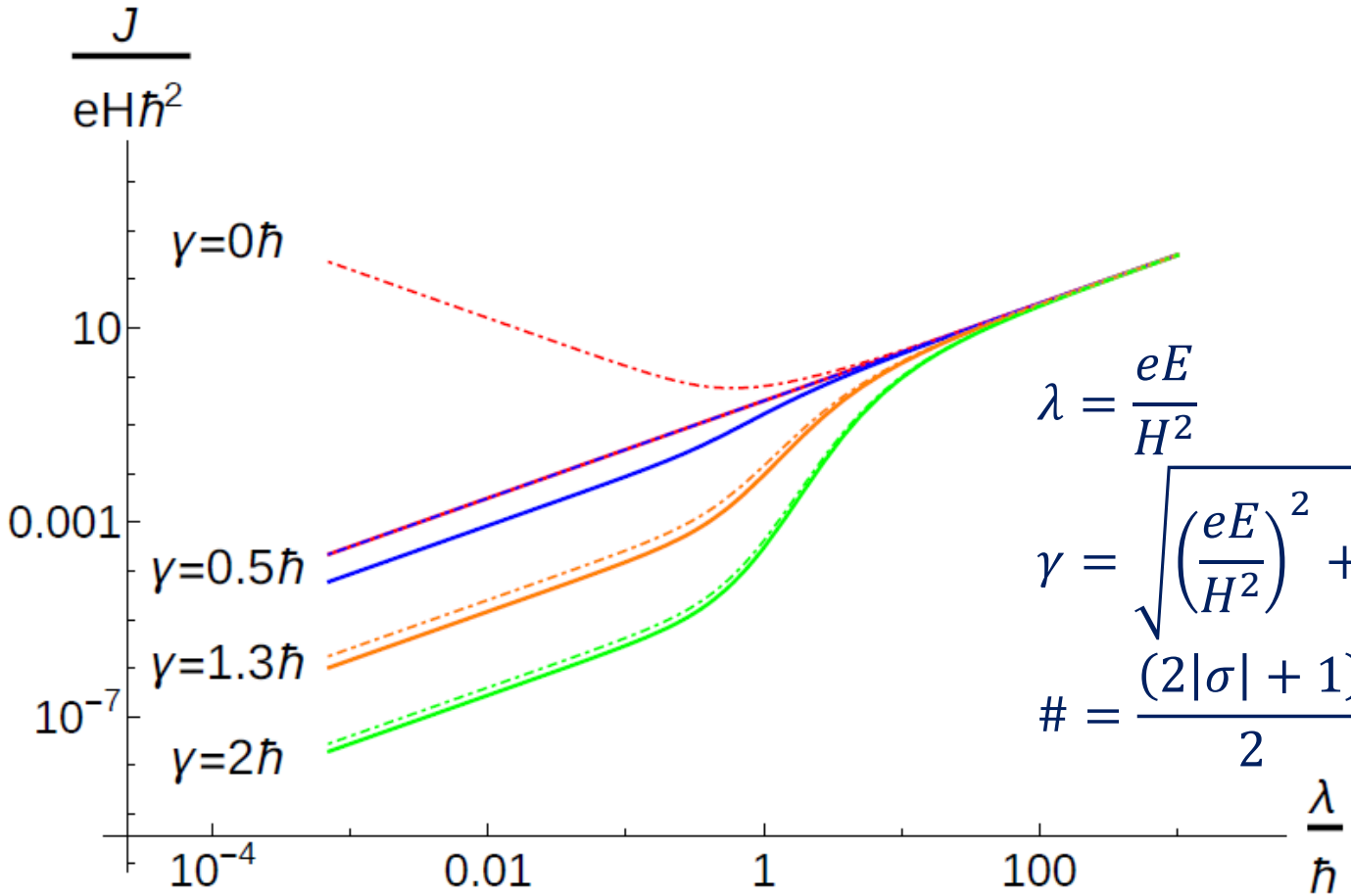
$$T_{KN} = T_U + \sqrt{T_U^2 + \frac{R}{8\pi^2}}, \quad \overline{T}_{KN} = T_U - \sqrt{T_U^2 + \frac{R}{8\pi^2}}$$

$$T_U = \frac{(qQ + pP)(Q^2 + P^2) - 2nar_0}{2\pi\overline{m}\left(r_0^2 + a^2\right)^2}, \quad R = -\frac{2}{r_0^2 + a^2}$$

$$\overline{m} = \sqrt{1 + \frac{\lambda + 1/4}{m^2\left(r_0^2 + a^2\right)}}$$

Conductivity & IR-Hyperconductivity in dS

Current in 1+1 D



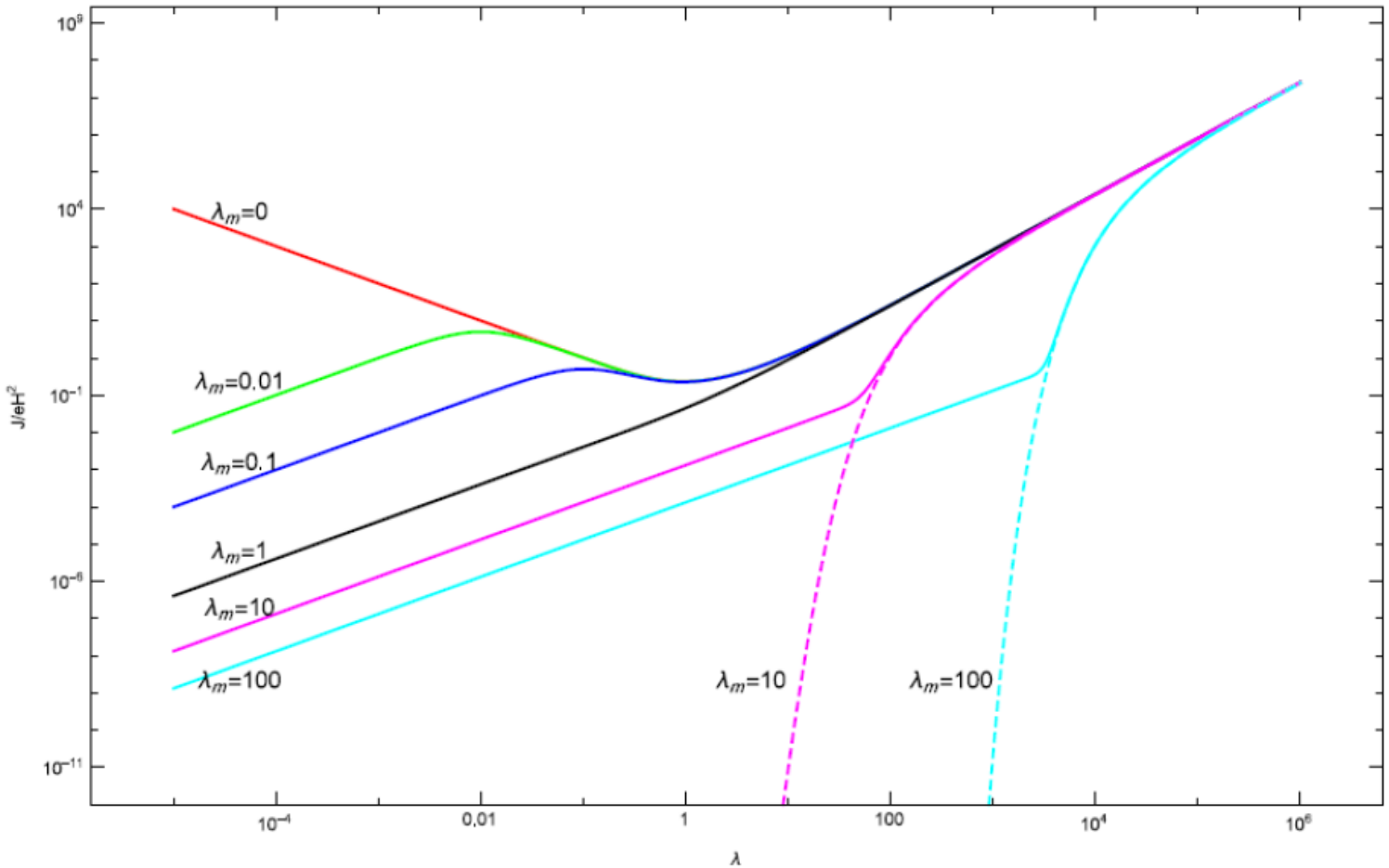
$$\lambda = \frac{eE}{H^2}$$

$$\gamma = \sqrt{\left(\frac{eE}{H^2}\right)^2 + \left(\frac{m}{H}\right)^2 - \left(\frac{D-1}{2}\right)^2}$$

$$\# = \frac{(2|\sigma|+1)}{2} H^2 \gamma \int \frac{d^{D-2}k_\perp}{(2\pi)^{D-2}}$$

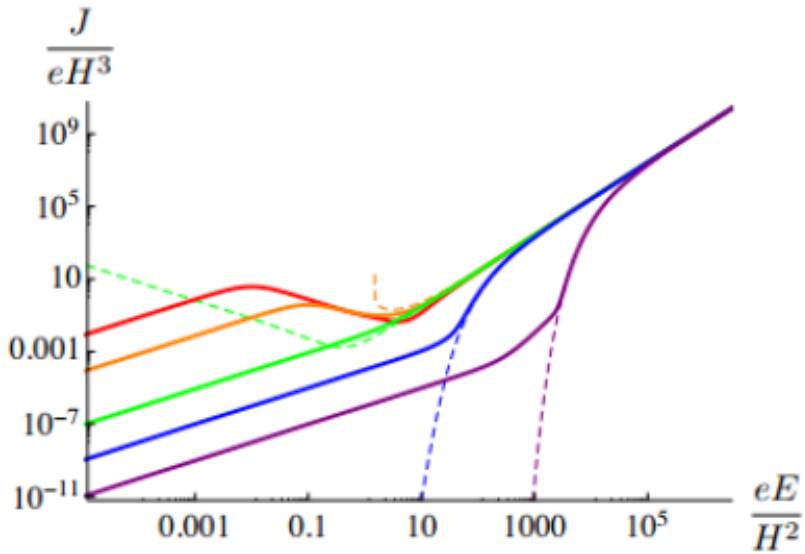
Bosons (Fermions): dotted (solid) lines; Fröb et al 1401.4137 (bosons), Stahl 1507.01686 (fermions)

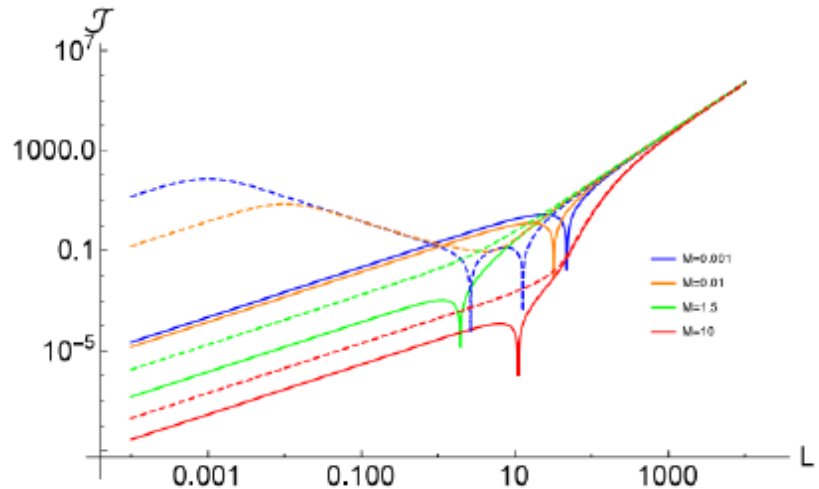
Current in 2+1 D



Bosonic current, Bavarsad 1602.06556

Current in 3+1 D





[Left] Bosonic current, Kobayashi 1602.06556
[Right] Fermionic current, Hayashinaka 1603.0465

Schwinger Mechanism in E-B & de Sitter Space

Bavarsad, SPK, Stahl, Xue, arXiv:1707.03975 & EPJ Web of Conferences (ICGAC13-IK15)

Schwinger Effect in E-B & dS_4

- Conformal metric for 4-dimensional dS and parallel E & B

$$ds^2 = \Omega^2(\tau)\left(-d\tau^2 + dx^2 + dy^2 + dz^2\right),\ \Omega(\tau) = -\frac{1}{H\tau},\ \tau \in (-\infty, 0)$$

$$A_\mu(x) = -\frac{E}{H^2\tau}\delta^3_\mu + By\delta^1_\mu$$

- Bogoliubov coefficients

$$\alpha = \frac{(2|\gamma|)^{1/2}\Gamma(2\gamma)}{\Gamma\left(\frac{1}{2}+\kappa+\gamma\right)} e^{i\frac{\pi}{2}(\kappa-\gamma)},\ \beta = -i\frac{(2|\gamma|)^{1/2}\Gamma(-2\gamma)}{\Gamma\left(\frac{1}{2}+\kappa-\gamma\right)} e^{i\frac{\pi}{2}(\kappa+\gamma)}$$

$$k = \sqrt{k_z^2 + qB(2n+1)},\ \mu = \frac{m}{H},\ \lambda = \frac{qE}{H^2},$$

$$\kappa = i\frac{\lambda k_z}{k},\ \gamma = \sqrt{\left(\frac{3}{2}\right)^2 - \left(\lambda^2 + \mu^2\right)},\ l = qB\tau^2$$

Schwinger Effect in E-B & dS_4

- Pair production rate

$$\Gamma = \frac{1}{\Omega^4(\tau) T L_y} \sum_{n=0}^{\infty} \frac{dk_z}{(2\pi)} \frac{dk_y}{(2\pi)} \left|\beta(k_z, n)\right|^2$$

- Under the semiclassical condition $\lambda^2 + \mu^2 \gg 1$

$$\Gamma = \left(\frac{qB\Omega^{-2}}{2\pi}\right)\left(\frac{H^2|\gamma|}{2\pi}\right)\sum_{n=0}^{\infty}\left[\frac{e^{2\pi|\kappa|}-1}{e^{2\pi|\gamma|}-e^{-2\pi|\gamma|}} + \frac{1}{e^{2\pi|\gamma|}-1}\right]$$

$$\kappa = \frac{\lambda|\gamma|}{\sqrt{|\gamma|^2 + l(2n+1)}}$$

- Riemann zeta-function prescription

$$\Gamma = \left(\frac{qB\Omega^{-2}}{2\pi}\right)\left(\frac{H^2|\gamma|}{2\pi}\right)\left(\frac{1}{e^{4\pi|\gamma|}-1}\right)\left[\frac{1}{2} + \sum_{n=0}^{\infty} e^{2\pi(|\kappa|+|\gamma|)}\right]$$

Induced Current

- Induced Current

$$J = \frac{qH^3 l}{(2\pi)^2} \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \frac{dk_z}{k} (\lambda - k_z \tau) e^{i\pi\kappa} \left| W_{\kappa,\lambda}(2ik\tau) \right|^2$$

- In the weak magnetic field regime $l \ll \min(1, \lambda, \mu)$

$$J_{reg} = \left(\frac{qH^3}{(2\pi)^2} \right) \frac{l\gamma \sinh(2\pi\lambda)}{\sin(2\pi\gamma)} \qquad 2\pi$$

- In the weak magnetic field and strong electric field

$$J_{reg} = \frac{q}{H} \left(\frac{qB\Omega^{-2}}{2\pi} \right) \left(\frac{qE}{2\pi} \right) e^{-\frac{\pi m^2}{|qE|}}$$

- Hyperconductivity in the infrared regime $l \ll \mu \ll \lambda \ll 1$

$$J_{reg} = \frac{9}{2} \left(\frac{qB\Omega^{-2}}{2\pi} \right) \left(\frac{qE}{2\pi} \right) \frac{qH^3}{(qE)^2 + (mH)^2}$$

Induced Current

- Hyperconductivity in the infrared regime $l \ll \mu \ll \lambda \ll 1$

$$J_{reg} = \frac{9}{2}\left(\frac{qB\Omega^{-2}}{2\pi}\right)\left(\frac{qE}{2\pi}\right)\frac{qH^3}{(qE)^2+(mH)^2}$$

- In the strong magnetic field regime $l \gg \max(1,\lambda,\mu)$

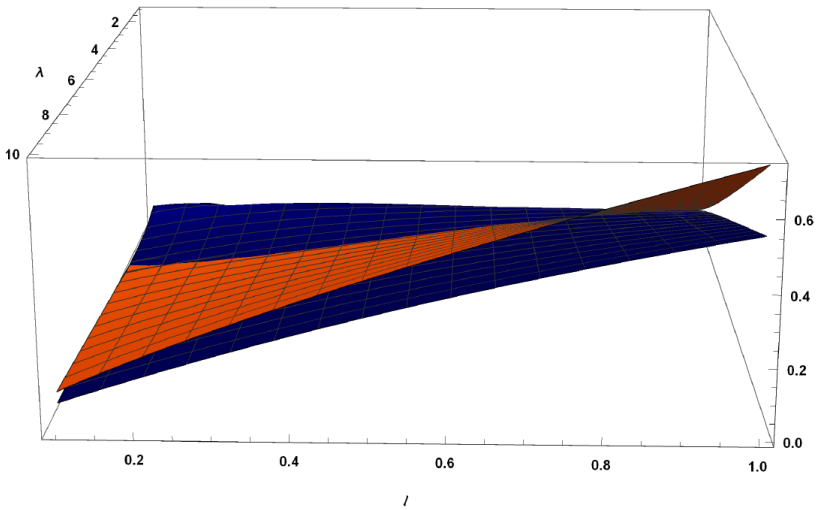
$$J_{reg} \cong \left(\gamma_{\text{Euler}} + \ln(4)\right)\frac{qH^3 l\lambda}{2(2\pi)^2}$$

$$\approx \frac{q}{H}\left(\frac{qB\Omega^{-2}}{2\pi}\right)\left(\frac{qE}{2\pi}\right)$$

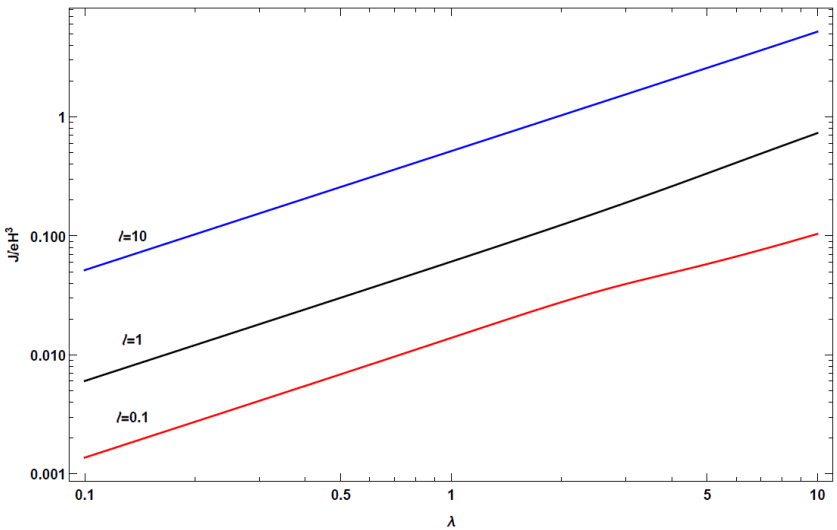
Induced Current

Induced current J/eH^3 vs

Semiclassical J_{sem}/eH^3 (blue)



Induced current J/eH^3

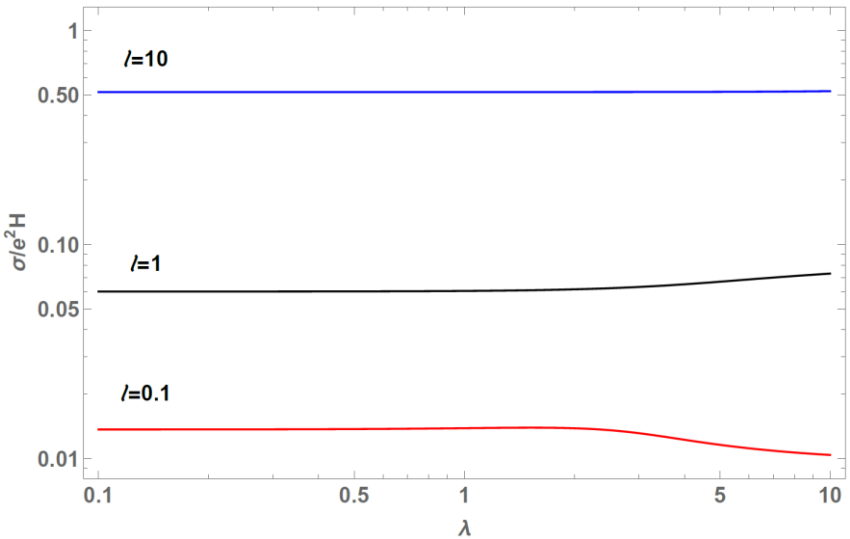


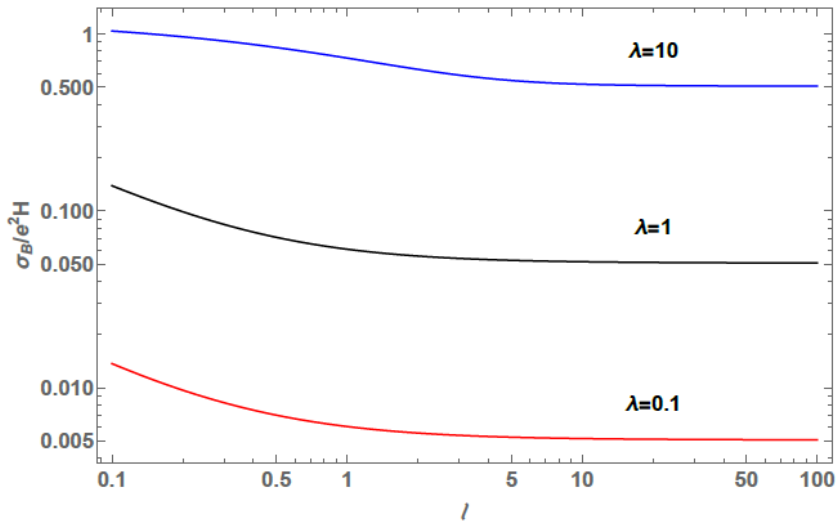
$l = eB\tau^2$, $\lambda = \frac{eE}{H^2}$ in the lowest Landau state

Induced Current

Electric conductivity $\sigma = J/E$
& Normalized one σ/e^2H

Magnetic conductivity $\sigma_B = J/B\Omega^{-2}$
& Normalized one σ_B/e^2H





$$l = eB\tau^2,\ \lambda = \frac{eE}{H^2}$$

Constraint on Primordial Magnetic Field Generation

Astrophysical/Cosmological Magnetic Fields

Astrophysical Magnetic Fields

- Magnetic fields of μG amplitude observed in galaxies, clusters and redshift objects (z <4).
- Observed correlation on scales of the order of the object size: hard to explain.
- To generate the galactic fields of μG amplitude by flux conservation during the formation of galaxies, seed fields of about nG are required.

Cosmological Magnetic Fields

- Lower bound on magnetic field amplitude in the intergalactic medium from observation of blazars with gamma ray telescopes

$$B_{Mpc} > 6 \times 10^{-18} G \quad \text{[Vovk 1112.2534]}$$

- The origin is not yet understood: after recombination (related to structure formation) or primordial?

Primordial Magnetic Field

- Assuming a primordial magnetic field in the universe could explain
  - Observations in all structure & at high redshift: astrophysical magnetic field
  - Lower bound in the intergalactic medium: cosmological magnetic field
- Many generation mechanisms proposed but none preferred:

Causal mechanism	Non causal mechanism
• Phase transition, MHD turbulence, charge & current density + vorticity, …	• Inflation: vacuum fluctuations generate EM field; after reheating, conductivity in the universe is very large, E-field dissipates away & B-field stays.
• Challenge: small correlation length & blue spectrum & too small seeds on cosmologically relevant scales	• Generation at all scales, spectrum can be red.

- Assume that a conformal symmetry breaking generates an EM-field during inflation.

Constraint on Primordial Magnetic Field

- Strong coupling problem
- Backreaction problem
- Schwinger effect

IFF model

- Need to break conformal invariance to generate EM-field
- A bound for the current magnetic field (Kobayashi 1408.4141)

$$B_0 \leq 10^{-28} G \left(\frac{k}{a_0} Mpc\right) \left(\frac{H_{inf}}{M_p}\right)^{1/2} \left(\frac{\sqrt{4\pi\alpha}}{|e|}\right)^3 I_{end}^2 Q$$

Tighter bound

- The result of Bavasard 1707.03975 updates that of Kobayashi

$$B_0 \leq 10^{-28} G \left(\frac{k}{a_0} Mpc\right)^2 \left(\frac{H_{inf}}{M_p}\right)^{1/2} \left(\frac{\sqrt{4\pi\alpha}}{|e|}\right)^3 I_{end}^2 \tilde{Q}$$

Conclusion

- Strong intertwinement of Maxwell theory (QED) and (quantum) gravity in (near-)extremal black holes
  - Spontaneous pair production
  - Vacuum polarization (one-loop action) via In-out formalism and Γ-regularization
- QED phenomena in (anti-) de Sitter space (cosmology)
  - Effective temperature interpretation of Schwinger mechanism
  - Current in (A)dS
  - Constraint on primordial magnetogenesis